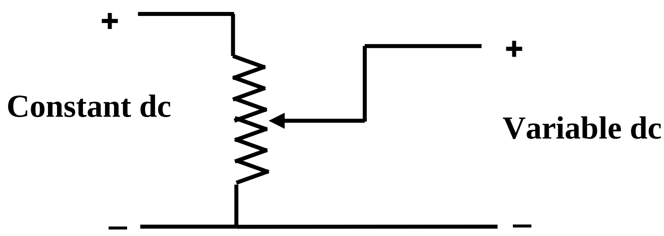
Chapter Four

DC choppers

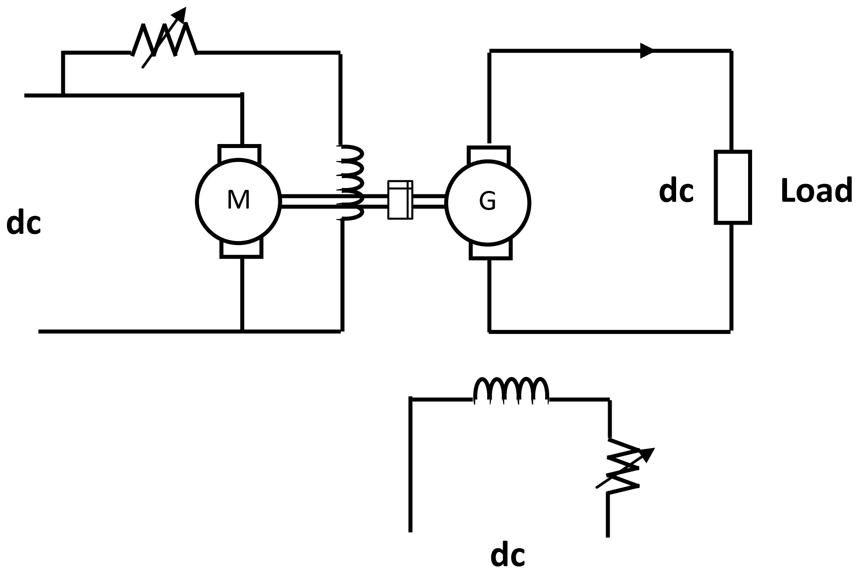
Introduction:

The classical methods of dc to dc converters are:

1- Resistance control: In this method a variable resistance is used between the fixed-voltage dc source and the load.



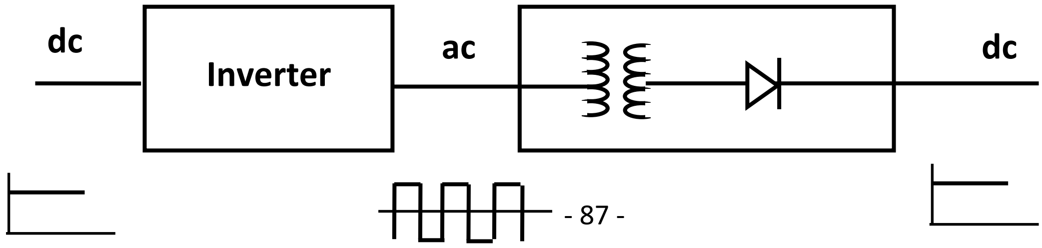
2- Motor-Generator set: A variable dc output voltage is obtained by controlling the field current of the dc generator.



The two broad types of dc-to-dc thyristor converters are:

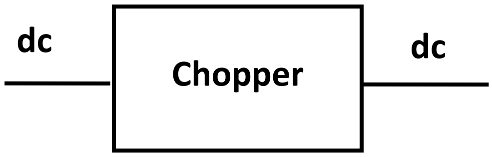
1- Inverter-rectifier:

The dc is first converter to a.c. , which is then stepped up or down by a transformer and then rectified back to d.c.



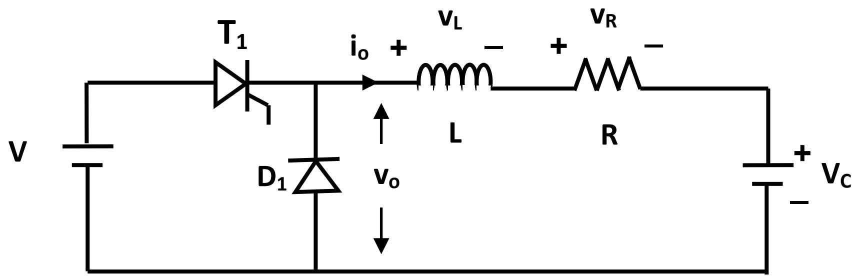
2- DC chopper:

It converters the dc directly to dc and is a relatively new technology. It may be visualized as a dc equivalent to an ac transformer.

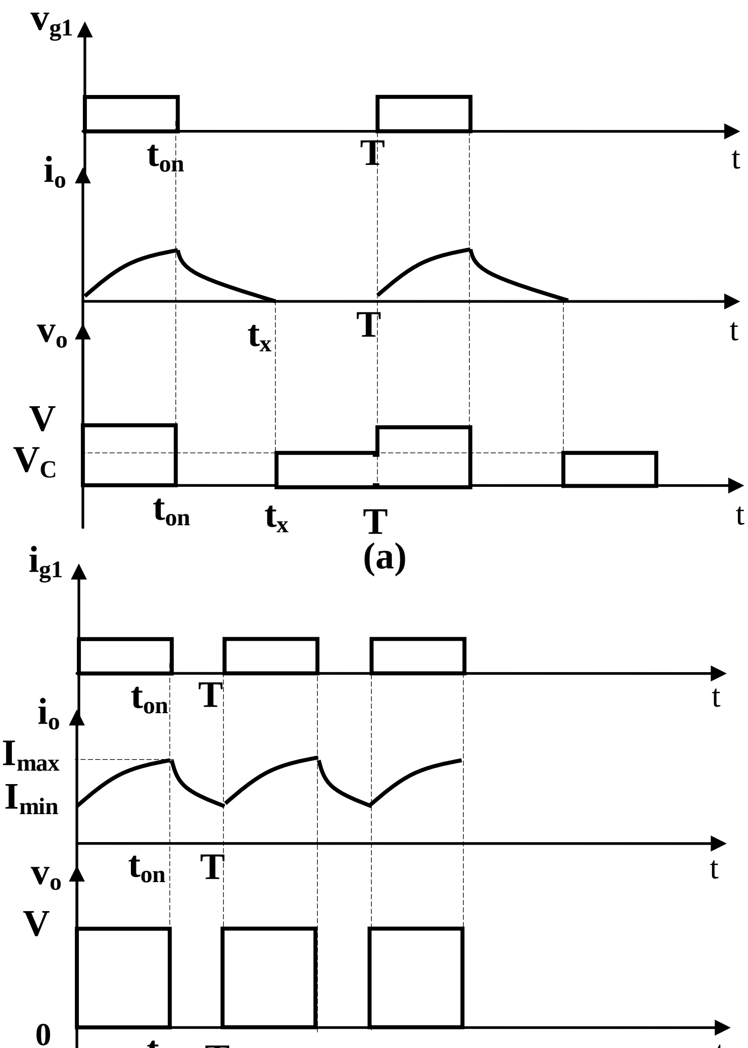


Principles of chopper operation:

The basic principles of dc chopper are shown below:



The commutation circuit is not shown.



For the figure-a, the load current is discontinuous, so that during the interval for which i_o is zero, $v_o=V_C$. In Fig. (b), the periodic time T has been reduced so that the current is continuous and there is no time of i_o is zero.

Analysis of the chopper circuit:

First when the current is continuous, i.e., figure-b:

$-v_o + v_R + v_L + V_C = 0$-------------------------------------(1)

$-v_o + L\frac{di_o}{dt} + +Ri_o + V_C = 0$

$\frac{di_o}{dt} + \frac{R}{L} i_o = \frac{v_o - V_C}{L}$ -------------------------------------(2)

Solving equation (2) and using the initial conditions,when thyristor T_1 is turned on at t=0, then at $t=0^+$, $v_o=V$ and $i_o=I_{min}$.

$i_o = \frac{V-V_C}{R}(1 - e^{\frac{-t}{\tau}}) + I_{min}\, e^{\frac{-t}{\tau}}$ $\quad 0 \leq t < t_{on}$-----------------(3)

where $\tau = \frac{L}{R}$ sec.

At $t=t_{on}$, when T_1 is commutated,

$i_o = I_{max} = \frac{V-V_C}{R}(1 - e^{\frac{-t_{on}}{\tau}}) + I_{min}\, e^{\frac{-t_{on}}{\tau}}$ -----------------(4)

when T_1 is commutated, v_o becomes zero due to the conduction of the free-wheeling diode D_1. From equation (2):

$\frac{di_o}{dt'} + \frac{R}{L} i_o = \frac{-V_C}{L}$----------(5)

where $t'=t-t_{on}$

At $t'=0^+$, $i_o=I_{max}$, and from equation (5):

$i_o = \frac{-V_C}{R}(1 - e^{\frac{-t'}{\tau}}) + I_{max}\, e^{\frac{-t'}{\tau}}$ $\quad t_{on} \leq t < T$ ---------------(6)

At t=T, i.e., $t'=T-t_{on}$, $\quad i_o=I_{min}$,

$$\therefore i_o = I_{min} = \frac{-V_C}{R}\left(1 - e^{\frac{-(T-t_{on})}{\tau}}\right) + I_{max}\, e^{\frac{-(T-t_{on})}{\tau}} \text{-------------------}(7)$$

The solution of equations (4) and (7) gives:

$$I_{max} = \frac{V}{R} * \frac{1-e^{\frac{-t_{on}}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{V_C}{R} \text{---------------------}(8)$$

$$I_{min} = \frac{V}{R} * \frac{e^{\frac{t_{on}}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{V_C}{R} \text{------------------------}(9)$$

when the thyristor T_1 is continuously turned on, so that

t_{on}=T, then:

$$I_{max} = I_{min} = \frac{V-V_C}{R} \text{------------------------}(10)$$

If t_{on} is decreased to the value t_{on}^* at which I_{min}=0, then the converter is operating at the point of changeover from continuous-current operation to discontinuous-current operation.

From equation (9)

$$I_{min} = 0 = \frac{V}{R} * \frac{e^{\frac{t_{on}^*}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{V_C}{R}$$

$$\frac{V_C}{V} = \frac{e^{\frac{t_{on}^*}{\tau}}-1}{e^{\frac{T}{\tau}}-1} \text{-------------------------------}(11)$$

For discontinuous-current operation:

I_{min}=0 in equation (4)

$$I_{max} = \frac{V-V_C}{R}\left(1 - e^{\frac{-t_{on}}{\tau}}\right) \qquad 0 < t_{on} < t_{on}^* \text{-----------------}(12)$$

Substituting in equation (6) gives:

$$i_o = \frac{-V_C}{R}\left(1 - e^{\frac{-t'}{\tau}}\right) + \frac{V-V_C}{R}\left(1 - e^{\frac{-t_{on}}{\tau}}\right)e^{\frac{-t'}{\tau}} \qquad 0 < t_{on} < t_{on}^* \text{-----------}(13)$$

This current will become zero at $t=t_x$, $t'=t_x-t_{on}$

$$i_o = 0 = \frac{-V_C}{R}\left(1 - e^{\frac{-(t_x - t_{on})}{\tau}}\right) + \frac{V - V_C}{R}\left(1 - e^{\frac{-t_{on}}{\tau}}\right)e^{\frac{-(t_x - t_{on})}{\tau}}$$

$$t_x = \tau \ln\left[e^{\frac{t_{on}}{\tau}}\left[1 + \frac{V - V_C}{V_C}\left(1 - e^{\frac{-t_{on}}{\tau}}\right)\right]\right] \text{------------------------(14)}$$

The output voltage can be expresses by the series:

$$v_o = a_o + \sum_{n=1}^{\infty}(a_n \sin n\omega t + b_n \cos n\omega t)$$

$$v_o = a_o + \sum_{n=1}^{\infty} c_n \sin(n\omega t + \theta_n)$$

where $\omega = \frac{2\pi}{T}$

$$a_o = \frac{1}{T}\int_0^T v_o\, dt = \frac{1}{T}\left[\int_0^{t_{on}} V dt + \int_{t_x}^T V_C\, dt\right] = V_{o(av)}$$

$$a_o = \frac{t_{on}}{T}V + \frac{T - t_x}{T}V_C \text{ -------------------------(15)}$$

$$a_n = \frac{2}{T}\int_0^T v_o \sin n\omega t\, dt$$

$$= \frac{2}{T}\left[\int_0^{t_{on}} V \sin n\omega t\, dt + \int_{t_x}^T V_C \sin n\omega t\, dt\right]$$

$$= \frac{2V}{Tn\omega}[1 - \cos n\omega t_{on}] - \frac{2V_C}{Tn\omega}[1 - \cos n\omega t_x]$$

$$\omega = \frac{2\pi}{T}$$

$$\therefore a_n = \frac{V}{\pi n}[1 - \cos n\omega t_{on}] - \frac{V_C}{\pi n}[1 - \cos n\omega t_x] \text{-----------------(16)}$$

$$b_n = \frac{2}{T}\int_0^T v_o \cos n\omega t\, dt$$

$$\therefore b_n = \frac{V}{\pi n}\sin n\omega t_{on} - \frac{V_C}{\pi n}\sin n\omega t_x \text{-------------------------(17)}$$

$$C_n = \sqrt{a_n^2 + b_n^2} \text{-----------------------------(18)}$$

$$\theta_n = \tan^{-1}\frac{b_n}{a_n}$$

Example-1: For the chopper shown previously, V=110 V, L=1 mH, R=0.25 Ω, V_C=11 V, T=2500 μs, and t_{on}=1000 μs.

a- Calculate the average output current I_o and the average output voltage V_o.

b- Calculate the maximum and minimum values of instantaneous output current I_{max} and I_{min}.

c- Calculate the rms values of the first harmonic (fundamental) output voltage and current.

Solution:

a- First we must determine the current is continuous or not.

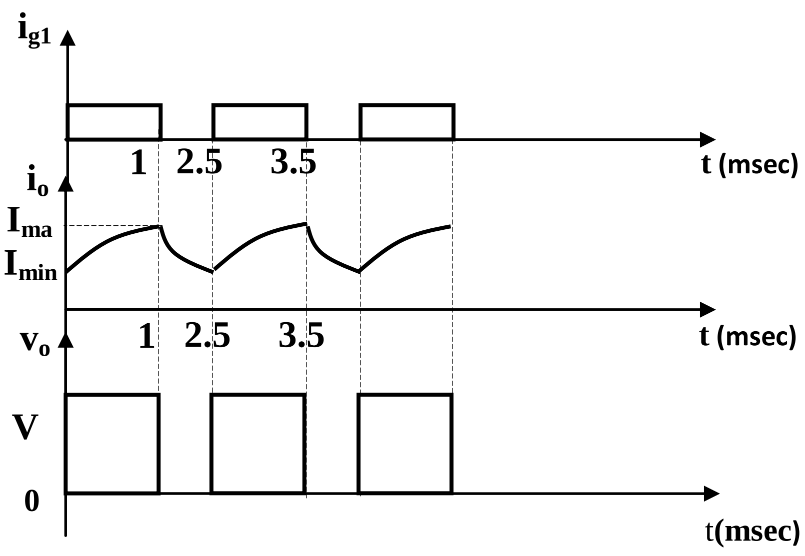
$$t_x = \tau \ln\left[e^{\frac{t_{on}}{\tau}}\left[1 + \frac{V - V_C}{V_C}\left(1 - e^{\frac{-t_{on}}{\tau}}\right)\right]\right]$$

$$\tau = \frac{L}{R} = \frac{1*10^{-3}}{0.25} = 0.004 \text{ sec}$$

$$\therefore t_x = 0.004\ln\left[e^{\frac{1000*10^{-6}}{0.004}}\left[1 + \frac{110-11}{11}\left(1 - e^{\frac{-1000*10^{-6}}{0.004}}\right)\right]\right]$$

=5382.1 μsec >T

∴ The current is continuous.



At I_{min} T=t_x

$$V_{o(av)} = \frac{t_{on}}{T}V + \frac{T-t_x}{T}V_C = \frac{t_{on}}{T}V = \frac{1000}{2500} * 110 = 44\ V$$

$$I_o = \frac{V_{o(av)}-V_C}{R} = \frac{44-11}{0.25} = 132\ A$$

b- $$I_{max} = \frac{V}{R} * \frac{1-e^{\frac{-t_{on}}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{V_C}{R}$$

$$I_{max} = \frac{110}{0.25} * \frac{1-e^{\frac{-1000*10^{-6}}{0.004}}}{1-e^{\frac{-2500*10^{-6}}{0.004}}} - \frac{11}{0.25} = 165.424\ \ A$$

$$I_{min} = \frac{V}{R} * \frac{e^{\frac{t_{on}}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{V_C}{R}$$

$$I_{min} = \frac{110}{0.25} * \frac{e^{\frac{1000*10^{-6}}{0.004}}-1}{e^{\frac{2500*10^{-6}}{0.004}}-1} - \frac{11}{0.25} = 99.935\ \ A$$

c- $$a_n = \frac{V}{\pi n}[1-\cos n\omega t_{on}] - \frac{V_C}{\pi n}[1-\cos n\omega t_x]$$

At T=t_x $\quad \therefore\ \omega t_x = \frac{2\pi}{T} * T = 2\pi$

$$\therefore a_n = \frac{V}{\pi n}[1-\cos n\omega t_{on}]$$

$$\therefore b_n = \frac{V}{\pi n}\sin n\omega t_{on} - \frac{V_C}{\pi n}\sin n\omega t_x = \frac{V}{\pi n}\sin n\omega t_{on}$$

$$\therefore C_n = \sqrt{a_n^2 + b_n^2} = \frac{V}{\pi n}[1 - 2\cos n\omega t_{on} + \cos^2 n\omega t_{on} + \sin^2 n\omega t_{on}]$$

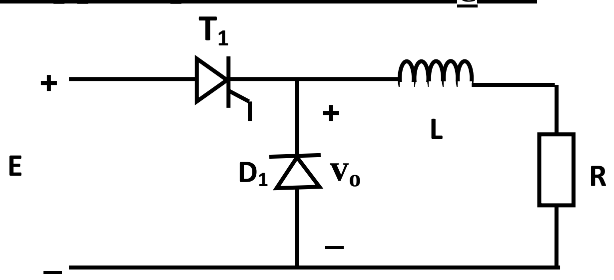
$$C_n = \frac{\sqrt{2}V}{n\pi}[1-\cos n\omega t_{on}]^{\frac{1}{2}}$$

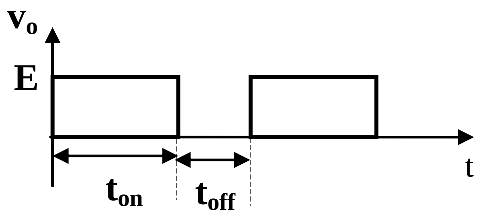
$$C_1 = \frac{\sqrt{2}*110}{1*\pi}\left[1-\cos(1*\frac{2\pi}{250*10^{-6}}*1000*10^{-6})\right]^{\frac{1}{2}} = 66.60075\ \ V$$

$$V_{1R} = \frac{C_1}{\sqrt{2}} = 47.0938\ \ V$$

$$I_{1R} = \frac{V_1}{\sqrt{R^2+(\omega L)^2}} = \frac{47.0938}{\sqrt{0.25^2+\left(\frac{2\pi*1*10^{-3}}{2500*10^{-6}}\right)^2}} = 18.646\ \ A$$

Chopper operation with V_C=0:





The average load voltage $= V_o = E * \frac{T_{on}}{T_{on} + T_{off}} = E * \frac{T_{on}}{T} = \alpha E$

where: T_{on}=on time.

T_{off}=off time.

T= T_{on}+ T_{off}= chopping period.

$\alpha = \frac{T_{on}}{T}$ =duty cycle.

The load voltage is controlled by controlling the duty cycle of the chopper. It can be varied in one of the following ways:

1- Constant frequency system: f=1/T

T is kept constant and T_{on} is varied. This may called pulse-width modulation.

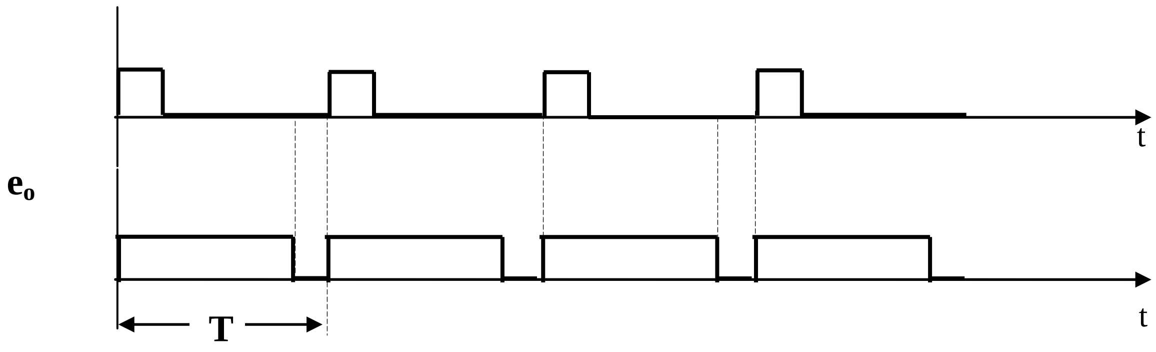
2- Variable frequency system:

T is varied and either T_{on} or T_{off} is kept constant. This may be called frequency modulation.

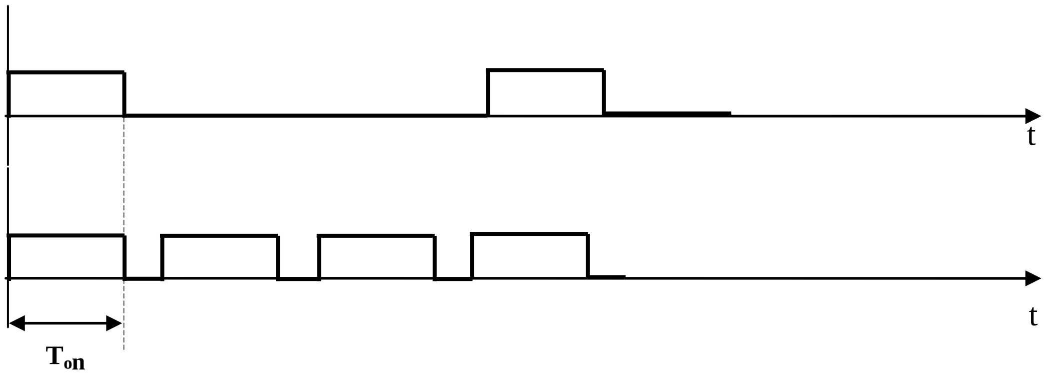
Disadvantages of variable-frequency system:

a- The frequency has to be varied over wide range to provide the full output voltage range. Filter design for variable frequency operation is difficult.

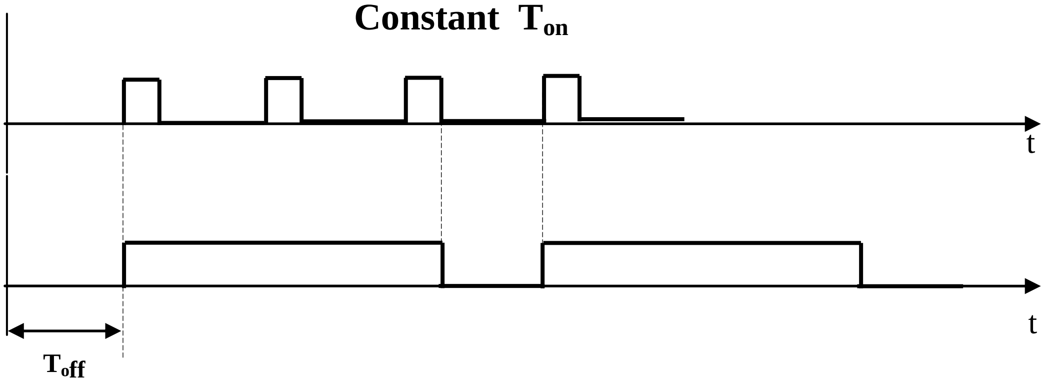
b- The large off-time at low output voltage will make the current in the load circuit to be discontinuous.



Constant frequency

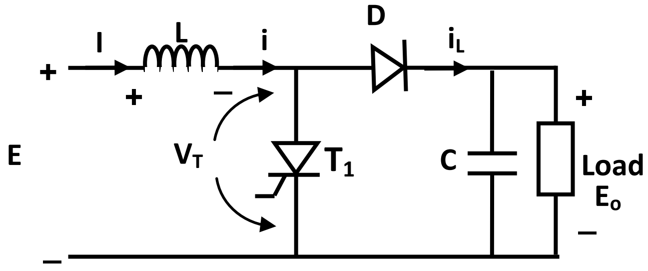


Constant T_{on}

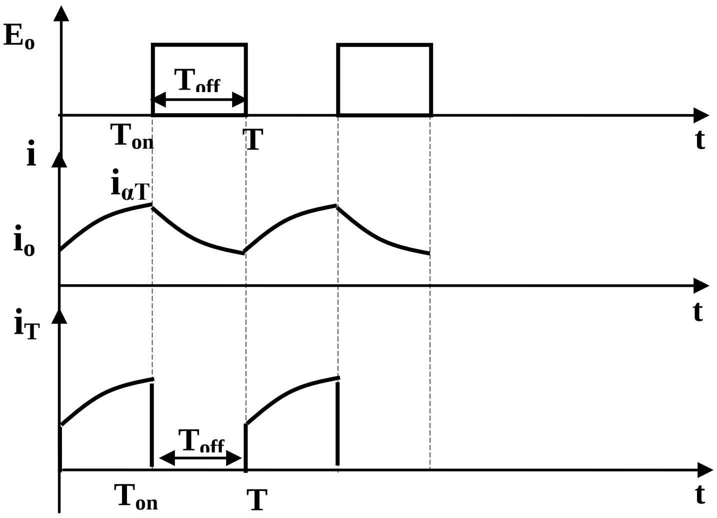


Constant T_{off}

Step-up choppers: (parallel choppers)



The commutation circuit is not shown.



When the thyristor T_1 is on, the current flows through L and T_1. An energy will be stored in L.

The energy stored in the inductor L during the time when the thyristor is on (T_{on}) is:

$W_i = E\ I\ T_{on}$

During the time when the thyristor is off (T_{off}), the energy in the inductor will be transferred to the load.

The capacitor C is used to smooth the output voltage. The diode is used to prevent the capacitor from being discharged through T_1 when it is on.

The energy transferred to the load during T_{off} is

$W_o = (E_o - E)\ I\ T_{off}$

For a lossless system, in the steady state, these two energies are equal

$E\ I\ T_{on} = (E_o - E)\ I\ T_{off}$

$$\therefore E_o = E * \frac{T_{on} + T_{off}}{T_{off}} = E * \frac{T}{T_{off}} = \frac{E}{1-\alpha}$$

This equation shows that the output voltage is greater than the input voltage.

$$\alpha = \frac{T_{on}}{T} = duty\ cycle$$

$$\therefore T_{on} = \alpha\ T$$

For $0 \leq t < \alpha T$, T_1 is on and D is off.

$i_L = 0$ $V_{T_1} = 0$

$E - L\frac{di}{dt} = 0$

$\therefore\ i = i_o + \frac{Et}{L}$

where i_o is the initial inductor current (t=0).

At t=αT

$i_{\alpha T} = i_o + \frac{E}{L}\alpha T$ --------------(1)

For $\alpha T \leq t < T$, T_1 is off and D is on.

$i_L = i$, $V_{T_1} = E_o$

$E - E_o = L\frac{di}{dt'}$

$t' = t - \alpha T = t - T_{on}$

$di = -\frac{E_o - E}{L}dt'$

$i = i_{\alpha T} - \frac{E_o - E}{L}(t - \alpha T)$

At t=T, $i=i_o$

$i_o = i_{\alpha T} - \frac{E_o - E}{L}(T - \alpha T)$---------------------(2)

From (1) and (2) gives:

$i_o = i_o + \frac{E}{L}\alpha T - \frac{E_o - E}{L}T(1 - \alpha)$

$\therefore E_o = \frac{E}{1 - \alpha}$

$\therefore E_o > E$

Commutation circuits:

Commutation circuits can be divided into two groups:

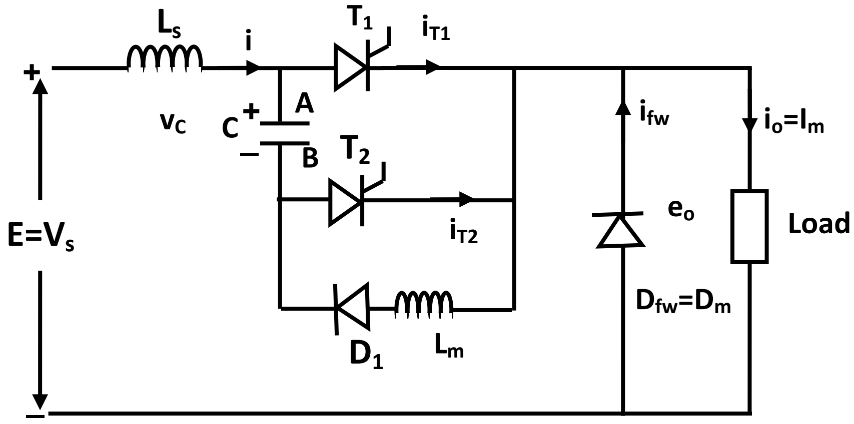
1- Load commutation.

2- Forced commutation.

a- Voltage commutation.

b- Current commutation.

Voltage commutation of a chopper:

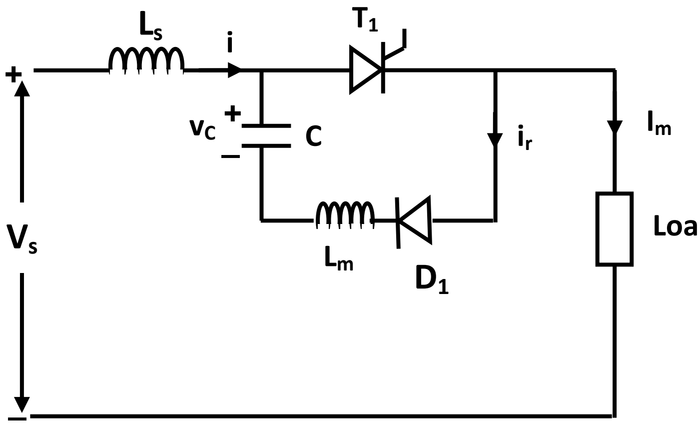


The chopper is T_1

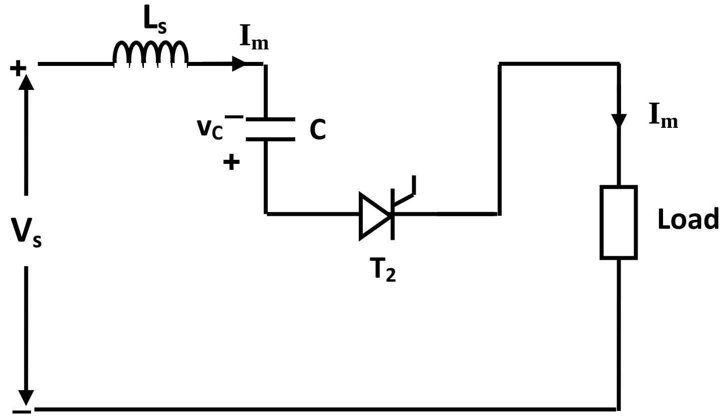
The commutation circuit consists of an auxiliary thyristor T_2, diode D_1, inductor L_m and capacitor C.

At the beginning of the operation, thyristor T_2 is fired and this causes the commutation capacitor C to charge through the load to voltage V_C, which should be the supply voltage, V_S , in the first cycle. The plate A becomes positive with respect to plate B.

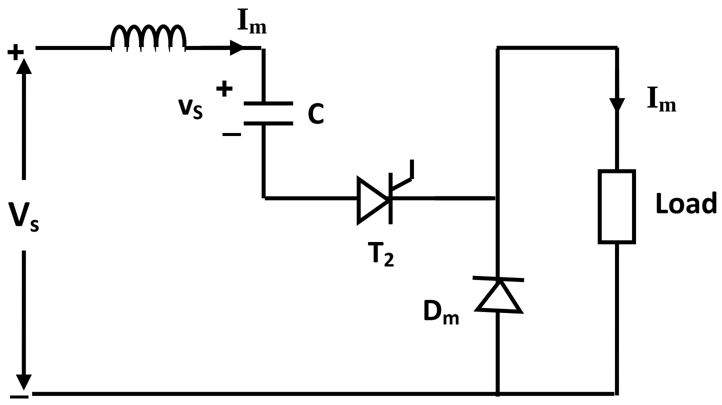
When the capacitor is fully charged, the current through T_2 ceases (become zero) and T_2 stops conducting. The circuit operation can be divided into five modes, and the equivalent circuits under steady-state conductions are:



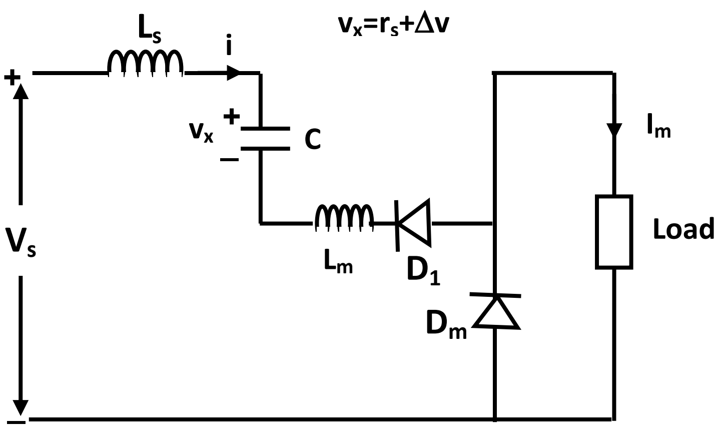
Mode-1-



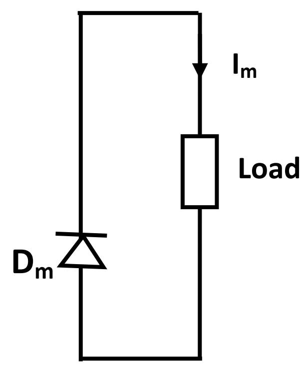
Mode-2-



Mode-3-



Mode-4-



Mode-5-

Mode 1 begins when T_1 is fired. The load is connected to the supply. The commutation capacitor C reverses its charge through the resonant reversing circuit formed by T_1, D_1, and L_m.

The resonant current is given by:

$$i_r = V_C \sqrt{\frac{C}{L_m}} \sin \omega_m t \text{ --------------------(1)}$$

The peak value of resonant reversal current is:

$$I_P = V_C \sqrt{\frac{C}{L_m}} \text{ ------------------------(2)}$$

The capacitor voltage is found from:

$$v_C(t) = V_C \cos \omega_m t \text{ -----------------(3)}$$

where $\omega_m = \frac{1}{\sqrt{L_m C}}$

After time $t = t_r = \pi\sqrt{L_m C}$, the capacitor voltage is reverse to - V_C.

Mode 2 begins when the commutation thyristor T_2 is fired. A reverse voltage is applied across the main thyristor T_1 and it is turned off. The capacitor C discharges through the load from $-V_C$ to zero and this discharging time, which is also called the available turn-off time, is given by:

$$t_q = \frac{V_C C}{I_m} \text{ ---------------------(4)}$$

where I_m is the peak load current. The circuit available turn-off time, t_q, must be greater than the turn-off time of the thyristor, t_{off}. t_q must be designed for maximum value of load current and minimum value of capacitor voltage.

The time required for the capacitor to recharged back to the supply is called the recharging time and is given by:

$$t_C = \frac{V_S C}{I_m} \text{ ---------------------(5)}$$

Thus the total time necessary for the capacitor to discharge and recharge is called the commutation time, which is:

$t_d= t_q+t_C$ ---------------------(6)

This mode ends at $t=t_d$ when the capacitor C recharges to V_S and the freewheeling diode D_m starts conducting.

Mode 3 begins when the freewheeling diode D_m starts conducting and the load current decays. The energy stored in the source inductance L_S (plus any stray inductance in the circuit) is transferred into the capacitor and the current is:

$i_S(t) = I_m \cos \omega_S t$ ---------------------(7)

And the instantaneous capacitor voltage is:

$v_C(t) = V_S + I_m\sqrt{\frac{L_S}{C}} \sin \omega_S t$ ---------------------(8)

where $\omega_S = \frac{1}{\sqrt{L_S C}}$

After time $t=t_S=0.5\pi\sqrt{L_S C}$, the overcharging current becomes zero and the capacitor is recharged to:

$V_x=V_S+\Delta V$ ---------------------(9)

where: V_x= The peak voltage of the commutation capacitor.

ΔV= The overvoltage of the commutation capacitor

From equation (8): $\Delta V=I_m\sqrt{\frac{L_S}{C}}$ ---------------------(10)

Mode 4 begins when the overcharging is complete and the load current continues to decay. It is important to note that this mode exists due to diode D_1 because it allows the resonant oscillation to continue through the circuit formed by D_m, D_1, C, and the supply. This will undercharge the commutation capacitor C and the undercharging current through the capacitor is given by:

$i_C(t) = -\Delta V \sqrt{\frac{C}{(L_S+L_m)}} \sin \omega_u t$ --------------------(11)

The commutation capacitor voltage is:

$v_C(t) = V_x - \Delta V(1 - \cos \omega_u t)$ ----------------------(12)

where $\omega_u = \frac{1}{\sqrt{C(L_S+L_m)}}$

After time $t = t_u = \pi\sqrt{C(L_S + L_m)}$, the undercharge current becomes zero and the diode D_1 stops conducting.

From equation (12):

$V_C = V_x - 2\Delta V = V_S - \Delta V$ ----------------------(13)

If there is no overcharging, there will not be any undercharge.

Mode 5 begins when the commutation process is complete and the load current continues to decay through diode D_m. This mode ends when the main thyristor T_1 is refired at the beginning of next cycle. The different waveform for the current and voltage are shown below:

The average output voltage of the chopper is:

$V_o = \frac{1}{T}\left[V_S kT \quad + t_d \frac{1}{2}(V_C + V_S)\right]$ ----------------------(14)

where $k = \alpha = \frac{T_{on}}{T}$ & $f = \frac{1}{T}$

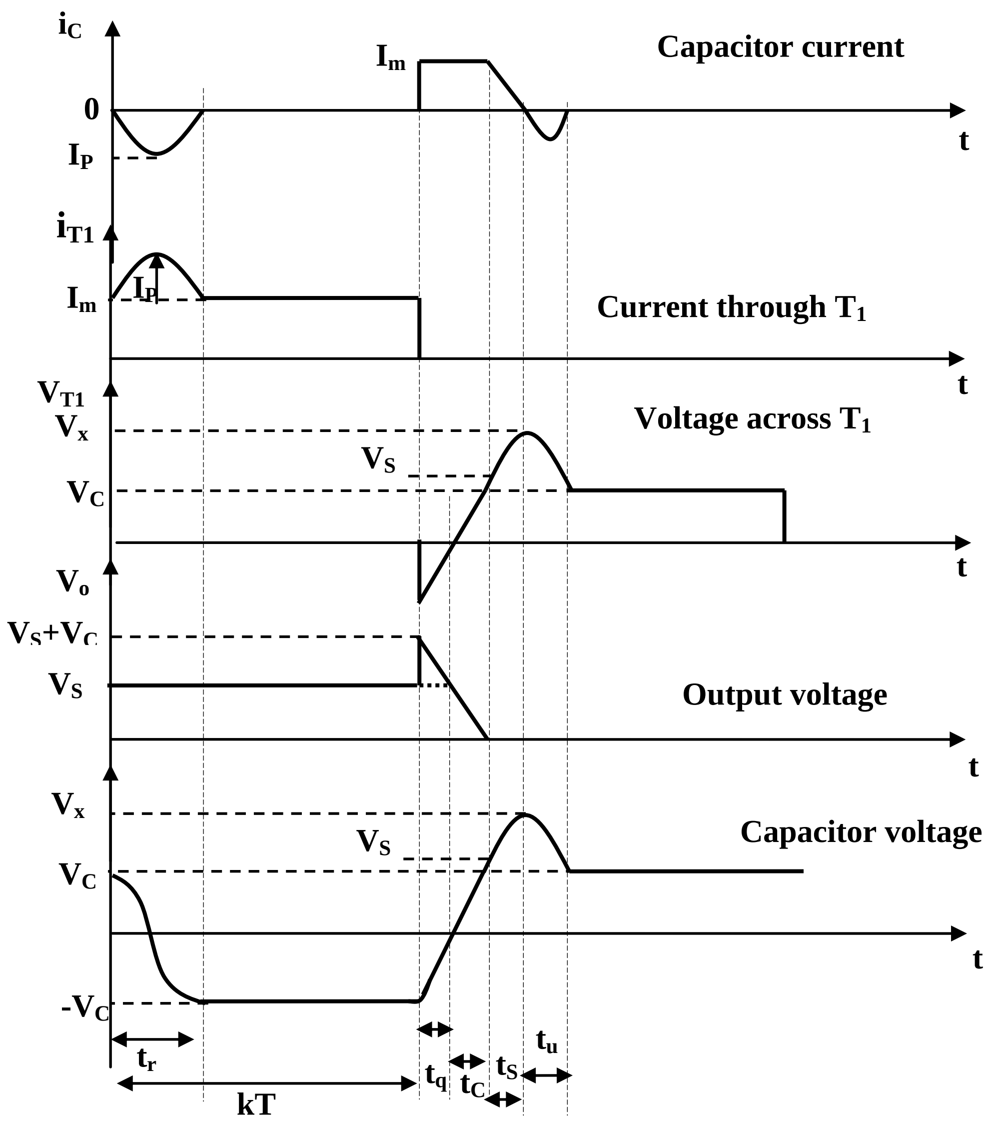
For k=0,

$V_o = 0.5 t_d (V_C + V_S) f$ ---------------------(15)

This limits the minimum output voltage of the chopper. However, the thyristor T_1 must be on for a minimum time of $t_r = \sqrt{L_m C}$ to allow the charge reversal of the capacitor. Therefore, the minimum duty cycle and minimum output voltage are:

$t_r = k_{min} T = \pi\sqrt{L_m C}$ ---------------------(16)

$$k_{min} = \alpha_{min} = \frac{t_r}{T} = t_r f = f\pi\sqrt{L_m C}$$ ---------------------(17)



$$V_{o(\min)} = k_{min} V_S + 0.5\, t_d (V_C + V) f$$

$$= f[V_S t_r + 0.5\, t_d (V_C + V_S)]$$ ---------------------(18)

The minimum output voltage ($V_{o(\min)}$) can be varied by controlling the chopping frequency (f).

The maximum value of the duty cycle is also limited to allow the commutation capacitor to discharge and recharge.

$k_{max} T = T - t_d - t_S - t_u$

$\therefore k_{max} = 1 - \frac{t_d + t_S + t_u}{T}$ ---------------------(19)

The maximum output voltage is:

$V_{o(\max)} = k_{max} V_S + 0.5\, t_d (V_C + V_S) f$ ---------------------(20)

The maximum thyristor current is $i_{T1_P} = I_m + I_P$ and this increases the peak current rating of the thyristor.

Example-2: A series chopper with voltage commutation circuit is used to supply an inductive load. The load requires an average current of I_a=425 A with peak current of I_m=450 A. The input supply voltage is V_S=220 V. The chopping frequency is f=400Hz and the turn-off time of the main thyristor is t_{off}=18 μs. If the peak current through the main thyristor is limited to 180 % of I_m and the source inductance is negligible (L_S=0), determine the (a) commutation capacitor, C; (b) inductance L_m; (c) minimum and maximum output voltage.

Solution:

a- when L_S=0 ;   Δv=0

$\therefore V_C = V_S = V_x = 220\, V$

$\frac{V_C C}{I_m} = t_q > t_{off}$

$\frac{220 C}{450} > 18 * 10^{-6}$

$C > 36.8182\, \mu F$

Let C=40 μF

b-

$i_{T_{1P}} = I_m + I_P$

$\therefore I_P = i_{T_{1P}} - I_m = 1.8 * 450 - 450 = 360\, A$

$= V_C\sqrt{\frac{C}{L_m}} = 220\sqrt{\frac{40*10^{-6}}{L_m}}$

$\therefore L_m = 14.9383\ \mu H$

c-

$V_{omin} = f[V_S t_r + 0.5\ t_d(V_C + V_S)]$

$t_r = \pi\sqrt{L_m C} = \pi\sqrt{14.9383 * 10^{-6} * 40 * 10^{-6}} = 76.795\ \mu sec.$

$t_q = \frac{V_C C}{L_m} = \frac{220*40*10^{-6}}{450} = 19.556\ \mu sec$

$t_C = \frac{V_S C}{L_m} = 19.556\ \mu sec$

$\therefore t_d = t_q + t_C = 39.111\ \mu sec$

$\therefore V_{omin} = 400[220 * 76.795 * 10^{-6} + 0.5 * 39.111 * 10^{-6} * 440] = 10.2\ V$

$V_{o(\max)} = k_{max} V_S + 0.5\ t_d(V_C + V_S) f$

$k_{max} = 1 - \frac{t_d + t_S + t_u}{T} = 1 - f(t_d + t_S + t_u)$

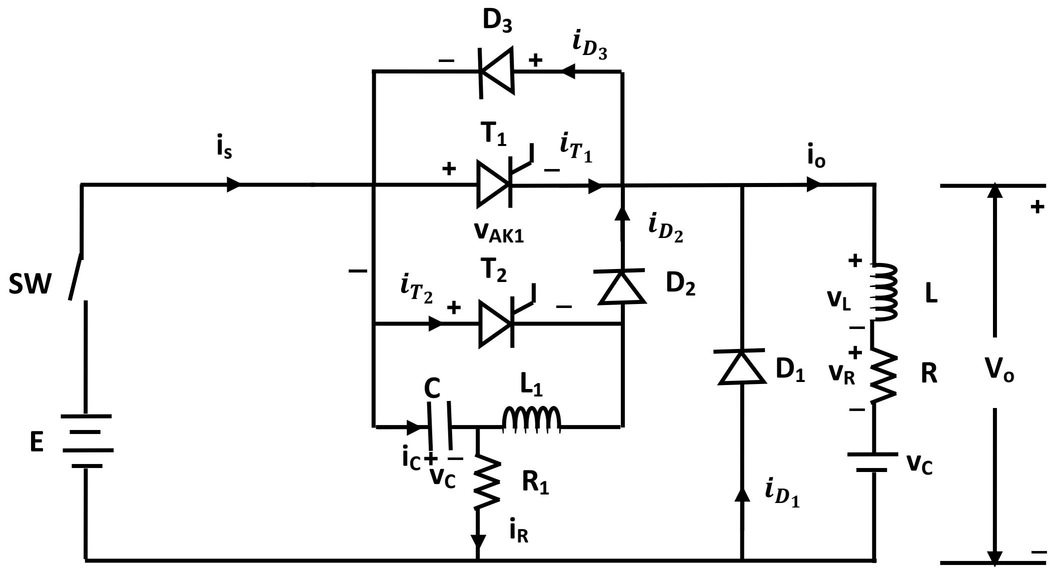
Since there is no overcharging, there will be no overcharging period;

$\therefore t_S = t_u = 0$

$\therefore k_{max} = 1 - 400(39.111 * 10^{-6}) = 0.9844$

$V_{o(\max)} = 0.9844 * 220 + 0.5 * 39.111 * 10^{-6}(220 + 220) * 400 = 220\ V$

Current commutation of a chopper:



The sequence of operation is as follows:

1- The converter is connected to the source by closing switch SW , and capacitor C is charged up to v_C=E volts via resistor R_1.

2- At t=0, when the capacitor is fully charged, thyristor T_1 is turned on, and load current i_o increases exponentially from zero to I_{max}

3- At $t=T_{on}$, thyristor T_2 is turned on, initiating the commutation cycle, and an oscillatory current flows in the ringing circuit comprising C, L_1, and T_2; i_C is initially negative. It is assumed that the commutation interval is so short that i_o remains constant at the value I_{max} throughout the interval. It is also assumed that R_1 is sufficiently large to permit i_R to be neglected in the analysis of the commutation circuit, but is yet small enough to permit v_C to decay to the value E before the next commutation cycle is initiated.

4- When i_C becomes positive, diode D_2 conducts, T_2 turns off and since i_o is assumed constant, i_C reduces i_{T_1} .

5- When i_{T_1} is reduced to zero by the increasing value of i_C, diode D_3 begins to conduct, and the forward voltage drop across this diode commutates thyristor T_1. Current i_C-I_{max} then flows through diode D_3.

6- After i_C has passed its maximum positive value and again become less than I_{max}, diode D_1 conducts. A new oscillatory circuit then exists, comprising C, L_1, D_2, D_1, and source E.

7- The oscillatory cycle of i_C is complete, and i_C becomes zero leaving v_C>E.

8- i_o decays exponentially through D_1 from the value I_{max}, and simultaneously v_C decays through R_1 to the value v_C=E.

9- At t=T, when $i_o=I_{min}$, T_1 is again turned on.

Commutation circuit analysis:

A new time scale is employed, such that t=0 when the commutation cycle is initiated by the application of the gating signal to thyristor T_2. For the circuit comprising C,L, and T_2.

$v_C + v_{L_1} = \frac{1}{C}\int i_C\, dt + L_1 \frac{di_C}{dt} = 0$ ------------------------(1)

$\frac{d^2 i_C}{dt^2} + \frac{1}{L_1 C} i_C = 0$ ------------------------(2)

Using the initial conditions:

$i_C = 0$ At $\quad t = 0^+$------------------------(3)

$v_{AK2} = 0$ and $v_C = E$ At $\quad t = 0^+$ ------------------------(4)

$v_C + v_{L_1} = 0$ At $\quad t = 0^+$ ------------------------(5)

From equations (4) & (5):

$\frac{di_C}{dt} = -\frac{E}{L_1}$ At $\quad t = 0^+$ ------------------------(6)

From equations (6) & (2):

$i_C = -\frac{E}{\omega_r L_1} \sin(\omega_r t)$ ------------------------(7)

where $\omega_r = \frac{1}{\sqrt{L_1 C}}$

$\therefore v_C = \frac{1}{C}\int i_C\, dt = \frac{1}{C}\int -\frac{E}{\omega_r L_1} \sin \omega_r t\, dt$

$v_C = E\ cos(\omega_r t)$ ------------------------(8)

Equations (7) and (8) apply throughout period I of the commutation interval, during which time the oscillatory elements C and L_1 are short-circuited through a succession of thyristors and diodes , i.e., during $0< t < t_2$.

At instant $t=\pi/\omega_r$, i_C reverses and T_2 turns off; i_C now flows through diode D_2, and since the main thyristor voltage $v_{AK1} = 0$, the oscillatory elements continue to be short-circuited. For this part of period I, i_C may be considered to flow in a negative direction through T_1.

$$i_{T_1} = I_{max} - i_C = I_{o1} - i_C \qquad t > \frac{\pi}{\omega_r} \text{------------------------(9)}$$

At instant $t = t_1$, $i_C = I_{max}$, $i_{T_1} = 0$, and diode D_3 begins to conduct, so that:

$i_{D_3} = i_C - I_{o1}$ ------------------------(10)

$v_{AK1} = -v_{D3}$ ------------------------(11)

Thus T_1 is commutated.

When T_1 is commutated:

$$i_C = I_{o1} = -\frac{E}{\omega_r L_1} \sin(\omega_r t_1) \text{ ------------------------(12)}$$

$$\therefore t_1 = \frac{1}{\omega_r} \sin^{-1}\left(-\frac{\omega_r L_1 I_{o1}}{E}\right) = \frac{\pi}{\omega_r} + \frac{1}{\omega_r} \sin^{-1}\left(\frac{\omega_r L_1 I_{o1}}{E}\right) \text{ ------------------------(13)}$$

At $t=t_2$, i_C has passed its positive maximum and fallen to the value I_{o1}. Thus:

$$t_2 = \frac{2\pi}{\omega_r} - \frac{1}{\omega_r} \sin^{-1}\left(\frac{\omega_r L_1 I_{o1}}{E}\right)$$

$$t_2 = \frac{2\pi}{\omega_r} - t_1 + \frac{\pi}{\omega_r} = \frac{3\pi}{\omega_r} - t_1 \text{ ------------------------(14)}$$

At this instant, which ends period I of the commutation interval, diode D_1 tends to begin to conduct, since i_o is constant and i_C is falling. The result of conduction of D_1 would be that v_o become zero, and since $v_C < E$, i_C would tend to rise turning off D_1.

For a short period the capacitor is charged at constant current $i_C = I_{o1}$ until $v_C = E$. During this charging interval, which is called period II of the commutation interval.

$v_o = E - v_C$ ------------------------(15)

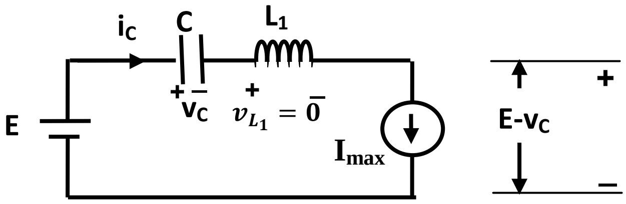
At $t=t_2$, D_3 ceases to conduct, so that the reverse voltage is removed from thyristor T_1:

$E = v_{AK1} + v_o$ ------------------------(16)

From (15) in (16) gives:

$v_{AK1} = v_C$ ------------------------(17)

that is, a forward voltage is applied to thyristor T_1.



Equivalent circuit for period II

$v_{L_1} = L_1 \frac{di_C}{dt'} = 0$ since $i_C = I_{max} = cons.$

$i_C = I_{o1} = C \frac{dv_C}{dt'}$ ------------------------(18)

where $t' = t - t_2$ ------------------------(19)

At $t' = t_2$, from equation (8):

$v_C = V_{C_2} = E \cos \omega_r t_2$ ------------------------(20)

Thus from equation (18) and (20):

$v_C = \frac{I_{o1}}{C} t' + v_{C_2}$ ------------------------(21)

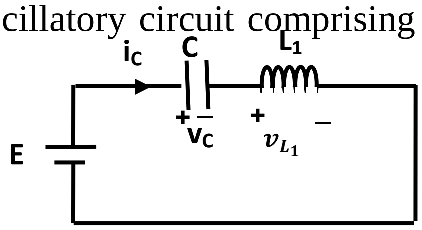
At $t' = t_1'$

$v_C = E = \frac{I_{o1}}{C} t_1' + v_{C_2}$ ------------------------(22)

From equation (20) and (22):

$t_1^{'} = \frac{CE}{I_{o1}}(1 - \cos \omega_r t_2)$ ------------------------(23)

At this instant v_o becomes zero, and diode D_1 begins to conduct; $i_C = I_{o1}$ now becomes the initial current in the oscillatory circuit comprising C, L_1, D_1, and source E as shown below:



The current in this circuit may be considered to flow in the reverse direction through D_1, that is:

$i_{D_1} = i_o - i_C > 0$ ------------------------(24)

This is during period III of the commutation interval,

$E = v_C + v_{L_1}$ ------------------------(25)

$\frac{1}{C}\int i_C \, dt^{''} + L_1 \frac{di_C}{dt^{''}} = E$ ------------------------(26)

where $t^{''} = t^{'} - t_1^{'} = t - t_2 - t_1^{'}$ ------------------------(27)

From equation (26):

$\frac{d^2 i_C}{dt^{''2}} + \frac{1}{L_1 C} i_C = 0$ ------------------------(28)

The initial conditions for the solution of this equation are:

$i_C = I_{o1}$, $\frac{di_C}{dt^{''}} = 0$, $t^{''} = 0$ ------------------------(29)

The solution of equation (28):

$i_C = I_{o1} \cos(\omega_r t^{''})$ ------------------------(30)

i_C becomes zero when $t^{''} = t_1^{''}$

$\therefore t_1^{''} = \frac{\pi}{2\omega_r}$ ------------------------(31)

At this instant, all of the energy stored in inductance L_1 at $t'' = 0$ has been transferred to capacitor C, and as result $v_C > E$.

Form equation (30):

$$v_C = \frac{1}{C}\int i_C \, dt'' = \frac{1}{C}\int I_{o1}\cos(\omega_r t'')\, dt'' \text{ ------------------------(32)}$$

with the initial values:

$$v_C = E \ \ at\ t'' = 0 \text{ ------------------------(33)}$$

the solution of equation (32) is:

$$v_C = \frac{I_{o1}}{\omega_r C}\sin(\omega_r t'') + E \text{ ------------------------(34)}$$

From (31) in (34):

$$v_{C_{max}} = E + \frac{I_{o1}}{\omega_r C} \text{ ------------------------(35)}$$

This capacitor voltage then decays through R_1 to v_C=E.

Also the current i_o decays exponentially from the value I_{max} through the load.

The commutation interval is:

$$t_C = t_2 + t_1' + t_1'' \text{ ------------------------(36)}$$

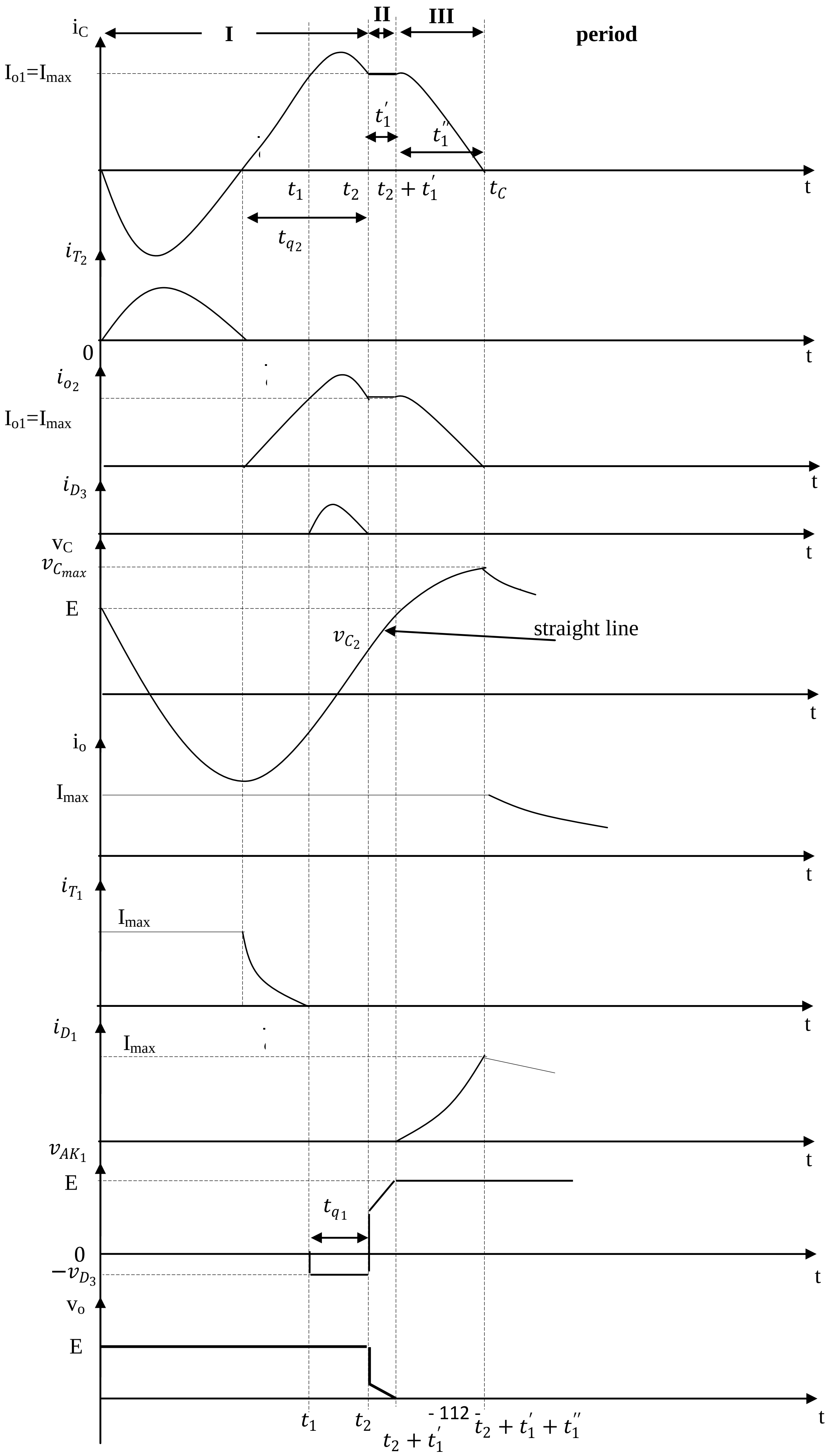
The time available for commutation of thyristor T_1 is:

$$t_{q_1} = t_2 - t_1 \text{ ------------------------(37)}$$

From equation (13) and (14) in (37) gives:

$$t_{q_1} = \frac{\pi}{\omega_r} - \frac{2}{\omega_r}\sin^{-1}\left(\frac{\omega_r L_1 I_{o1}}{E}\right) \text{ ------------------------(38)}$$

From this equation it is clear that the time available for turn-off of the main thyristor decreases with increase of load current.


I
II
III
period
i_C
$I_{o1}=I_{max}$
t_1'
t_1''
t_1
t_2
$t_2 + t_1'$
t_C
t_{q_2}
t
i_{T_2}
0
i_{o_2}
$I_{o1}=I_{max}$
i_{D_3}
v_C
$v_{C_{max}}$
E
v_{C_2}
straight line
i_o
I_{max}
i_{T_1}
I_{max}
i_{D_1}
I_{max}
v_{AK_1}
E
t_{q_1}
0
$-v_{D_3}$
v_o
E
t_1
t_2
$t_2 + t_1'$
$t_2 + t_1' + t_1''$

This detailed analysis of the commutation circuit gives a complete information about the rating of each component in the circuit because it shows the current and voltage variation across all the commutation circuit components.

<u>Design of the current Commutation circuit:</u>

Let the maximum capacitor voltage be "x" times the supply voltage, i.e.,

$V_{C_{max}} = xE$ ------------------------(39)

substituting for $V_{C_{max}}$ in equation (35) gives:

$$xE = E + \frac{I_{o1}}{\frac{1}{\sqrt{L_1 C}} C}$$

$$\therefore \sqrt{\frac{L_1}{C}} = \frac{E(x-1)}{I_{o1}} \text{ ------------------------(40)}$$

From equation (38):

$$t_{q_1} = \frac{\pi}{\omega_r} - \frac{2}{\omega_r} \sin^{-1}\left(\frac{\frac{1}{\sqrt{L_1 C}} L_1 I_{o1}}{E}\right) \geq t_{off}$$

$$t_{q_1} = \frac{\pi}{\omega_r} - \frac{2}{\omega_r} \sin^{-1}\left(\frac{\sqrt{\frac{L_1}{C}} I_{o1}}{E}\right) \geq t_{off} \text{ ------------------------(41)}$$

From (40) in (41) gives:

$$t_{q_1} = \frac{\pi}{\omega_r} - \frac{2}{\omega_r} \sin^{-1}(x-1) \geq t_{off} \text{ ------------------------(42)}$$

From this equation it is possible to calculate ω_r

From equations (35) and (39):

$$C = \frac{I_{o1}}{E(x-1)\omega_r} \text{ ------------------------(43)}$$

$\omega_r = \frac{1}{\sqrt{L_1 C}}$, $\omega_r{}^2 = \frac{1}{L_1 C}$, $C = \frac{1}{L_1 \omega_r{}^2}$ in (43)

$L_1 = \frac{E(x-1)}{\omega_r I_{o1}}$ ------------------------(44)

<u>Example-3:</u> For the chopper in example-1, if the chopper is current commutated with L_1=4μH and C=40 μF:

a- Calculate the time available for turn-off thyristor T_1.

b- Calculate the commutation interval.

c- Determine the time available for turn-off thyristor T_2.

<u>Soultion:</u>

a- From example-1 , $I_{o1}=I_{max}$=165.424 A,   E=110 V

$\omega_r = \frac{1}{\sqrt{L_1 C}}$ $\omega_r = \frac{1}{\sqrt{4*10^{-6}*40*10^{-6}}} = 79056.941 \frac{rad}{sec}$.

$t_{q_1} = \frac{\pi}{\omega_r} - \frac{2}{\omega_r}\sin^{-1}\left(\frac{\omega_r L_1 I_{o1}}{E}\right)$

$$t_{q_1} = \frac{\pi}{79056.941} - \frac{2}{79056.941}\sin^{-1}\left(\frac{79056.9414 * 4 * 10^{-6} * 165.424}{110}\right) = 27.2\ \mu sec$$

b- $t_C = t_2 + t_1^{'} + t_2^{''}$

$t_2 = \frac{2\pi}{\omega_r} - \frac{1}{\omega_r}\sin^{-1}\left(\frac{\omega_r L_1 I_{o1}}{E}\right)$

$t_2 = \frac{2\pi}{79056.9414} - \frac{1}{79056.9414}\sin^{-1}\left(\frac{79056.9414*4*10^{-6}*165.424}{110}\right) = 73.207\mu sec$

$t_1^{'} = \frac{CE}{I_{o1}}(1 - \cos\omega_r t_2)$

$= \frac{40*10^{-6}*110}{165.424}(1 - \cos(79056.9414 * 73.207 * 10^{-6})) = 3.201\ \mu sec$

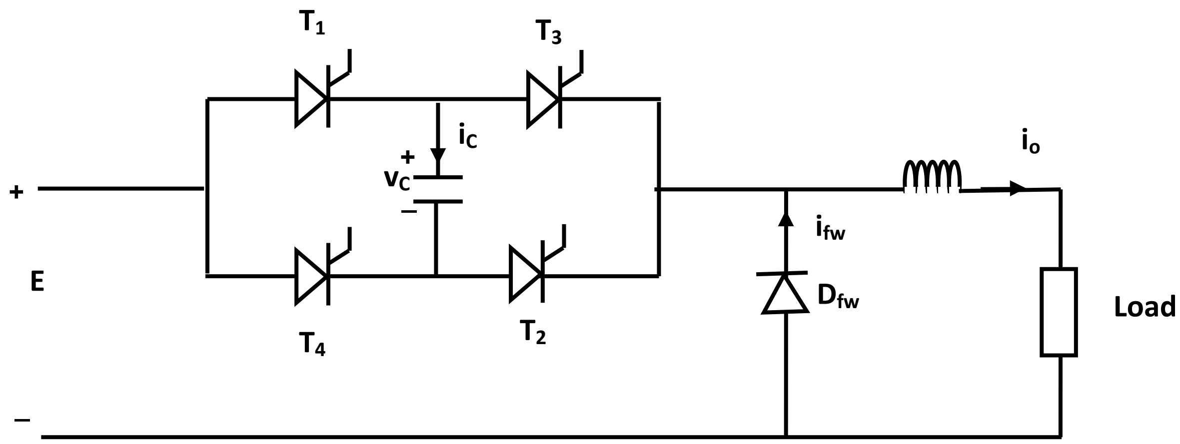
$t_2^{''} = \frac{\pi}{2\omega_r} = \frac{\pi}{2*79056.9414} = 19.869\ \mu sec$

$t_C = t_2 + t_1^{'} + t_2^{''} = 96.277\ \mu sec$

c- Thyristor T_2 ceases to conduct at instant $t = \frac{\pi}{\omega_r}$, and a forward voltage is again applied to it at the end of period I of the commutation interval.

$$\therefore t_{q_2} = t_2 - \frac{\pi}{\omega_r} = 73.207 * 10^{-6} - \frac{\pi}{79056.9414} = 33.468\,\mu sec$$

Load commutation of a chopper:



$$i_C = C\frac{dv_C}{dt} \qquad \Rightarrow \qquad T_{on} = \frac{2EC}{I_o}$$

$$E_o = \frac{1}{2}\,T_{on} * 2E * \frac{1}{T} = ET_{on}f = \frac{2E^2Cf}{I_o}$$

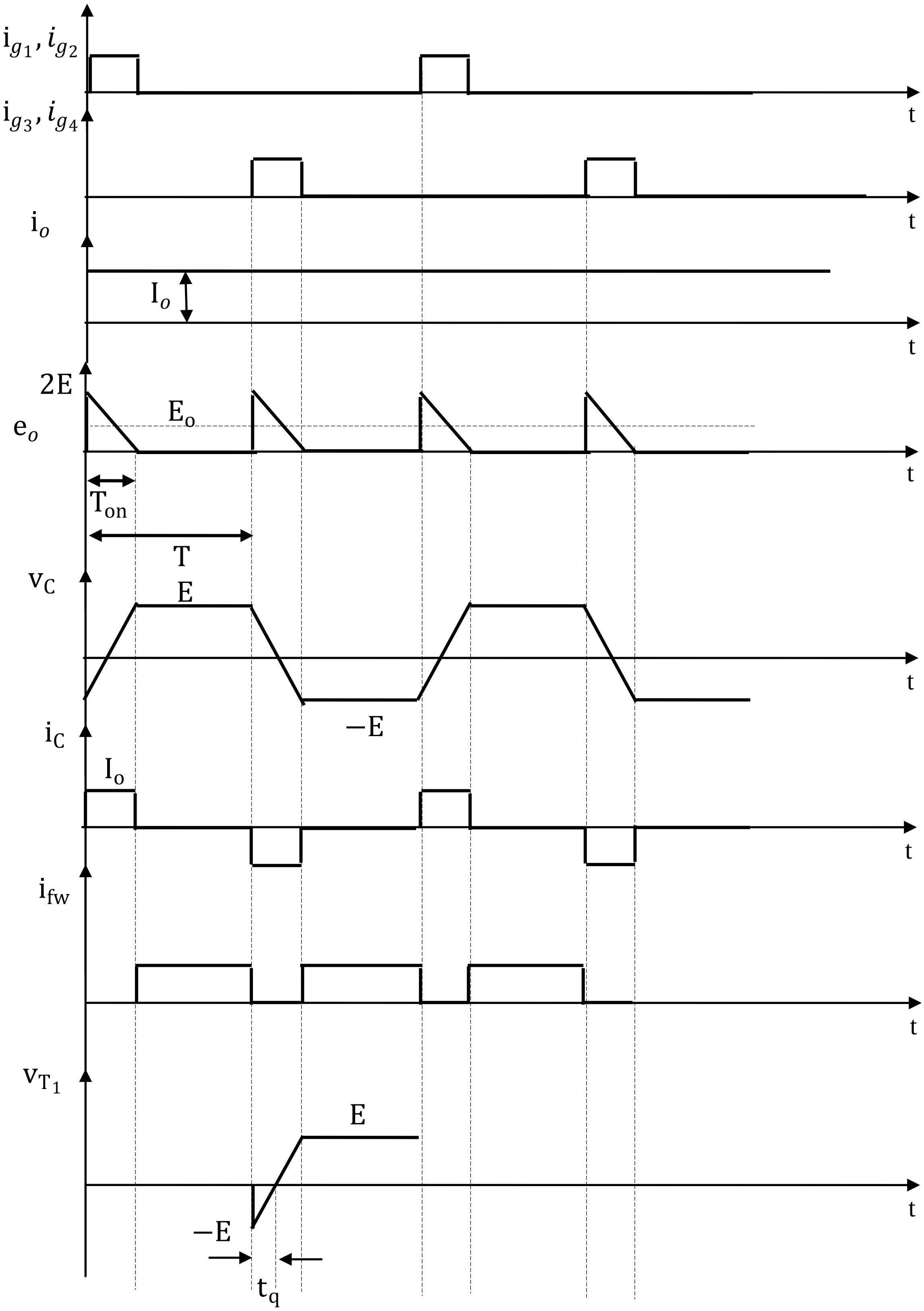
where f=Frequency of the chopper.

At $f=f_{max}$, $E_{o\,max}=E$

$$\therefore E_{omax} = E = \frac{2E^2Cf_{max}}{I_o} \qquad \therefore f_{max} = \frac{I_o}{2EC}$$

The value of C is chosen for maximum load current I_{om}:

$$C = \frac{I_{om}}{2Ef_{max}}$$



i_{g_1}, i_{g_2}
t
i_{g_3}, i_{g_4}
t
i_o
I_o
t
2E
e_o
E_o
t
T_{on}
T
v_C
E
t
−E
i_C
I_o
t
i_{fw}
t
v_{T_1}
E
t
−E
t_q

Sheet No.3

1- A series chopper is connected to 110 V dc supply and a load of L=0.2 mH, R=0.25 Ω, v_C=40 V. The chopper has T=2500 μsec, and t_{on}=1250 μsec.

a- Calculate the average output current I_o and the average output voltage V_o.

b- Calculate the maximum and minimum values of instantaneous output current I_{max} and I_{min}.

c- Calculate the rms values of the first harmonic(fundamental) output voltage and current.

2- A series chopper with voltage commutation circuit is used to supply an inductive load. The load requires an average current of I_a=425 A with peak current of I_m=450 A. The input supply voltage is V_S=220V. The chopping frequency is f=400Hz and the turn-off time of the main thyristor is t_{off}=18 μsec. If the peak current through the main thyristor is limited to 180% of I_m and the source inductance is L_S=4 μH, determine:

a- The peak capacitor voltage.

b- The available turn-off time , t_q.

c- The total commutation time, t_d.

3- A series chopper is connected to 110V dc supply and a load of L=0.2 mH, R=0.25 Ω, v_C=40 V. The chopper has T=2500 μsec, and t_{on}=1250 μsec. The chopper is designed to operate with current commutation with t_q=50 μsec and x=1.6667, determine:

a- The commutation inductance L_1 and capacitance C.

b- The maximum capacitor voltage V_{Cmax}.

c- The peak current in the commutating thyristor.

d- Repeat a to C for x=1.4.

4- Repeat Example-1 for v_C=0 and L of so large a value that the output current may be assumed constant or ripple-free at the value V_o/R.

5- If the chopper in Example-1 is connected to 600 V dc and has L=4 mH, v_C=200 V, R=1.5 Ω, T=4000 μsec, and t_{on}=2500 μsec. Show that the output current is continuous.

6- the chopper in Example-1 has 600 V, v_C=350 V, R=0.1 Ω, T=1800 μsec, and L is of so large value that the output current may be assumed constant at I_o=100 A. Calculate the required value of t_{on}.

7- A step-up chopper is used to control power flow from a dc voltage E=110 V to a battery voltage of 220 V. The power transferred to the battery is 35 kW. The current ripple of the inductor is negligible. Determine:

a- The duty cycle.

b- The effective load resistance.

c- The average input current.

8- Design the values of commutation components L_m and C to provide a turn-off time t_q=20 μsec for a voltage commutated chopper. The specifications for the circuit are V_S=600 V, I_m=350 A, and L_S=6 μH. The peak current through T_1 is not to exceed 2 I_m.

9- Design the values of commutation capacitor C to provide a turn-off-time requirement of t_q=20 μsec for a load commutated chopper if E=600 V, I_m=350 A.

10- A current commutated chopper with E=600 V, I_{max}=150 A t_{off}=25 μsec, Δt=6 μsec (t_q- t_{off}=Δt). Calculate the values of L_1 and C if the maximum capacitor voltage is not to exceed 1000 V.
