Chapter Two

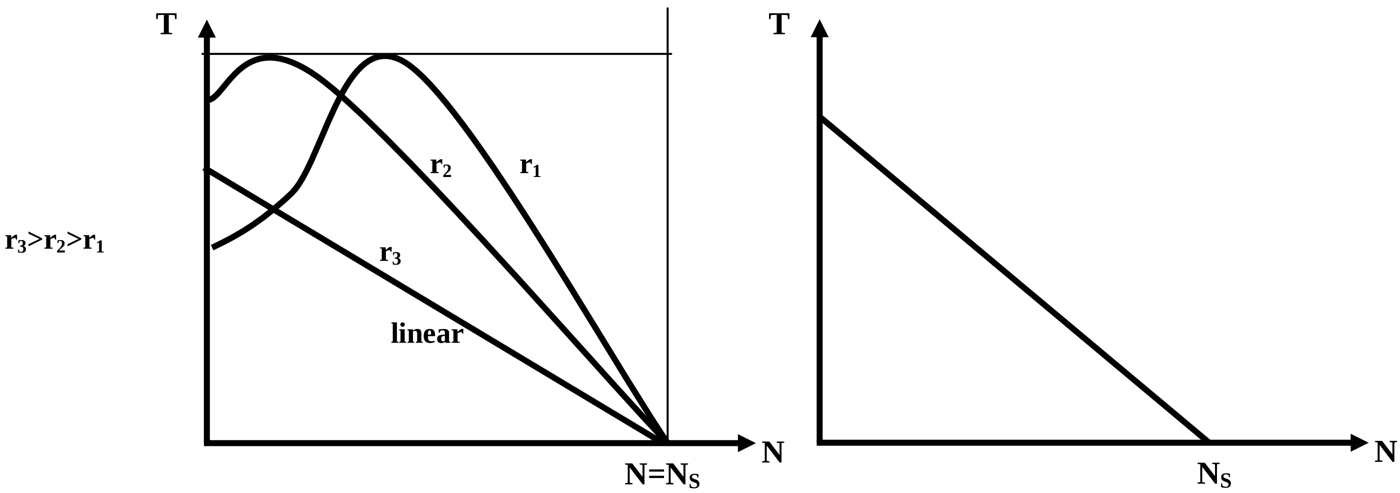
Servomotors

Servomotors:

Servomotors (control motors) are motors which are designed and built for use in feedback control systems. They have high speed of response and they are made for d.c. as well as for a.c. operation. These motors usually fractional horsepower motors having low efficiency.

Two-phase servomotors:

An a.c. servomotor is an induction motor with two primary windings mutually displaced in magnetic position from one another by 90^{o} electrical degrees. It has low inertia and high-resistance rotor, thus, giving a speed-torque curve that is linear in shape from no load speed to stand-still.



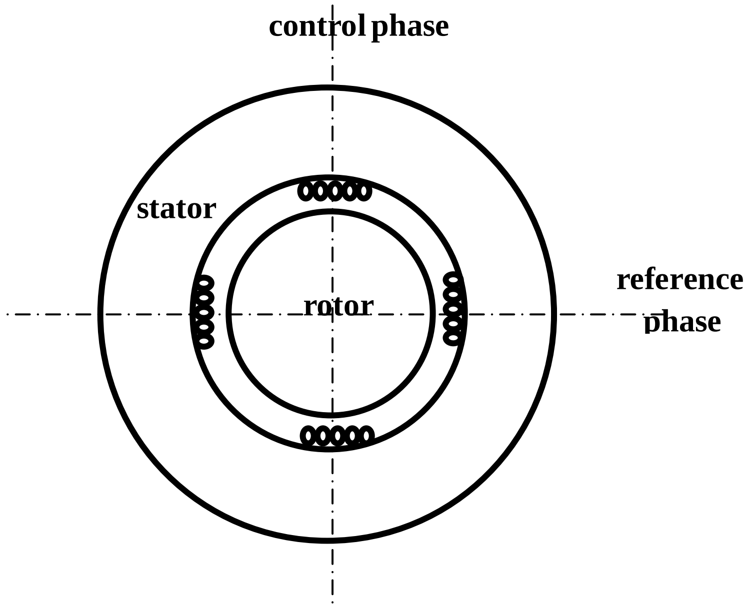
It is designed for operation with a constant voltage applied to one of the windings , called the fixed phase , while a time-displaced adjustable voltage is impressed on the other winding which is called the control winding.

The two phase induction motor is the most important machine which is used as a servomotor. This is because:

1- No brushes and slip-rings are used. Thus, less maintenance is required and the motor is rugged and robust in construction.

2- The motor requires only a simple control amplifier.

Construction:

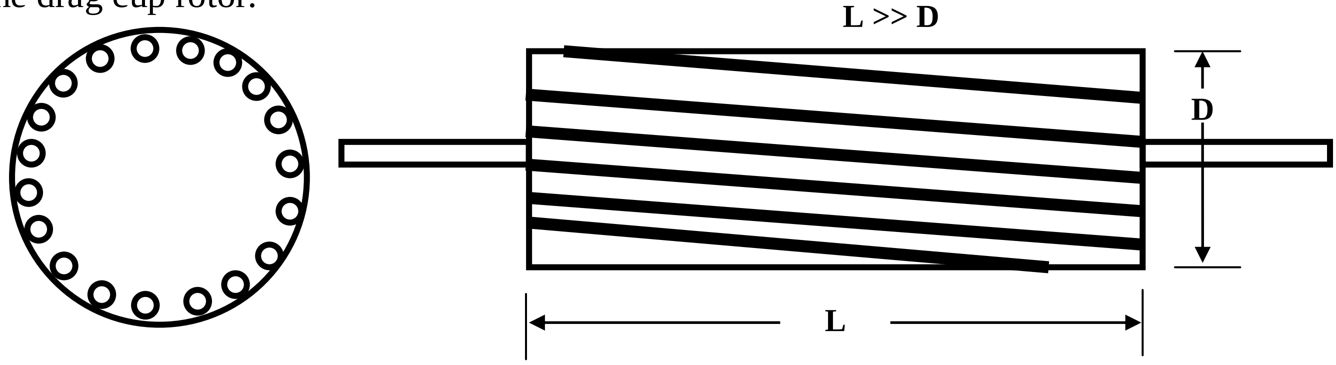


Stator:

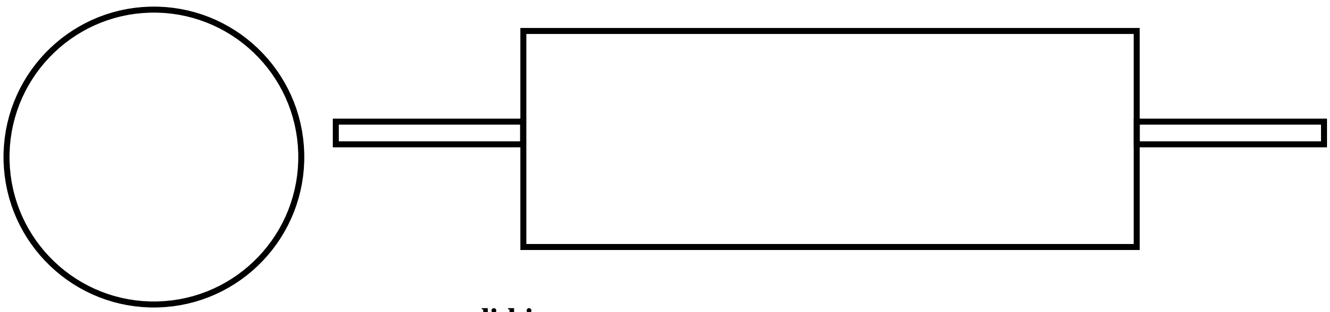
The stator is similar to that of the split-phase induction motor. It has two windings called control winding and reference winding displaced by 90^{o} electrical angle with each other. The two windings may be identical or not depending on the applications.

Rotor:

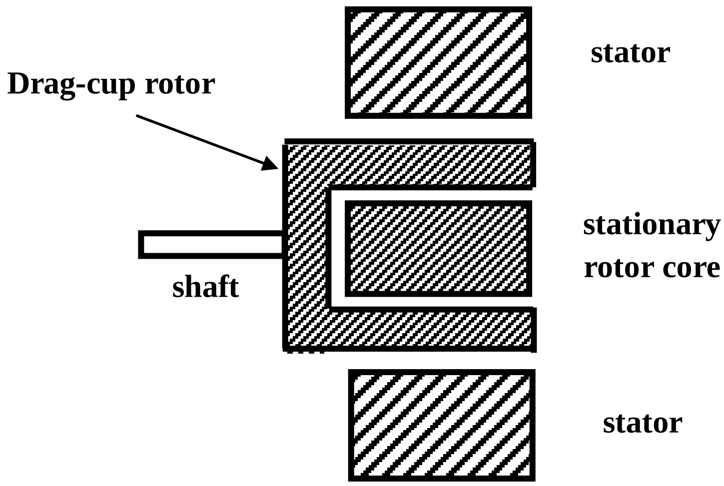
There are three different rotor types. The squirral cage rotor, the solid iron rotor and the drag cup rotor.



squirral cage rotor



solid iron rotor



The squirral-cage rotor is usually small in diameter to keep the mechanical inertia as low as possible and the rotor has high resistance in order to obtain linear speed-torque characteristics. The rotor winding is skewed in order to:

1- make the motor run quietly by reducing the magnetic hum.

2- reduce the locking tendency of the rotor, the tendency of the rotor teeth to remain under the stator teeth due to direct magnetic attraction between them.

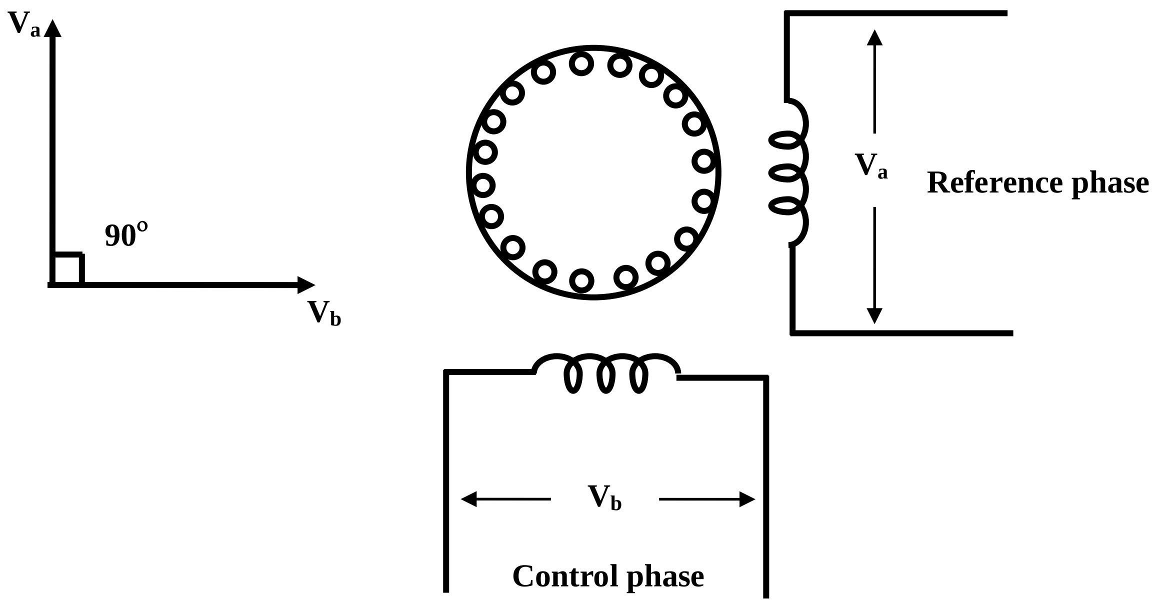
The solid iron rotor consists of a solid cylinder of steel without any conductors or slots. The motor operation depends upon the production of eddy currents in the steel rotor. Thus , the rotor must have, in addition to good magnetic properties, high conductivity so that sufficient eddy current can flow in the rotor. No skewing is required since there are no teeth. The torque developed by this motor is lower than that of the squirral-cage rotor.

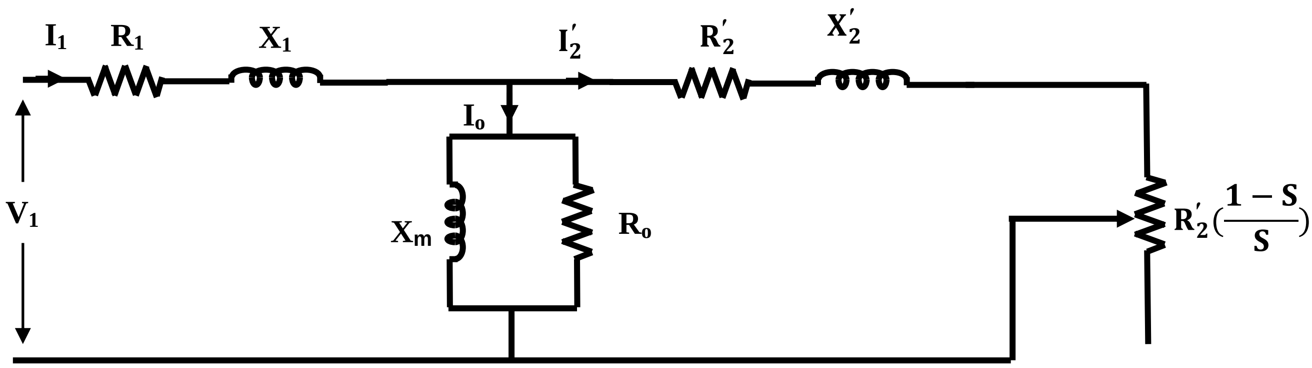
The drag-cup rotor consists of a cup of a nonmagnetic conducting material such as copper. This rotor is used for low output motors (few watts) in order to minimize the moment of inertia. The drag-cup rotor can be described as a special from of squirral-cage rotor in which the rotor teeth are removed, the rotor core is held stationary and the squirral cage bars and the end rings are replaced by a cylinderical cup,.

<u>Principles of operation:</u>

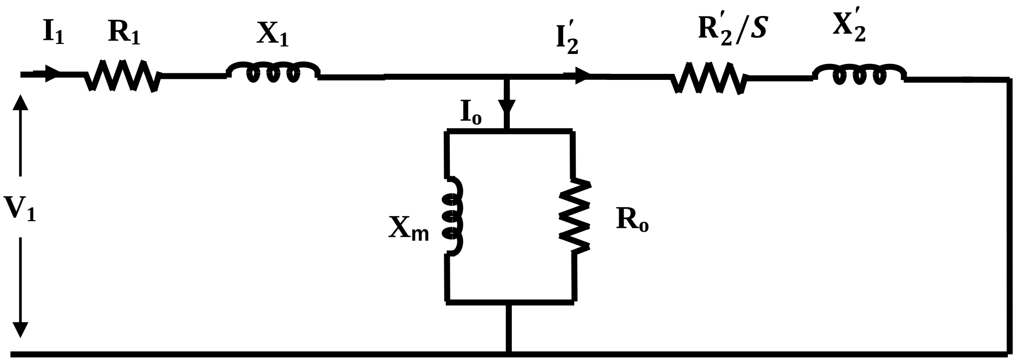
<u>Two phase balanced operation:</u>

The two windings have equal number of turns of the same cross section, i.e., symmetrical two-phase windings. Balanced two-phase supply is applied.





equivalent circuit



$Z_1 = R_1 + jX_1$ = stator impedance.

$Z_2 = \frac{R'_2}{S} + jX'_2$ = rotor impedance.

$Z_o = \frac{R_o\, jX_m}{R_o + jX_m}$ =magnetising branch impedance.

$$Z_{ph} = Z_1 + \frac{Z_o\, Z_2}{Z_o + Z_2}$$

$$I_1 = \frac{V_1}{Z_{ph}}$$

$$I_2' = I_1\, \frac{Z_o}{Z_o + Z_2}$$

rotor copper loss $=2I_2'^2 R_2'$
rotor input$=P_g = 2I_2'^2 R_2'/S$

mechanical power developed $=P_d = 2I_2'^2 R_2'(\frac{1-S}{S})$

output power $=P_o = P_d -$(mechanical losses +rotor iron losses)

power input $=2V_1 I_1 \cos\varphi_1 = P_i$

$$\zeta\% = \frac{P_o}{P_i} * 100$$

developed torque = $T_d = \frac{P_d}{\omega}$ = gross-torque

where: $\omega = 2\pi \frac{N}{60} = 2\pi \frac{(1-S)N_s}{60}$= (1-S)$\omega_s$

T= output torque (useful torque)=T_d - T_{loss}

where: T_{loss} is the friction and windage torque

$$T_d = \frac{2I_2'^2 R_2'(\frac{1-S}{S})}{\omega}$$

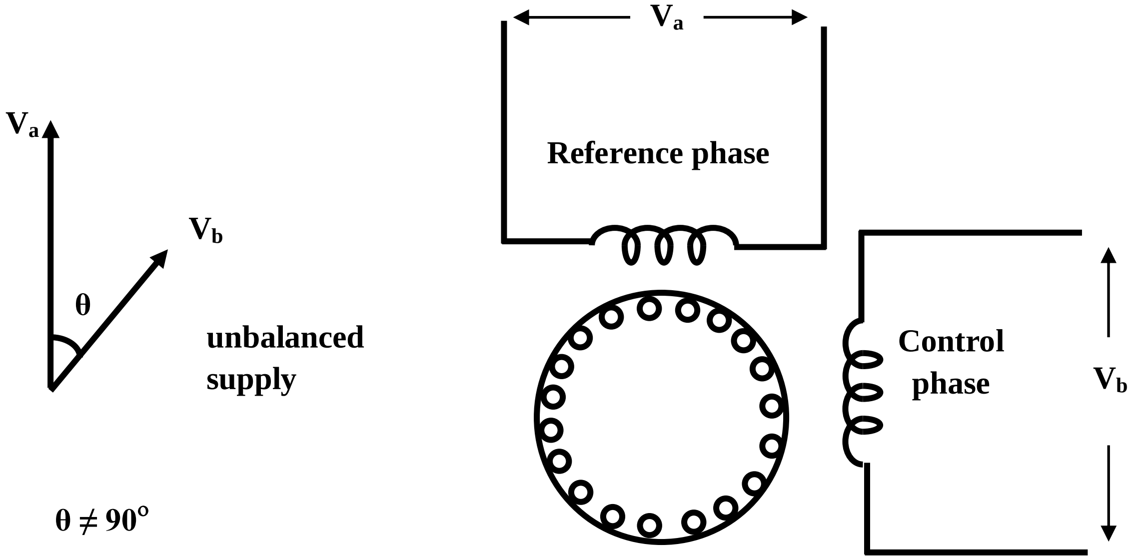
$$= \frac{2\, R_2'(\frac{1-S}{S})}{(1-S)\omega_s} [\frac{V_1}{Z_1 + \frac{Z_o\, Z_2}{Z_o + Z_2}}\, \frac{Z_o}{Z_o + Z_2}]^2$$

$$\boxed{\therefore T_d = \frac{2}{\omega_s}\, \frac{R_2'}{S} [\frac{Z_o}{Z_1 Z_2 + Z_1 Z_o + Z_2 Z_o}]^2 V_1^2}$$

Unbalanced operation of 2-phase servomotor:

The 2-phase windings are assumed to be balanced. i.e., the two windings have the same number of turns with the same conductors cross section area and they are displaced by 90^{o} electrical angle in space.

Unbalanced voltages are applied:



$\theta \neq 90^{o}$

$V_a \neq V_b$

This unbalanced supply can be analysed into positive and negative balanced voltages:

$|V_{a1}| = |V_{b1}|$
+ve sequence voltages

$|V_{a2}| = |V_{b2}|$
-ve sequence voltages

The negative sequence voltages will produce rotating field in the backward direction and thus producing backward torque.

$V_a = V_{a1} + V_{a2}$ ------------------------(1)

$V_b = V_{b1} + V_{b2}$ ------------------------(2)

$V_{a1} = j\, V_{b1}$ --------------------------(3)

$V_{b2} = j\, V_{a2}$ --------------------------(4)

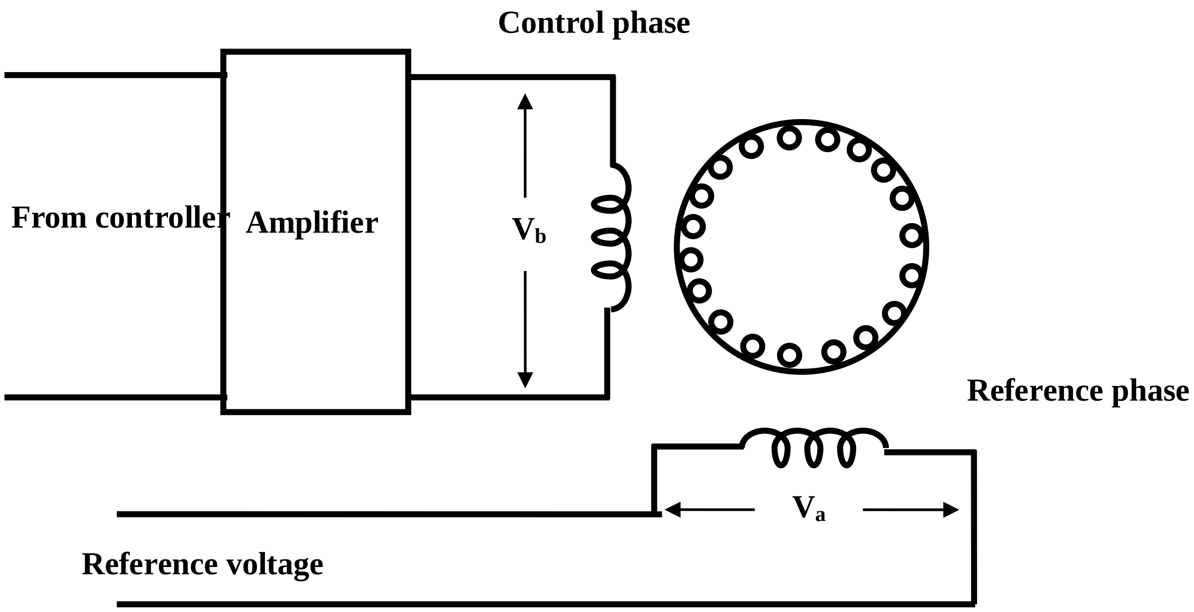
$$\boxed{V_{a1} = \frac{V_a + jV_b}{2}}$$

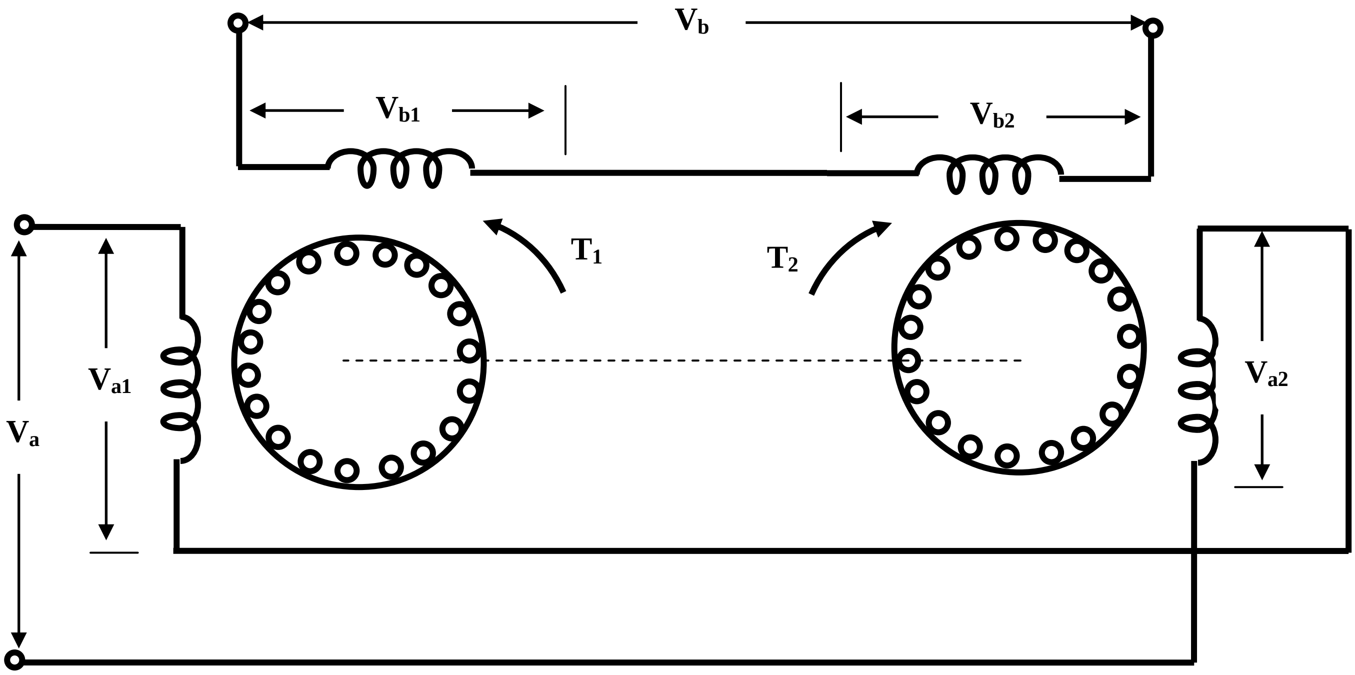
$$\boxed{V_{a2} = \frac{V_a - jV_b}{2}}$$

$$\boxed{V_{b1} = -j\, V_{a1}}$$

$$\boxed{V_{b2} = j\, V_{a2}}$$

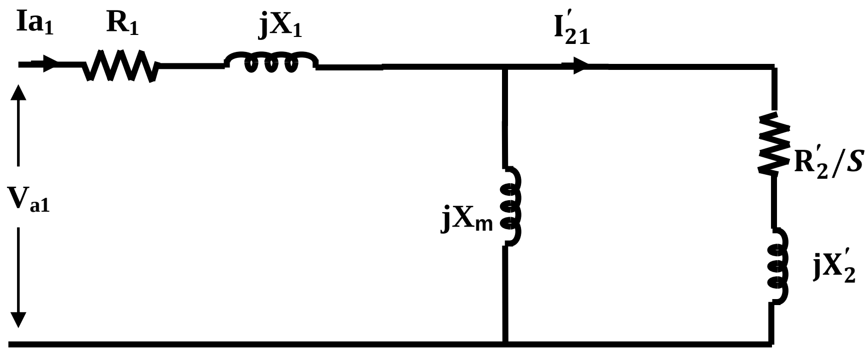
Determination of machine performance using the sequence components of voltages:



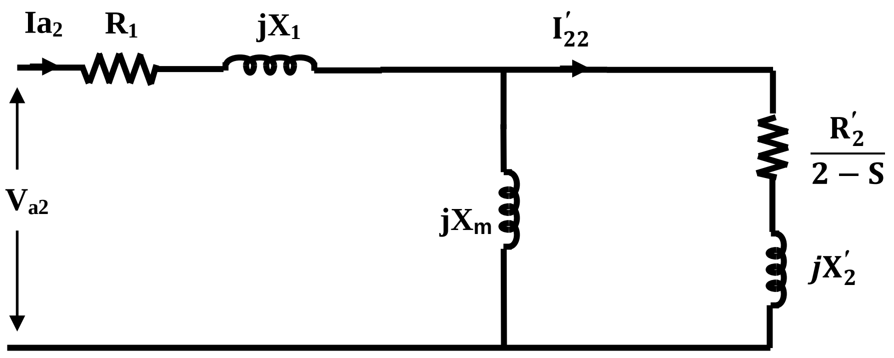


Using symmetrical components to analyse the 2-phase servomotor leads to a very useful and interesting physical interpretation of the analysis of the unbalanced two-phase servomotor. The unbalanced voltages are replaced by positive and negative sequence voltages. The positive sequence components in each phase (V_{a1} and V_{b1}) produce revolving magnetic field which interacts with the rotor winding to produce a positive sequence torque T_1. At the same time the negative-sequence components in each phase (V_{a2} and V_{b2}) produces revolving magnetic field which interacts with the rotor winding to produce a negative sequence torque T_2. The rotor responds to the resultant torque which is the difference between T_1 and T_2. The techniques of balanced operation can now be used to determine T_1 and T_2 and the results can then be superposed to obtain the resultant.

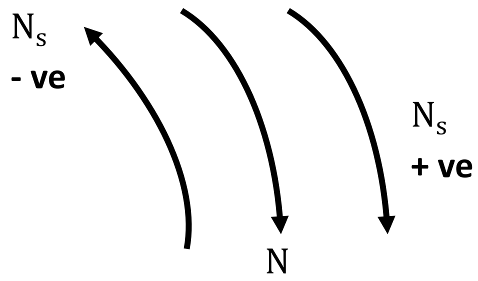
Equivalent circuit:



+ve sequence equivalent circuit



- ve sequence equivalent circuit



$S = \frac{N_s - N}{N_s}$ + ve sequence

$S_2 = \frac{N_s + N}{N_s}$ - ve sequence

$$= \frac{N_s + (N_s - SN_s)}{N_s} = \frac{2N_s - SN_s}{N_s}$$

$$= 2 - S$$

<u>+ ve sequence response:</u>

$$Z_{ph\,1} = R_1 + jX_1 + \frac{jX_m\left(\frac{R_2'}{S} + jX_2'\right)}{jX_m + \frac{R_2'}{S} + jX_2'}$$

$$I_{a1} = \frac{V_{a1}}{Z_{ph\,1}}$$

$$I_{21}' = I_{a1} \cdot \frac{jX_m}{jX_m + \frac{R_2'}{S} + jX_2'}$$

$$P_{g1} = 2I_{21}'^{2}\,\frac{R_2'}{S}$$

$$\therefore T_1 = \frac{P_{g1}}{\omega_s} = \frac{P_{g1}}{2\pi\frac{N_s}{60}}$$

- ve sequence response:

$$Z_{ph\,2} = R_1 + jX_1 + \frac{jX_m\left(\frac{R_2'}{2-S} + jX_2'\right)}{jX_m + \frac{R_2'}{2-S} + jX_2'}$$

$$I_{a2} = \frac{V_{a2}}{Z_{ph\,2}}$$

$$I_{22}' = I_{a2} \cdot \frac{jX_m}{jX_m + \frac{R_2'}{2-S} + jX_2'}$$

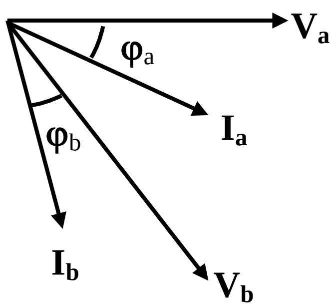
$$P_{g2} = 2I_{22}'^{2}\, \frac{R_2'}{2-S}$$

$$\therefore\ T_2 = \frac{P_{g2}}{\omega_s}$$

$$\therefore\ T = T_1 - T_2$$

input power = input to phase a + input to phase b.

$= V_a\, I_a \cos \varphi_a + V_b\, I_b \cos \varphi_b$



Output power = ω T

$$\zeta\% = \frac{\text{output power}}{\text{input power}} * 100$$

Example:

A 5 watt, 60 Hz, two-pole, two-phase servomotor has the following parameters:

$$R_1 = 285\,\Omega \quad , \quad R_2' = 850\,\Omega$$

$$X_1 = 60\,\Omega \quad , \quad X_2' = 60\,\Omega$$

$$X_m = 995\,\Omega$$

when it is operating at a slip of 0.6 , determine:

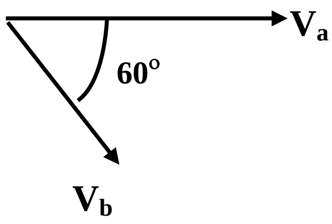
(a) the resultant torque in synchronous watts.

(b) the stator phase currents.

(c) the efficiency.

Assume the servomotor operates with the following unbalanced two-phase voltage : $V_a = 120\ \angle 0^o$ volts and $V_b = 75\angle{-60^o}$ volts. Neglect mechanical losses.

(a)



$$V_{a1} = \frac{V_a + j\,V_b}{2}$$

$$= \frac{120\angle 0^o + j\,75\angle{-60^o}}{2} = 94.358\ \angle 11.42\ ^o\ V$$

$$V_{a2} = \frac{V_a - j\,V_b}{2}$$

$$= \frac{120\angle 0^o -\ 75\angle{-60^o}}{2} = 33.304\ \angle{-34.264}\ ^o\ V$$

$$Z_{ph\,1} = R_1 + jX_1 + \frac{jX_m\,(\frac{R_2'}{S} + jX_2')}{jX_m + \frac{R_2'}{S} + jX_2'}$$

$$Z_{ph\,1} = 285 + j60 + \frac{j995(\frac{850}{0.6} + j60)}{j995 + \frac{850}{0.6} + j60}$$

$$=1028.723\ \angle 44.437\ ^o\ \Omega$$

$$I_{a1} = \frac{V_{a1}}{Z_{ph\,1}} = \frac{94.358\ \angle 11.42\ ^o}{1028.723\ \angle 44.437\ ^o} = 0.0917\ \angle{-32.975}\ ^o\ A$$

$$I_{21}' = I_{a1}.\ \frac{jX_m}{jX_m + \frac{R_2'}{S} + jX_2'}$$

$$= 0.0917\ \angle{-32.975}\ ^o.\ \frac{j995}{j995 + \frac{850}{0.6} + j60} = 0.0517\ \angle 20.35\ ^o\ A$$

$$P_{g1} = 2I_{21}'^2\ \frac{R_2'}{S} = 2(0.0517)^2\,\frac{850}{0.6}$$

$$= 7.5732\ \text{watts.}$$

$$T_1 = \frac{P_{g1}}{\omega_s} = \frac{7,5732}{2\pi\frac{3600}{60}} = 20.089\ mN.m.$$

$$N_s = \frac{120f}{2p} = \frac{120 * 60}{2} = 3600\ r.p.m.$$

$$Z_{ph\,2} = R_1 + jX_1 + \frac{jX_m\left(\frac{R_2'}{2-S} + jX_2'\right)}{jX_m + \frac{R_2'}{2-S} + jX_2'}$$

$$= 285 + j60 + \frac{j995\left(\frac{850}{2-0.6} + j60\right)}{j995 + \frac{850}{2-0.6} + j60}$$

$$= 774.332\ \angle 26.877\ ^{o}\ \Omega$$

$$I_{a2} = \frac{V_{a2}}{Z_{ph\,2}} = \frac{33.304\ \angle{-34.264}\ ^{o}}{774.332\ \angle 26.877\ ^{o}} = 0.04301\ \angle{-61.141}\ ^{o}\ A$$

$$I_{22}' = I_{a2}.\ \frac{jX_m}{jX_m + \frac{R_2'}{2-S} + jX_2'}$$

$$= 0.04301\ \angle{-61.141}\ ^{o}.\ \frac{j995}{j995 + \frac{850}{2-0.6} + j60} = 0.0352\ \angle{-31.221}\ ^{o}\ A$$

$$P_{g2} = 2I_{22}'^{2}\ \frac{R_2'}{2-S} = \ 2(0.0352)^2\frac{850}{2-0.6} = 1.5045\ watts.$$

$$T_2 = \frac{P_{g2}}{\omega_s} = \frac{1.5045}{2\pi\frac{3600}{60}} = 3.991\ mN.m.$$

The resultant torque in synchronous watts= P_{g1} - P_{g2}

= 7.5732 - 1.5045= 6.0687

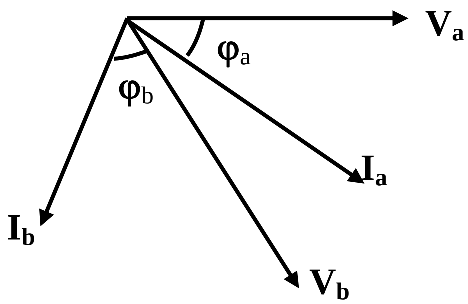
(b) $I_a = I_{a1} + I_{a2} = 0.0917\angle{-32.975^o} + 0.04301\angle{-61.141^o}$

$= 0.1312\angle{-41.877^o}$ A

$I_b = I_{b1} + I_{b2} = -j\ I_{a1} + j\ I_{a2}$

$= -j\ 0.0917\angle{-32.975^o} + j\ 0.04301\angle{-61.141^o}$

$= 0.0575\angle -102.295^o$ A



(c)

$\varphi_a = 41.877^o$

$\varphi_a = 102.295^o - 60^o$

$= 42.295^o$

input power = $V_a I_a \cos\varphi_a + V_b I_b \cos\varphi_b$

$= 120 * 0.1312 \cos 41.877^o + 75 * 0.0575 \cos 42.295^o$

$= 14.9126$ watts.

$T = T_1 - T_2 = 20.089 * 10^{-3} - 3.991 * 10^{-3}$

$= 16.098$ m N.m.

output torque = T (neglecting T_{loss})

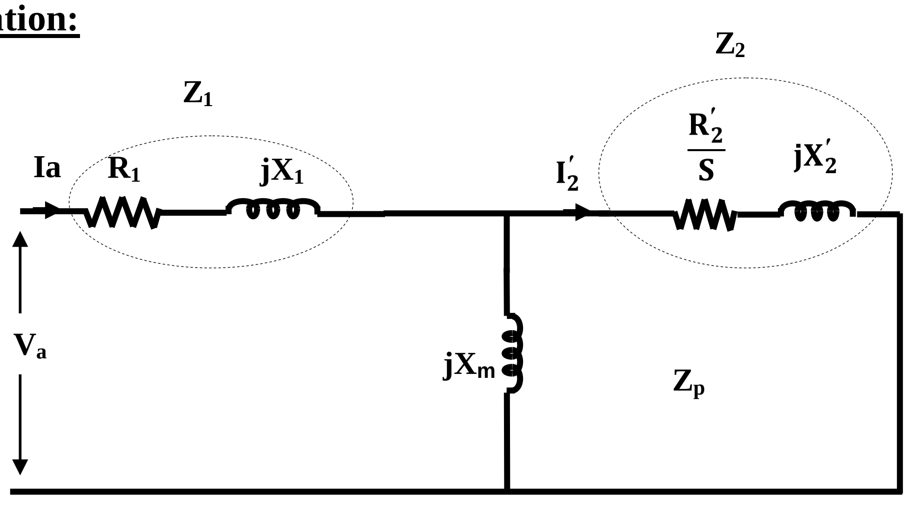
output power= $T\omega = 16.089 * 10^{-3} * 2\pi \frac{(1-0.6)*3600}{60}$

$= 2.4275$ watts

$$\zeta\% = \frac{\text{output power}}{\text{input power}} * 100 = \frac{2.4275}{14.9126} * 100$$

$= 16.2782$ %

Balanced operation:



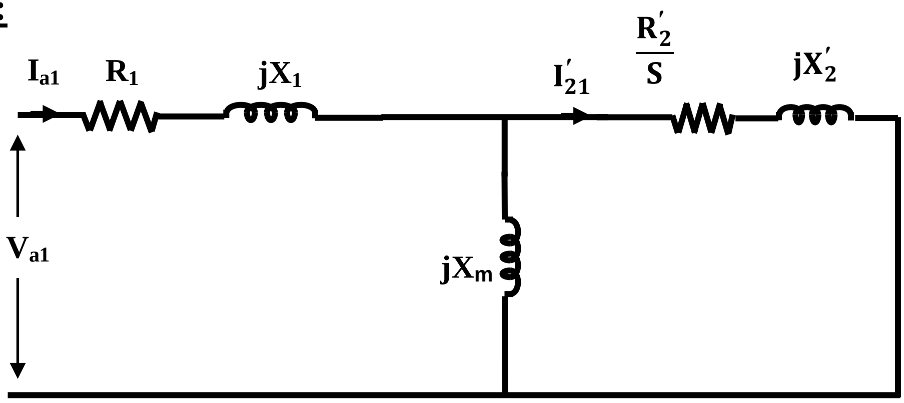
$$I_a = \frac{V_a}{Z_{ph}}$$

$$\therefore I_2' = I_a . \frac{Z_p}{Z_2} = \frac{V_a}{Z_{ph}} . \frac{Z_p}{Z_2}$$

$$\therefore P_g = 2[\frac{V_a}{Z_{ph}} . \frac{Z_p}{Z_2}]^2 \frac{R_2'}{S}$$

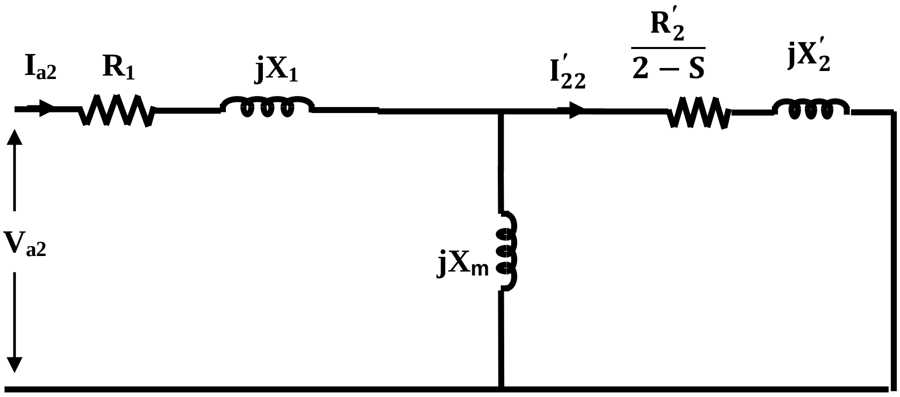
Unbalanced operation:

+ ve sequence:



$$P_{g1} = 2[\frac{V_{a1}}{Z_{ph\,1}} . \frac{Z_{p1}}{Z_{21}}]^2 \frac{R_2'}{S}$$

- ve sequence:



$$P_{g2} = 2[\frac{V_{a2}}{Z_{ph\,2}} . \frac{Z_{p2}}{Z_{22}}]^2 \frac{R_2'}{2 - S}$$

$$P_{g1} = [\frac{V_{a1}}{V_a}]^2 P_g$$

$$P_{g2} = [\frac{V_{a2}}{V_a}]^2 P_g'$$

where:

$$P_g' = 2[\frac{V_a}{Z_{ph\,2}} \cdot \frac{Z_{p2}}{Z_{22}}]^2 \frac{R_2'}{2-S}$$

$$T = \frac{1}{\omega_s}[\left(\frac{V_{a1}}{V_a}\right)^2 P_g - \left(\frac{V_{a2}}{V_a}\right)^2 P_g']$$

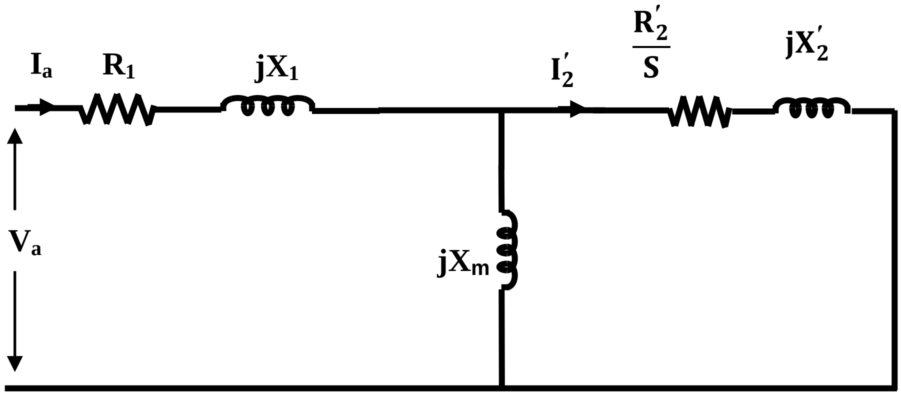
$$\therefore T = \left(\frac{V_{a1}}{V_a}\right)^2 T_{B1} - \left(\frac{V_{a2}}{V_a}\right)^2 T_{B2}$$

where: $T_{B1} = \frac{P_g}{\omega_s}$ denotes the developed torque at slip S for balanced two-phase operation at rated voltage.

$T_{B2} = \frac{P_g'}{\omega_s}$ denotes the developed torque computed at slip 2-S for balanced two-phase operation at rated voltage.

The importance of this equation lies in the fact that the resultant torque at any slip and for any condition of unbalanced two-phase voltages may be computed in terms of the <u>torque for full-voltage balanced operation.</u> This information is usually supplied by the motor manufacturer. Hence, the torque-speed characteristic for various values of the control winding voltage can be determined without needing the equivalent circuit parameters.

<u>Example:</u> Compute the resultant torque of the servomotor of the previous example using the last equation.



$$I_a = \frac{V_a}{Z_{ph}}$$

$$= \frac{120 \angle 0^{\circ}}{1028.723 \angle 44.437^{\circ}} = 0.1167 \angle -44.437^{\circ} A$$

$$I_2' = I_a . \frac{jX_m}{jX_m + \frac{R_2'}{S} + jX_2'} = 0.1167 \angle -44.437^{\circ} . \frac{j995}{j995 + \frac{850}{0.6} + j60}$$

$$= 0.0657 \angle 8.8877^{\circ} A$$

$$P_g = 2I_2'^2 \frac{R_2'}{S} = 2(0.0657)^2 \frac{850}{0.6} = 12.2301 \, watts$$

$$P_{g1} = [\frac{V_{a1}}{V_a}]^2 P_g = [\frac{94.358}{120}]^2 * 12.2301 = 7.5618 \text{ watts}$$

To find P_g' , the motor is assumed to be operating at a slip of 2-S=1.4 with balanced two-phase voltages applied.

$$I_a = \frac{V_a}{Z_{ph\,2}} = \frac{120}{774.332 \angle 26.877^{\circ}} = 0.15497 \angle -26.877^{\circ} A$$

$$I_2' = I_a . \frac{jX_m}{jX_m + \frac{R_2'}{2-S} + jX_2'}$$

$$= 0.15497 \angle -26.877^{\circ} . \frac{j995}{j995 + \frac{850}{1.4} + j60} = 0.12668 \angle 3.043^{\circ} A$$

$$\therefore P_g{}' = 2I_2'^2 \frac{R_2'}{2-S} = 2(0.12668)^2 \frac{850}{1.4} = 19.4866 \, watts.$$

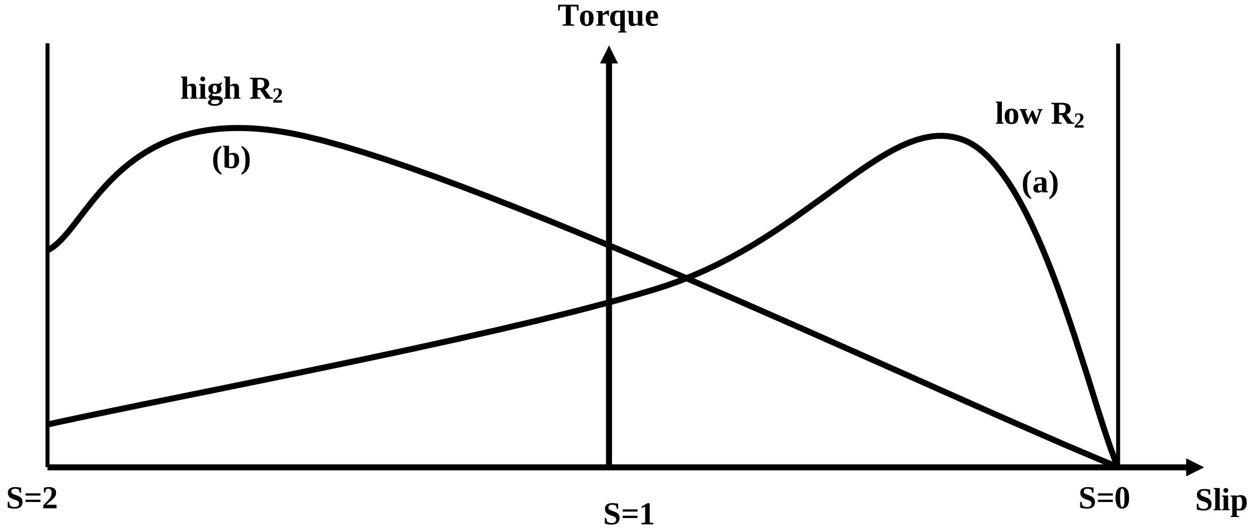
$$P_{g2} = [\frac{V_{a2}}{V_a}]^2 P_g' = [\frac{33.304}{120}]^2 * 19.4866 = 1.501 \text{ watts}$$

$$T = \frac{1}{\omega_s}\left[P_{g1} - P_{g2}\right] = \frac{1}{2\pi \frac{3600}{60}} [7.5618 - 1.501]$$

$$= 0.01608 \text{ N.m.}$$

Servomotor torque-speed curves:

The torque-speed curve of a two-phase induction motor supplied by a balanced two-phase voltage has a shape similar to that of the three-phase induction motor as shown in curve "a".



This characteristic is not suitable for control systems because of the positive slope over most of the operating speed range. The positive slope represents negative damping in the control system which in turn can lead to a condition of instability. Therefore for control systems applications the motor must be modified in a way that ensures positive damping over the full speed range. The usual way to achieve this result is to design the motor with very high rotor resistance. The torque-speed characteristic then will take the shape shown by curve "b".

In many applications of the servomotor in feedback control systems , phase a is energized with fixed rated voltage (reference voltage), while phase b is energized with a varying control voltage that is usually obtained from a control amplifier. The control voltage is usually adjusted to be exactly 90^o out of phase with the reference voltage. Thus, the unbalance operation is only because of the unbalance in voltage magnitude. Assume 90^o phase shift between V_a and V_b, we can define the quality K such that:

$$K = \left|\frac{V_b}{V_a}\right|$$

$$\therefore V_b = -j\,K\,V_a$$

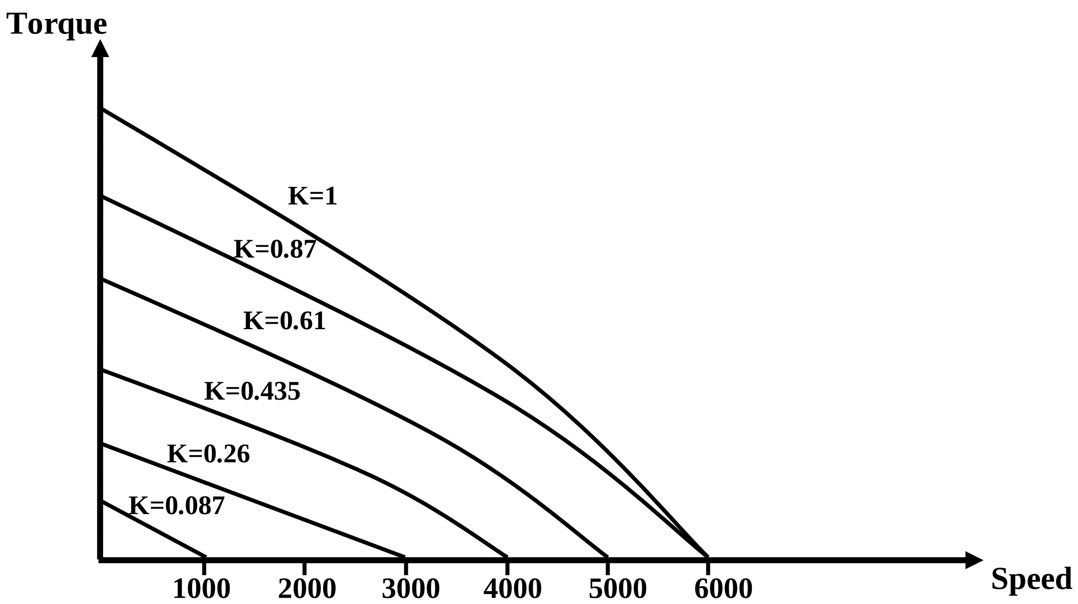
$$V_{a1} = \frac{V_a + jV_b}{2} = \frac{V_a + j(-jKV_a)}{2}$$

$$\therefore \boxed{V_{a1} = \frac{V_a}{2}(1 + K)}$$

similarly:

$$V_{a2} = \frac{V_a - jV_b}{2} = \frac{V_a - j(-jKV_a)}{2}$$

$$\therefore \boxed{V_{a2} = \frac{V_a}{2}(1 - K)}$$

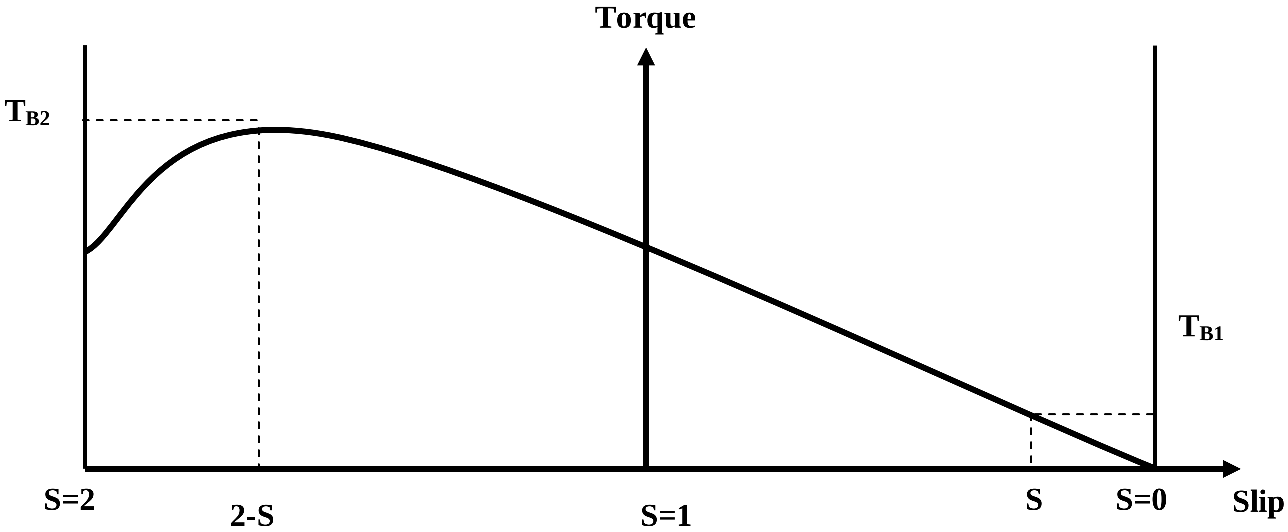


The above figure represents the torque-speed curves for a two-phase servomotor for various values of the control voltage expressed as a fraction (K) of the reference voltage. K=1 means balanced operation.

If the information about the torque-speed characteristic of the servomotor is supplied by the manufacturer for balanced rated operation over ther full range of slip from zero to 2, then the torque-speed characteristic for any value of K can be easily determined without using the equivalent circuit by the following equation:

$$T = \left(\frac{V_{a1}}{V_a}\right)^2 T_{B1} - \left(\frac{V_{a2}}{V_a}\right)^2 T_{B2}$$

$$\therefore \boxed{T = \left(\frac{1+K}{2}\right)^2 T_{B1} - \left(\frac{1-K}{2}\right)^2 T_{B2}}$$

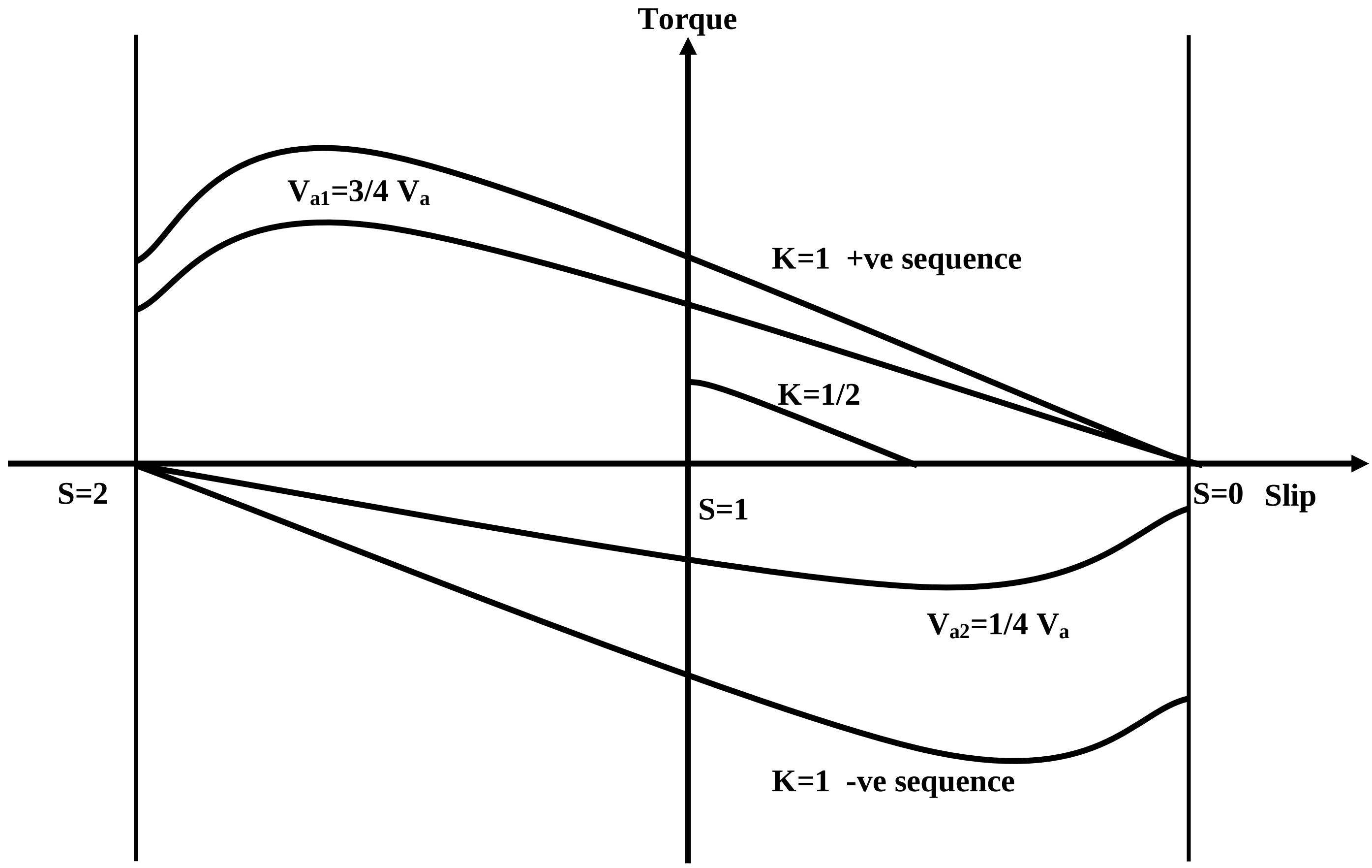


Thus, to calculate the speed-torque characteristic for K=1/2:

$$T = \left(\frac{1+\frac{1}{2}}{2}\right)^2 T_{B1} - \left(\frac{1-\frac{1}{2}}{2}\right)^2 T_{B2} = \left(\frac{3}{4}\right)^2 T_{B1} - \left(\frac{1}{4}\right)^2 T_{B2}$$

$$T = \frac{9}{16} T_{B1} - \frac{1}{16} T_{B2}$$

Taking a set of values for S and evaluating T_{B1} and T_{B2} from the given speed torque characteristic, when K=1 , it is possible to determine T for these values of S and thus to draw the torque-speed characteristic.



T_{B1} and T_{B2} can be calculated as follows using the equivalent circuit parameters:

$$T_d = \frac{2}{\omega_s}.\frac{R_2'}{S}\,[\frac{Z_0}{Z_1Z_2 + Z_1Z_0 + Z_2Z_0}]^2 V_1^2$$

$$\therefore T_{B1} = \frac{2}{\omega_s}.\frac{R_2'}{S}\,[\frac{Z_0}{Z_2(Z_1 + Z_0) + Z_1Z_0}]^2 V_a^2$$

$$= \frac{2}{\omega_s}.\frac{R_2'}{S}\,[\frac{Z_0\ V_a}{Z_1 + Z_0}]^2[\frac{1}{Z_2 + \frac{Z_1Z_0}{Z_1 + Z_0}}]^2$$

let $\frac{Z_1Z_0}{Z_1+Z_0} = R' + jX'$

$$\therefore T_{B1} = \frac{2}{\omega_s}.\frac{R_2'}{S}\,[\frac{Z_0\ V_a}{Z_1 + Z_0}]^2[\frac{1}{\frac{R_2'}{S} + jX_2' + R' + jX'}]^2$$

$$\therefore T_{B1} = C\frac{\frac{1}{S}}{(\frac{R_2'}{S} + R')^2 + (X_2' + X')^2}$$

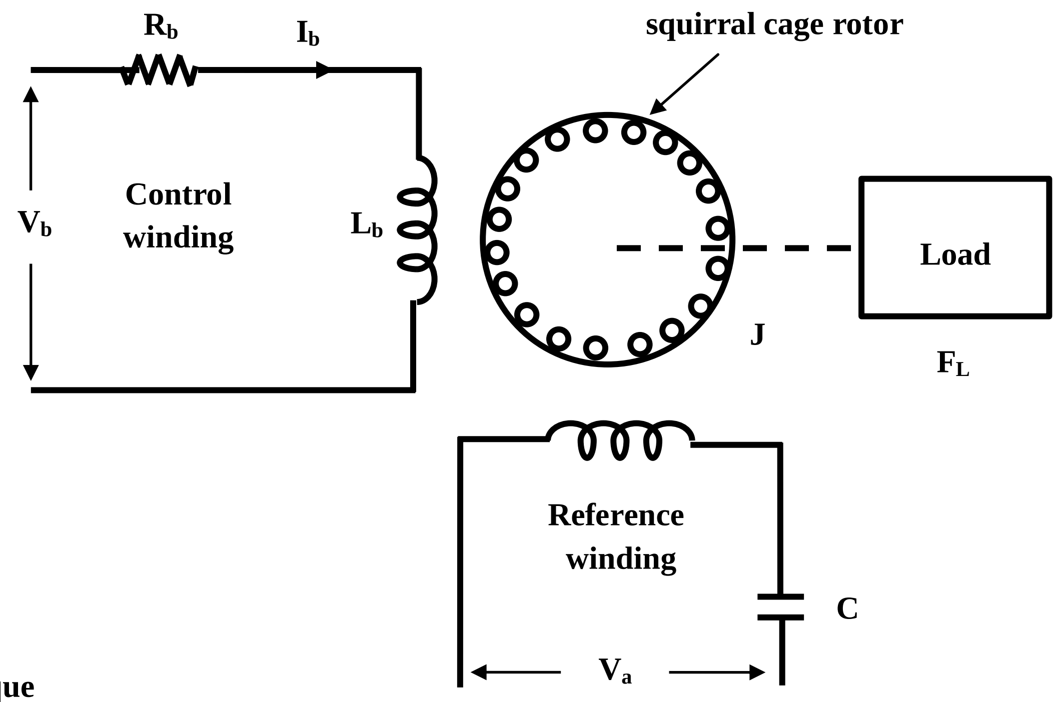
similarly:

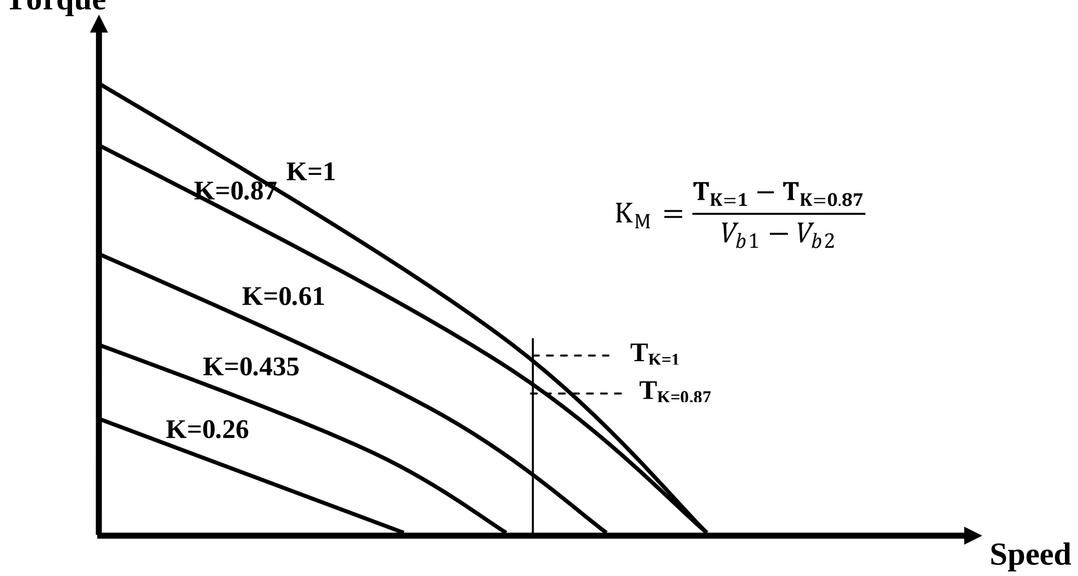
$$\therefore T_{B2} = C\frac{\frac{1}{2-S}}{(\frac{R_2'}{2-S}+R')^2+(X_2'+X')^2}$$

where:

$$C = \frac{2}{\omega_s}.R_2' \left[\frac{Z_0\ V_a}{Z_1+Z_0}\right]^2$$

Two-phase servomotor transfer functions:





Typical torque-speed characteristics of servomotor

The servomotor is assumed to be operated at a fixed reference voltage. The variable input is the control winding voltage V_b. The torque-speed curves are assumed to be linear. Thus, an increase in motor torque results either by an increase in control winding voltage or by decrease in speed for fixed V_b.

Thus,

$$T = K_M V_b - F_M \frac{d\theta}{dt}$$

where: T=motor developed torque.

K_M = motor torque constant (Nm/volt).

F_M =motor equivalent viscous-friction constant (Nm/rad/sec).

It can be seen from the torque-speed curves that F_M is the slope of the torque-speed curve at constant V_b. Also K_M is the change in torque per unit of change in control voltage V_b at constant speed.

Equating this torque to load torque gives:

$$K_M V_b - F_M \frac{d\theta}{dt} = J \frac{d^2\theta}{dt^2} + F_L \frac{d\theta}{dt}$$

where: J=rotor inertia + load inertia referred to motor shaft.

F_L =viscous friction of load referred to motor shaft.

viscous friction = friction which is proportional with speed.

$$K_M V_b = J \frac{d^2\theta}{dt^2} + (F_M + F_L) \frac{d\theta}{dt}$$

$$K_M V_b = J \frac{d^2\theta}{dt^2} + F \frac{d\theta}{dt}$$

where: $F = F_M + F_L$

Taking Laplace transform yields:

$$K_M V_b(s) = Js^2\theta(s) + Fs\theta(s)$$

$$\frac{\theta(s)}{V_b(s)} = \frac{K_M/F}{s(1 + s\,J/F)}$$

$$\therefore \quad \boxed{\frac{\theta(s)}{V_b(s)} = \frac{K_m}{s(1+s\tau_m)}}$$

where: $\quad K_m = \frac{K_M}{F}$

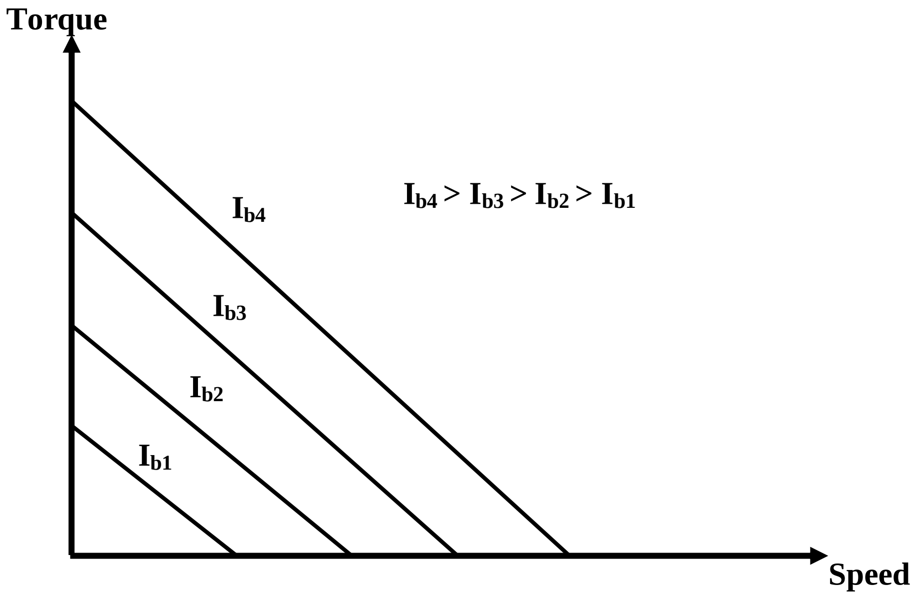
$$\tau_m = \frac{J}{F} = \text{motor time constant.}$$

$$V_b(s) \longrightarrow \boxed{\frac{K_m}{s(1+s\tau_m)}} \longrightarrow \theta(s)$$

<u>Effect of the control winding time constant:</u>

In the above derivation of control motor transfer function the control winding time constant was neglected. A more exact description of the servomotor transfer function can be obtained by knowing that the control winding voltage produces control winding field current I_b which in turn produces the flux that enables the motor to develop torque. Thus,

$$T = K_b I_b - F_M \frac{d\theta}{dt}$$



$$\therefore K_b I_b - F_M \frac{d\theta}{dt} = J \frac{d^2\theta}{dt^2} + F_L \frac{d\theta}{dt}$$

$$K_bI_b = J\frac{d^2\theta}{dt^2} + (F_M + F_L)\frac{d\theta}{dt} = J\frac{d^2\theta}{dt^2} + F\frac{d\theta}{dt}$$

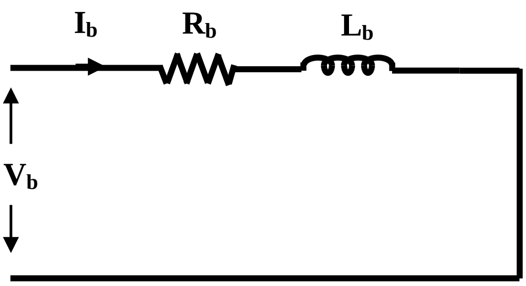
where: $F=F_M + F_L$

Taking Laplace transform yields:

$$K_bI_b(s) = Js^2\theta(s) + Fs\theta(s)$$

$$\frac{\theta(s)}{I_b(s)} = \frac{K_b/F}{s(1+\tau_m s)} \quad -------(1)$$

where: $\tau_m = \frac{J}{F}$



$$v_b = R_bi_b + L_b\frac{di_b}{dt}$$

$$V_b(s) = R_bI_b(s) + L_bsI_b(s)$$

$$\frac{I_b(s)}{V_b(s)} = \frac{1}{R_b(1+\tau_b s)} \quad -------(2)$$

where: $\tau_b = \frac{L_b}{R_b}$ = control winding time constant.

Multiplying equations (1) and (2):

$$\frac{\theta(s)}{V_b(s)} = \frac{\frac{K_b}{F\,R_b}}{s(1+\tau_m s)(1+\tau_b s)}$$

From this equation $\frac{K_b}{R_b} = K_M$

$$\therefore \boxed{\frac{\theta(s)}{V_b(s)} = \frac{K_m}{s(1+\tau_m s)(1+\tau_b s)}}$$

$$T = K_Mv_b - F_M\frac{d\theta}{dt}$$

$$T = K_bv_b - F_M\frac{d\theta}{dt}$$

$$K_Mv_b = K_bI_b$$

$$\frac{V_b}{I_b} = R_b \text{ (steady state)}$$

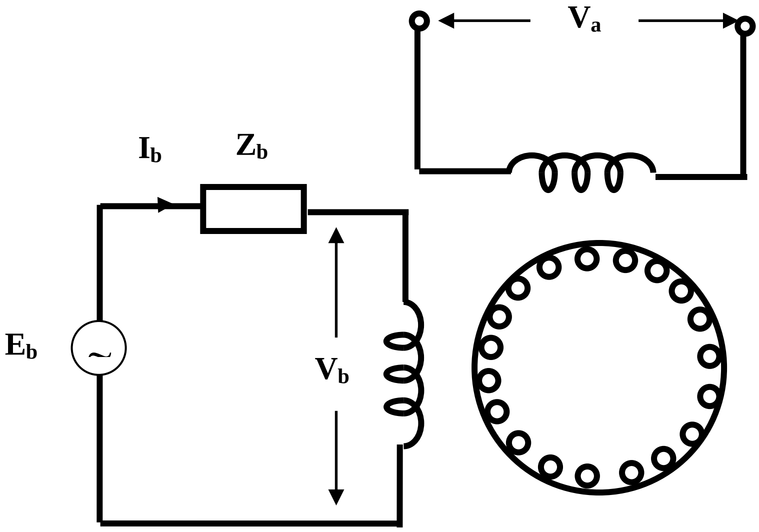
$$K_M = \frac{K_b}{R_b}$$

Motor characteristics in the presence of finite control-impedance:

$E_b = V_b + I_b Z_b$

Assuming that

$E_b = -j\,KV_a$



$$I_b = I_{b1} + I_{b2} = \frac{V_{b1}}{Z_1} + \frac{V_{b2}}{Z_2}$$

Z_1 = + ve sequence impedance (at slip s)

Z_2 = - ve sequence impedance (at slip 2-s)

$$\therefore I_b = \frac{-jV_{a1}}{Z_1} + \frac{jV_{a2}}{Z_2}$$

$$\because V_a = V_{a1} + V_{a2} \qquad \therefore V_{a2} = V_a - V_{a1}$$

$$\therefore I_b = \frac{-jV_{a1}}{Z_1} + \frac{jV_a}{Z_2} - \frac{jV_{a1}}{Z_2}$$

$$E_b = V_b + I_b Z_b$$

$$V_b = V_{b1} + V_{b2} = -jV_{a1} + jV_{a2} = -jV_{a1} + j(V_a - V_{a1})$$

$$= -2jV_{a1} + jV_a$$

$$\therefore E_b = -jKV_a = -2jV_{a1} + jV_a + Z_b\left[\frac{-jV_{a1}}{Z_1} + \frac{jV_{a2}}{Z_2} - \frac{jV_{a1}}{Z_2}\right]$$

$$V_a\left(-K - 1 - \frac{Z_b}{Z_2}\right) = V_{a1}\left(-2 - \frac{Z_b}{Z_1} - \frac{Z_b}{Z_2}\right)$$

$$\therefore \quad \boxed{\frac{V_{a1}}{V_a} = \frac{1 + K + \frac{Z_b}{Z_2}}{2 + \frac{Z_b}{Z_1} + \frac{Z_b}{Z_2}}}$$

$$V_{a2} = V_a - V_{a1} = V_a - V_a\left[\frac{1 + K + \frac{Z_b}{Z_2}}{2 + \frac{Z_b}{Z_1} + \frac{Z_b}{Z_2}}\right]$$

$$\therefore \quad \boxed{\frac{V_{a2}}{V_a} = \frac{1 - K + \frac{Z_b}{Z_1}}{2 + \frac{Z_b}{Z_1} + \frac{Z_b}{Z_2}}}$$

$$\therefore \quad T = \left(\frac{V_{a1}}{V_a}\right)^2 T_{B1} - \left(\frac{V_{a2}}{V_a}\right)^2 T_{B2}$$

$$= \left(\frac{1 + K + \frac{Z_b}{Z_2}}{2 + \frac{Z_b}{Z_1} + \frac{Z_b}{Z_2}}\right)^2 T_{B1} - \left(\frac{1 - K + \frac{Z_b}{Z_1}}{2 + \frac{Z_b}{Z_1} + \frac{Z_b}{Z_2}}\right)^2 T_{B2}$$

if $Z_b=0$

$$\therefore \quad T = \left(\frac{1 + K}{2}\right)^2 T_{B1} - \left(\frac{1 - K}{2}\right)^2 T_{B2}$$

Damped AC servomotors:

servomotors are used in feedback control systems. The servomotors are used to actuate something directly or indirectly. Now, for successful operation of a control system, the servomotor must fulfill at least the following two things:

1- The motor must have high response to changes in control phase voltage.

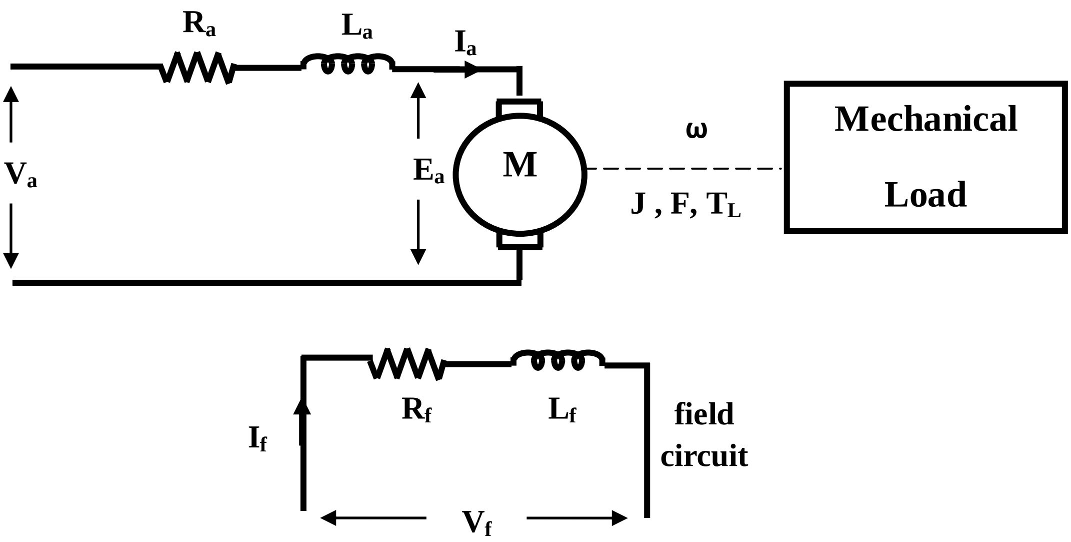
2- The motor must be stable, i.e., it must not oscillate or overshoot.

Fast response can be achieved by having high torque to inertia ratio. Overshooting is minimized and stability is achieved by the use of damping or retarding torque that increases with speed. The damping torque can be developed by the motor itself or viscous damper or an inertia damper or a tachometer.

D.C. servomotors:

D.C. motors are used in the power output stage of large class of servomechanism. In high power applications, the d.c. motors are preferred over a.c. motors because of the ease of control of the speed and direction of rotation.

1- Armature-controlled d.c. motor:



where:

J = The combined inertia of the load and the rotor of the motor.

F = The equivalent viscous friction of the motor and the load.

T_L = The opposite load torque.

L_a = The armature leakage inductance.

R_a = The armature winding resistance.

ω = The angular speed of the rotor.

R_f = The field resistance.

L_f = The field leakage inductance.

Assume we wish to find the manner in which the motor speed responds to changes in the applied armature winding voltage for constant field current and negligible armature leakage inductance.

The electromagnetic torque of the motor T is equal to:

$T = K \, . \, \Phi \, . \, I_a$

where Φ= airgap flux

$\because I_f$ = constant

$\therefore \Phi$ = constant

$\therefore T = K_t \, . \, I_a$

$$I_a = \frac{T}{K_t} ------ (1)$$

$$V_a = E_a + I_a R_a$$

$$E_a = K. \Phi. \omega = K_\omega \, . \, \omega$$

where $K_\omega = K_t$

$$\therefore I_a R_a = V_a - K_\omega \, . \, \omega --- (2)$$

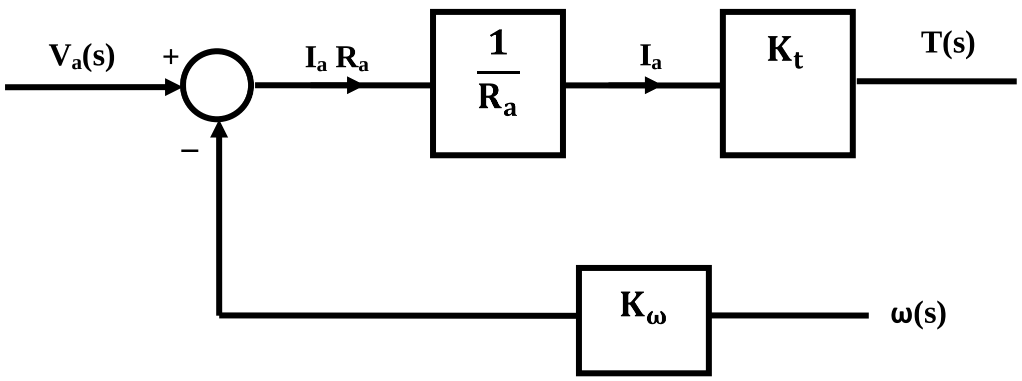
from (1) in (2)

$$T \frac{R_a}{K_t} = V_a - K_\omega \, . \, \omega$$

$$\therefore T = \frac{K_t}{R_a} V_a - \frac{K_\omega \, . \, K_t}{R_a} \omega$$

Taking Laplace transform for this equation gives:

$$T(s) = \frac{K_t}{R_a} V_a(s) - \frac{K_\omega \, . \, K_t}{R_a} \omega(s) ----(3)$$



At the motor shaft, the electromagnetic torque at the motor must be equal to the sum of the opposing torques. Thus,

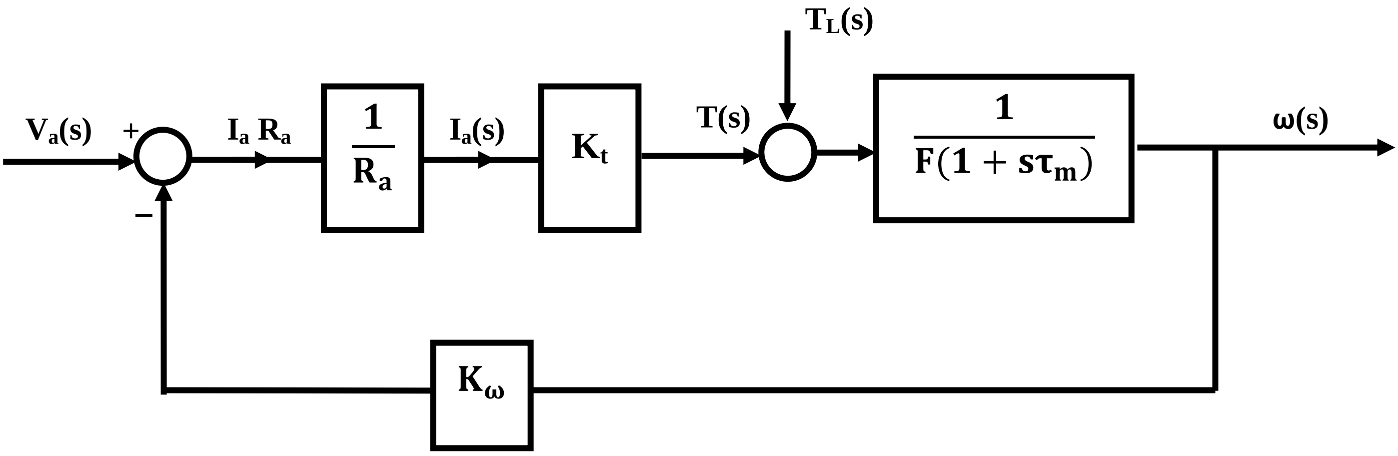
$$T = J\frac{d\omega}{dt} + F\,\omega + T_L$$

Taking Laplace transform gives:

$$T(s) - T_L(s) = Js\omega(s) + F\,\omega(s)$$

$$\therefore \frac{\omega(s)}{T(s) - T_L(s)} = \frac{1}{F(1 + s\tau_m)} - - - - -(4)$$

where $\tau_m = \frac{J}{F}$ = mechanical time constant.



From this block diagram and for T_L equal zero:

$$\omega(s) = (V_a(s) - K_\omega\,\omega(s))\frac{K_t/R_a}{F(1 + s\tau_m)}$$

$$\omega(s)\frac{F(1 + s\tau_m)}{K_t/R_a} + K_\omega\,\omega(s) = V_a(s)$$

$$\omega(s)[\frac{F(1 + s\tau_m) + \frac{K_t\,K_\omega}{R_a}}{\frac{K_t}{R_a}}] = V_a(s)$$

$$\frac{\omega(s)}{V_a(s)} = \frac{K_t/R_a}{F(1 + s\tau_m) + \frac{K_t\,K_\omega}{R_a}} = \frac{K_t/R_a}{F + s\tau_m F + \frac{K_t\,K_\omega}{R_a}}$$

$$= \frac{K_t}{FR_a + K_t\,K_\omega + s\tau_m FR_a}$$

$$= \frac{K_t}{(FR_a + K_t K_\omega)(1 + s\frac{\tau_m FR_a}{FR_a + K_t K_\omega})}$$

$$\frac{\omega(s)}{V_a(s)} = \frac{K_t}{FR_a + K_t K_\omega} \cdot \frac{1}{1 + s\tau_m'}$$

where $\tau_m' = \frac{\tau_m FR_a}{FR_a + K_t K_\omega} = \frac{J R_a}{FR_a + K_t K_\omega}$

It is possible to derive the desired transfer function without first developing the block diagram.

From equation (3) and (4) for T_L=0.0 gives:

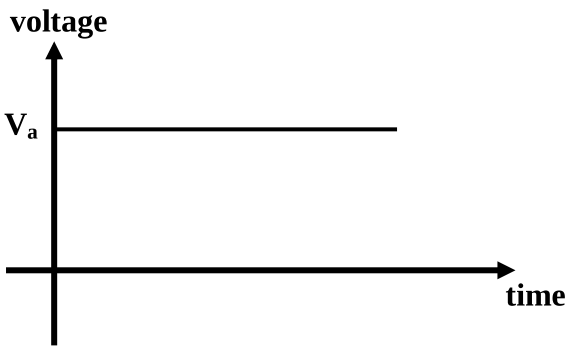
$$\omega(s)[F(1 + s\tau_m)] = \frac{K_t}{R_a} V_a(s) - \frac{K_t K_\omega}{R_a} \omega(s)$$

$$\omega(s)\left[F(1 + s\tau_m) + \frac{K_t K_\omega}{R_a}\right] = \frac{K_t}{R_a} V_a(s)$$

$$\frac{\omega(s)}{V_a(s)} = \frac{K_t}{FR_a(1 + s\tau_m) + K_t K_\omega} = \frac{K_t}{FR_a + K_t K_\omega + s\tau_m FR_a}$$

$$\boxed{\frac{\omega(s)}{V_a(s)} = \frac{K_t}{FR_a + K_t K_\omega} \cdot \frac{1}{1 + s\tau_m'}}$$

What is the dynamic response of the motor speed to a step change in the applied armature voltage V_a? .



$$V_a(s) = \frac{V_a}{s}$$

$$\therefore \omega(s) = \frac{V_a K_t}{FR_a + K_t K_\omega} \cdot \frac{1}{s} \cdot \frac{1}{1 + s\tau_m'}$$

$$= \frac{V_a K_t}{\tau_m'(FR_a + K_t K_\omega)} \cdot \frac{1}{s} \cdot \frac{1}{s + \frac{1}{\tau_m'}}$$

$$= \left[\frac{C_1}{s} - \frac{C_2}{s + \frac{1}{\tau_m'}}\right] = \frac{C_1 s + C_1 \frac{1}{\tau_m'} - C_2 s}{s(s + \frac{1}{\tau_m'})}$$

$$C_1 s - C_2 s = 0$$

$$\therefore C_1 = C_2$$

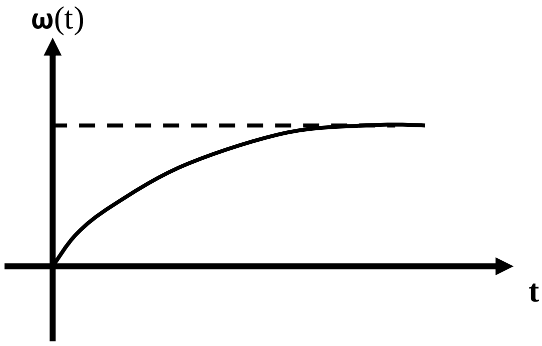
$$C_1 \frac{1}{\tau_m'} = \frac{V_a K_t}{\tau_m'(FR_a + K_t K_\omega)}$$

$$\therefore C_1 = C_2 = \frac{V_a K_t}{FR_a + K_t K_\omega}$$

$$\therefore \omega(s) = \frac{V_a K_t}{FR_a + K_t K_\omega}\left[\frac{1}{s} - \frac{1}{s + \frac{1}{\tau_m'}}\right]$$

The time solution for this equation is:

$$\omega(t) = \frac{V_a K_t}{FR_a + K_t K_\omega}[1 - e^{-t/\tau_m'}]$$



Response of the motor speed to a step change in motor voltage V_a.

At $L_a \neq 0$

$$I_a R_a + L_a \frac{dI_a}{dt} + E_a = V_a$$

$$E_a = K_\omega \omega$$

$$I_a R_a + L_a \frac{dI_a}{dt} + K_\omega \omega = V_a$$

Taking Laplace transform:

$$I_a(s) R_a + L_a s I_a(s) + K_\omega \omega(s) = V_a(s)$$

$$I_a(s)(R_a + L_a s) + K_\omega \omega(s) = V_a(s)$$

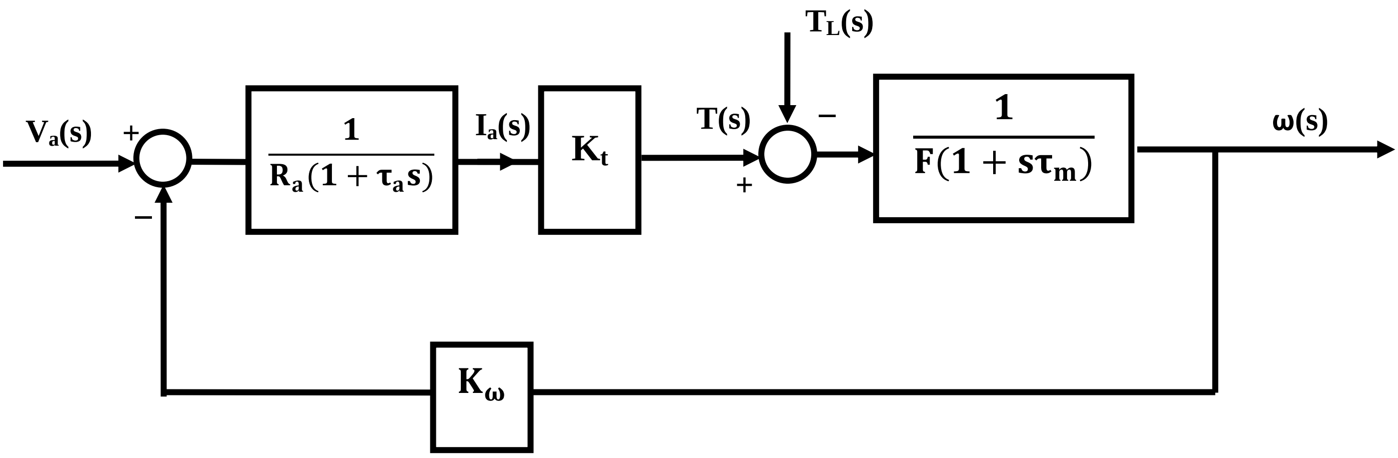
Substituting for $I_a(s)$ yields:

$$\frac{T(s)}{K_t}(R_a + L_a s) + K_\omega \omega(s) = V_a(s)$$

$$\therefore T(s) = \frac{[V_a(s) - K_\omega \omega(s)]K_t}{R_a(1 + \tau_a s)} - - - - - (5)$$

where $\tau_a = \frac{L_a}{R_a}$=armature time constant.

From equations (4) and (5):



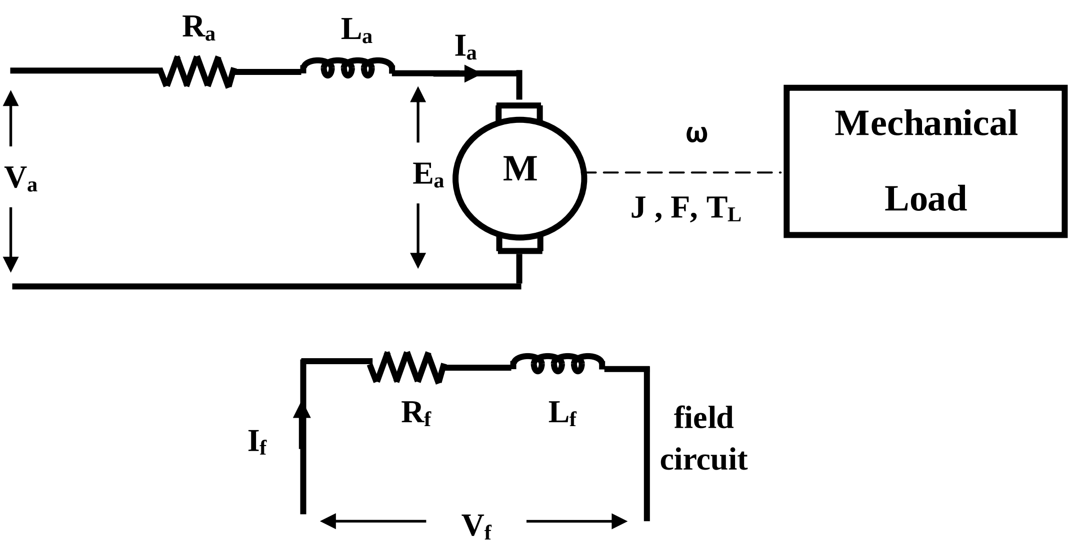
From equations (4) and (5) and neglecting T_L yields:

$$\omega(s)F(1+s\tau_m) = \frac{[V_a(s) - K_\omega\,\omega(s)]K_t}{R_a(1+s\tau_a)}$$

$$\omega(s)[FR_a(1+s\tau_a)(1+s\tau_m) + K_t K_\omega\,] = K_t V_a(s)$$

$$\boxed{\frac{\omega(s)}{V_a(s)} = \frac{K_t/FR_a}{\frac{K_t K_\omega}{FR_a} + (1+s\tau_m)(1+s\tau_a)}}$$

2-Field controlled d.c. motors:



Armature applied voltage & armature current are assumed to be constants. The armature current is applied from a suitable current source.

$$V_f = I_f R_f + L_f \frac{dI_f}{dt}$$

$$V_f(s) = I_f(s)R_f + L_f s I_f(s) = I_f(s)(R_f + sL_f)$$

$$\therefore \frac{I_f(s)}{V_f(s)} = \frac{1}{R_f(1 + s\tau_f)} ----(1)$$

where $\tau_f = \frac{L_f}{R_f}$ =field time constant.

$$T = K\,.\,\Phi\,.\,I_a = K_a I_f$$

$$\therefore \frac{T(s)}{I_f(s)} = K_a ----(2)$$

$$T = J\frac{d\omega}{dt} + F\omega$$

$$\therefore T(s) = Js\omega(s) + F\omega(s)$$

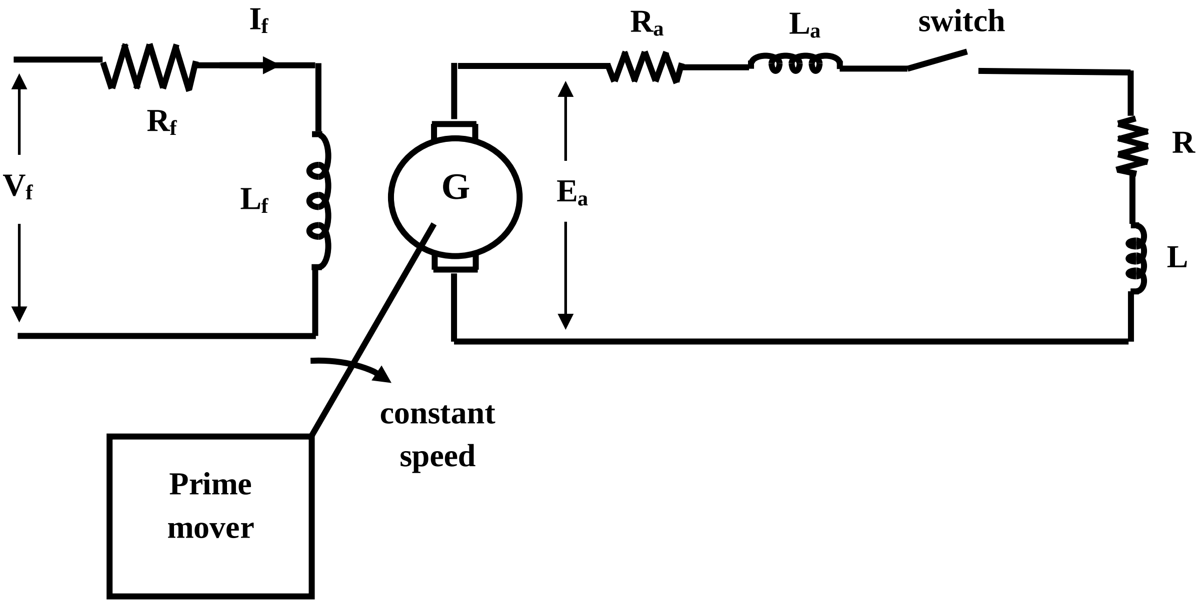
$$\therefore \frac{\omega(s)}{T(s)} = \frac{1}{F(1 + s\tau_m)} ----(3)$$

multiplying equation (1), (2), (3) yields

$$\frac{I_f(s)}{V_f(s)} \cdot \frac{T(s)}{I_f(s)} \cdot \frac{\omega(s)}{T(s)} = \frac{\omega(s)}{V_f(s)}$$

$$\boxed{\therefore \frac{\omega(s)}{V_f(s)} = \frac{K_a/FR_f}{(1 + s\tau_f)(1 + s\tau_m)}}$$

D.C. Generator Dynamics:



How does the armature induced e.m.f. of a d.c. generator respond to a change in field current? How does the armature winding current of the d.c. generator respond to changes in field current? These are questions that are treated here.

a-No external generator load:

$$V_f = R_f I_f + L_f \frac{dI_f}{dt}$$

$$V_f(s) = R_f I_f(s) + L_f s I_f(s)$$

$$\therefore \frac{I_f(s)}{V_f(s)} = \frac{1/R_f}{(1 + s\tau_f)} ----(1)$$

where $\tau_f = \frac{L_f}{R_f}$

$E_a = K . \Phi . \omega = K_f . I_f$ (neglecting saturation)

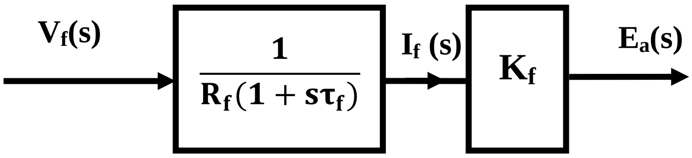
$$E_a(s) = K_f . I_f(s)$$

$$\frac{E_a(s)}{I_f(s)} = K_f ----(2)$$

from (1) and (2)

$$\frac{E_a(s)}{I_f(s)} \cdot \frac{I_f(s)}{V_f(s)} = \frac{K_f/R_f}{1 + s\tau_f}$$

$$\boxed{\frac{E_a(s)}{V_f(s)} = \frac{K_f}{R_f(1+s\tau_f)}} - - - - (3)$$



<u>b-Generator dynamics including the effect of the load:</u>

$$E_a = I_a R_a + L_a \frac{dI_a}{dt} + I_a R + L \frac{dI_a}{dt}$$

$$E_a(s) = I_a(s)(R_a + R)(1 + s\tau_A)$$

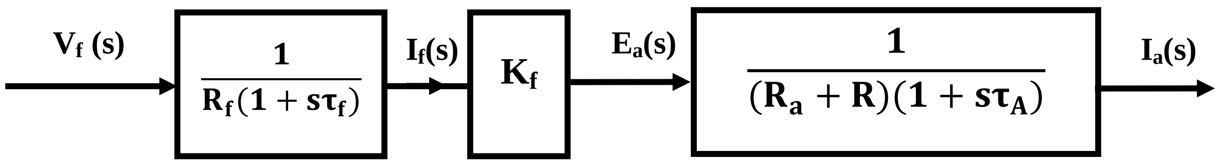
where $\tau_A = \frac{L_a + L}{R_a + R}$ = armature circuit time constant.

$$\therefore \frac{I_a(s)}{E_a(s)} = \frac{1}{(R_a + R)(1 + s\tau_A)} - - - - - - (4)$$

from (3) and (4)

$$\frac{I_a(s)}{E_a(s)} \cdot \frac{E_a(s)}{V_f(s)} = \frac{I_a(s)}{V_f(s)}$$

$$\boxed{\frac{I_a(s)}{V_f(s)} = \frac{K_f}{R_f(R_a + R)(1 + s\tau_A)(1 + s\tau_f)}}$$



Example: A d.c. generator has a field winding resistance of 40 Ω and a field inductance of 8 H. The generated e.m.f. per field ampere is 100 and the magnetization curve is linear. The armature winding resistance and leakage inductance are respectively 0.1 Ω and 0.2 H. The load resistance and inductance are respectively 5 Ω and 2.35 H. Determine the time solution for the armature current when a field voltage of 102 V is applied to the field winding. Assume that the prime mover is running at rated speed and that the load switch is closed.

$$\frac{I_a(s)}{V_f(s)} = \frac{K_f}{R_f(R_a + R)(1 + s\tau_A)(1 + s\tau_f)}$$

$$K_f = 100$$

$$\tau_f = \frac{L_f}{R_f} = \frac{8}{40} = \frac{1}{5}$$

$$\tau_A = \frac{L_a + L}{R_a + R} = \frac{0.2 + 2.35}{0.1 + 5} = \frac{1}{2}$$

$$V_f(s) = \frac{V_f}{s} = \frac{102}{5}$$

$$\therefore I_a(s) = \frac{102}{5} \cdot \frac{100}{40(0.1 + 5)} \cdot \frac{1}{1 + \frac{s}{2}} \cdot \frac{1}{1 + \frac{s}{5}}$$

$$= \frac{500}{5(2 + s)(5 + s)} = \frac{C_1}{s} + \frac{C_2}{2 + s} + \frac{C_3}{5 + s}$$

$$C_1 = [\frac{500}{(2 + s)(5 + s)}]_{s=0} = 50$$

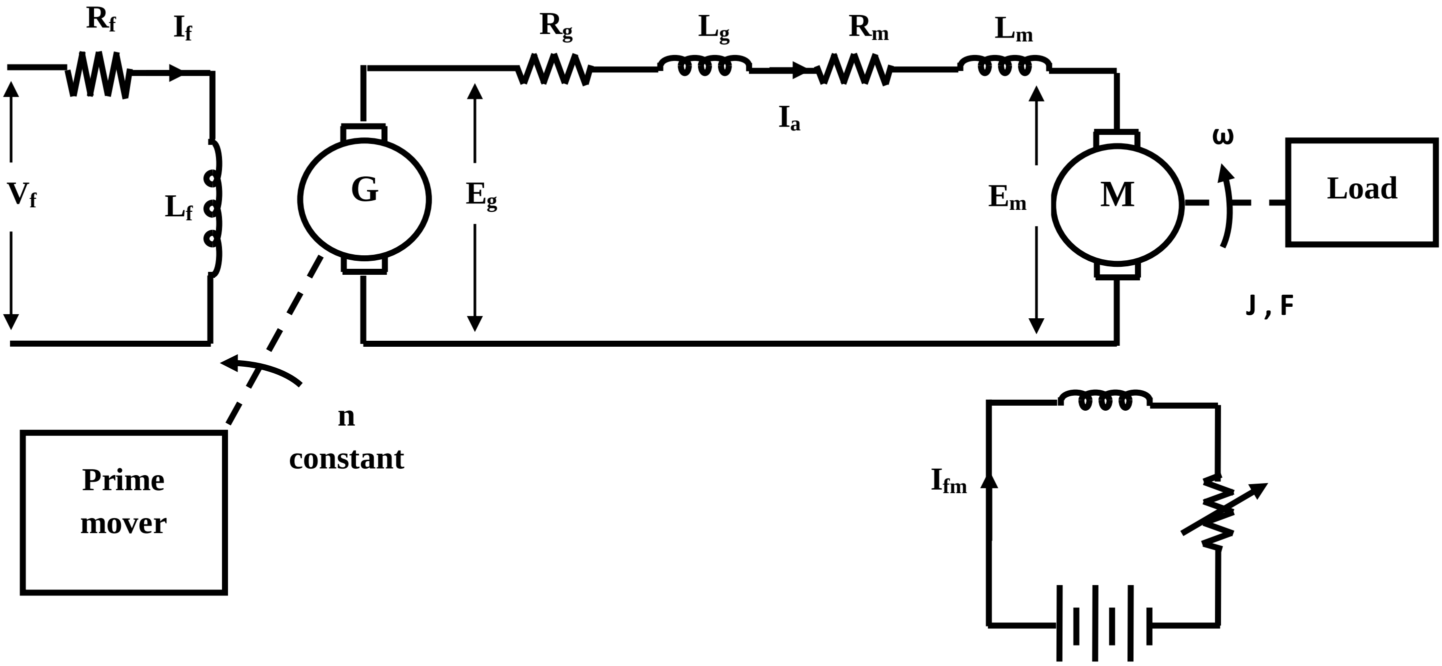
$$C_2 = [\frac{500}{s(5 + s)}]_{s=-2} = -83.333$$

$$C_3 = [\frac{500}{s(2 + s)}]_{s=-5} = 33.333$$

$$I_a(s) = \frac{50}{s} + \frac{33.333}{5 + s} - \frac{83.333}{2 + s}$$

$$\therefore I_a(t) = 50 + 33.333e^{-5t} - 83.333e^{-2t}$$

Generator-motor set dynamics:



A method of speed adjustment that is employed in industry whenever control over a wide range is required is shown above. The armature voltage is controlled by controlling the field current of the generator.

$$V_f = R_f I_f + L_f \frac{dI_f}{dt}$$

$$\therefore \frac{I_f(s)}{V_f(s)} = \frac{1}{R_f(1 + s\tau_f)} ----(1)$$

where $\tau_f = \frac{L_f}{R_f}$

$$\frac{E_g(s)}{I_f(s)} = K_f \quad (V/A) ------(2)$$

$$E_g = I_a R_g + L_g \frac{dI_a}{dt} + I_a R_m + L_m \frac{dI_a}{dt} + E_m$$

$$E_m = K_\omega \omega$$

$$E_g - K_\omega \omega = I_a\left(R_g + R_m\right) + (L_g + L_m)\frac{dI_a}{dt}$$

$$E_g(s) - K_\omega \omega(s) = I_a(s)R + (L_g + L_m)sI_a(s)$$

$$\therefore \frac{I_a(s)}{E_g(s) - K_\omega \omega(s)} = \frac{1}{R(1 + s\tau_A)} - - - - (3)$$

where $\tau_A = \frac{L_g + L_m}{R_g + R_m}$

$T = K_t I_a$ (constant motor field current)

$$\therefore \frac{T(s)}{I_a(s)} = K_t - - - - - - - (4)$$

$$T = J\frac{d\omega}{dt} + F\omega$$

$$\therefore \frac{\omega(s)}{T(s)} = \frac{1}{F(1 + s\tau_m)} - - - - (5)$$

where $\tau_m = \frac{J}{F}$ = mechanical time constant.

from equation (1) and (2):

$$\frac{E_g(s)}{V_f(s)} = \frac{K_f}{R_f(1 + s\tau_f)} - - - - (6)$$

from equation (3), (4) and (5):

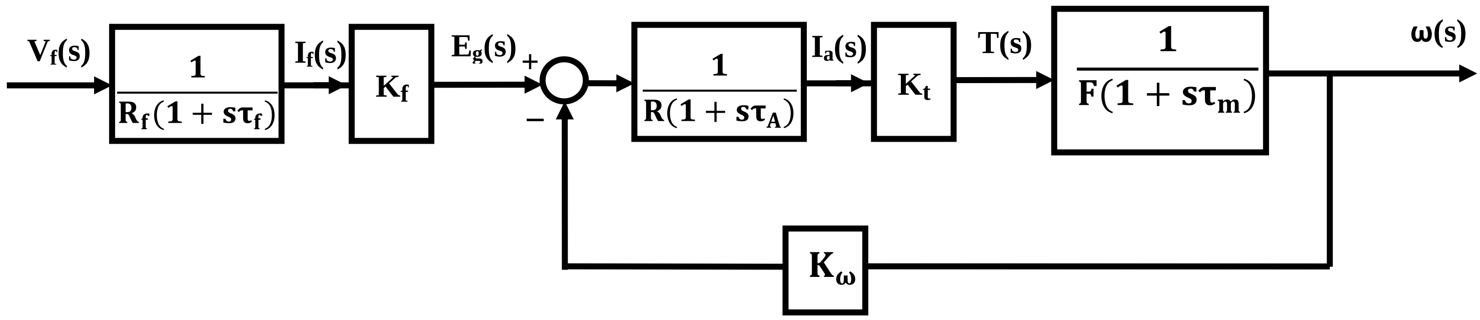
$$\frac{\omega(s)}{E_g(s) - K_\omega \omega(s)} = \frac{K_t}{FR(1 + s\tau_A)(1 + s\tau_m)}$$

$$\omega(s)[K_t K_\omega + FR(1 + s\tau_A)(1 + s\tau_m)] = K_t E_g(s)$$

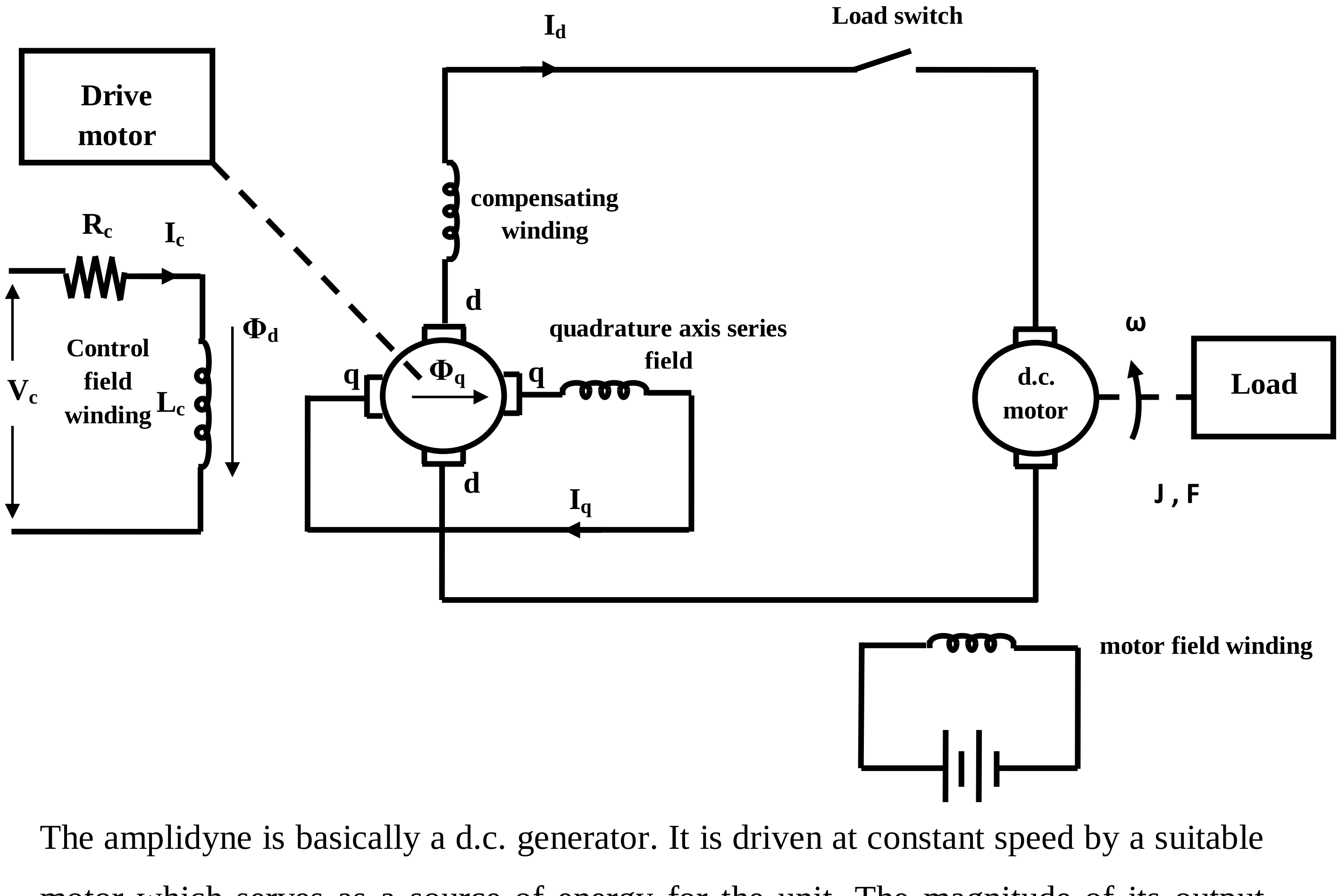
$$\therefore \frac{\omega(s)}{E_g(s)} = \frac{K_t}{[K_t K_\omega + FR(1 + s\tau_A)(1 + s\tau_m)]} - - - - - - (7)$$

from equation (6) and (7):

$$\boxed{\frac{\omega(s)}{V_f(s)} = \frac{K_t K_f}{R_f(1 + s\tau_f)[K_t K_\omega + FR(1 + s\tau_A)(1 + s\tau_m)]}}$$



The Amplidyne generator:



The amplidyne is basically a d.c. generator. It is driven at constant speed by a suitable motor which serves as a source of energy for the unit. The magnitude of its output voltage is controlled by the amount of field current through the control winding. The principles of operation is as follows: A small current flowing in the control winding creates a flux Φ_d with the direction shown. This flux will induce e.m.f. E_q between brushes qq due to armature rotation. This e.m.f. will cause large short circuit current I_q to flow producing flux Φ_q which in turn will produce e.m.f. E_d between brushes dd.

The transfer function of the amplidyne without load: E_d is the output and V_c is the input.

$$V_c = R_c I_c + L_c \frac{dI_c}{dt}$$

$$\therefore \frac{I_c(s)}{V_c(s)} = \frac{1}{R_c(1 + s\tau_c)} ----(1)$$

where $\tau_c = \frac{L_c}{R_c}$ = the control winding time constant.

$E_q = K_1 \Phi_d$ $\qquad\qquad$ $E_q = K\Phi_d \omega$

where K_1 = A function of the armature speed.

$\therefore E_q = K_q I_c$ (neglecting saturation & assuming complete compensation for the armature reaction in the d-axis)

$$\therefore \frac{E_q(s)}{I_c(s)} = K_q ------(2)$$

$$E_q = I_q R_q + L_q \frac{dI_q}{dt}$$

$$\therefore \frac{I_q(s)}{E_q(s)} = \frac{1}{R_q(1 + s\tau_q)} ----(3)$$

where: R_q = total resistance in the quadrature axis circuit.

L_q = total inductance in the quadrature axis circuit.

I_q = current flowing in the quadrature axis circuit.

$\tau_q = \frac{L_q}{R_q}$

$$E_d = K_d I_q$$

$$\therefore \frac{E_d(s)}{I_q(s)} = K_d ------(4)$$

Multiplying equation (1) , (2) , (3) and (4) gives:

$$\boxed{\frac{E_d(s)}{V_c(s)} = \frac{K_q K_d}{R_c R_q} \cdot \frac{1}{(1 + s\tau_c)(1 + s\tau_q)}} - - - - - - - - (5)$$

Amplidyne transfer function with connected load:

$$E_d = I_d R_d + L_d \frac{dI_d}{dt} + E_b$$

where: R_d = total resistance in the direct-axis circuit.

L_d = total inductance in the direct-axis circuit.

$$E_d = I_d R_d + L_d \frac{dI_d}{dt} + K_\omega \omega$$

$$\therefore \frac{I_d(s)}{E_d(s) - K_\omega \omega(s)} = \frac{1}{R_d(1 + s\tau_d)} - - - - (6)$$

where $\tau_d = \frac{L_d}{R_d}$ = direct axis path time constant.

$$T = K_t I_d$$

$$\frac{T(s)}{I_d(s)} = K_t - - - - - - - (7)$$

$$T = J\frac{d\omega}{dt} + F\omega$$

$$\frac{\omega(s)}{T(s)} = \frac{1}{F(1 + s\tau_m)} - - - - (8)$$

Multiplying equation (6) , (7) and (8) gives:

$$\frac{\omega(s)}{E_d(s) - K_\omega \omega(s)} = \frac{K_t}{FR_d(1 + s\tau_d)(1 + s\tau_m)}$$

$$\omega(s)[K_t K_\omega + FR_d(1 + s\tau_d)(1 + s\tau_m)] = K_t E_d(s)$$

$$\therefore \frac{\omega(s)}{E_d(s)} = \frac{K_t}{K_t K_\omega + FR_d(1 + s\tau_d)(1 + s\tau_m)} - - - - - (9)$$

Multiplying equation (9) and (5) gives:

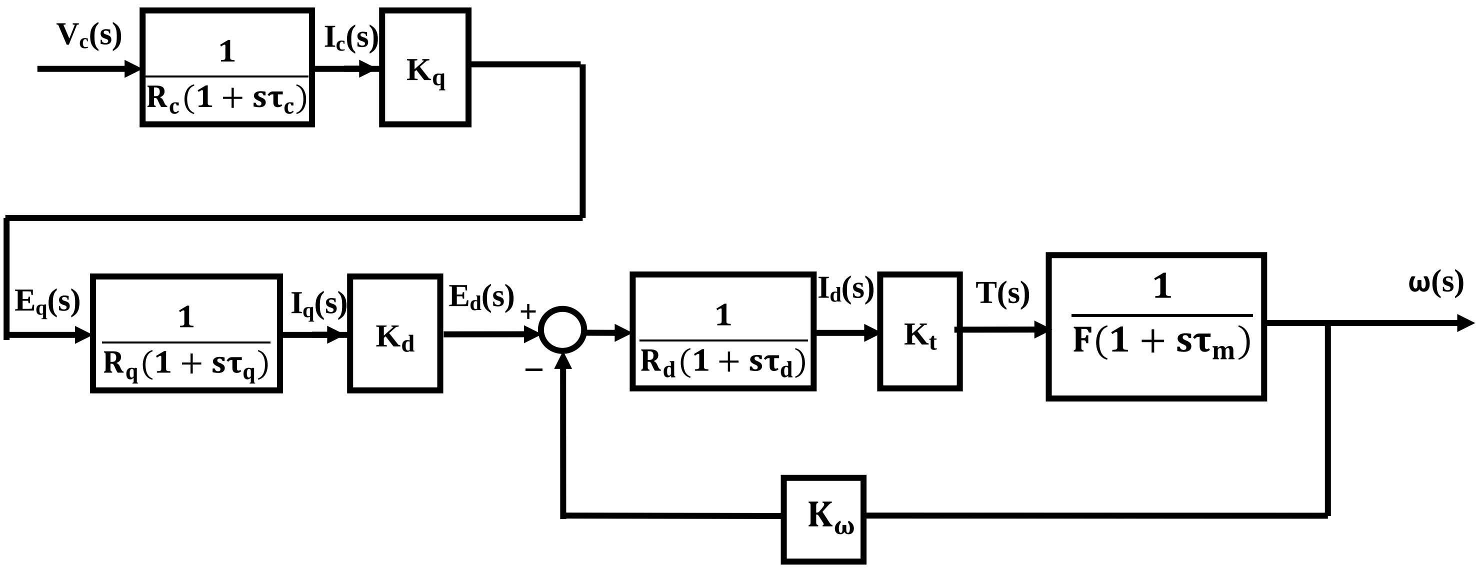
$$\frac{\omega(s)}{V_f(s)} = \frac{K_q K_d K_t / R_c R_q R_d F}{(1+s\tau_c)(1+s\tau_q)\left[(1+s\tau_d)(1+s\tau_m) + \frac{K_t K_\omega}{F R_d}\right]}$$

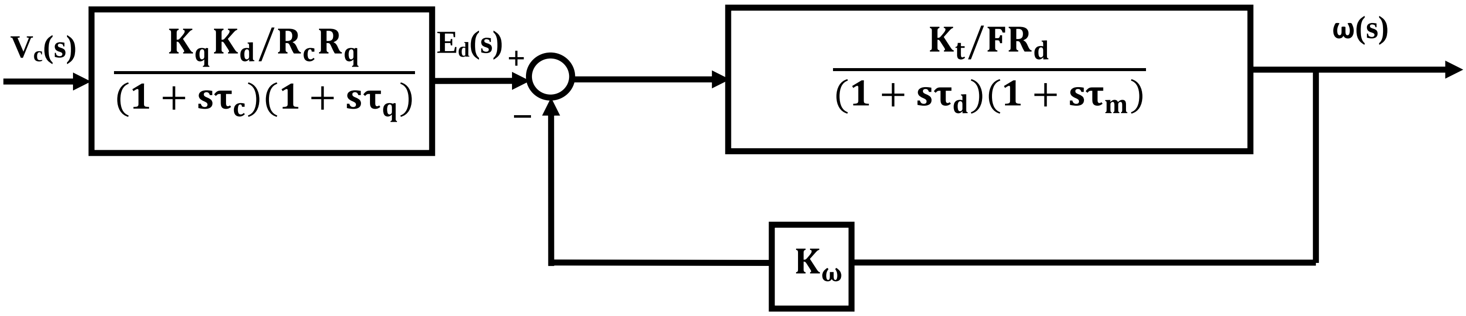
In applications required displacement rather than velocity:

$$\omega = \frac{d\theta}{dt}$$

$$\omega(s) = s\,\theta(s)$$

$$\therefore \frac{\theta(s)}{\omega(s)} = \frac{1}{s}$$





Servomotors & T.F.

Sheet NO.5

1- A two-phase, four-pole servomotor is energized by an auxiliary winding voltage that is 90^o out of phase with the main winding voltage. The main winding voltage rating at 60 Hz is 75 V. The motor parameters are as follows:

$r_1 = 50\ \Omega$ $\qquad r_2' = 100\ \Omega$

$x_1 = 120\ \Omega$ $\qquad x_m = 100\ \Omega$

The rotor leakage reactance is negligible. Compute the developed torque at a slip of 0.4 when the auxiliary voltage is 37.5 V.

(17.89 m Nm)

2- Appropriate no-load tests on a two-phase servomotor rated at 115 V, 60 Hz, two-poles, give the following parameters:

$r_1 = 302\ \Omega$ $\qquad r_2' = 1380\ \Omega$

$x_1 = 385\Omega$ $\qquad x_2' = 385\ \Omega$

$x_m = 695\Omega$

This servomotor is operated with $V_a=115\angle 0^o$ and $V_b=80\angle -90^o$ V, and a rotor slip of 0.25.

a- Draw the appropriate equivalent circuit for the positive- and negative- sequence voltage sets. Show the proper applied voltage in each case.

b- Compute the developed torque, expressed in synchronous watts and Nm.

c- What is the r.m.s. value of the current that flows through the control winding?

d- Compute the torque developed by this motor at a slip of 0.25 when it operates as a balanced two-phase motor, i.e., $V_a=115\angle 0^o$ and $V_b=115\angle -90^o$ V.

e- Compute the developed torque when the motor operates as a balanced two-phase motor at a slip of 1.75.

f- By using the results of parts (d) and (e), compute the torque developed by this motor at a slip of 0.25 when $V_a=115\angle 0^o$ and $V_b=80\angle -90^o$ V.

g- When this motor operates at a slip of 0.5, the positive-sequence current is found to be $80.5\angle -51.1^{o}$ mA and the negative-sequence current is $38.5\angle -98.4^{o}$ mA. Determine the voltages applied to the main and control windings. Find the control winding current.

(97.5 V; 17.5 V; 1.10506 W; 2.9312 m Nm; 0.0682 A; 4.6207 m Nm; 16.8546 m Nm; 2.9311 m Nm; $115.209\angle -0.6777^{o}$ V; $75.52062\angle -50.838^{o}$ V; $61.31\angle -113.6164^{o}$ mA)

3- The servomotor is problem (2) is operated with the main and control winding voltages always in quadrature.

a- Compute the starting torque when the control winding voltage equals the main winding voltage, i.e. , K=1.

b- Determine the starting torque for K=0.5.

(13.8734 m Nm; 6.9367 m Nm)

4- A two-phase servomotor has $115\angle 0^{o}$V applied to the main winding. At a particular point of operation corresponding to a control winding voltage of $75\angle -80^{o}$V, the positive- and negative-sequence impedances are $220\angle 67^{o}\Omega$ and $175\angle 57^{o}\Omega$ respectively.

a- Determine the positive and negative sequence components of the main and control winding currents.

b- Find the total values of the main and control winding currents.

($0.4302479\angle -63.055192^{o}$A; $0.1232904\angle -74.56636^{o}$A; $0.4302479\angle -153.05519^{o}$A; $0.1232904\angle 15.43364^{o}$A; $0.55161\angle -65.61165^{o}$A; $0.3104138\angle -148.50905^{o}$A)

5- A two-phase, two-pole, a.c. servomotor equipped with a drag-cup rotor has the following parameters at 60 Hz:

$r_1 = 360\ \Omega$ $\qquad$ $r_2' = 260\ \Omega$

$x_1 = 50\Omega$ $\qquad$ $x_2' = 50\ \Omega$

$$x_m = 890\,\Omega$$

The main winding is identical to the control winding , and both are rated at 115 V. The source of the control winding voltage is arranged to provide a quadrature voltage at all times. The internal impedance of this source is negligibly small.

a- Determine the developed torque in synchronous watts for K=0.8 and S=0.3.

b- Calculate the power supplied to the main winding in part (a).

c- Calculate the power supplied to the control winding in part (a).

d- Calculate the motor efficiency for the conditions in part (a).

e- Determine the developed torque in synchronous watts for K=1 and S=0.3.

f- Repeat part (e) for K=1 and S=1.7.

g- Find the developed torque in Nm for K=0.7 and a rotor slip of 0.3.

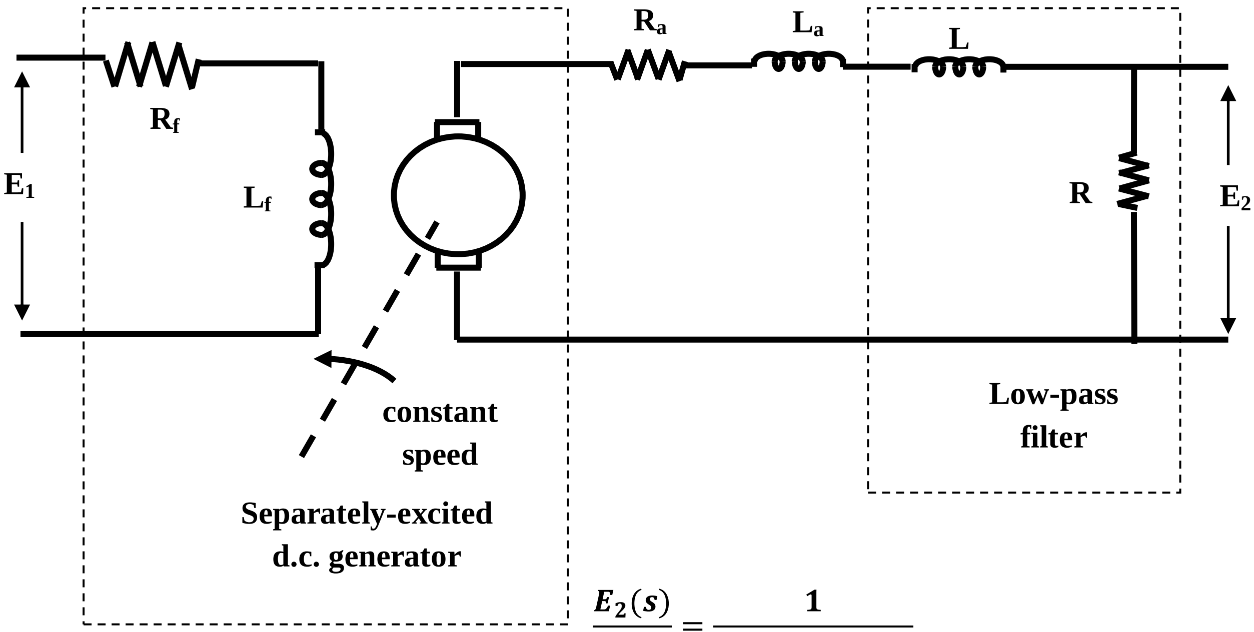
(10.534639 W; 13.5433 W; 6.7832835 W; 36.2788 %; 13.175 W; 13.70481 W; 24.4317 m Nm)

6- Determine the transfer function that relates the voltage E_2 to the input voltage E_1 in the system shown below. The system parameters are as follows:

Generator: L_f = 50 H, R_f = 50 Ω , R_a = 1 Ω , L_a = 1 H.

e.m.f. constant: K_g=100 V/field ampere.

Low-pass filter: L=1 H, R=1 Ω.



$$\frac{E_2(s)}{E_1(s)} = \frac{1}{(1+s)(1+s)}$$

7- The motor shown below is operated at constant field current. The motor parameters are : R_a = 0.5 Ω ; L_a = 0 ; K_ω = 2 V/rad/s ; K_t =6 .78 Nm/A , and F=1.356 Nm/rad/s. The equivalent inertia is 56.952 N.m.s^2.

a- What is the numerical expression for the transfer function that relates the motor speed in radians per second to the voltage applied to the armature circuit?

b- If the applied armature voltage is a step of 200 V magnitude, find the steady state speed of the motor.

c- Obtain the complete expression for the motor speed from the instant the voltage of part(a) is applied to the time that steady-state reached.

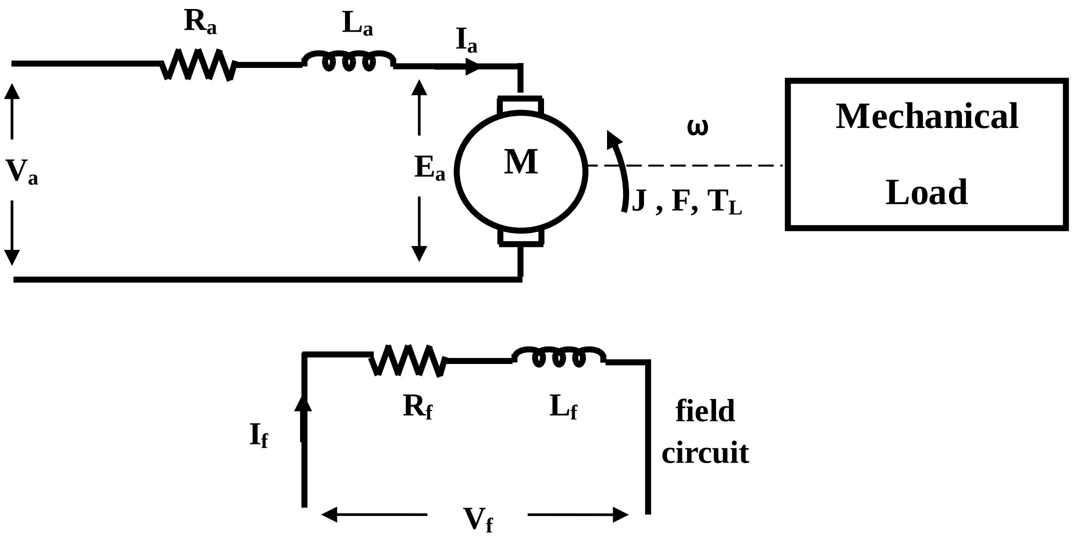
d- Draw the block diagram of this system using the numerical values.

e- The application of 100 V to the armature of the motor at no load gives a speed of 47.6 rad/s.

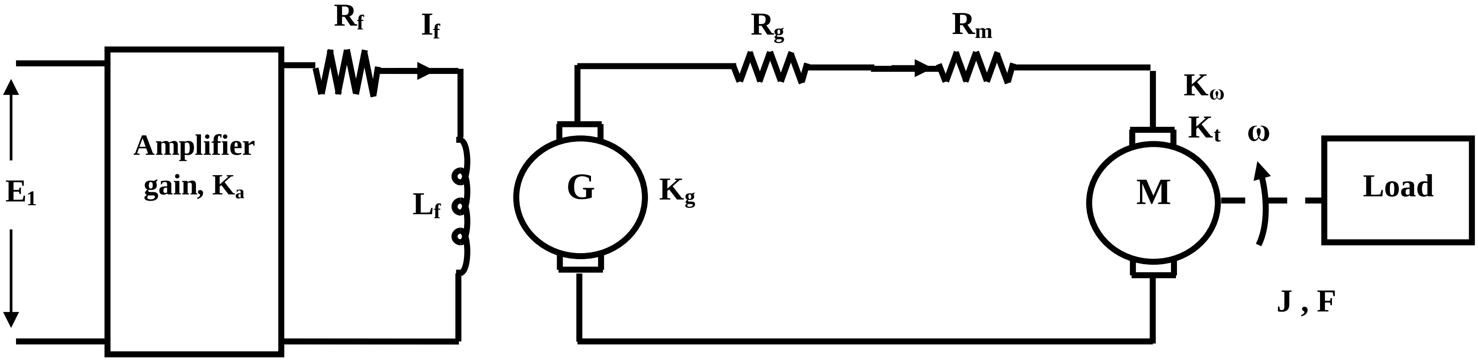
1- If a load torque of 54.24 N.m. is applied to the motor shaft, find the new operating speed.

2- How long does it take for the new speed to be reached?

f- Repeat parts (a) to (e) using L_a=0.05 H.



8- Determine the different equation of the system shown below in terms of the system parameters. Consider that the armature leakage inductance of the generator and motor are both negligible. Take the generator voltage constant as K_g in volts per field ampere, the motor speed constant as K_ω in volts per radian per speed, and the motor torque constant as K_t in Nm per armature ampere.



9- A 10 h.p. armature controlled d.c. motor drives a load whose viscous friction coefficient is 2.712 Nm/rad/s and whose angular inertia is 23.73 Nm.s^2. The field winding is separately excited and maintained fixed. The corresponding motor parameters are then as follows:

$R_a = 0.3\ \Omega$

$K_\omega = 1$ V/rad/s

$K_t = 2.034$ Nm/arm. amp.

The armature winding inductance is negligible.

a- Compute the steady-state speed corresponding to a step applied armature voltage of 210 V.

b- How long does the motor take to reach within 95% of the steady-state speed of part (a)?

10- The motor of problem (9) is now used as a field-controlled d.c. machine. The armature is assumed to be energized from a constant current source. The field winding parameters are: $R_f = 50\ \Omega$; L_f =20 H. The motor torque constant $K_f = 81.36$ Nm/field amp.

a- Draw the block diagram of the transfer function.

b- Calculate the value of steady-state speed for a step applied field voltage of 100 V.

c- Approximately how long does it take the motor to reach within 95% of the final speed in part (b).

11- A two-servomotor has a standstill torque at full control winding voltage of $28.248*10^{-3}$ Nm. Also , at full control voltage the no load speed is 100 rad/s. The motor torque constant is $0.7062*10^{-3}$ Nm/V or $28.248*10^{-3}$ Nm/control ampere. The control winding inductance is 6 H. Assume linear torque-speed curves and a rotor inertia of 3.531 $Nm.s^2$.

a- Determine the complete numerical expressions relating motor displacement to control winding voltage.

b- If the control winding voltage is taken from an amplifier having a voltage gain of 200 , write the total transfer function expression.

c- Place a unity feedback loop around the transfer function of part (b) and find the closed loop transfer function.

d- what is the time equation of the system of part (b).

e- what is the time equation of the system of part (c).

12- An amplidyne generator is used as a power stage that drives the d.c. motor of problem (9). The amplidyne has a control winding resistance of 400 Ω , a quadrature-axis circuit resistance of 0.1 Ω, and a direct-axis circuit resistance of 0.5 Ω. The voltage induced in the cross-axis for each milliampere of control current is 0.1 V, and the voltage induced in the direct axis for each ampere in the quadrature axis is 5 V. The inductance of the control winding is 50 H and that of the quadrature axis is 2 mH.

a- Determine the transfer function that relates motor output speed to amplidyne control winding voltage.

b- For a control voltage of 5 V, compute the steady-state speed of the motor.

c- Identify the predominant factor in establishing the dynamic response of this amplidyne-motor system.