Ohm's Law, Power, and Energy

OBJECTIVES

After studying this chapter, you will be able to

- compute voltage, current, and resistance in simple circuits using Ohm's law,
- use the voltage reference convention to determine polarity,
- describe how voltage, current, and power are related in a resistive circuit,
- compute power in dc circuits,
- use the power reference convention to describe the direction of power transfer,
- compute energy used by electrical loads,
- determine energy costs,
- determine the efficiency of machines and systems,

4.1 Ohm's Law

Consider the circuit of Figure 4–1. Using a circuit similar in concept to this, Ohm determined experimentally that *current in a resistive circuit is directly proportional to its applied voltage and inversely proportional to its resistance*. In equation form, Ohm's law states

$$I = \frac{E}{R} \quad \text{[amps, A]} \tag{4–1}$$

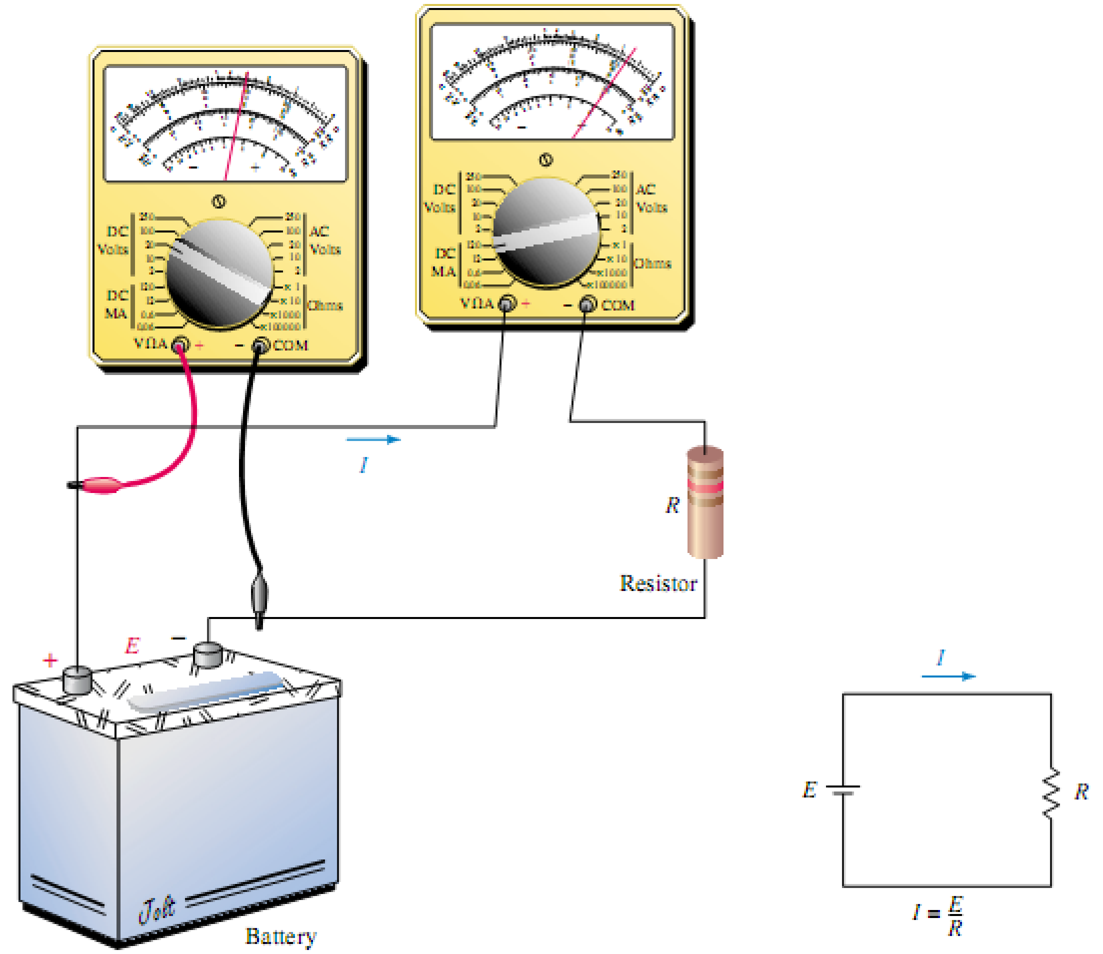
where

E is the voltage in volts,

R is the resistance in ohms,

I is the current in amperes.

From this you can see that the larger the applied voltage, the larger the current, while the larger the resistance, the smaller the current.



(a) Test circuit

(b) Schematic, meters not shown

EXAMPLE 4–1 A 27-Ω resistor is connected to a 12-V battery. What is the current?

Solution Substituting the resistance and voltage values into Ohm's law yields

$$I = \frac{E}{R} = \frac{12\text{ V}}{27\ \Omega} = 0.444\text{ A}$$

EXAMPLE 4–2 The lamp of Figure 4–4 draws 25 mA when connected to a 6-V battery. What is its resistance?

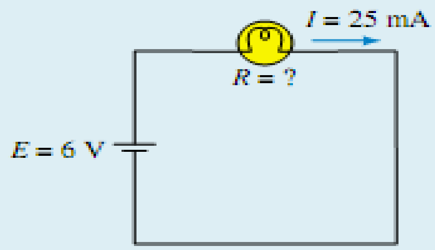


FIGURE 4–4

Solution Using Equation 4–3,

$$R = \frac{E}{I} = \frac{6\text{ V}}{25 \times 10^{-3}\text{ A}} = 240\ \Omega$$

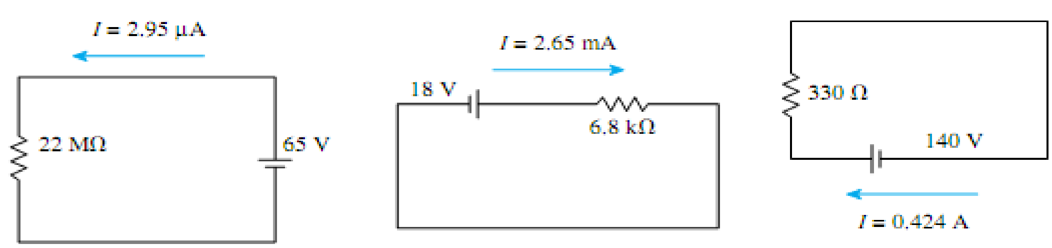
EXAMPLE 4–3 If 125 μA is the current in a resistor with color bands red, red, yellow, what is the voltage across the resistor?

Solution Using the color code of Chapter 3, $R = 220\text{ k}\Omega$. From Ohm's law, $E = IR = (125 \times 10^{-6}\text{ A})(220 \times 10^{3}\ \Omega) = 27.5\text{ V}$.

EXAMPLE 4–4 A resistor with the color code brown, red, yellow is connected to a 30-V source. What is I?

Solution When E is in volts and R in kΩ, the answer comes out directly in mA. From the color code, $R = 120\text{ k}\Omega$. Thus,

$$I = \frac{E}{R} = \frac{30\text{ V}}{120\text{ k}\Omega} = 0.25\text{ mA}$$



(a) $I = \frac{65\text{ V}}{22\text{ M}\Omega} = 2.95\ \mu\text{A}$ (b) $I = \frac{18\text{ V}}{6.8\text{ k}\Omega} = 2.65\text{ mA}$ (c) $I = \frac{140\text{ V}}{330\ \Omega} = 0.424\text{ A}$

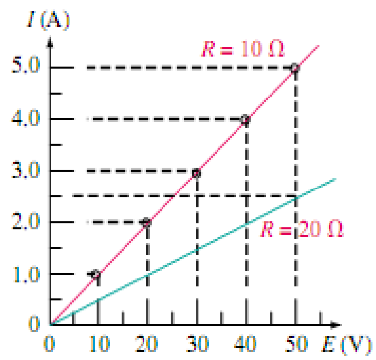


FIGURE 4–7 Graphical representation of Ohm's law. The red plot is for a 10-Ω resistor while the green plot is for a 20-Ω resistor.

Ohm's Law in Graphical Form

The relationship between current and voltage described by Equation 4–1 may be shown graphically as in Figure 4–7. The graphs, which are straight lines, show clearly that the relationship between voltage and current is linear, i.e., that current is directly proportional to voltage.

Open Circuits

Current can only exist where there is a conductive path (e.g., a length of wire). For the circuit of Figure 4–8, I equals zero since there is no conductor between points a and b. We refer to this as an *open circuit*. Since $I = 0$, substitution of this into Equation 4–3 yields

$$R = \frac{E}{I} = \frac{E}{0} \Rightarrow \infty \text{ ohms}$$

Thus, an open circuit has infinite resistance.

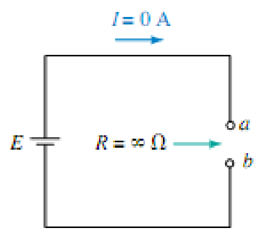


FIGURE 4–8 An open circuit has infinite resistance.

Voltage Symbols

Two different symbols are used to represent voltage. For sources, use uppercase E; for loads (and other components), use uppercase V. This is illustrated in Figure 4–9.

Using the symbol V, Ohm's law may be rewritten in its several forms as

$$I = \frac{V}{R} \quad \text{[amps]} \tag{4–4}$$

$$V = IR \quad \text{[volts]} \tag{4–5}$$

$$R = \frac{V}{I} \quad \text{[ohms]} \tag{4–6}$$

These relationships hold for every resistor in a circuit, no matter how complex the circuit. Since $V = IR$, these voltages are often referred to as *IR drops*.

EXAMPLE 4–5 The current through each resistor of Figure 4–10 is $I = 0.5$ A. Compute V_1 and V_2.

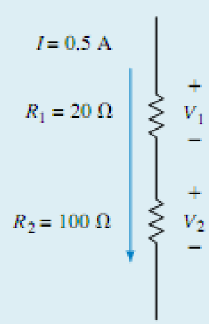
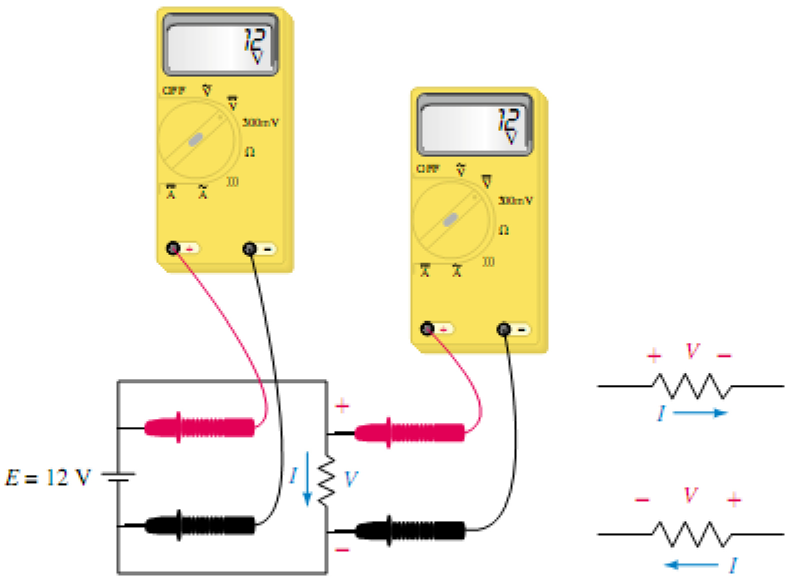


FIGURE 4–10 Ohm's law applies to each resistor.

Solution $V_1 = IR_1 = (0.5\text{ A})(20\ \Omega) = 10$ V. Note, I is also the current through R_2. Thus, $V_2 = IR_2 = (0.5\text{ A})(100\ \Omega) = 50$ V.

4.2 Voltage Polarity and Current Direction

So far, we have paid little attention to the polarity of voltages across resistors. However, polarity is of extreme importance; fortunately, there is a simple relationship between current direction and voltage polarity. To get at the idea, consider Figure 4–11(a). Here the polarity of V is obvious since the resistor is connected directly to the source. This makes the top end of the resistor positive with respect to the bottom end, and $V = E = 12$ V as indicated by the meters.



(a) Defining convention (b) Examples

For each resistor of Figure 4–12, compute V and show its polarity.

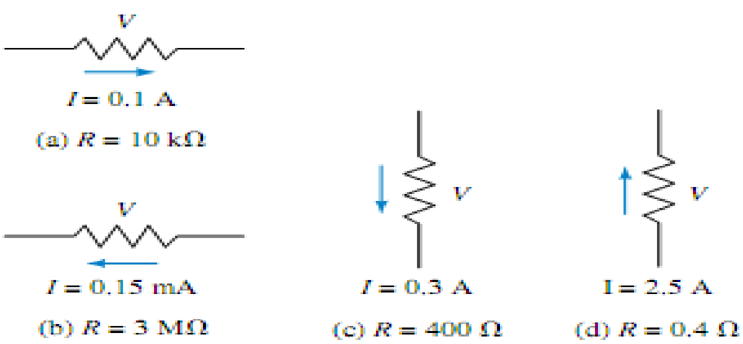
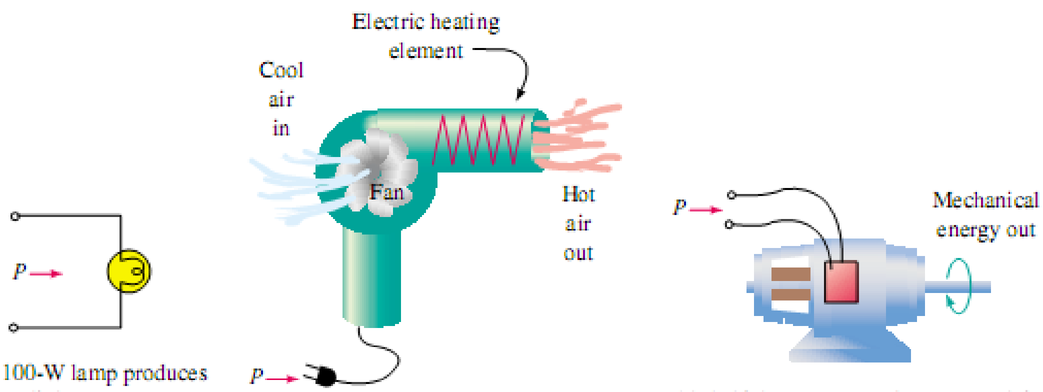


FIGURE 4–12

Answers:
a. 1000 V, + at left b. 450 V, + at right c. 120 V, + at top d. 1 V, + at bottom

4.3 Power

Power is familiar to all of us, at least in a general sort of way. We know, for example, that electric heaters and light bulbs are rated in watts (W) and that motors are rated in horsepower (or watts), both being units of power as discussed in Chapter 1. We also know that the higher the watt rating of a device, the more energy we can get out of it per unit time. Figure 4–15 illustrates the idea. In (a), the greater the power rating of the light, the more light energy that it can produce per second. In (b), the greater the power rating of the heater, the more heat energy it can produce per second. In (c), the larger the power rating of the motor, the more mechanical work that it can do per second.



(a) A 100-W lamp produces more light energy per second than a 40-W lamp

(b) Hair dryer

(c) A 10-hp motor can do more work in a given time than a ½-hp motor

FIGURE 4–15 Energy conversion. Power P is a measure of the rate of energy conversion.

As you can see, power is related to energy, which is the capacity to do work. Formally, **power** is defined as the rate of doing work or, equivalently, as the rate of transfer of energy. The symbol for power is P. By definition,

$$P = \frac{W}{t} \quad \text{[watts, W]} \tag{4–7}$$

where W is the work (or energy) in joules and t is the corresponding time interval of t seconds.

The SI unit of power is the watt. From Equation 4–7, we see that P also has units of joules per second. If you substitute $W = 1$ J and $t = 1$ s you get $P = 1$ J/1 s $= 1$ W. From this, you can see that *one watt equals one joule per second.* Occasionally, you also need power in horsepower. To convert, recall that 1 hp = 746 watts.

Power in Electrical and Electronic Systems

Since our interest is in electrical power, we need expressions for P in terms of electrical quantities. Recall from Chapter 2 that voltage is defined as work per unit charge and current as the rate of transfer of charge, i.e.,

$$V = \frac{W}{Q} \tag{4–8}$$

and

$$I = \frac{Q}{t} \tag{4–9}$$

From Equation 4–8, $W = QV$. Substituting this into Equation 4–7 yields $P = W/t = (QV)/t = V(Q/t)$. Replacing Q/t with I, we get

$$P = VI \quad \text{[watts, W]} \tag{4–10}$$

and, for a source,

$$P = EI \quad \text{[watts, W]} \tag{4–11}$$

Additional relationships are obtained by substituting $V = IR$ and $I = V/R$ into Equation 4–10:

$$P = I^2R \quad \text{[watts, W]} \tag{4–12}$$

and

$$P = \frac{V^2}{R} \quad \text{[watts, W]} \tag{4–13}$$

EXAMPLE 4–6 Compute the power supplied to the electric heater of Figure 4–16 using all three electrical power formulas.

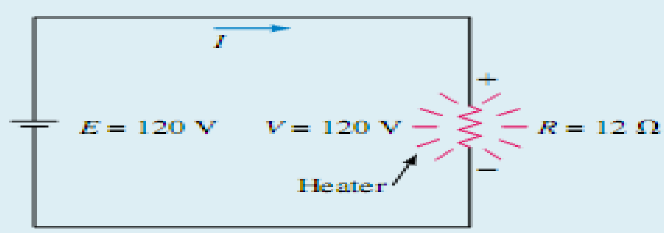


FIGURE 4–16 Power to the load (i.e., the heater) can be computed from any of the power formulas.

Solution $I = V/R = 120\ \text{V}/12\ \Omega = 10\ \text{A}$. Thus, the power may be calculated as follows:

a. $P = VI = (120\ \text{V})(10\ \text{A}) = 1200\ \text{W}$
b. $P = I^2R = (10\ \text{A})^2(12\ \Omega) = 1200\ \text{W}$
c. $P = V^2/R = (120\ \text{V})^2/12\ \Omega = 1200\ \text{W}$

Note that all give the same answer, as they must.

EXAMPLE 4–7 Compute the power to each resistor in Figure 4–17 using Equation 4–13.

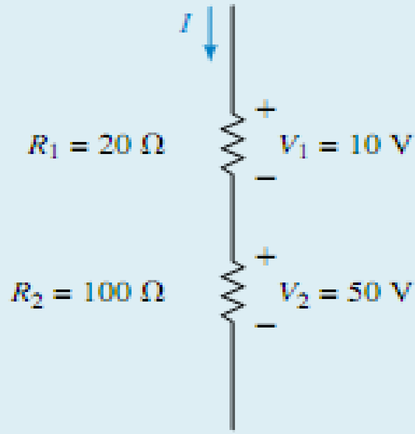


FIGURE 4–17

Solution You must use the appropriate voltage in the power equation. For resistor R_1, use V_1; for resistor R_2, use V_2.

a. $P_1 = V_1^2/R_1 = (10\text{ V})^2/20\ \Omega = 5\text{ W}$

b. $P_2 = V_2^2/R_2 = (50\text{ V})^2/100\ \Omega = 25\text{ W}$

EXAMPLE 4–8 If the dc motor of Figure 4–15(c) draws 6 A from a 120-V source,

a. Compute its power input in watts.

b. Assuming the motor is 100% efficient (i.e., that all electrical power supplied to it is output as mechanical power), compute its power output in horsepower.

Solution

a. $P_{in} = VI = (120\text{ V})(6\text{ A}) = 720\text{ W}$

b. $P_{out} = P_{in} = 720$ W. Converting to horsepower, $P_{out} = (720\text{ W})/(746\text{ W/hp}) = 0.965$ hp.

EXAMPLE 4–9 Use the above convention to describe power transfer for the electric vehicle of Figure 4–20.

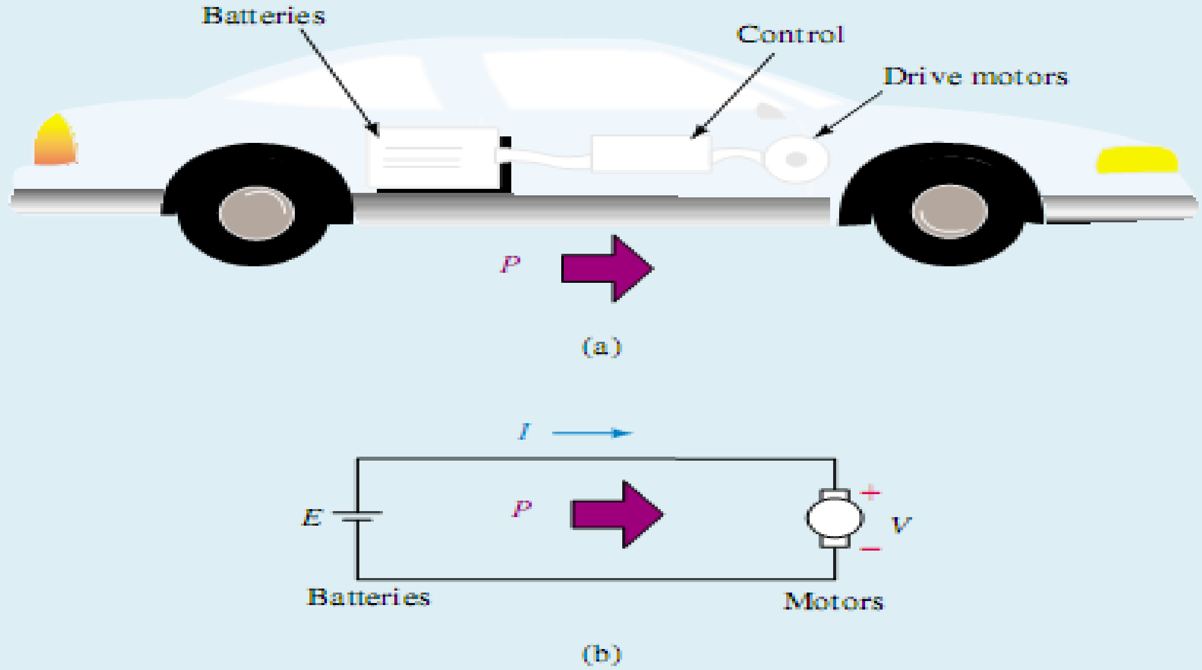


FIGURE 4–20

Solution During normal operation, the batteries supply power to the motors, and current and power are both positive, Fig. 4–20(b). However, when the vehicle is going downhill, its motors are driven by the weight of the car and they act as generators. Since the motors now act as the source and the batteries as the load, the actual current is opposite in direction to the reference arrow shown and is thus negative (recall Figure 4–14). Thus, $P = VI$ is negative. The interpretation is, therefore, that power transfer is in the direction opposite to the power reference arrow. For example, if $V = 48$ volts and $I = -10$ A, then $P = VI = (48\text{ V})(-10\text{ A}) = -480$ W. This is consistent with what is happening, since minus 480 W *into* the motors is the same as plus 480 W *out*. This 480 W of power flows from the motors to the batteries, helping to charge them as the car goes downhill.

4.5 Energy

Earlier (Equation 4–7), we defined power as the rate of doing work. When you transpose this equation, you get the formula for **energy:**

$$W = Pt \tag{4–14}$$

If t is measured in seconds, W has units of watt-seconds (i.e., joules, J), while if t is measured in hours, W has units of watthours (Wh). Note that in Equation 4–14, P must be constant over the time interval under consideration. If it is not, apply Equation 4–14 to each interval over which P is constant as described later in this section. (For the more general case, you need calculus.)

The most familiar example of energy usage is the energy that we use in our homes and pay for on our utility bills. This energy is the energy used by the lights and electrical appliances in our homes. For example, if you run a 100-W lamp for 1 hour, the energy consumed is $W = Pt = (100\text{ W})(1\text{h}) = 100\text{ Wh}$, while if you run a 1500-W electric heater for 12 hours, the energy consumed is $W = (1500\text{ W})(12\text{ h}) = 18\,000\text{ Wh}$.

The last example illustrates that the watthour is too small a unit for practical purposes. For this reason, we use **kilowatthours** (kWh). By definition,

$$\text{energy}_{(\text{kWh})} = \frac{\text{energy}_{(\text{Wh})}}{1000} \tag{4–15}$$

Thus, for the above example, W = 18 kWh. In most of North America, the kilowatthour (kWh) is the unit used on your utility bill.

For multiple loads, the total energy is the sum of the energy of individual loads.

EXAMPLE 4–10 Determine the total energy used by a 100-W lamp for 12 hours and a 1.5-kW heater for 45 minutes.

Solution Convert all quantities to the same set of units, e.g., convert 1.5 kW to 1500 W and 45 minutes to 0.75 h. Then,

$$W = (100\text{ W})(12\text{ h}) + (1500\text{ W})(0.75\text{ h}) = 2325\text{ Wh} = 2.325\text{ kWh}$$

Alternatively, convert all power to kilowatts first. Thus,

$$W = (0.1\text{ kW})(12\text{ h}) + (1.5\text{ kW})(0.75\text{ h}) = 2.325\text{ kWh}$$

EXAMPLE 4–11 Suppose you use the following electrical appliances: a 1.5-kW heater for 7½ hours; a 3.6-kW broiler for 17 minutes; three 100-W lamps for 4 hours; a 900-W toaster for 6 minutes. At $0.09 per kilowatthour, how much will this cost you?

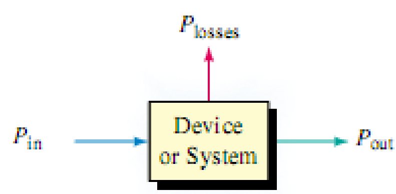
Solution Convert time in minutes to hours. Thus,

$$W = (1500)(7\tfrac{1}{2}) + (3600)\left(\frac{17}{60}\right) + (3)(100)(4) + (900)\left(\frac{6}{60}\right)$$
$$= 13\,560 \text{ Wh} = 13.56 \text{ kWh}$$
$$\text{cost} = (13.56 \text{ kWh})(\$0.09/\text{kWh}) = \$1.22$$

4.6 Efficiency

Poor efficiency results in wasted energy and higher costs. For example, an inefficient motor costs more to run than an efficient one for the same output. An inefficient piece of electronic gear generates more heat than an efficient one, and this heat must be removed, resulting in increased costs for fans, heat sinks, and the like.

Efficiency can be expressed in terms of either energy or power. Power is generally easier to measure, so we usually use power. The efficiency of a device or system (Figure 4–22) is defined as the ratio of power output P_{out} to power input P_{in}, and it is usually expressed in percent and denoted by the Greek letter η (eta). Thus,



$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

In terms of energy,

$$\eta = \frac{W_{out}}{W_{in}} \times 100\%$$

Since $P_{in} = P_{out} + P_{losses}$, efficiency can also be expressed as

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}} \times 100\% = \frac{1}{1 + \dfrac{P_{losses}}{P_{out}}} \times 100\%$$

EXAMPLE 4–12 A 120-V dc motor draws 12 A and develops an output power of 1.6 hp.

a. What is its efficiency?

b. How much power is wasted?

Solution

a. $P_{in} = EI = (120 \text{ V})(12 \text{ A}) = 1440$ W, and $P_{out} = 1.6 \text{ hp} \times 746 \text{ W/hp} =$ 1194 W. Thus,

$$\eta = \frac{P_{out}}{P_{in}} = \frac{1194 \text{ W}}{1440 \text{ W}} \times 100 = 82.9\%$$

b. $P_{losses} = P_{in} - P_{out} = 1440 - 1194 = 246$ W

EXAMPLE 4–13 The efficiency of a power amplifier is the ratio of the power delivered to the load (e.g., speakers) to the power drawn from the power supply. Generally, this efficiency is not very high. For example, suppose a power amplifier delivers 400 W to its speaker system. If the power loss is 509 W, what is its efficiency?

Solution

$$P_{in} = P_{out} + P_{losses} = 400\text{ W} + 509\text{ W} = 909\text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{400\text{ W}}{909\text{ W}} \times 100\% = 44\%$$

EXAMPLE 4–14

a. For a certain system, $\eta_1 = 95\%$, $\eta_2 = 85\%$, and $\eta_3 = 75\%$. What is η_T?

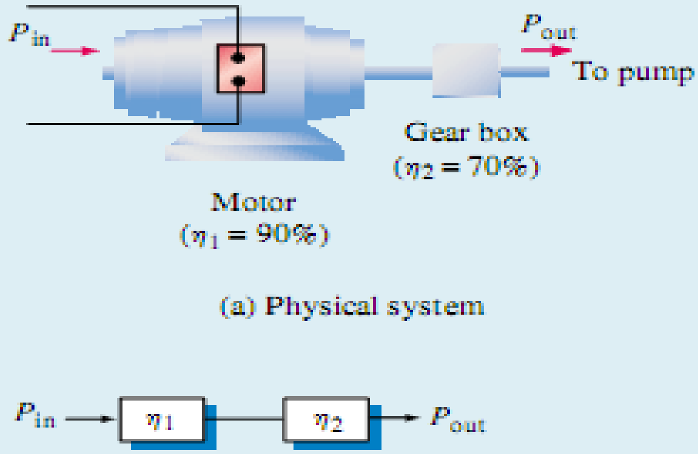
b. If $\eta_T = 65\%$, $\eta_2 = 80\%$, and $\eta_3 = 90\%$, what is η_1?

Solution

a. Convert all efficiencies to a decimal value, then multiply. Thus, $\eta_T = \eta_1\eta_2\eta_3 = (0.95)(0.85)(0.75) = 0.61$ or 61%.

b. $\eta_1 = \eta_T/(\eta_2\eta_3) = (0.65)/(0.80 \times 0.90) = 0.903$ or 90.3%

EXAMPLE 4–15 A motor drives a pump through a gearbox (Figure 4–24). Power input to the motor is 1200 W. How many horsepower are delivered to the pump?



(a) Physical system

(b) Block diagram

FIGURE 4–24 Motor driving pump through a gear box.

Solution The efficiency of the motor-gearbox combination is $\eta_T = (0.90)(0.70) = 0.63$. The output of the gearbox (and hence the input to the pump) is $P_{out} = \eta_T \times P_{in} = (0.63)(1200\text{ W}) = 756$ W. Converting to horsepower, $P_{out} = (756\text{ W})/(746\text{ W/hp}) = 1.01$ hp.