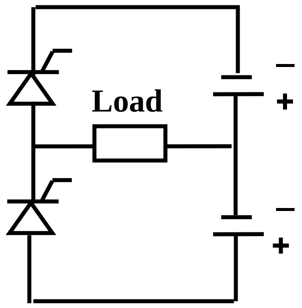
Chapter Three

Inverters

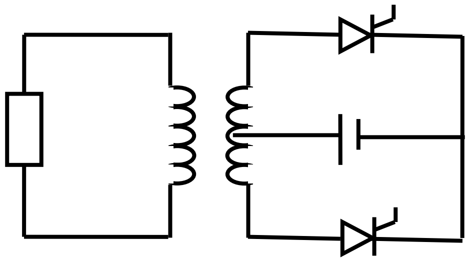
Introduction:

DC to AC converters are known as inverters. The function of an inverter is to change a DC input voltage to a symmetrical AC output voltage of desired magnitude and frequency. A variable output voltage can be obtained by varying the input DC voltage keeping the gain of the inverter constant. On the other hand, if the DC input voltage is fixed, a variable output voltage can be obtained by varying the gain of the inverter, which is normally accomplished by pulse-width- modulation (PWM) control within the inverter.

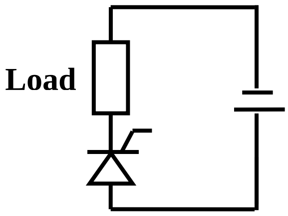
Types of inverter circuits:



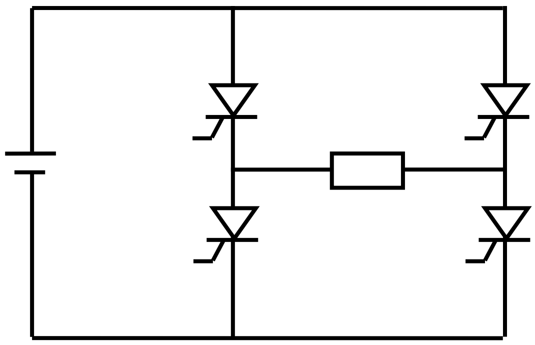
1- Centre-tapped supply inverter.



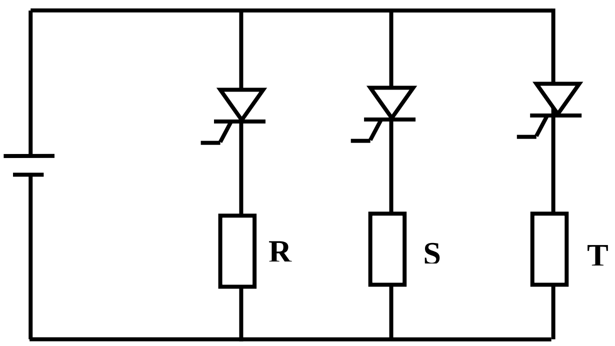
2- Centre-tapped lead inverter.



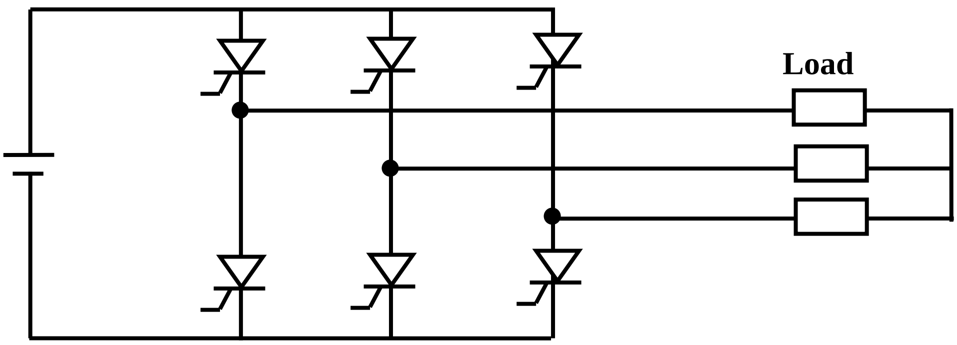
3- Section-type inverter.



4- Single-phase bridge inverter.



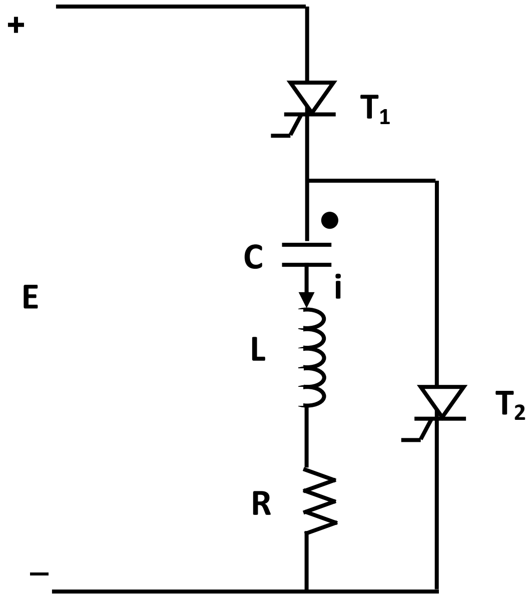
5- Half-wave 3-phase inverter.



6- Three-phase bridge inverter.

The output voltages in the section-type inverter and half-wave 3-ph inverter contain dc components.

<u>Series inverters:</u>



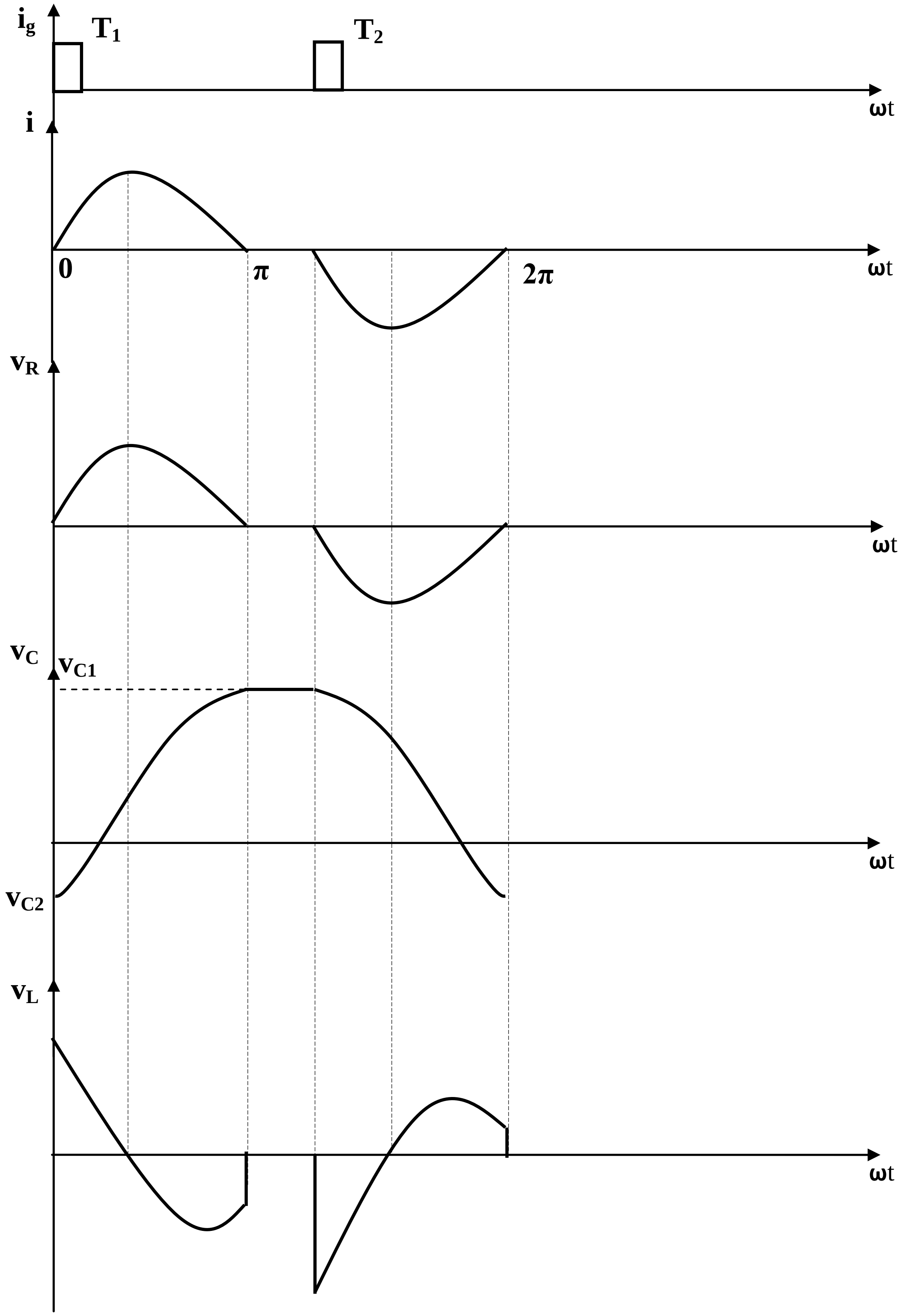
When T_1 is fired, capacitor C will start charging. When the capacitor C is completely charged, the current i will be zero.

Then, when T_2 is on, the capacitor C will start discharging through R &L, i.e., causing negative current in the load.

For the first period, T_1 is on:

$$E = v_R + v_L + v_C = Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + V_{C_2} \text{ ------------------------------------(1)}$$

$$0 = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C}i$$



$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

The solution of this equation is:

$$i = e^{-\alpha t}(A_1 \sin\omega t + A_2 \cos \omega t)$$

where:

$$\alpha = \frac{R}{2L}$$

$$\omega = \sqrt{\omega_O^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$\omega_O = \frac{1}{\sqrt{LC}}$$

Form the initial conditions:

At t=0, i=0 $\therefore A_2 = 0$

$$\therefore i = e^{-\alpha t} A_1 \sin \omega t$$

$$\frac{di}{dt} = -\alpha e^{-\alpha t} A_1 \sin \omega t + e^{-\alpha t} A_1\, \omega \cos \omega t$$

At t=0 $\left.\frac{di}{dt}\right|_{t=0} = A_1 \omega$

At t=0 in equation (1)

$$E = L\frac{di}{dt} + V_{C_2} = A_1 \omega L + V_{C_2}$$

$$A_1 = \frac{E - V_{C2}}{\omega L}$$

$$\boxed{\therefore i = \frac{E - V_{C2}}{\omega L} e^{-\alpha t} \sin \omega t}$$

$$\therefore v_R = iR = \frac{E - V_{C_2}}{\omega L} R e^{-\alpha t} \sin \omega t$$

$$v_L = L\frac{di}{dt} = \frac{E - V_{C_2}}{\omega} e^{-\alpha t} (\omega \cos \omega t - \alpha \sin \omega t)$$

$$v_C = E - v_L - v_R$$

For the second period, T_2 is on:

$$v_L + v_R + v_C = 0$$

$L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + V_{C_1} + Ri = 0$ -----------------------------------(2)

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

The solution of this equation and using the initial condition gives:

$i = Ae^{-\alpha t}\sin\omega t$-----------------------------------(3)

where

$$\alpha = \frac{R}{2L}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

At t=0, i=0 in equation (2) gives:

$\left.\frac{di}{dt}\right|_{t=0} = -\frac{V_{C_1}}{L}$

From equation (3):

$$\frac{di}{dt} = -\alpha e^{-\alpha t} A\sin\omega t + e^{-\alpha t} A\,\omega\cos\omega t$$

$\left.\frac{di}{dt}\right|_{t=0} = A\omega = -\frac{V_{C_1}}{L}$

$\therefore A = -\frac{V_{C_1}}{\omega L}$ in (3)

$$\boxed{\therefore i = -\frac{V_{C_1}}{\omega L}e^{-\alpha t}\sin\omega t}$$

$$v_R = Ri = -\frac{V_{C_1}}{\omega L}Re^{-\alpha t}\sin\omega t$$

$$v_L = L\frac{di}{dt} = -\frac{V_{C_1}}{\omega} e^{-\alpha t}(-\alpha \sin \omega t + \omega \cos \omega t)$$

$$v_C = -(v_R + v_L)$$

Note: T_1 is on:

$$v_C = E - v_L - v_R$$

$$= E - \frac{E - V_{C_2}}{\omega} e^{-\alpha t}\,(\omega \cos \omega t - \alpha \sin \omega t) - \frac{E - V_{C_2}}{\omega L} R e^{-\alpha t} \sin \omega t$$

At t=π/ω $v_C = V_{C_1}$

$$V_{C_1} = E - \frac{E - v_{C_2}}{\omega} e^{\frac{-\alpha\pi}{\omega}}(-\omega)$$

$V_{C_1} = E\left(1 + e^{\frac{-\alpha\pi}{\omega}}\right) - V_{C_2} e^{\frac{-\alpha\pi}{\omega}}$ ------------------(1)

T_2 is on:

$v_C = -(v_R + v_L)$

$$= \frac{V_{C_1}}{\omega L} R e^{-\alpha t} \sin \omega t + \frac{V_{C_1}}{\omega} e^{-\alpha t}(-\alpha \sin \omega t + \omega \cos \omega t)$$

At t=π/ω $v_C = V_{C_2}$

$V_{C_2} = \frac{V_{C_1}}{\omega} e^{\frac{-\alpha\pi}{\omega}}(-\omega) = -V_{C_1} e^{\frac{-\alpha\pi}{\omega}}$ ------------------(2)

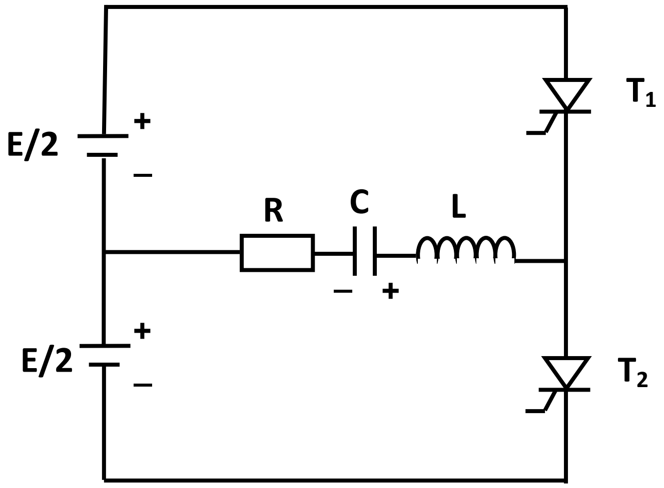
From (2) in (1):

$V_{C_1} = E\left(1 + e^{\frac{-\alpha\pi}{\omega}}\right) + V_{C_1} e^{\frac{-2\alpha\pi}{\omega}}$

$$V_{C_1} = \frac{E(1 + e^{\frac{-\alpha\pi}{\omega}})}{1 - e^{\frac{-2\alpha\pi}{\omega}}} = \frac{E}{1 - e^{\frac{-\alpha\pi}{\omega}}} \qquad V_{C_1} > E$$

Centre-tapped supply inverter:

single-phase half-bridge:



T_1 is on, C is charged > E/2, then T_1 is off before i try to reverse.

when T_2 is on, C is charged in reverse direction.

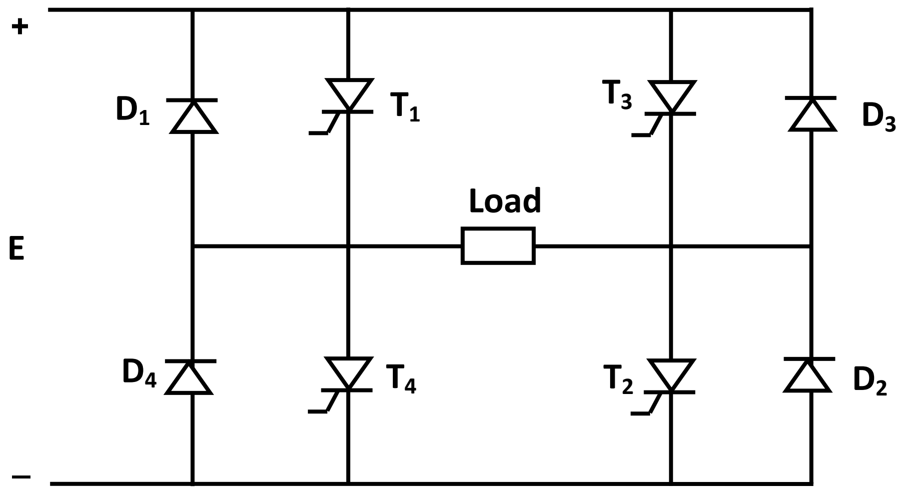
when T_1 is on:

$$i = \frac{E - 2V_{C_2}}{2\omega L} e^{-\alpha t} \sin \omega t$$

when T_2 is on:

$$i = -\frac{E + 2V_{C_1}}{2\omega L} e^{-\alpha t} \sin \omega t$$

Bridge inverters:

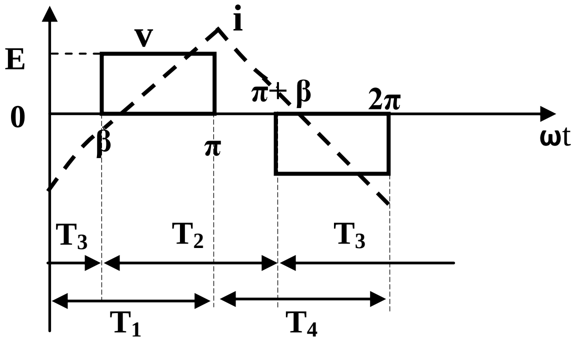


At $\omega t=0$ T_1 ON

$\omega t=\beta$ T_2 ON

$\omega t=\pi$ T_4 ON

$\omega t=\pi+\beta$ T_3 ON



$$V_{rms} = E\sqrt{1-\frac{\beta}{\pi}}$$

For R-L load:

(1) $0 < \omega t < \beta$, v=0 , D_1 & T_3 on.

$$i = i_o e^{\frac{-R}{L}t} = i_o e^{\frac{-\omega t}{Q}} \quad , \quad Q = \frac{\omega L}{R} \quad , \quad i_o < 0$$

(2) $\beta < \omega t < \pi$, v=E , T_1 & T_2 <u>or</u> D_1+D_2 on.

$$i = \frac{E}{R} + \left(i_\beta - \frac{E}{R}\right) e^{\frac{-(\omega t-\beta)}{Q}} \quad , \quad i_\beta < 0$$

(3) $\pi < \omega t < \pi+\beta$, v=0 , D_4 & T_2 on.

$$i = -i_o e^{\frac{-(\omega t-\pi)}{Q}}$$

(4) $\pi+\beta < \omega t < 2\pi$, v=-E , T_3 & T_4 <u>or</u> D_4+D_3 on.

$$i = -\frac{E}{R} - \left(i_\beta - \frac{E}{R}\right) e^{\frac{-(\omega t-\pi-\beta)}{Q}}$$

$$i = -\frac{E}{R} - \left(i_\beta - \frac{E}{R}\right) e^{\frac{-\left(t - \frac{\pi}{\omega} - \frac{\beta}{\omega}\right)}{\tau}}$$

where $\tau = \frac{L}{R}$

At $\omega t = \beta$ $i = i_\beta$

$$i_\beta = i_o e^{\frac{-\beta}{Q}} \text{ -------------------------------(1)}$$

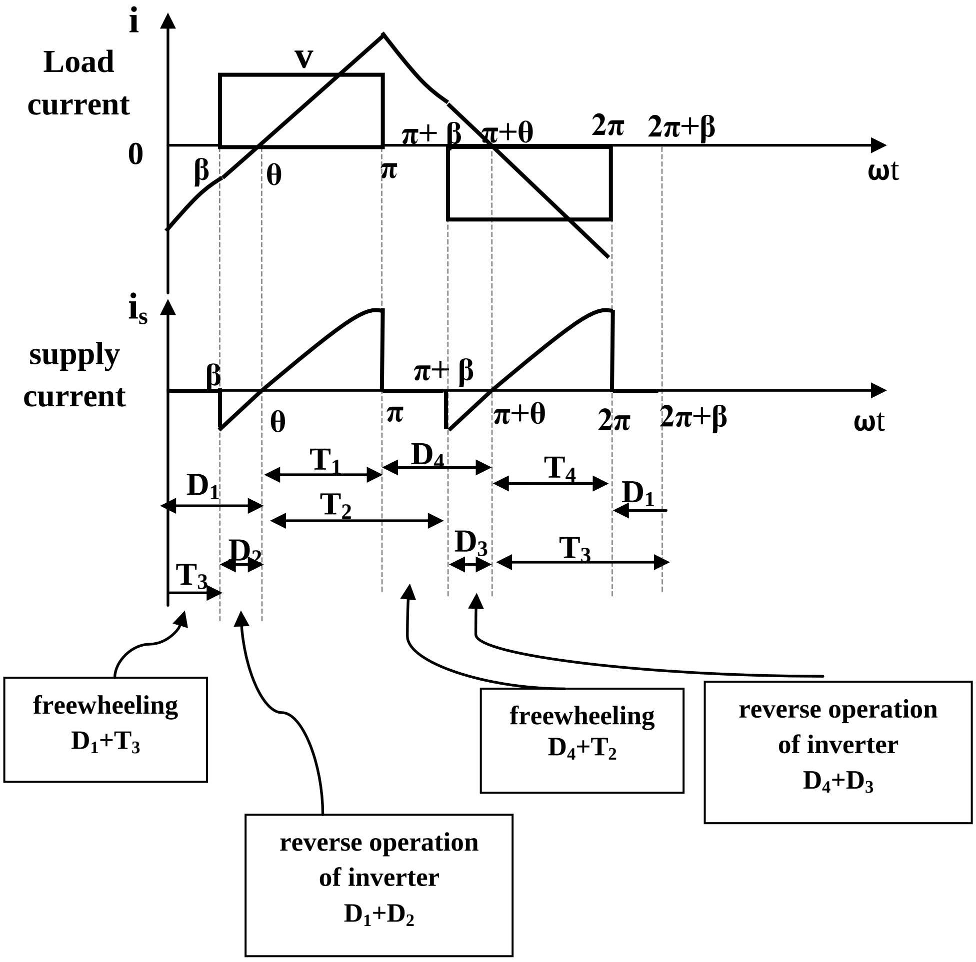
At $\omega t = \pi$ $i = -i_o$

$$-i_o = \frac{E}{R} + \left(i_\beta - \frac{E}{R}\right) e^{\frac{-(\pi-\beta)}{Q}} \text{ ---------------------(2)}$$

Solving (1) & (2) gives:

$$i_\beta = -\frac{E}{R} * \frac{e^{\frac{-\beta}{Q}} - e^{\frac{-\pi}{Q}}}{1 + e^{-\frac{\pi}{Q}}}$$

$$i_o = -\frac{E}{R} * \frac{1 - e^{\frac{-(\pi-\beta)}{Q}}}{1 + e^{\frac{-\pi}{Q}}}$$



Load commutation of the half-bridge inverter:

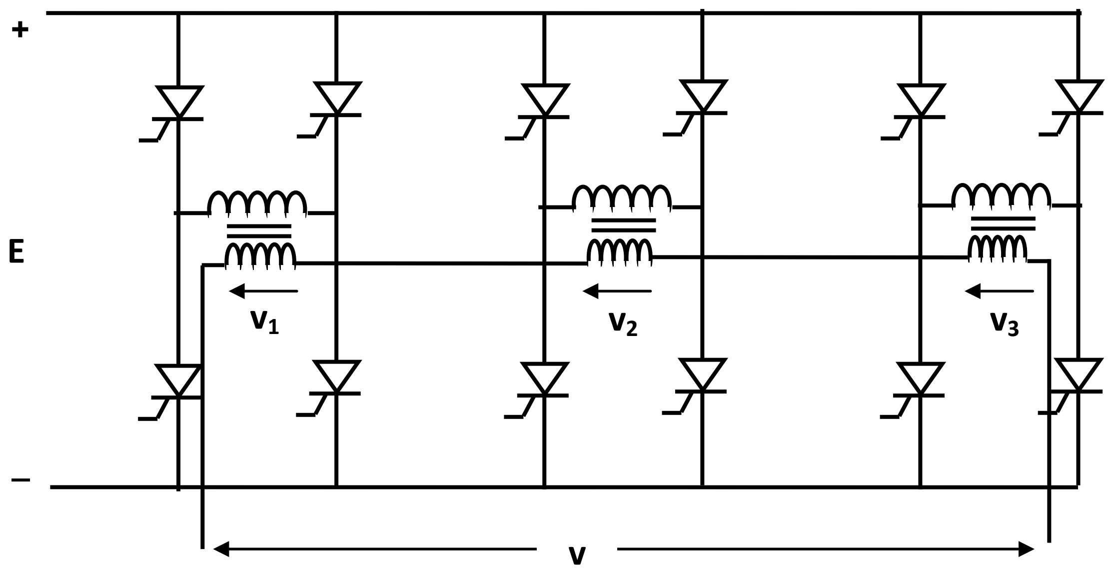
Let the current i goes to zero at time $t=t_x$ during the first half-cycle.

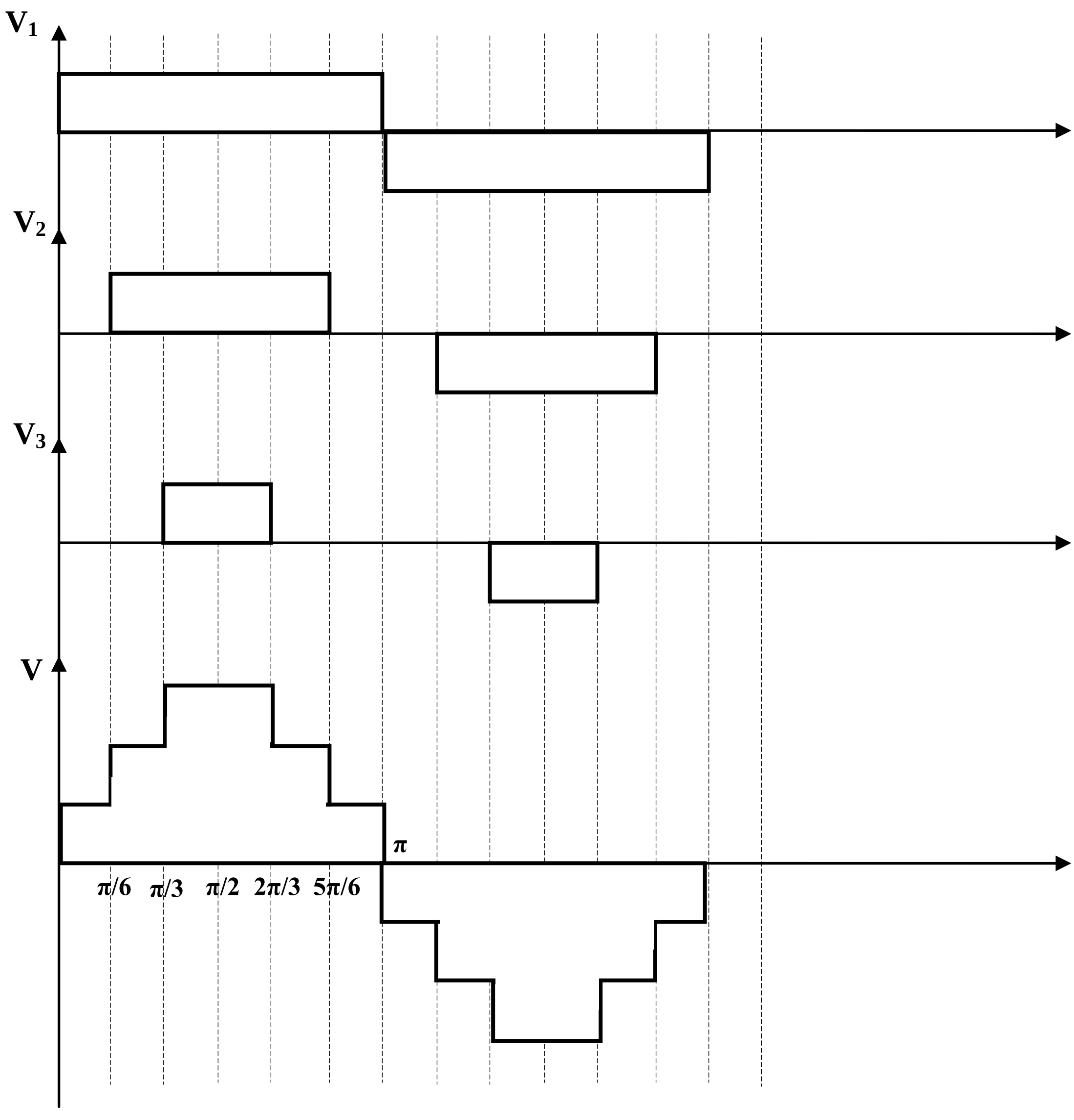
Let t_{off} is the turn-off time of thyristors T_1 and T_2.

$$t_q = \frac{T}{2} - t_x > t_{off} \quad sec$$

When the gating signal is removed from T_1 and T_2 is turned on at $t=T/2$, T_1 will not conduct. Then the inverter is load-commutate. If $t_q < t_{off}$ then the thyristors must be forced-commutated.

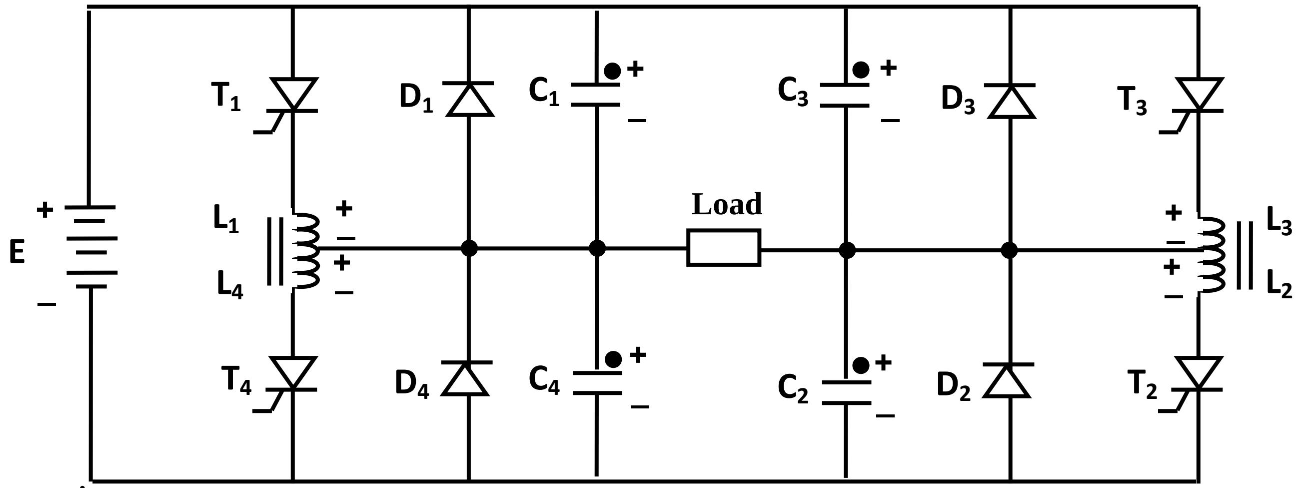
Sinewave single-phase inverters:

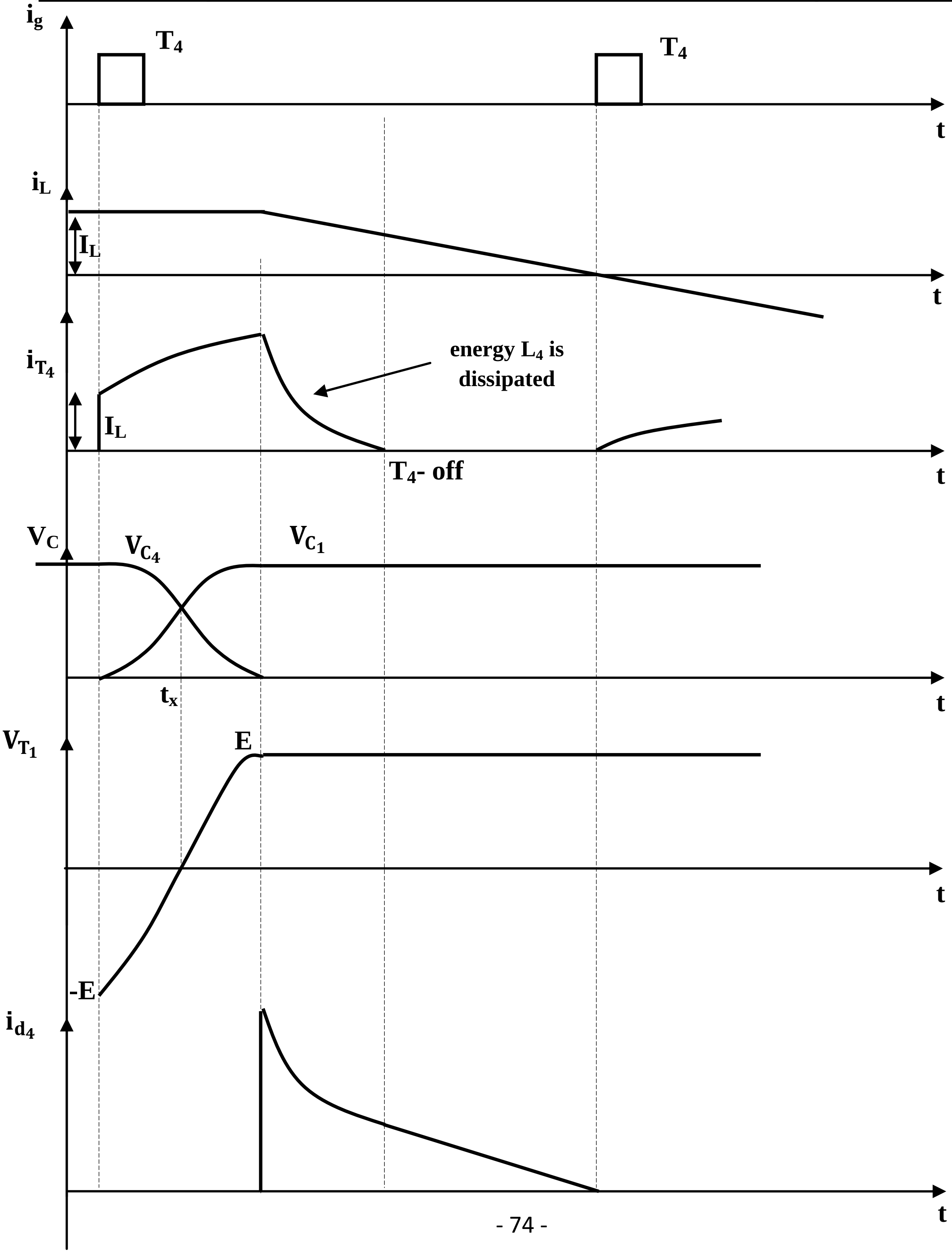




The commutation circuit is not shown

McMurray-Bedford bridge inverter:





The operation of this circuit can be summarized as follows:

1- when T_1 and T_2 are fired the current will flow through T_1, L_1, the load, L_2, and T_2.

2- The firing of T_3 will commutate T_2 and T_4 will commutate T_1. This commutation is called complete commutation.

3- This commutation process can be explained as follows:

a- The load is assumed to be highly inductive and the voltage drop across L_1 is zero. $V_{C_1} = 0, V_{T_1} = 0$.

b- When T_4 is fired, the voltage $V_{T_4} = 0$ and the voltage $V_{C_4} = E$ is completely applied to L_4. This will induce an equal voltage on L_1.

c- This will apply a reverse voltage (E) on the thyristor T_1, thus it will be turned-off.

4- The current through T_1 is reduced to zero.

5- The current through L_4 is equal to the current through L_1 because $L_1 = L_4$.

6- C_4 is discharged and C_1 is charged. The charging current is flowing through the load and through L_4 & T_4. The load current remains constant during this time (charging and discharging) due to the high load inductance.

7- Due to C_4 discharging the current through L_4 & T_4 is increased. The voltage on L_4 is reduced and this will reduced the induced voltage on L_1.

8- The voltage across T_1 is equal to the net voltage in the closed circuit T_1, L_1 & C_1, i.e., $V_{T_1} = V_{C_1} - V_{L_1}$.

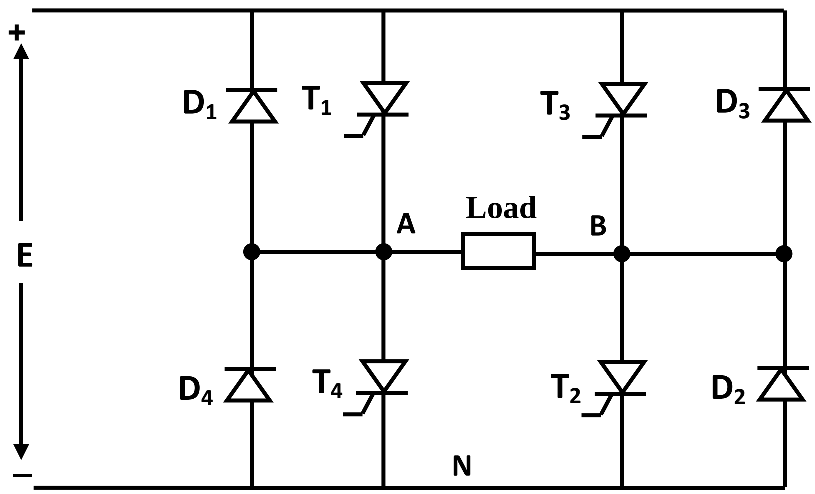
9- At $t=t_x$ (when $V_{T_1} = 0$, i.e., $V_{C_1} = V_{L_1}$), T_1 will be forward biased and thus it must be turned-off before t_x.

10- At the end of C_1 charging, $V_{C_1} = E$. The energy stored in the load inductance is returned back to the supply through D_3 & D_4 and the load current is reduced to zero. The thyristor T_4 must be fired again to carry the negative load current.

11- The energy stored in L_4 is dissipated through T_4 & D_4 & L_4.The energy stored in L_1, L_2, L_3 and L_4 increases with load current and output frequency. This dissipated energy reduces the inverter efficiency and this energy is called the trapped energy.

<u>Pulse width modulation inverters:</u>

The output voltage of an inverter can be changed by using pulse width modulation technique and it is also possible to reduce the harmonic contents through the use of modified sinusoidal pulse-width modulation.



Let T_1 is fired at:

$\omega t=0,\ \alpha_2,\ \pi-\alpha_1,\ \pi+\alpha_1,\ 2\pi-\alpha_2$

and T_4 is fired at:

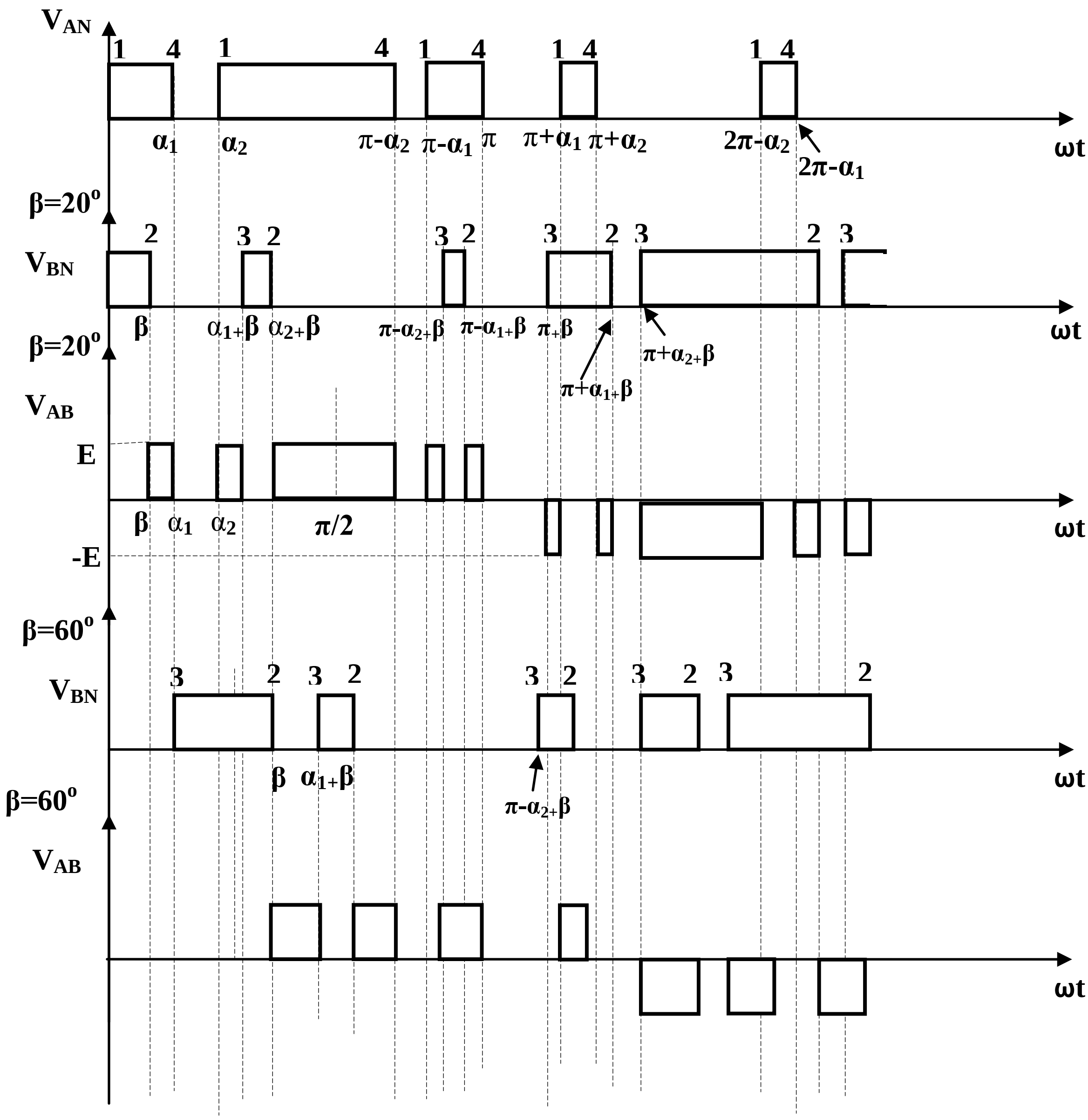
$\omega t=\alpha_1,\ \pi-\alpha_2,\ \pi,\ \pi+\alpha_2,\ 2\pi-\alpha_1$

This firing of T_2 is delayed by an angle β from T_1 and similarly for T_3 and T_4.

Fourier series analysis:

$$V_o = \frac{a_o}{2} + \sum_{n=1}^{\infty}(a_n \sin n\omega t + b_n \cos n\omega t)$$

$$\frac{a_o}{2} = \frac{1}{2\pi}\int_0^{2\pi} V_o\, d\omega t = 0$$



1-2=β

& 3-4 =β

$$a_n = \frac{1}{\pi}\int_0^{2\pi} V_o \sin n\omega t \, d\omega t = \frac{2}{\pi}\int_0^{\pi} V_o \sin n\omega t \, d\omega t$$

$$= \frac{4}{\pi} E\left[\int_{\beta}^{\alpha_1} \sin n\omega t \, d\omega t + \int_{\alpha_2}^{\beta+\alpha_1} \sin n\omega t \, d\omega t + \int_{\beta+\alpha_2}^{\frac{\pi}{2}} \sin n\omega t \, d\omega t\right]$$

$$= \frac{4}{\pi}\frac{E}{n}[\cos(n\beta) - \cos(n\alpha_1) + \cos(n\alpha_2) - \cos(n\beta + n\alpha_1) + \cos(n\beta + n\alpha_2)]$$

if $\beta=0$

$$a_n = \frac{4}{\pi}\frac{E}{n}[1 - 2\cos n\alpha_1 + 2\cos n\alpha_2]$$

Let $a_3=a_5=0$

$$1 - 2\cos(3\alpha_1) + 2\cos(3\alpha_2) = 0$$

$$1 - 2\cos(5\alpha_1) + 2\cos(5\alpha_2) = 0$$

$$\therefore\ \alpha_2 = 33.3^o \ \ \& \ \ \alpha_1 = 23.62^o$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} V_o \cos(n\omega t)\, d\omega t = 0 \ \ \text{[No cosine harmonic in the output]}$$

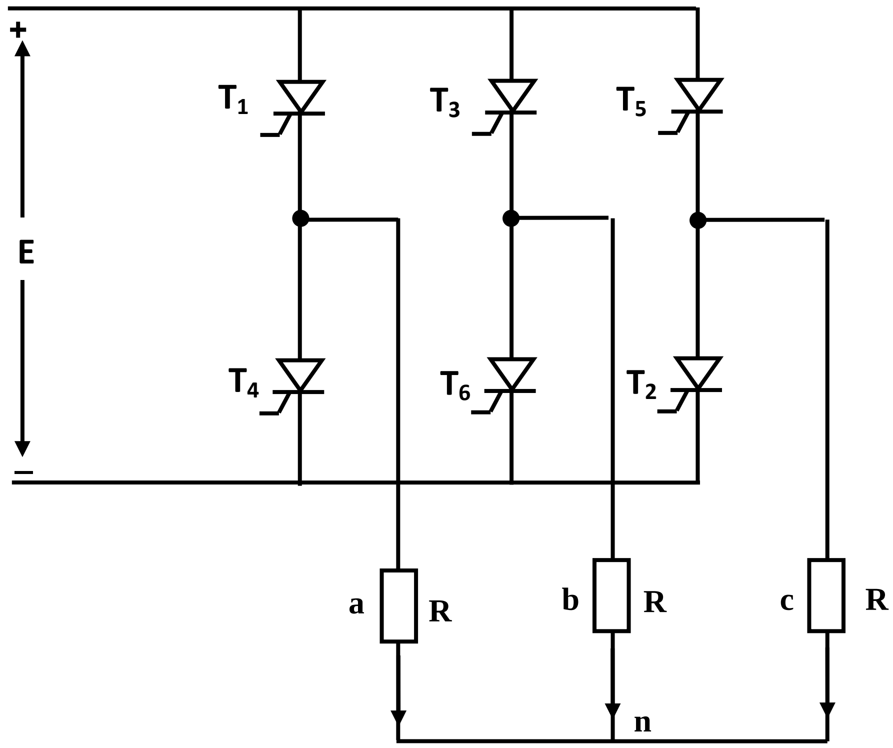
$$v_1 = \frac{4}{\pi}E[\cos(\beta) - \cos(\alpha_1) + \cos(\alpha_2) - \cos(\beta + \alpha_1) + \cos(\beta + \alpha_2)]\sin(\omega t)$$

$v_3=0$, $v_5=0$

$$v_7 = \frac{4}{\pi}\frac{E}{7}[\cos(7\beta) - \cos(7\alpha_1) + \cos(7\alpha_2) - \cos(7\beta + 7\alpha_1) + \cos(7\beta + 7\alpha_2)]\sin(7\omega t)$$

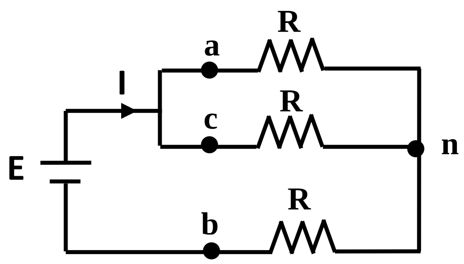
etc.

Three-phase inverters:



There are three modes of operation in a half-cycle:

1- During mode "1" for $0 \leq \omega t < \pi/3$, thyristors 1,6, and 5 are on:

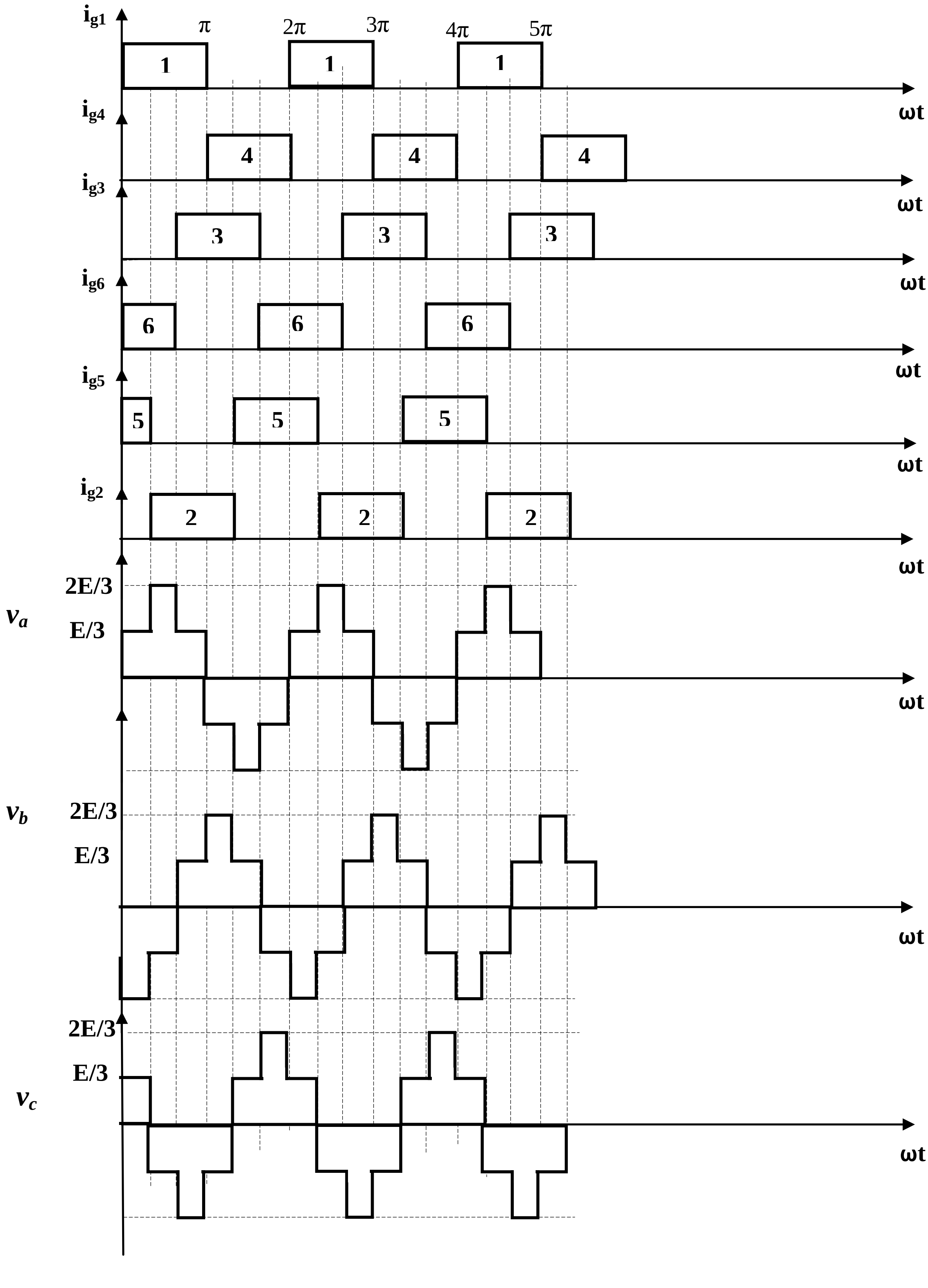


$$R_{eq} = R + \frac{R}{2} = \frac{3}{2}R$$

$$\therefore I = \frac{E}{\frac{3}{2}R} = \frac{2E}{3R}$$

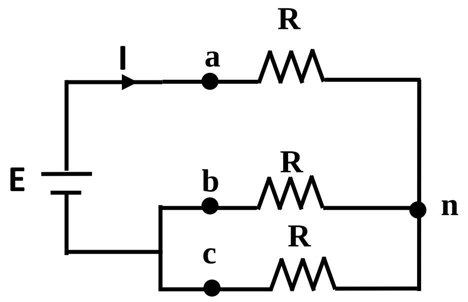
$$v_{an} = I\frac{R}{2} = \frac{E}{3} = v_{cn}$$

$$v_{bn} = -IR = -\frac{2E}{3}$$



i_{g1}
π
2π
3π
4π
5π
1
1
1
ωt
i_{g4}
4
4
4
ωt
i_{g3}
3
3
3
ωt
i_{g6}
6
6
6
ωt
i_{g5}
5
5
5
ωt
i_{g2}
2
2
2
ωt
v_a
2E/3
E/3
ωt
v_b
2E/3
E/3
ωt
v_c
2E/3
E/3
ωt

2- During mode "2" for $\pi/3 \le \omega t < 2\pi/3$, thyristors 1,6, and 2 are on:



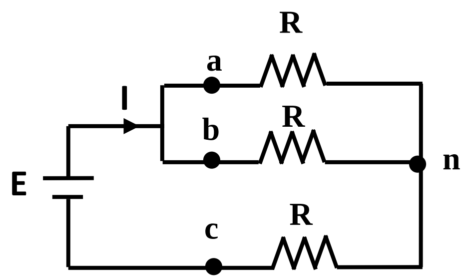
$$R_{eq} = R + \frac{R}{2} = \frac{3}{2}R$$

$$\therefore I = \frac{E}{\frac{3}{2}R} = \frac{2E}{3R}$$

$$v_{an} = IR = \frac{2E}{3}$$

$$v_{bn} = v_{cn} = -I * \frac{R}{2} = -\frac{E}{3}$$

3- During mode "3" for $2\pi/3 \le \omega t < \pi$, thyristors 1,3, and 2 are on:



$$R_{eq} = R + \frac{R}{2} = \frac{3}{2}R$$

$$\therefore I = \frac{E}{\frac{3}{2}R} = \frac{2E}{3R}$$

$$v_{an} = v_{bn} = I\frac{R}{2} = \frac{E}{3}$$

$$v_{cn} = -IR = -\frac{2E}{3}$$

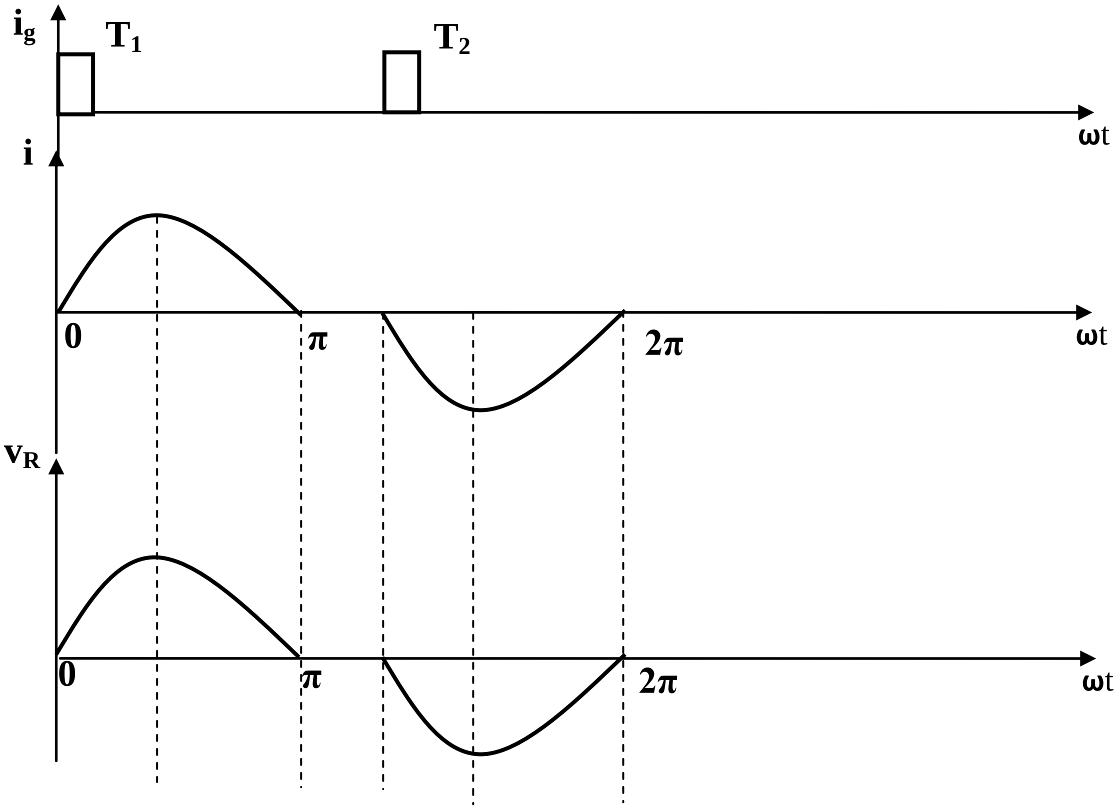
<u>Example:</u> A series inverter is connected to 100 V dc supply. The RLC load of the inverter has the following values: R=1.5 Ω, L=20μH, and C=10μF. The turn-off time of the used thyristors is 10μsec. Calculate:

a- The maximum frequency that this inverter can give.

b- The voltage range across the capacitor.

c- Maximum current.

<u>Solution:</u>



(a) $f_m = \frac{1}{t_{1on} + t_{q1} + t_{2on} + t_{q2}} = \frac{1}{T}$

$t_{1on} = t_{2on} = \frac{\pi}{\omega}, \qquad t_{q_1} = t_{q_2} = 10\mu sec$

$\omega = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\frac{1}{20*10^{-6}*10*10^{-6}} - \frac{1.5^2}{4(20*10^{-6})}}$ =59947.894 rad/sec.

$$\therefore f_m = \frac{1}{2t_{on} + 2t_q} = \frac{1}{\frac{2\pi}{\omega} + 2t_q} = \frac{1}{\frac{2\pi}{59947.894} + 2*10*10^{-6}} = 8012.129\ Hz$$

$$\alpha = \frac{R}{2L} = \frac{1.5}{2*20*10^{-6}} = 37500$$

(b) For the first period:

$$i = \frac{E - V_{C_2}}{\omega L} e^{-\alpha t} \sin \omega t$$

$$v_C = E - v_L - v_R = E - \frac{E - V_{C_2}}{\omega} e^{-\alpha t} (\omega \cos \omega t - \alpha \sin \omega t) - \frac{E - V_{C_2}}{\omega L} e^{-\alpha t} \sin \omega t$$

At $t = \frac{\pi}{\omega} \Rightarrow \ \omega t = \pi \quad v_C = V_{C_1}$

$v_C = V_{C_1} = E + (E - V_{C_2}) e^{\frac{-\pi\alpha}{\omega}}$------------------(1)

For the second period:

$v_C = -(v_R + v_C)$

$$i = -\frac{V_{C_1}}{\omega L} e^{-\alpha t} \sin \omega t$$

$$= \frac{V_{C_1}}{\omega L} R e^{-\alpha t} \sin \omega t + \frac{V_{C_1}}{\omega} e^{-\alpha t} (-\alpha \sin \omega t + \omega \cos \omega t)$$

At $t = \frac{\pi}{\omega} \Rightarrow \quad v_C = v_{C_2}$

$v_{C_2} = \frac{V_{C1}}{\omega} e^{\frac{-\pi\alpha}{\omega}} (-\omega) = -V_{C_1} e^{\frac{-\pi\alpha}{\omega}}$-------------------(2)

From (2) in (1)

$$V_{C_1} = \frac{E}{1 - e^{\frac{-\pi\alpha}{\omega}}} = \frac{100}{1 - e^{-37500*\pi/59947.894}} = 116.296\ V$$

$$V_{C_2} = -116.296 e^{-37500*\pi/59947.894} = -16.2963\ V$$

(c)

$$i = \frac{E - V_{C_2}}{\omega L} e^{-\alpha t} \sin \omega t$$

$$\frac{di}{dt} = 0 = \frac{E - V_{C_2}}{\omega L} e^{-\alpha t} (-\alpha \sin \omega t + \omega \cos \omega t) = 0$$

$$\alpha \sin \omega t = \omega \cos \omega t$$

$$\tan \omega t = \frac{\omega}{\alpha}$$

$$\tan(59947.894t) = \frac{59947.894}{37500}$$

$$\therefore \omega t = 1.01181$$

$$\therefore t = 16.878 \;\; \mu sec$$

$$\therefore i = \frac{100 - (-16.2963)}{59947.894 * 20 * 10^{-6}} e^{-37500 * 16.878 * 10^{-6}} \sin(1.01181) = 43.6693 \, A$$

<u>Sheet No.2</u>

1- A single-phase half-bridge inverter with RLC load for which R=0.8 Ω, ωL=10 Ω,1/ωC=10 Ω, E/2=25 V. The gating signals are arranged to have a rectangular voltage wave of amplitude E/2 and periodic time T.

a- Find the equation of the fundamental load current.

b- Show that the power delivered by the two sources is equal to the power delivered to the load circuit.

c- State whether this circuit will require forced commutation.

2- Repeat example-1 with 1/ωC=9.2 Ω.

3- A single-phase bridge inverter with freewheeling diodes is connected to 300 V supply. The load is an RLC series circuit in which R=0.5 Ω, ωL=10 Ω and 1/ωC=10.5 Ω. T=1 msec.

The output voltage is a rectangular wave at 300 V amplitude and periodic time T.

a- Repeat parts a and b of example-1.

b- Calculate the time available for turn-off of the thyristors with this load circuit.

4- A single-phase bridge inverter with freewheeling diodes is connected to RLC load. The gate signals of T_1 and T_4 are delayed by β=π/3 rad. from the gate signals of T_2 and T_3 respectively.

a- Draw one cycle of the required gate current waveforms for the four thyristors.

b- If the load circuit is such that i_L passes through zero value at ωt equal to:

(i) 0 and π rad.

(ii) π/3 and π+π/3 rad.

(iii) -π/3 and π-π/3 rad.

draw for each one cycle of the current waveform and indicate on it which devices are conducting in the various intervals throughout the cycle.

5- A single-phase bridge inverter with freewheeling diodes has a positive gate voltage on G_1 and G_2 between π/6 to 5π/6 and a positive gate voltage on G_3 and G_4 between 7π/6 to 11π/6 rad. At the end of each gating signals the two thyristors are commutated. Draw one cycle of the load-circuit fundamental current and voltage waveforms if the load is resistive. Indicate with current waveform which devices are conducting in the various intervals throughout the cycle.

6- A series interval is connected to 300 V supply and R=3 Ω, L=40 μH, and C=5 μF. The output frequency is to be 8000 Hz. Determine:

a- The minimum time provided for turn-off t_{qmin}.

b- The peak thyristor current.

c- The power output.

d- The rms thyristor current.

e- Sketch the waveforms of the output current.

7- Repeat example-6 for $R=1\ \Omega$.

8- A single-phase half-bridge inverter with freewheeling diodes has a resistive load of $R=2.4\ \Omega$ and the dc input voltage is $E=48$ V. The output voltage is a rectangular wave. Determine:

a- The rms output voltage at the fundamental frequency.

b- The output power.

c- The average and peak currents of each thyristor.

d- The reverse blocking voltage of each thyristor.

9- Repeat example-8 for a single-phase bridge inverter with freewheeling diodes.

10- The inverter in example-9 has an RLC load with $R=10\ \Omega$, $L=31.5$ mH, and $C=112\ \mu$F. The inverter frequency , $f_o=60$ Hz and the dc input voltage $E=220$ V.

a- Express the instantaneous load current in Fourier series.

Calculate the,

b- The rms load current at fundamental frequency.

c- Output power.

d- Average current of the dc supply.

e- The rms and peak current of each thyristor.

f- Draw the waveform of fundamental load current and show the conduction intervals of the thyristors and diodes. Calculate the conduction time of the thyristors and diodes.
