Resistance

OBJECTIVES

After studying this chapter, you will be able to

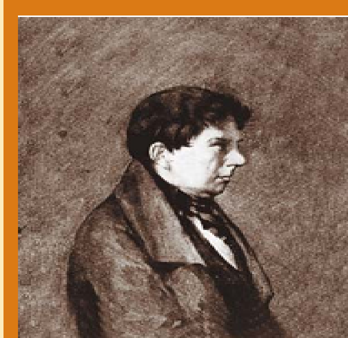
- calculate the resistance of a section of conductor, given its cross-sectional area and length,
- convert between areas measured in square mils, square meters, and circular mils,
- use tables of wire data to obtain the cross-sectional dimensions of various gauges of wire and predict the allowable current for a particular gauge of wire,
- use the temperature coefficient of a material to calculate the change in resistance as the temperature of the sample changes,
- use resistor color codes to determine the resistance and tolerance of a given fixed-composition resistor,
- demonstrate the procedure for using an ohmmeter to determine circuit continuity and to measure the resistance of both an isolated component and one which is located in a circuit,
- develop an understanding of various ohmic devices such as thermistors and photocells,
- develop an understanding of the resistance of nonlinear devices such as varistors and diodes,
- calculate the conductance of any resistive component.

Georg Simon Ohm and Resistance

ONE OF THE FUNDAMENTAL RELATIONSHIPS of circuit theory is that between voltage, current, and resistance. This relationship and the properties of resistance were investigated by the German physicist Georg Simon Ohm (1787–1854) using a circuit similar to that of Figure 3–1. Working with Volta's recently developed battery and wires of different materials, lengths, and thicknesses, Ohm found that current depended on both voltage and resistance. For example, for a fixed resistance, he found that doubling the voltage doubled the current, tripling the voltage tripled the current, and so on. Also, for a fixed voltage, Ohm found that the opposition to current was directly proportional to the length of the wire and inversely proportional to its cross-sectional area. From this, he was able to define the resistance of a wire and show that current was inversely proportional to this resistance; e.g., when he doubled the resistance; he found that the current decreased to half of its former value.

These two results when combined form what is known as Ohm's law. (You will study Ohm's law in great detail in Chapter 4.) Ohm's results are of such fundamental importance that they represent the real beginnings of what we now call electrical circuit analysis.

PUTTING IT IN PERSPECTIVE



3.1 Resistance of Conductors

As mentioned in the chapter preview, conductors are materials which permit the flow of charge. However, conductors do not all behave the same way. Rather, we find that the resistance of a material is dependent upon several factors:

- Type of material
- Length of the conductor
- Cross-sectional area
- Temperature

If a certain length of wire is subjected to a current, the moving electrons will collide with other electrons within the material. Differences at the atomic level of various materials cause variation in how the collisions affect resistance. For example, silver has more free electrons than copper, and so the resistance of a silver wire will be less than the resistance of a copper wire having the identical dimensions. We may therefore conclude the following:

The resistance of a conductor is dependent upon the type of material.

If we were to double the length of the wire, we can expect that the number of collisions over the length of the wire would double, thereby causing the resistance to also double. This effect may be summarized as follows:

The resistance of a metallic conductor is directly proportional to the length of the conductor.

The resistance of a metallic conductor is inversely proportional to the cross-sectional area of the conductor.

The factors governing the resistance of a conductor at a given temperature may be summarized mathematically as follows:

$$R = \frac{\rho \ell}{A} \quad [\text{ohms}, \Omega] \tag{3–1}$$

where

ρ = resistivity, in ohm-meters (Ω-m)

ℓ = length, in meters (m)

A = cross-sectional area, in square meters (m^2).

In the above equation the lowercase Greek letter rho (ρ) is the constant of proportionality and is called the **resistivity** of the material. Resistivity is a physical property of a material and is measured in ohm-meters (Ω-m) in the SI system. Table 3–1 lists the resistivities of various materials at a temperature of 20°C. The effects on resistance due to changes in temperature will be examined in Section 3.4.

Since most conductors are circular, as shown in Figure 3–2, we may determine the cross-sectional area from either the radius or the diameter as follows:

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} \qquad \textbf{(3–2)}$$

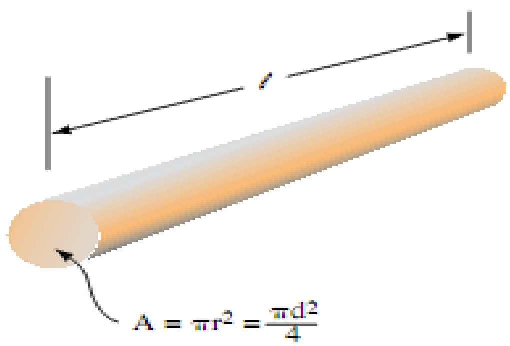


TABLE 3–1 Resistivity of Materials, ρ

Material	Resistivity, ρ, at 20°C (Ω-m)
Silver	1.645×10^{-8}
Copper	1.723×10^{-8}
Gold	2.443×10^{-8}
Aluminum	2.825×10^{-8}
Tungsten	5.485×10^{-8}
Iron	12.30×10^{-8}
Lead	22×10^{-8}
Mercury	95.8×10^{-8}
Nichrome	99.72×10^{-8}
Carbon	3500×10^{-8}
Germanium	20–2300*
Silicon	≅500*
Wood	10^{8}–10^{14}
Glass	10^{10}–10^{14}
Mica	10^{11}–10^{15}
Hard rubber	10^{13}–10^{16}
Amber	5×10^{14}
Sulphur	1×10^{15}
Teflon	1×10^{16}

*The resistivities of these materials are dependent upon the impurities within the materials.

EXAMPLE 3–1 Most homes use solid copper wire having a diameter of 1.63 mm to provide electrical distribution to outlets and light sockets. Determine the resistance of 75 meters of a solid copper wire having the above diameter.

Solution We will first calculate the cross-sectional area of the wire using equation 3–2.

$$A = \frac{\pi d^2}{4}$$
$$= \frac{\pi(1.63 \times 10^{-3}\text{ m})^2}{4}$$
$$= 2.09 \times 10^{-6}\text{ m}^2$$

Now, using Table 3–1, the resistance of the length of wire is found as

$$R = \frac{\rho \ell}{A}$$
$$= \frac{(1.723 \times 10^{-8}\ \Omega\text{-m})(75\text{ m})}{2.09 \times 10^{-6}\text{ m}^2}$$
$$= 0.619\ \Omega$$

EXAMPLE 3–2 Bus bars are bare solid conductors (usually rectangular) used to carry large currents within buildings such as power generating stations, telephone exchanges, and large factories. Given a piece of aluminum bus bar as shown in Figure 3–3, determine the resistance between the ends of this bar at a temperature of 20°C.

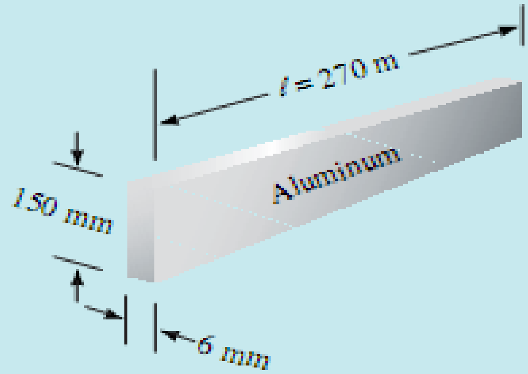


FIGURE 3–3 Conductor with a rectangular cross section.

Solution The cross-sectional area is

$$A = (150 \text{ mm})(6 \text{ mm})$$
$$= (0.15 \text{ m})(0.006 \text{ m})$$
$$= 0.0009 \text{ m}^2$$
$$= 9.00 \times 10^{-4} \text{ m}^2$$

The resistance between the ends of the bus bar is determined as

$$R = \frac{\rho \ell}{A}$$
$$= \frac{(2.825 \times 10^{-8} \ \Omega\text{-m})(270 \text{ m})}{9.00 \times 10^{-4} \text{ m}^2}$$
$$= 8.48 \times 10^{-3} \ \Omega = 8.48 \text{ m}\Omega$$

Homework :

1. Given two lengths of wire having identical dimensions. If one wire is made of copper and the other is made of iron, which wire will have the greater resistance? How much greater will the resistance be?
2. Given two pieces of copper wire which have the same cross-sectional area, determine the relative resistance of the one which is twice as long as the other.
3. Given two pieces of copper wire which have the same length, determine the relative resistance of the one which has twice the diameter of the other.

TABLE 3–2 Standard Solid Copper Wire at 20°C

Size	Diameter		Area		Resistance	Current Capacity
(AWG)	(inches)	(mm)	(CM)	(mm^2)	(Ω/1000 ft)	(A)
56	0.0005	0.012	0.240	0.000122	43 200	
54	0.0006	0.016	0.384	0.000195	27 000	
52	0.0008	0.020	0.608	0.000308	17 000	
50	0.0010	0.025	0.980	0.000497	10 600	
48	0.0013	0.032	1.54	0.000779	6 750	
46	0.0016	0.040	2.46	0.00125	4 210	
45	0.0019	0.047	3.10	0.00157	3 350	
44	0.0020	0.051	4.00	0.00243	2 590	
43	0.0022	0.056	4.84	0.00245	2 140	
42	0.0025	0.064	6.25	0.00317	1 660	
41	0.0028	0.071	7.84	0.00397	1 320	
40	0.0031	0.079	9.61	0.00487	1 080	
39	0.0035	0.089	12.2	0.00621	847	

38	0.0040	0.102	16.0	0.00811	648	
37	0.0045	0.114	20.2	0.0103	521	
36	0.0050	0.127	25.0	0.0127	415	
35	0.0056	0.142	31.4	0.0159	331	
34	0.0063	0.160	39.7	0.0201	261	
33	0.0071	0.180	50.4	0.0255	206	
32	0.0080	0.203	64.0	0.0324	162	
31	0.0089	0.226	79.2	0.0401	131	
30	0.0100	0.254	100	0.0507	104	
29	0.0113	0.287	128	0.0647	81.2	
28	0.0126	0.320	159	0.0804	65.3	
27	0.0142	0.361	202	0.102	51.4	
26	0.0159	0.404	253	0.128	41.0	0.75*
25	0.0179	0.455	320	0.162	32.4	
24	0.0201	0.511	404	0.205	25.7	1.3*
23	0.0226	0.574	511	0.259	20.3	
22	0.0253	0.643	640	0.324	16.2	2.0*
21	0.0285	0.724	812	0.412	12.8	
20	0.0320	0.813	1 020	0.519	10.1	3.0*
19	0.0359	0.912	1 290	0.653	8.05	
18	0.0403	1.02	1 620	0.823	6.39	5.0†
17	0.0453	1.15	2 050	1.04	5.05	
16	0.0508	1.29	2 580	1.31	4.02	10.0†
15	0.0571	1.45	3 260	1.65	3.18	
14	0.0641	1.63	4 110	2.08	2.52	15.0†
13	0.0720	1.83	5 180	2.63	2.00	
12	0.0808	2.05	6 530	3.31	1.59	20.0†
11	0.0907	2.30	8 230	4.17	1.26	
10	0.1019	2.588	10 380	5.261	0.998 8	30.0†
9	0.1144	2.906	13 090	6.632	0.792 5	
8	0.1285	3.264	16 510	8.367	0.628 1	
7	0.1443	3.665	20 820	10.55	0.498 1	
6	0.1620	4.115	26 240	13.30	0.395 2	
5	0.1819	4.620	33 090	16.77	0.313 4	
4	0.2043	5.189	41 740	21.15	0.248 5	
3	0.2294	5.827	52 620	26.67	0.197 1	
2	0.2576	6.543	66 360	33.62	0.156 3	
1	0.2893	7.348	83 690	42.41	0.123 9	

EXAMPLE 3–3 Calculate the resistance of 200 feet of AWG 16 solid copper wire at 20°C.

Solution From Table 3–2, we see that AWG 16 wire has a resistance of 4.02 Ω per 1000 feet. Since we are given a length of only 200 feet, the resistance will be determined as

$$R = \left(\frac{4.02\ \Omega}{1000\ \text{ft}}\right)(200\ \text{ft}) = 0.804\ \Omega$$

EXAMPLE 3–4 If AWG 14 solid copper wire is able to handle 15 A of current, determine the expected current capacity of AWG 24 and AWG 8 copper wire at 20°C.

Solution Since AWG 24 is ten sizes smaller than AWG 14, the smaller cable will be able to handle about one tenth the capacity of the larger-diameter cable.

AWG 24 will be able to handle approximately 1.5 A of current.

AWG 8 is six sizes larger than AWG 14. Since current capacity doubles for an increase of three sizes, AWG 11 would be able to handle 30 A and AWG 8 will be able to handle 60 A.

EXAMPLE 3–6

a. Determine the cross-sectional area in square mils and in circular mils of a copper bus bar having cross-sectional dimensions of 0.250 inch × 6.00 inch.

b. If this copper bus bar were to be replaced by AWG 2/0 cables, how many cables would be required?

Solution

a.
$$A_{\text{sq. mil}} = (250\text{ mils})(6000\text{ mils}) = 1\,500\,000\text{ sq. mils}$$

The area in circular mils is found by applying Equation 3–4, and this will be

$$\begin{aligned} A_{\text{CM}} &= (250\text{ mils})(6000\text{ mils}) \\ &= (1\,500\,000\text{ sq. mils})\left(\frac{4}{\pi}\text{ CM/sq. mil}\right) \\ &= 1\,910\,000\text{ CM} \\ &= 1910\text{ MCM} \end{aligned}$$

b. From Table 3–2, we see that AWG 2/0 cable has a cross-sectional area of 133.1 MCM (133 100 CM), and so the bus bar is equivalent to the following number of cables:

$$n = \frac{1910\text{ MCM}}{133.1\text{ MCM}} = 14.4$$

This example illustrates that 15 cables would need to be installed to be equivalent to a single 6-inch by 0.25-inch bus bar. Due to the expense and awkwardness of using this many cables, we see the economy of using solid bus bar. The main disadvantage of using bus bar is that the conductor is not covered with an insulation, and so the bus bar does not offer the same protection as cable. However, since bus bar is generally used in locations where only experienced technicians are permitted access, this disadvantage is a minor one.

EXAMPLE 3–7 Determine the resistance of an AWG 16 copper wire at 20°C if the wire has a diameter of 0.0508 inch and a length of 400 feet.

Solution The diameter in mils is found as

$$d = 0.0508 \text{ inch} = 50.8 \text{ mils}$$

Therefore the cross-sectional area (in circular mils) of AWG 16 is

$$A_{CM} = 50.8^2 = 2580 \text{ CM}$$

Now, by applying Equation 3–1 and using the appropriate units, we obtain the following:

$$\begin{aligned} R &= \frac{\rho \ell}{A_{CM}} \\ &= \frac{\left(10.36 \dfrac{\text{CM-}\Omega}{\text{ft}}\right)(400 \text{ ft})}{2580 \text{ CM}} \\ &= 1.61\ \Omega \end{aligned}$$

3.4 Temperature Effects

Section 3.1 indicated that the resistance of a conductor will not be constant at all temperatures. As temperature increases, more electrons will escape their orbits, causing additional collisions within the conductor. For most conducting materials, the increase in the number of collisions translates into a relatively linear increase in resistance, as shown in Figure 3–6.

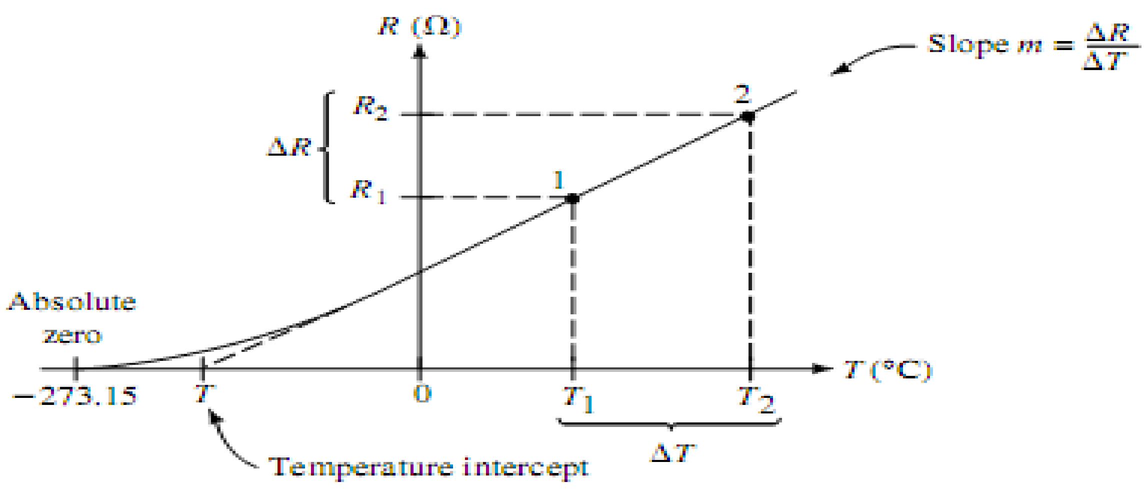


FIGURE 3–6 Temperature effects on the resistance of a conductor.

The rate at which the resistance of a material changes with a variation in temperature is called the **temperature coefficient** of the material and is assigned the Greek letter alpha (α). Some materials have only very slight changes in resistance, while other materials demonstrate dramatic changes in resistance with a change in temperature.

Any material for which resistance increases as temperature increases is said to have a **positive temperature coefficient.**

For semiconductor materials such as carbon, germanium, and silicon, increases in temperature allow electrons to escape their usually stable orbits and become free to move within the material. Although additional collisions do occur within the semiconductor, the effect of the collisions is minimal when compared with the contribution of the extra electrons to the overall flow of charge. As the temperature increases, the number of charge electrons increases, resulting in more current. Therefore, an increase in temperature results in a decrease in resistance. Consequently, these materials are referred to as having **negative temperature coefficients.**

Table 3–4 gives the temperature coefficients, α per degree Celsius, of various materials at 20°C and at 0°C.

TABLE 3–4 Temperature Intercepts and Coefficients for Common Materials

	T (°C)	α $(°C)^{-1}$ at 20°C	α $(°C)^{-1}$ at 0°C
Silver	−243	0.003 8	0.004 12
Copper	−234.5	0.003 93	0.004 27
Aluminum	−236	0.003 91	0.004 24
Tungsten	−202	0.004 50	0.004 95
Iron	−162	0.005 5	0.006 18
Lead	−224	0.004 26	0.004 66
Nichrome	−2270	0.000 44	0.000 44
Brass	−480	0.002 00	0.002 08
Platinum	−310	0.003 03	0.003 23
Carbon		−0.000 5	
Germanium		−0.048	
Silicon		−0.075	

If we consider that Figure 3–6 illustrates how the resistance of copper changes with temperature, we observe an almost linear increase in resistance as the temperature increases. Further, we see that as the temperature is decreased to **absolute zero** ($T = -273.15$°C), the resistance approaches zero.

In Figure 3–6, the point at which the linear portion of the line is extrapolated to cross the abscissa (temperature axis) is referred to as the **temperature intercept** or the **inferred absolute temperature** T of the material.

The value of α is typically given in chemical handbooks. In the above expression, α is measured in $(°C)^{-1}$, R_1 is the resistance in ohms at a temperature, T_1, and m is the slope of the linear portion of the curve ($m = \Delta R/\Delta T$). It is left as an end-of-chapter problem for the student to use Equations 3–6 and 3–7 to derive the following expression from Figure 3–6.

$$R_2 = R_1[1 + \alpha_1(T_2 - T_1)] \tag{3–8}$$

EXAMPLE 3–8 An aluminum wire has a resistance of 20 Ω at room temperature (20°C). Calculate the resistance of the same wire at temperatures of −40°C, 100°C, and 200°C.

Solution From Table 3–4, we see that aluminum has a temperature intercept of −236°C.

At T = −40°C:
The resistance at −40°C is determined using Equation 3–6.

$$R_{-40°C} = \frac{-40°C - (-236°C)}{20°C - (-236°C)} 20\ \Omega = \frac{196°C}{256°C} 20\ \Omega = 15.3\ \Omega$$

At T = 100°C:

$$R_{100°C} = \frac{100°C - (-236°C)}{20°C - (-236°C)} 20\ \Omega = \frac{336°C}{256°C} 20\ \Omega = 26.3\ \Omega$$

At T = 200°C:

$$R_{200°C} = \frac{200°C - (-236°C)}{20°C - (-236°C)} 20\ \Omega = \frac{436°C}{256°C} 20\ \Omega = 34.1\ \Omega$$

The above phenomenon indicates that the resistance of conductors changes quite dramatically with changes in temperature. For this reason manufacturers generally specify the range of temperatures over which a conductor may operate safely.

EXAMPLE 3–9 Tungsten wire is used as filaments in incandescent light bulbs. Current in the wire causes the wire to reach extremely high temperatures. Determine the temperature of the filament of a 100-W light bulb if the resistance at room temperature is measured to be 11.7 Ω and when the light is on, the resistance is determined to be 144 Ω.

Solution If we rewrite Equation 3–6, we are able to solve for the temperature T_2 as follows

$$T_2 = (T_1 - T)\frac{R_2}{R_1} + T$$

$$= [20°\text{C} - (-202°\text{C})]\frac{144\ \Omega}{11.7\ \Omega} + (-202°\text{C})$$

$$= 2530°\text{C}$$

A HVDC (high-voltage dc) transmission line must be able to operate over a wide temperature range. Calculate the resistance of 900 km of 1843 MCM aluminum conductor at temperatures of −40°C and +40°C.

Answers: 20.8 Ω; 29.3 Ω

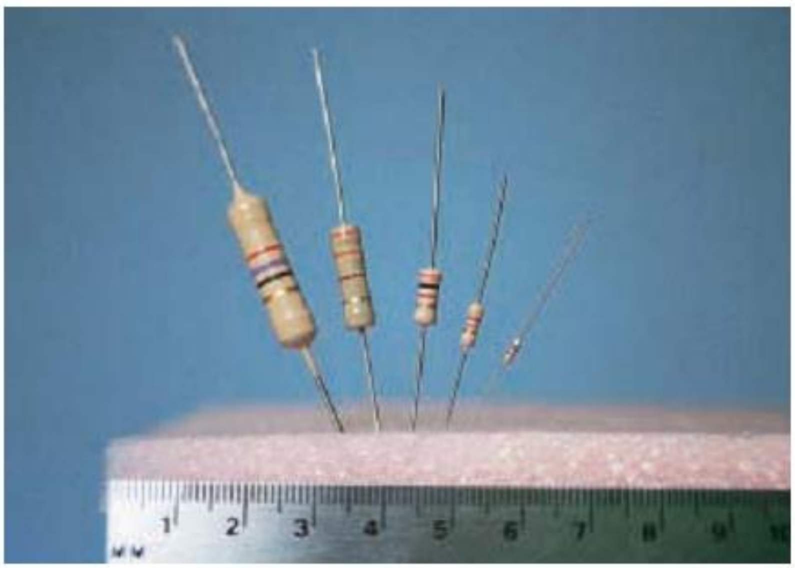


FIGURE 3–8 Actual size of carbon resistors (2 W, 1 W, ½ W, ¼ W, ⅛ W).

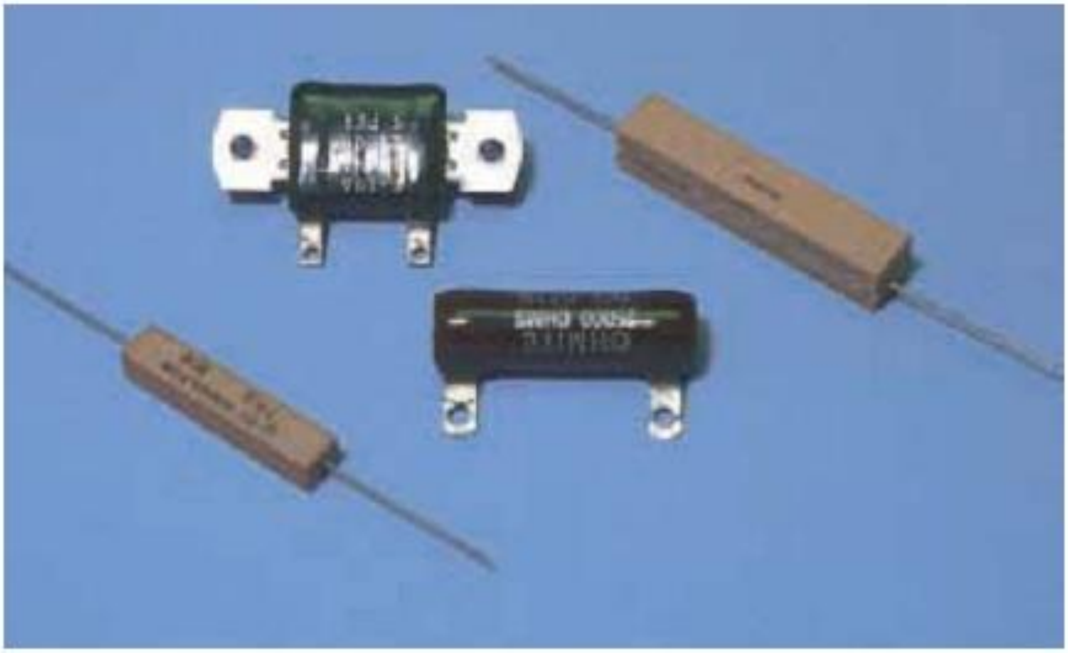


FIGURE 3–10 Power resistors.

(a) External view of variable resistors.

(b) Internal view of variable resistor.

FIGURE 3–12 Variable resistors. *(Courtesy of Bourns, Inc.)*

Resistor Color code

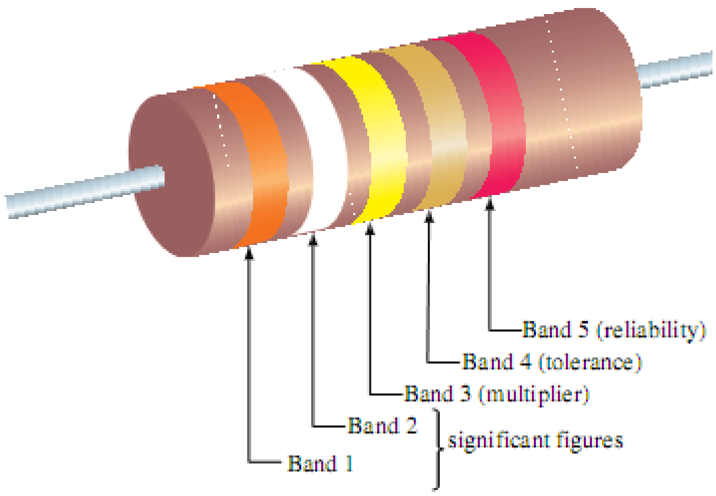


FIGURE 3–15 Resistor color codes.

EXAMPLE 3–10 Determine the resistance of a carbon film resistor having the color codes shown in Figure 3–16.

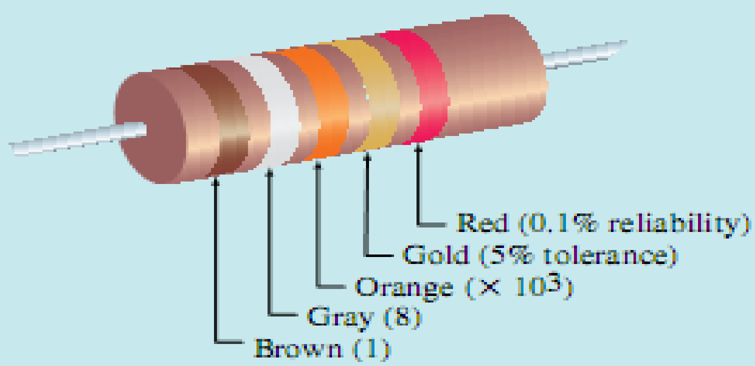


FIGURE 3–16

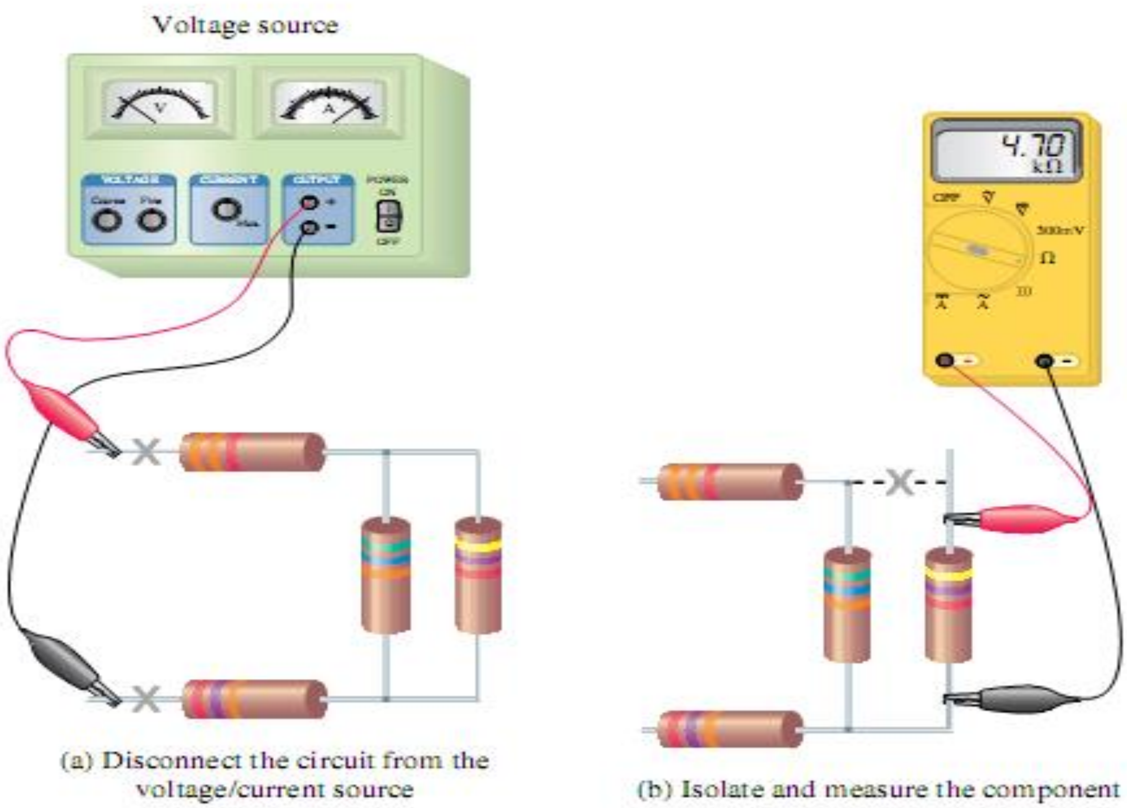
Solution From Table 3–5, we see that the resistor will have a value determined as

$$\begin{aligned} R &= 18 \times 10^3 \ \Omega \pm 5\% \\ &= 18 \ \text{k}\Omega \pm 0.9 \ \text{k}\Omega \text{ with a reliability of } 0.1\% \end{aligned}$$

This specification indicates that the resistance will fall between 17.1 kΩ and 18.9 kΩ. After 1000 hours, we would expect that no more than 1 resistor in 1000 would fall outside the specified range.

TABLE 3–5 Resistor Color Codes

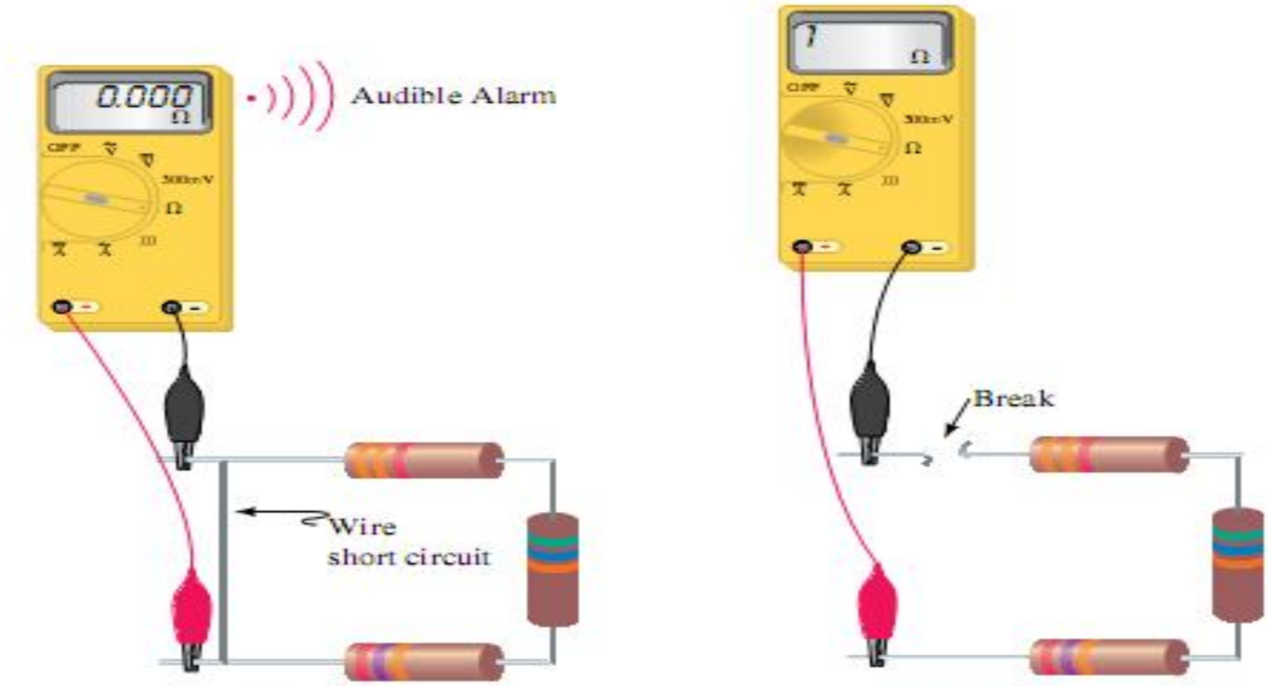
Color	Band 1 Sig. Fig.	Band 2 Sig. Fig.	Band 3 Multiplier	Band 4 Tolerance	Band 5 Reliability
Black		0	$10^0 = 1$		
Brown	1	1	$10^1 = 10$		1%
Red	2	2	$10^2 = 100$		0.1%
Orange	3	3	$10^3 = 1\,000$		0.01%
Yellow	4	4	$10^4 = 10\,000$		0.001%
Green	5	5	$10^5 = 100\,000$		
Blue	6	6	$10^6 = 1\,000\,000$		
Violet	7	7	$10^7 = 10\,000\,000$		
Gray	8	8			
White	9	9			
Gold			0.1	5%	
Silver			0.01	10%	
No color				20%	



(a) Disconnect the circuit from the voltage/current source

(b) Isolate and measure the component

FIGURE 3–19 Using an ohmmeter to measure resistance in a circuit.



(a) Short circuit (b) Open circuit

FIGURE 3–20

3.11 Conductance

Conductance, *G,* is defined as the measure of a material's ability to allow the flow of charge and is assigned the SI unit the siemens (S). A large conductance indicates that a material is able to conduct current well, whereas a low value of conductance indicates that a material does not readily permit the flow of charge. Mathematically, conductance is defined as the reciprocal of resistance. Thus

$$G = \frac{1}{R} \quad \text{[siemens, S]} \tag{3–10}$$

where R is resistance, in ohms (Ω).