Related Rates of Changes:

Related Rate Problems Strategy:

1. *Draw a picture and name the variables and constant.* Use t for time. Assume all variables are differentiable functions of t.
2. *Write down the numerical information* (in terms of symbols you have chosen).
3. *Write down what you are asked to find* (usually a rate, expressed as a derivative).
4. *Write an equation that relates the variables.* You may have to combine two or more equations to get single equation that relates the variable whose rate you want to the variables whose rates you know.
5. *Differentiate with respect to* t. Then express the rate you want in terms of the rate and variables whose values you know.
6. *Evaluate.* Use known values to find the unknown rate.

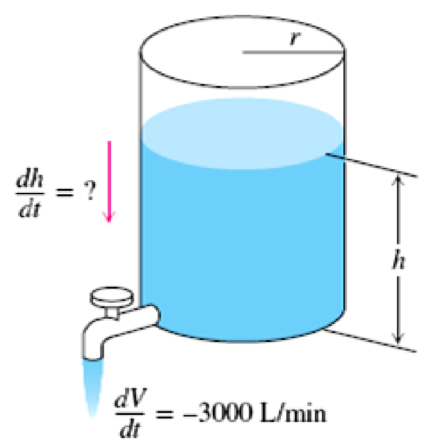
Example 1: How rapidly will the fluid level inside a vertical cylindrical tank if we pump the fluid out at the rate of 3000 L/min.?

Sol.: **Step 1:** *Draw a picture and name the variables and constant.*

Assume the radius of the tank $= r$ (constant),

height of the fluid $= h$ (variable),

and the volume of the fluid $= V$ (variable).



Step 2: *Write down the numerical information.*

$$\frac{dV}{dt} = -1000 \text{ L/min.}$$

Step 3: *Write down what you are asked to find.*

$$\frac{dh}{dt} = ?$$

Step 4: *Write an equation that relates the variables.*

$$V = \pi . r^2 . h$$

Step 5: *Differentiate with respect to t.*

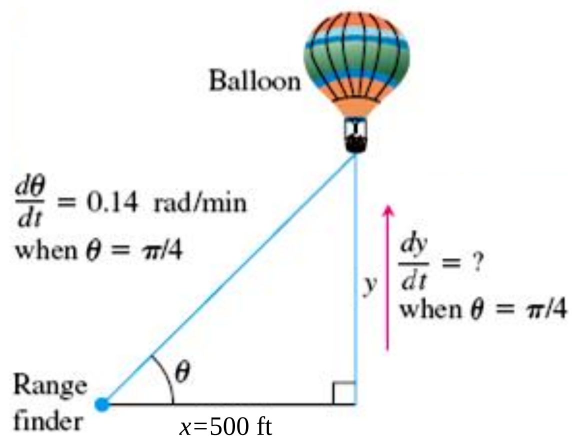
$$\frac{dV}{dt} = \pi . r^2 . \frac{dh}{dt}$$ (where r is constant with respect to t)

Step 6: *Evaluate.*

$$\frac{dh}{dt} = \frac{dV/dt}{\pi . r^2} = \frac{-3000}{1000\pi . r^2} = \frac{-3}{\pi . r^2}$$ m/min. (where 1m^3=1000 L).

If we assume r=1m $\therefore \frac{dh}{dt} = \frac{-3}{\pi . r^2} = \frac{-3}{\pi .(1)^2} = \frac{-3}{\pi}$ m/min=$\frac{-3}{100\pi} \approx -95$ cm/min.

Example 2: A hot air balloon rising straight up from a level field is tracked by a range finder 500 ft from liftoff point. At the moment when the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?



Sol.: Step 1: *Draw a picture and name the variables and constant.*

θ = the angle in radians the range finder makes with the ground (variable).

y = the height in feet of the balloon (variable).

x= the distance between the liftoff point and the range finder (constant = 500 ft)

Step 2: *Write down the numerical information.*

$$\frac{d\theta}{dt} = 0.14 \text{ rad/min.} \qquad \text{when } \theta = \frac{\pi}{4}$$

Step 3: *Write down what you are asked to find.*

$$\frac{dy}{dt} = ? \qquad \text{when } \theta = \frac{\pi}{4}$$

Step 4: *Write an equation that relates the variables.*

$\tan\theta = \frac{y}{x} = \frac{y}{500} \quad \Rightarrow \quad y = 500\tan\theta$ (where x is constant with respect to t)

Step 5: *Differentiate with respect to t.*

$$\frac{dy}{dt} = 500\sec^2\theta.\frac{d\theta}{dt}$$

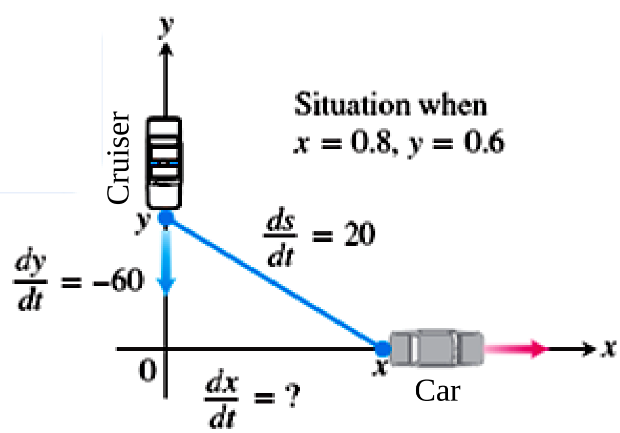
Step 6: *Evaluate* with $\theta = \frac{\pi}{4}$ and $\frac{d\theta}{dt} = 0.14$ to find $\frac{dy}{dt}$.

$$\frac{dy}{dt} = 500(\sqrt{2})^2 * 0.14 = 140 \qquad (\sec\frac{\pi}{4} = \sqrt{2})$$

At the moment in the question, the balloon is rising at a rate of 140 ft/min.

Example 3: A police cruiser, approaching a right-angled intersection from the north is chasing a speeding car that has turned corner and now is moving east. When the cruiser is 0.6 mi north of intersection and the car 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

Sol.: Step 1: *Draw a picture and name the variables and constant.*



x = position of car at time t.

y = position of cruiser at time t.

s = distance between car and cruiser at time t.

Step 2: *Write down the numerical information.*

x = 0.8 mi, y = 0.6 mi, $\frac{dy}{dt} = -60$ mph and $\frac{ds}{dt} = 20$ mph

Step 3: *Write down what you are asked to find.*

$$\frac{dx}{dt} = ?$$

Step 4: *Write an equation that relates the variables.*

$$s^2 = x^2 + y^2 \quad \Rightarrow \quad x^2 = s^2 - y^2$$

Step 5: *Differentiate with respect to t.*

$$2x\frac{dx}{dt} = 2s\frac{ds}{dt} - 2y\frac{dy}{dt} \quad \Rightarrow$$

$$\frac{dx}{dt} = \frac{1}{x}\left(s\frac{ds}{dt} - y\frac{dy}{dt}\right) = \frac{1}{x}\left(\sqrt{x^2+y^2}\,\frac{ds}{dt} - y\frac{dy}{dt}\right)$$

Step 6: *Evaluate* with $x = 0.8$ mi, $y = 0.6$ mi, $\frac{dy}{dt} = -60$ mph and $\frac{ds}{dt} = 20$ mph to find $\frac{dx}{dt}$.

$$\frac{dx}{dt} = \frac{1}{x}\left(\sqrt{x^2+y^2}\,\frac{ds}{dt} - y\frac{dy}{dt}\right) = \frac{1}{0.8}\left(\sqrt{(0.8)^2+(0.6)^2} * 20 - 0.6(-60)\right) = 70$$

At the moment in the question, the car's speed is 70 mph.

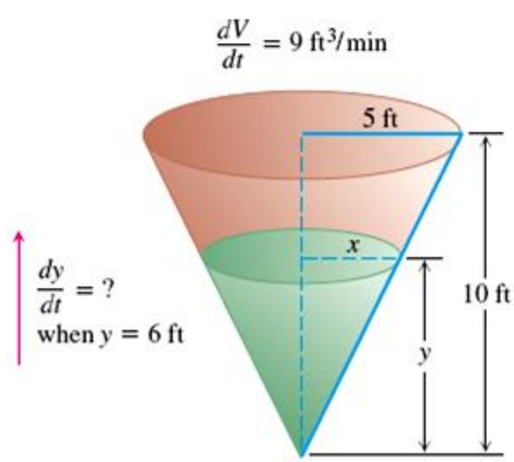
Example 4: Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?

Sol.: **Step 1:** *Draw a picture and name the variables and constant.*

V = volume (ft^3) of the water in the tank at time t (min).

x = radius (ft) of the surface of the water at time t.

y = depth (ft) of the surface of the water at time t.



Step 2: *Write down the numerical information.*

$$y = 6 \text{ ft} \text{ and } \frac{dV}{dt} = 9 \text{ ft}^3/\text{min}.$$

Step 3: *Write down what you are asked to find*.

$$\frac{dy}{dt} = ?$$

Step 4: *Write an equation that relates the variables.*

$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3} x^2 y$$

This equation involves x as well as V and y. Because no information is given about x and dx/dt at the time in the question, we need to eliminate x. the similar triangle give us away to express x in term of y.

$$\frac{x}{y} = \frac{5}{10} \quad \Rightarrow \quad x = \frac{y}{2}$$

Therefore,

$$V = \frac{\pi}{3} x^2 y = \frac{\pi}{3} (\frac{y}{2})^2 y = \frac{\pi}{12} y^3$$

Step 5: *Differentiate with respect to t.*

$$\frac{dV}{dt} = \frac{\pi}{12} * 3y^2 \frac{dy}{dt} = \frac{\pi}{4} y^2 \frac{dy}{dt} \quad \Rightarrow$$

Step 6: *Evaluate* with $y = 6$ ft and $\frac{dV}{dt} = 9 \text{ ft}^3/\text{min}$ to find $\frac{dy}{dt}$.

$$9 = \frac{\pi}{4} (6)^2 \frac{dy}{dt} \quad \Rightarrow \quad \frac{dy}{dt} = \frac{1}{\pi} \approx 0.32$$

At the moment in the question, the water level is rising at about 0.32 ft/min.

Homework:

1. When a circular plate of metal is heated in an oven its radius increases at a rate of 0.01 cm/min. At what rate is the plate's area increasing when its radius is 50 cm.

2. The length (l) of rectangle is decreasing at the rate of 2 cm/sec while the width (w) is increasing at the rate of 2 cm/sec. When $l=12$ cm and $w=5$ cm, find the rate of change of:

   (*a*) the area (*b*) the perimeter (*c*) the length of the diagonal of the rectangle.

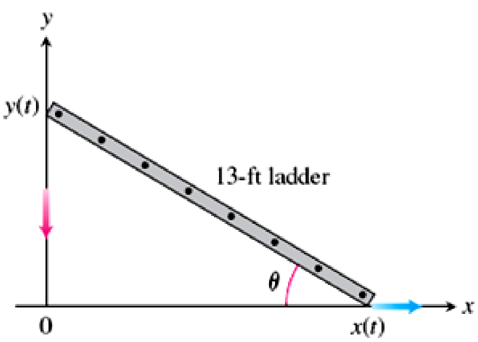
3. The commercial jets at 40 000 ft are flying at 520 mph along straight-line courses that cross at right angles. How fast is the distance between the planes closing when plane *A* is 5 mi from the intersection point and plane *B* is 12 mi from the intersection point.

4. A 13-ft ladder is leaning against a house when its base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

   (a) How fast is the top of the ladder sliding down the wall then?

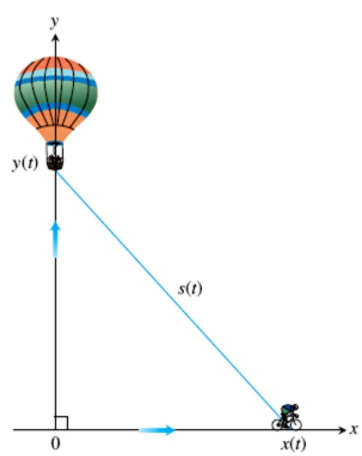
   (b) How fast is the area of the triangle formed by the ladder, wall and the ground changing then?

   (c) At what rate is the angle θ between the ladder and the ground changing then?

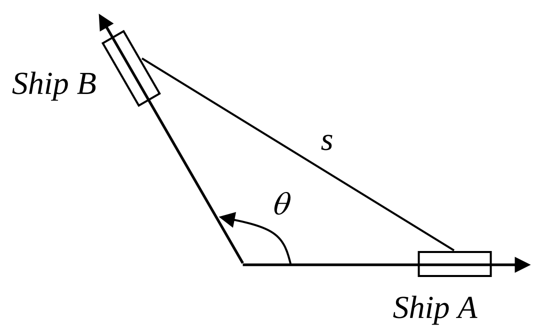


5. Sand falls from a conveyer belt at the rate of 10 ft^3/min onto a conical pile. The radius of the base is always equal to half the pile's height. How fast is the height growing when the pile is 5 ft high?

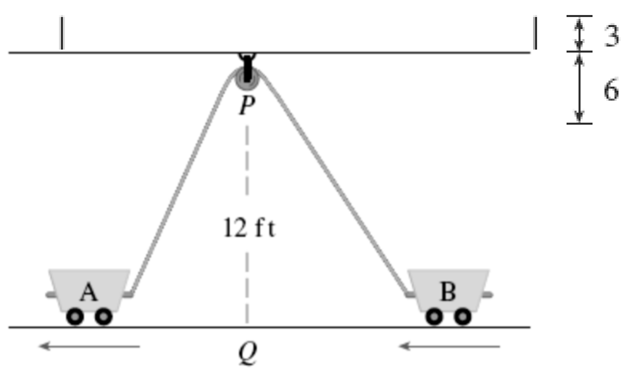
6. Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's radius increases at a constant rate.



7. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle passes under it, going 17 ft/sec. How fast is the distance between the bicycle and balloon increasing 3 sec later?
8. A spherical balloon is inflated with helium 0f 100π ft^3/min. How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast is the surface area increasing?
9. A man 6 ft tall walks at the rate of 5 ft/sec towards a streetlight that is 16 ft above the ground. At what rate is the tip of his shadow moving? At what rate is the length of his shadow when he is 10 ft from the base of the light?
10. Two ships are steaming straight away from a point O along routes that makes 120^o angle. Ship A moves at 14 mph. Ship B moves at 21 mph. How fast are the ships moving apart when $OA = 5$ mi and $OB = 3$mi?

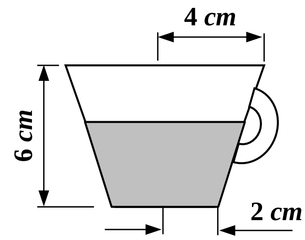


11. A swimming pool is 20 ft wide, 40 ft long, 3 ft deep at the shallow end, and 9 ft deep at its deepest point. A cross-section is shown in the figure. If the pool is being filled at a rate of 0.8 ft^3/min, how fast is the water level rising when the depth at the deepest point is 5 ft?
12. Two carts, A and B, are connected by a rope 39 ft long that passes over a pulley P (see the figure). The point



Q is on the floor 12 ft directly beneath P and between the carts. Cart A is being pulled away from Q at a speed of 2 ft/s. How fast is cart B moving toward Q at the instant when cart A is 5 ft from Q?

13. Coffee is poured at a uniform rate of 20 cm^3/sec. into a cup whose inside shaped like a truncated cone (as shown below). If the upper and lower radii of the cup are 4 cm and 2 cm and the height of the cup is 6cm, how fast will the coffee level be rising when the coffee halfway up?



[***Hint:*** Extend the cup downward to form a cone].

3. Optimization:

To optimize something means to make it as useful or effective as possible. In the mathematical models in which we use differentiable functions to describe things that interest us, this usually means finding where some function has its greatest or smallest value. What is the size of the most profitable production run? What is the best shape for an oil can? What is the stiffest beam we can cut from a 12-in. log?

In this section we show where such functions come from and how to find their extreme values.

Critical Points and Endpoints:

Our basic tool is the observation we made in previous section about local maxima and minima. There we discovered that the extreme values of any function f whatever can occur only at:

$$\left.\begin{array}{l}\text{1. Interior points where } f`=0, \\ \text{2. Interior points where } f` \text{ does not exist,}\end{array}\right\} \text{ named critical points.}$$

3. Endpoints of the function's domain.

Strategy for Solving Max-Min Problems

1. *Draw a picture.* Label the parts are important in the problem.
2. *Write an equation.* Write an equation for the quantity whose maximum or minimum value you want. If you can, express the quantities as a function of single variable, say $y=f(x)$. This may require some algebra and use of information from the statement of the problem. Note the domain in which the values of x are to be found.

3. *Test the critical points and end points.* The extreme value of f will be found among the values f takes on at the endpoints of the domain and at the points where $f`$ is zero or fails to exist. List the values of $f$ at these points. If $f$ has an absolute maximum or minimum on its domain, it will appear on the list. You may have to examine the sign pattern of $f`$ or the sign of $f``$ to decide whether a given value represents a maximum, a minimum, or neither.

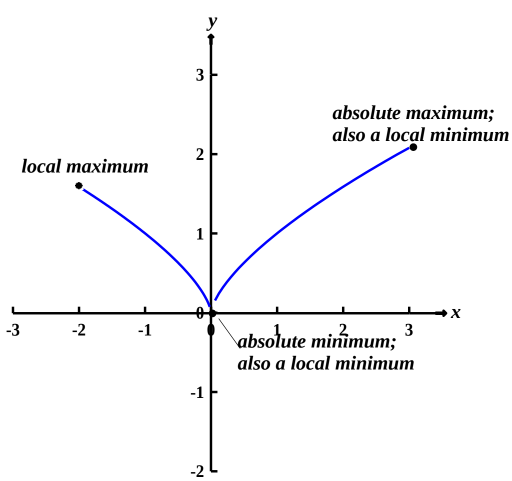
Example 1: Find the absolute maximum and minimum values of $y = x^{2/3}$ on the interval $-2 \le x \le 3$.

Sol.: We evaluate the function at the critical points and endpoints and take the largest and smallest of these values.

$$y = x^{2/3}$$

$$y` = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

has no zeroes but is undefined at $x=0$. The values of the function at this one critical point and the endpoints are:



Critical points value: $f(0) = 0$

Endpoint values: $f(-2) = (-2)^{2/3} = 4^{1/3}$,

$f(3) = (3)^{2/3} = 9^{1/3}$

We conclude that the function's maximum value is $9^{1/3}$, taken on at $x=3$.

The minimum value is 0, taken on at $x=0$

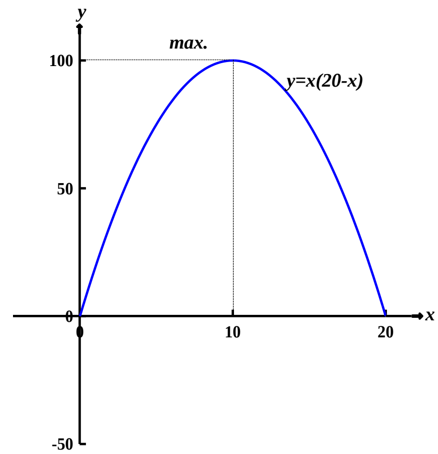
Example 2: Find two positive numbers whose sum is 20 and whose product is as large as possible.

Sol.: assume the first number is x

So, the second number is $20-x$.

$\therefore\ f(x) = x(20-x)$; $0 \le x \le 20$ $(Positive)$

$= 20x - x^2$



Critical points can be found from first derivative:

$f`(x) = 20 - 2x$

Put $f`(x) = 0 \Rightarrow 20 - 2x = 0 \Rightarrow x = \dfrac{20}{2} = 10$

- Critical point value:

$f(10) = 10(20-10) = 10*10 = 100$

- Endpoints values: $f(0) = 0(20-0) = 0$;

$f(20) = 20(20-20) = 0$

We conclude that the max. value is $f(10) = 100$

So the first number is 10

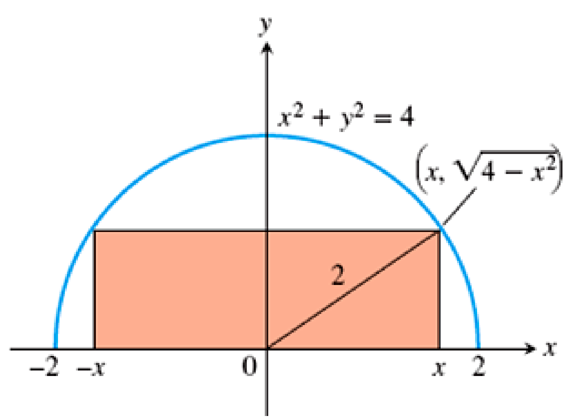
and the second number is 20-10=10

Example 3: A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area that the rectangle can have and what are the dimensions?

Sol.: Step 1: *Draw a picture:*

$\therefore$ the length of rectangle $= 2x$;

and the height $= y$.



Step 2: *Write an equation*:

$Area = A(x) = 2x.y$; where $y = \sqrt{r^2 - x^2}$

$= \sqrt{4 - x^2}$

$\therefore A(x) = 2x\sqrt{4-x^2}$; ${}_{positive}0 \le x \le r$

$0 \le x \le 2$

Step 3: *Test the critical points and end points*:

$$\frac{dA}{dx} = 2x(4-x^2)^{-1/2}\left(\frac{1}{2}\right)(-2x) + 2\sqrt{4-x^2}$$

$$= \frac{-2x^2}{\sqrt{4-x^2}} + 2\sqrt{4-x^2}$$

$$= \frac{-2x^2 + 2(4-x^2)}{\sqrt{4-x^2}} = \frac{-2x^2+8-2x^2}{\sqrt{4-x^2}} = \frac{8-4x^2}{\sqrt{4-x^2}}$$

- Put $\frac{dA}{dx} = 0 \Rightarrow \frac{8-4x^2}{\sqrt{4-x^2}} = 0 \Rightarrow 8-4x^2 = 0 \Rightarrow x^2 = \frac{8}{4} = 2$

$\therefore x = \mp\sqrt{2}$

We should neglect the negative root because it is out of the domain.

$\therefore x = \sqrt{2}$

- And $\frac{dA}{dx}$ is not defined at $\sqrt{4-x^2} = 0 \Rightarrow 4-x^2 = 0$

$\therefore x = \mp 2$

We also should neglect the negative root

$\therefore x = 2$ (it is also end point)

- Critical point value: $A(\sqrt{2}) = 2\sqrt{2}\sqrt{4-(\sqrt{2})^2} = 2\sqrt{2}\sqrt{2} = 4$
- Endpoints values: $A(0) = 2*0\sqrt{4-(0)^2} = 0$;

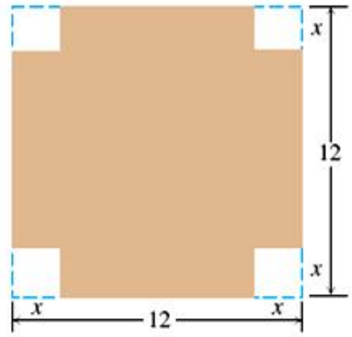
$A(2) = 2*2\sqrt{4-(2)^2} = 0$

$\therefore$ The max. area is 4 square units when the rectangle has

Length $= 2x = 2\sqrt{2}$ unit length;

and height $= \sqrt{4-(\sqrt{2})^2} = \sqrt{4-2} = \sqrt{2}$ unit length.

Example 4: An open-top-box is to be made by cutting small congruent squares from the corners of a 12-in-by 12-in sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?



(a)

Sol.: **Step 1:** *Draw a picture:*

Step 2: *Write an equation*:

$$V(x) = x(12-2x)^2$$

$$= 144x - 48x^2 + 4x^3; \qquad {}_{positive} 0 \le x \le \frac{12}{2}$$

$$0 \le x \le 6$$

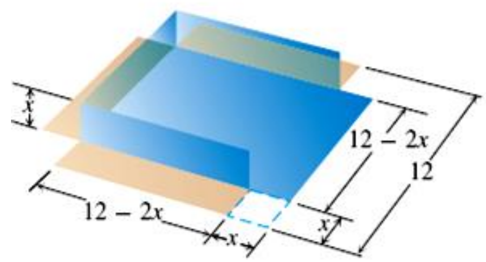
Step 3: *Test the critical points and end points*:

$$\frac{dV}{dx} = 144 - 96x + 12x^2$$

$$= 12(12 - 8x + x^2) = 12(2-x)(6-x)$$

Put $\frac{dV}{dx} = 0 \Rightarrow 12(2-x)(6-x) = 0$

either $x = 2$ or $x = 6$ (endpoint)



(b)

- Critical point value:

$$V(2) = 2(12 - 2*2)^2 = 128 \ \ in^3$$

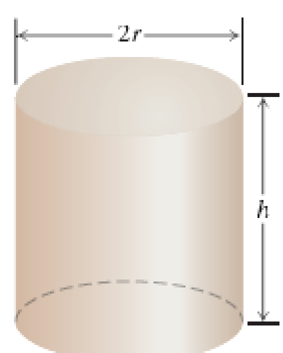
- Endpoints values: $V(0) = 0(12 - 2*0)^2 = 0$;

$$V(6) = 2(12 - 2*6)^2 = 0$$

∴ The max. volume is 128 in^3. The cut out squares should be 2 in on a side.

Example 5: You have been asked to design a 1-liter can shaped like a right circular cylinder. What dimensions will use the least material?

Solution: *Volume of can*: If r and h are measured in centimeters, then the volume of the can in cubic centimeters is



$\pi r^2 h = 1000.$ 1 liter = 1000 cm^3

Surface area of can: $A = 2\pi r^2 + 2\pi rh$

Circular ends Circular wall

How can we interpret the phrase "least material"? First, it is customary to ignore the thickness of the material and the waste in manufacturing. Then we ask for dimensions r and h that make the total surface area as small as possible while satisfying the constraint $\pi r^2 h = 1000$.

To express the surface area as a function of one variable, we solve for one of the variables in $\pi r^2 h = 1000$ and substitute that expression into the surface area formula. Solving for h is easier:

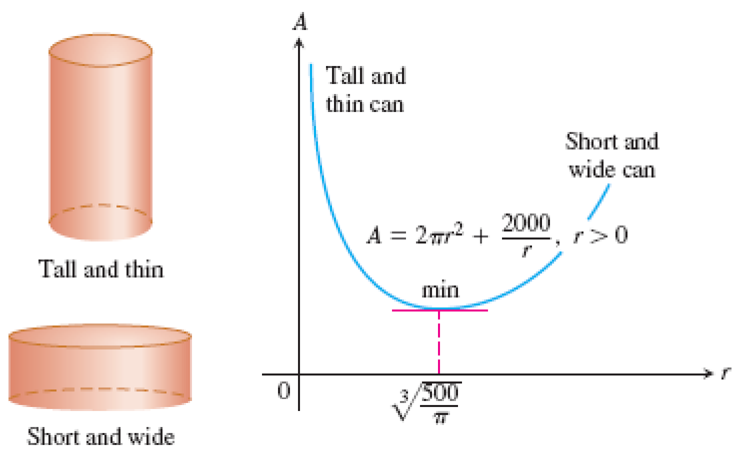
$$h = \frac{1000}{\pi r^2}$$

Thus

$$A = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{2000}{r}$$



The graph of $A = 2\pi r^2 + 2000/r$ is concave up.

Our goal is to find a value of $r > 0$ that minimizes the value of A. Figure below suggests that such a value exists. Notice from the graph that for small r (a tall thin container, like a piece of pipe), the term $2000/r$ dominates and A is large. For large r (a short wide container, like a pizza pan), the term $2\pi r^2$ dominates and A again is large.

Since A is differentiable on $r > 0$, an interval with no endpoints, it can have a minimum value only where its first derivative is zero.

$$\frac{dA}{dr} = 4\pi r - \frac{2000}{r^2}$$

$$0 = 4\pi r - \frac{2000}{r^2} \qquad \text{(set } dA/dr = 0\text{)}$$

$$4\pi r^3 = 2000 \qquad \text{(Multibly by } r^2\text{)}$$

$$r = \sqrt[3]{\frac{500}{\pi}} \approx 5.24 \text{ cm}$$

The second derivative:

$$\frac{d^2A}{dr^2} = 4\pi + \frac{4000}{r^3}$$

is positive throughout the domain of A. The graph is therefore everywhere concave up and the value of A at $r = \sqrt[3]{\frac{500}{\pi}}$ an absolute minimum.

The corresponding value of h (after a little algebra) is

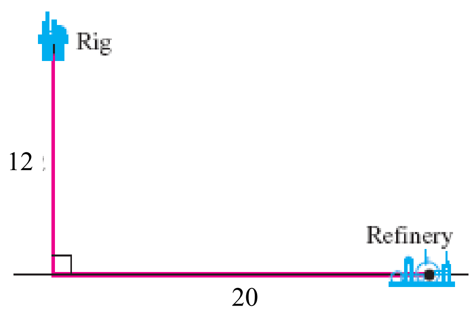
$$h = \frac{1000}{\pi r^2} = 2\sqrt[3]{\frac{500}{\pi}} = 2r.$$

The 1-L can that uses the least material has height equal to the diameter, here with $r = \approx 5.24$ cm and $h = \approx 10.48$ cm.

***Example 6*:** A drilling rig 12 mi offshore is to be connected by pipe to a refinery onshore, 20 mi straight down the coast from the rig. If underwater pipe costs \$500,000 per mile and land based pipe costs \$300,000 per mile, what combination of the two will give the least expensive connection?

***Solution*:** We try a few possibilities to get a feel for the problem:

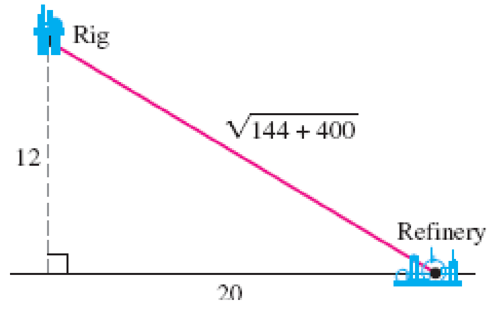
(a) *Smallest amount of underwater pipe*



Underwater pipe is more expensive, so we use as little as we can. We run straight to shore 12 mi and use land pipe for 20 mi to the refinery.

$$\begin{aligned}\text{Dollar cost} &= 12(500{,}000) + 20(300{,}000)\\ &= 12{,}000{,}000\end{aligned}$$

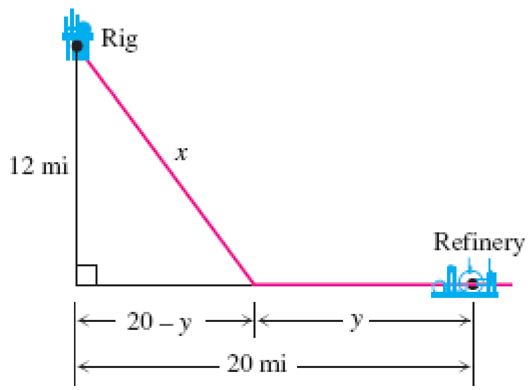
(b) *All pipe underwater (most direct route)*



We go straight to the refinery underwater.

$$\text{Dollar cost} = 500(\sqrt{144+400})$$
$$\approx 11{,}661{,}900$$

This is less expensive than plan (a).



Now we introduce the length x of underwater pipe and the length y of land-based pipe as variables. The right angle opposite the rig is the key to expressing the relationship between x and y, for the Pythagorean theorem gives

$$x^2 = 12^2 + (20-y)^2$$
$$x = \sqrt{144+(20-y)^2}$$

Only the positive root has meaning in this model.

The dollar cost of the pipeline is

$$c = 500{,}000x + 300{,}000y$$

To express c as a function of a single variable, we can substitute for x,

$$c(y) = 500{,}000\sqrt{144+(20-y)^2} + 300{,}y$$

Our goal now is to find the minimum value of $c(y)$ on the interval $0 \le y \le 20$. The first derivative of $c(y)$ with respect to y according to the Chain Rule is

$$c`(y) = 500{,}000.\frac{1}{2}.\frac{2(20-y)(-1)}{\sqrt{144+(20-y)^2}} + 300{,}000$$

$$= -500{,}000.\frac{20-y}{\sqrt{144+(20-y)^2}} + 300{,}000$$

Setting $c`$ equal to zero gives

$$500{,}000(20-y) = 300{,}000\sqrt{144+(20-y)^2}$$

$$\frac{5}{3}(20-y) = \sqrt{144+(20-y)^2}$$

$$\frac{25}{9}(20-y)^2 = 144+(20-y)^2$$

$$\frac{16}{9}(20-y)^2 = 144$$

$$(20-y) = \pm\frac{3}{4}.12 = \pm 9$$

$$y = 20 \pm 9$$

$$y = 11 \text{ or } \quad y = 29$$

Only $y = 11$ lies in the interval of interest. The values of c at this one critical point and at the endpoints are

$c(11) = 10{,}800{,}000$

$c(0) = 11{,}661{,}900$

$c(20) = 12{,}000{,}000$

The least expensive connection costs $10,800,000, and we achieve it by running the line underwater to the point on shore 11 mi from the refinery.

Example 7: Suppose a manufacturer can sell x-items a week for a revenue of $r(x) = 200\,x\text{-}0.01x^2$ cents, and it costs $c(x)\text{=}50x\text{+}20000$ cents to make x-items. Is there a most profitable number of items to make each week? If so, what is it?

Sol.: profit = revenue – cost

$p(x) = r(x) - c(x)$

$= (200\,x-0.01x^2) - (50x+20000)$

$=200\,x-0.01x^2-50x-20000$

$=150\,x-0.01x^2-20000$

To find the critical point put $\frac{dp}{dx}=0$

$\therefore \frac{dp}{dx}=150-0.02x=0 \quad \Rightarrow \quad x=\frac{150}{0.02}=7500$ items

and $\frac{d^2p}{dx^2}=-0.02<0$ for all values of x.

$\therefore$ The graph is concave down, so the critical point $x=7500$ is the location of an absolute max.

To answer the question, then, there is a production level for max. profit, and that level is $x=7500$ item per week.

Home work:

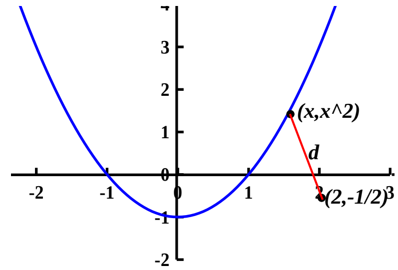
1. The sum of two non-negative numbers is 20. Find the numbers:

   (a) If the sum of their squares is to be as large as possible.

   (b) If one number plus to square root of the other is to be as large as possible.

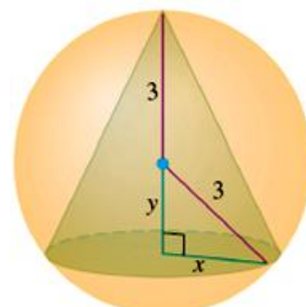
2. What is the largest possible area of a right triangle whose hypotenuse is 5 cm long?
3. What is the smallest perimeter possible for a rectangle whose area is 16 in^2?
4. You are planning to close off a corner of the first quadrant with a line segment 20 units long running from $(a,0)$ to $(0,b)$. Show that the area of the triangle enclosed by the segment is largest when $a=b$.
5. A rectangle plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose?
6. A 216-m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

7. Show that the value of x minimizes the square of the distance, and hence the distance, between point (x, x^2) and $(2,-1/2)$ in figure below is a solution of the equation $x = \frac{1}{x^2+1}$.



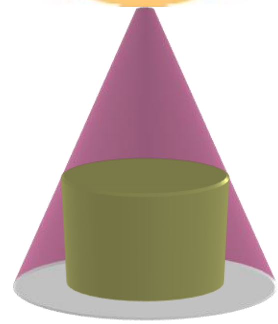
8. You are planning to makes an open top rectangular box from an $(8*15)\ in^2$ piece of cardboard by cutting squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way?

9. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3?

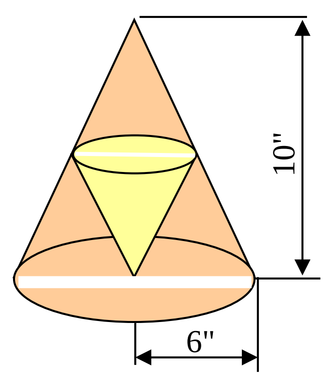


10. What are the dimensions of the lightest open-top right cylindrical can be holed a volume of $1000\ cm^3$?

11. Show that the right circular cylinder of greatest volume that can be inscribed in a right circular cone has volume that is 4/9 the volume of the cone.

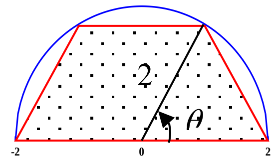


12. Find the dimensions of the largest cone can be inscribed in the cone whose radius is $6in$ and height is $10in$ as shown in figure nearby.

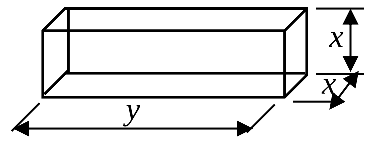


13. A trapezoid is inscribed in a semicircle of radius **2** so that one side is along the diameter (as shown below). Find the maximum possible area for trapezoid.

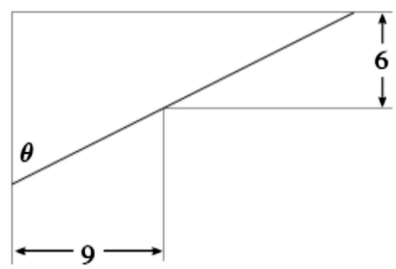
[***Hint:*** Express the area of the trapezoid in term of θ].



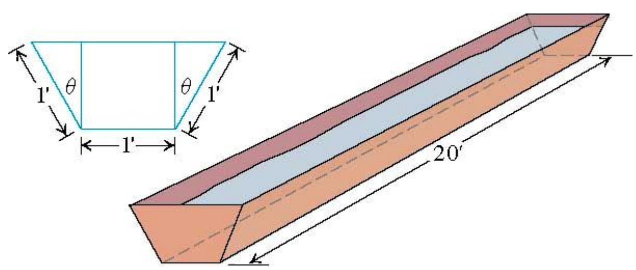
14. A box shaped wire-frame consists of two identical wire squares whose vertices are connected by four straight wires of equal lengths. If the frame of length (***L=96 cm***), what should the dimensions be to obtain a box of greatest volume.



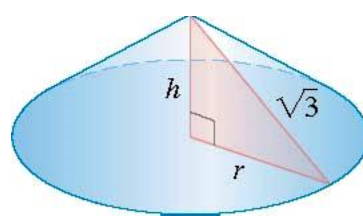
15. A steel pipe is being carried down a hallway 9 ft wide. At the end of the hall there is a right-angled turn into a narrower hallway 6 ft wide. What is the length of the longest pipe that can be carried horizontally around the corner?



16. The trough in figure nearby is to be made to the dimensions shown. Only the angle θ can be varied. What value of will maximize the trough's volume?

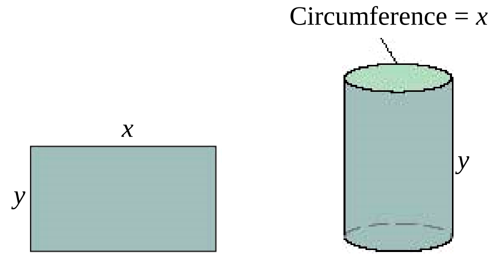


17. A right triangle whose hypotenuse $\sqrt{3}$ long is revolved about one of its legs to generate a right circular cone. Find the radius, height, and volume of the cone of greatest volume that can be made this way.



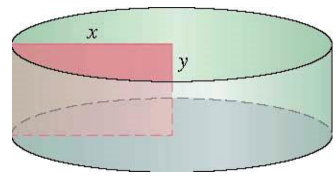
18. Compare the answers to the following two construction problems .

a . A rectangular sheet of perimeter 36 cm and dimensions x cm by y cm to be rolled into a cylinder as shown in part (a) of the figure. What values of x and y give the largest volume?



(a)

b . The same sheet is to be revolved about one of the sides of length y to sweep out the cylinder as shown in part (b) of the figure. What values of x and y give the largest volume?



(b)