

# Chapter two

## AC/DC Converter

### **Introduction:**

It is called also phase controlled rectifier. It is classified according to the number of thyristors in the circuit and the type of the supply (single phase or three phase).

The main types of AC/DC converters are:

#### **1. Half-wave phase controlled rectifier:**

In this circuit only one thyristor is used. The positive cycle of the input supply can be controlled only, so the output voltage will be part of the positive cycle only. Hence it is called half wave converter.

#### **2- Single phase full wave phase controlled converter**

Two thyristors are used with centre-tap transformer.

#### **3- Single phase full-wave controlled converter using the bridge principle**

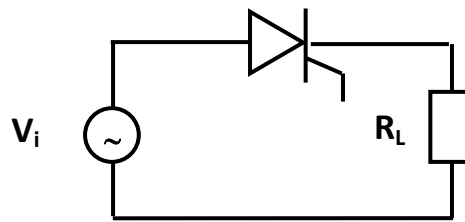
In this circuit, if two thyristors and two diodes in the form of bridge are used then it is called half-controlled converter. If four thyristors are used the bridge is called full- controlled converter.

#### **4- Three-phase bridge converter**

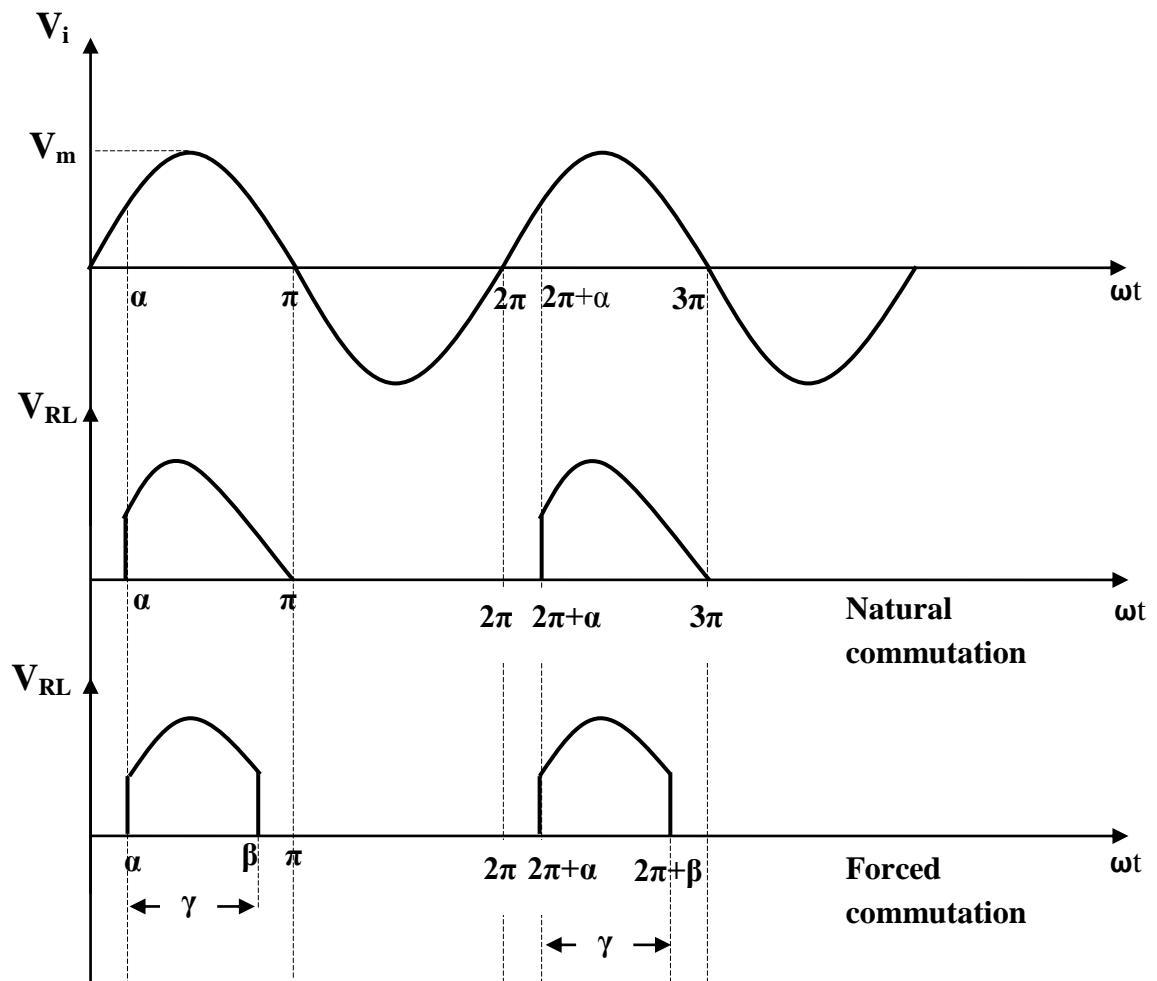
In this bridge, if the circuit consists of three thyristors and three diodes it is called half-controlled converter, but if six thyristors are used, it is called full-controlled converter. The difference between these two types is in the characteristics of them which will be shown later.

The following figures show the construction and characteristics of the main types of AC/DC converters.

## Half-wave phase controlled rectifier:



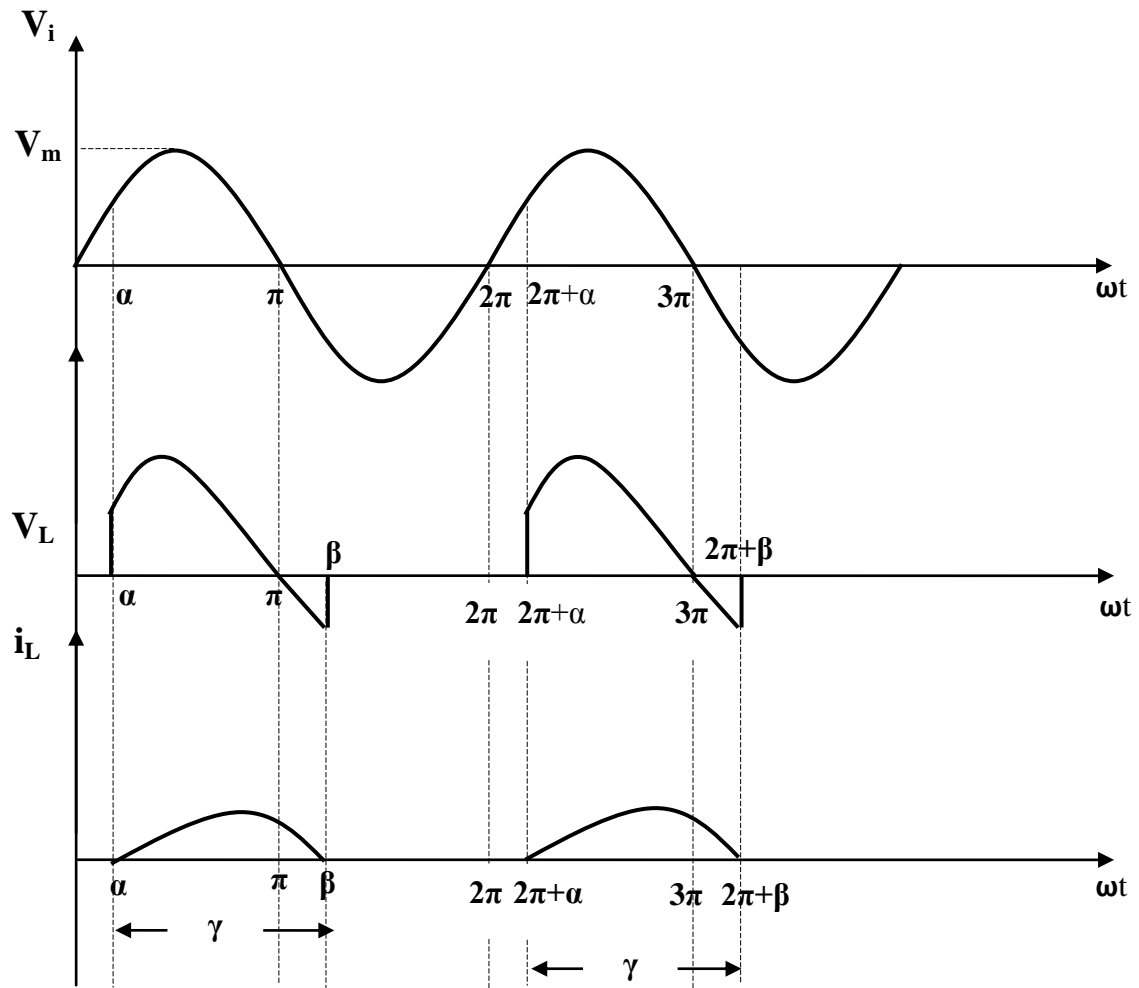
a- with R- load:



$\gamma$ : conduction angle

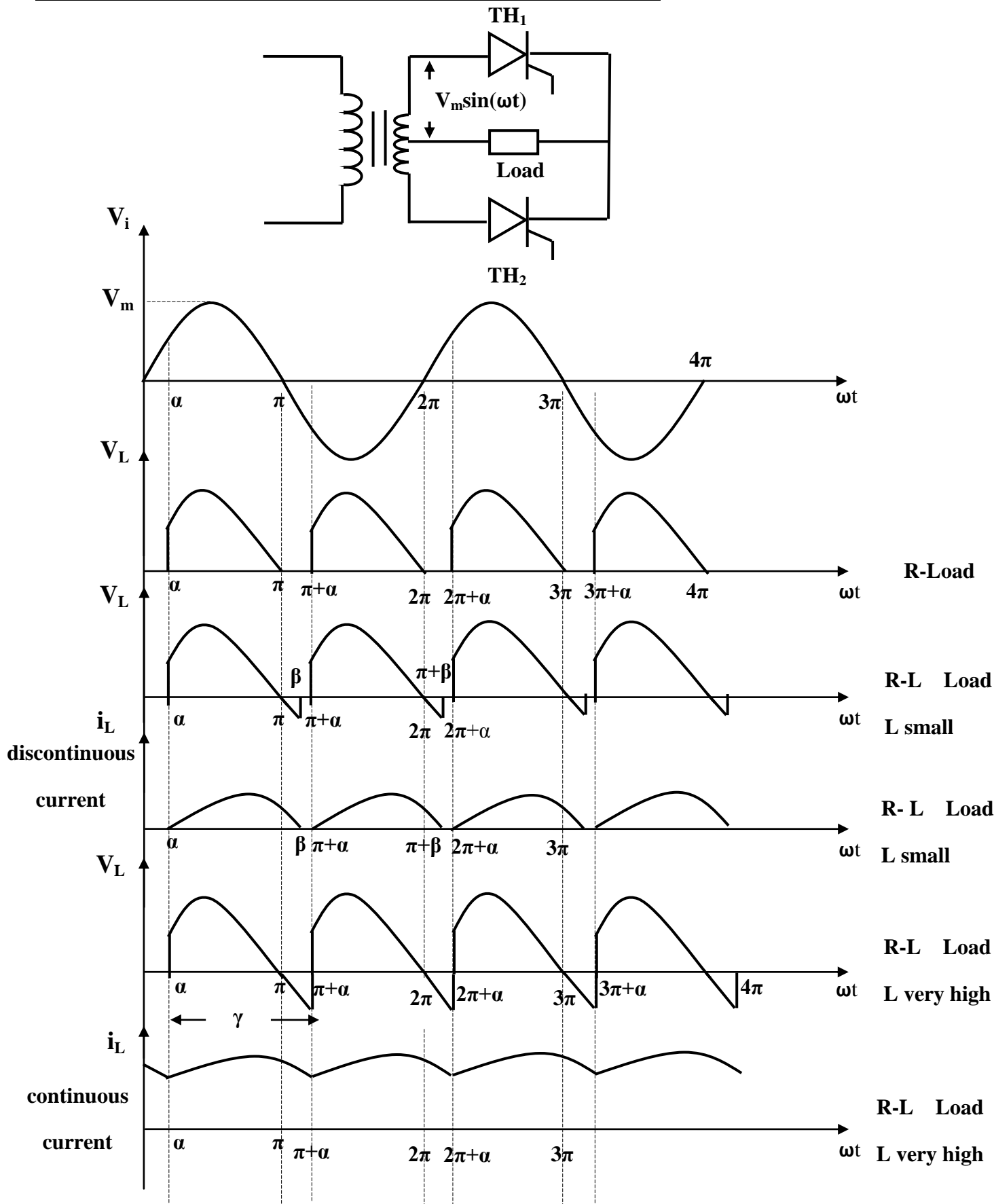
$$V_{RL(av)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t = \frac{V_m}{2\pi} (1 + \cos(\alpha)) \quad \text{for R load only}$$

b- with R-L load:



$\gamma$ : conduction angle

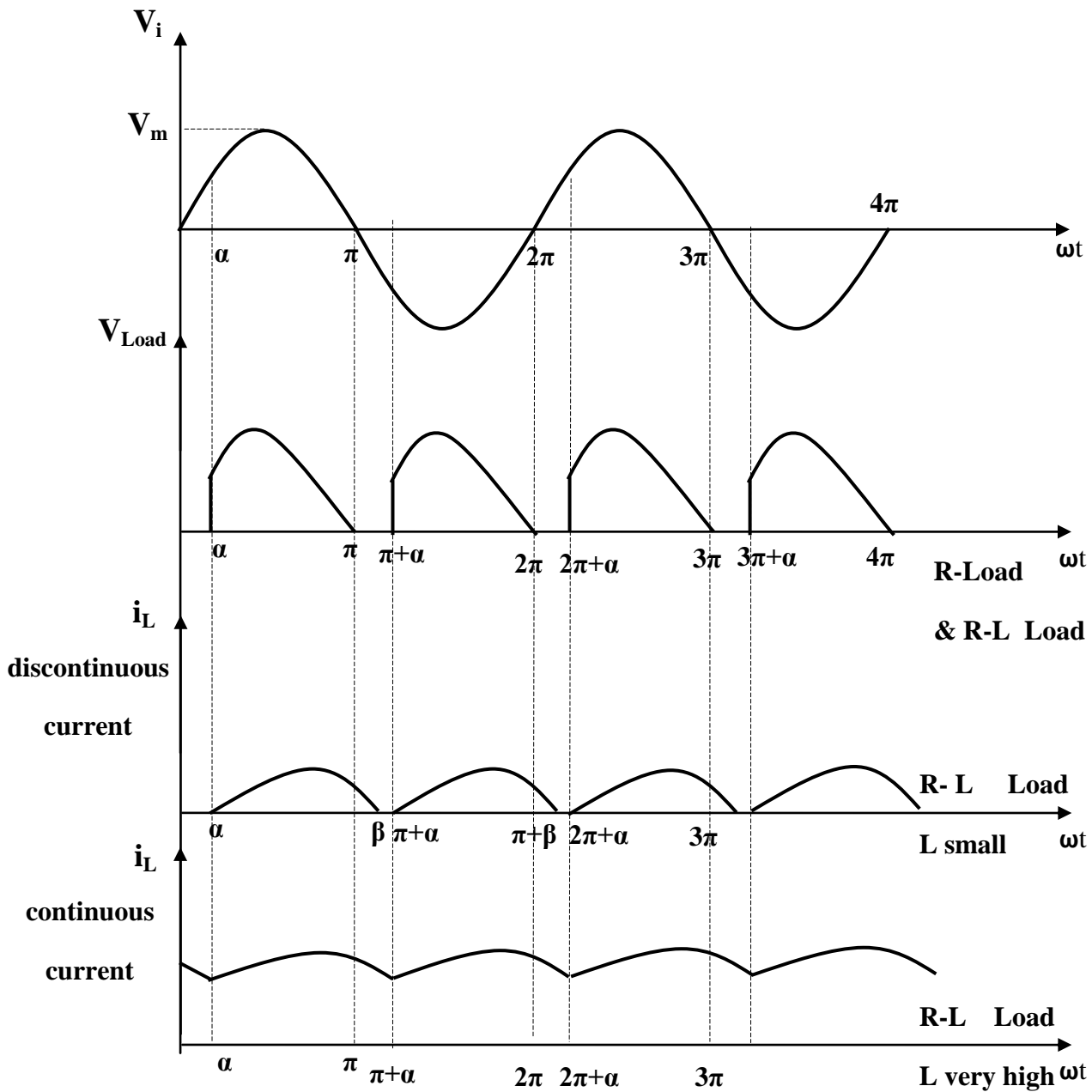
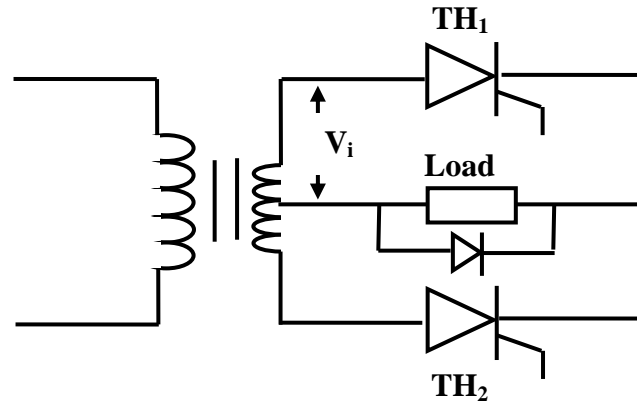
## Single phase full-wave phase-controlled converter:



$$V_{L(av)} = \frac{V_m}{\pi} (1 + \cos(\alpha)) \quad \text{for R load only}$$

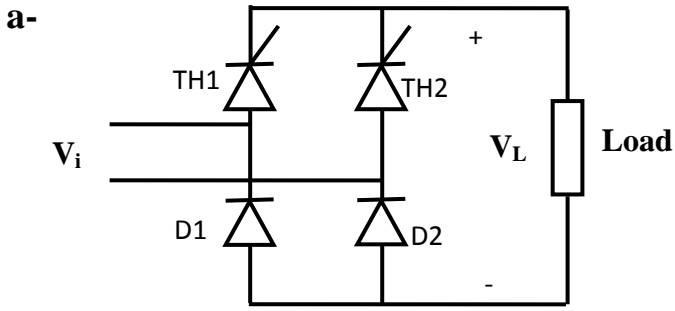
# Single phase full-wave phase-controlled converter with freewheeling diode:

diode:

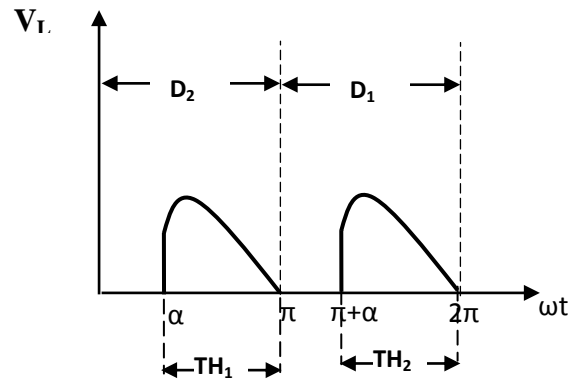


$$V_{L(av)} = \frac{V_m}{\pi} (1 + \cos(\alpha))$$

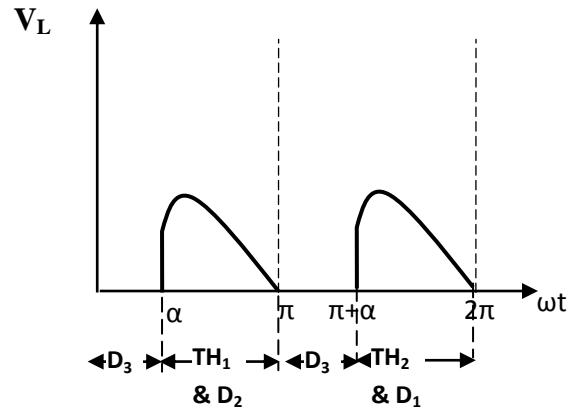
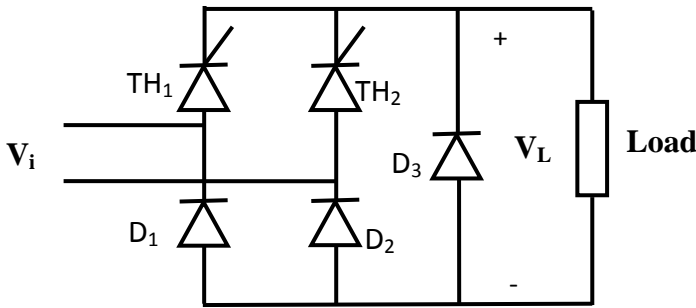
**Single phase full-wave controlled converter using the bridge principle:**



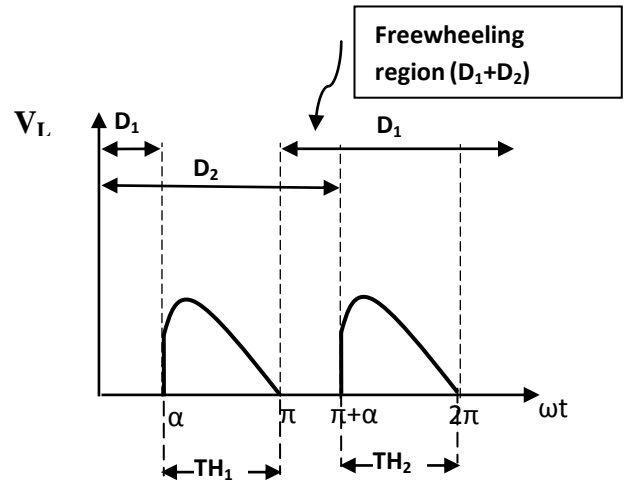
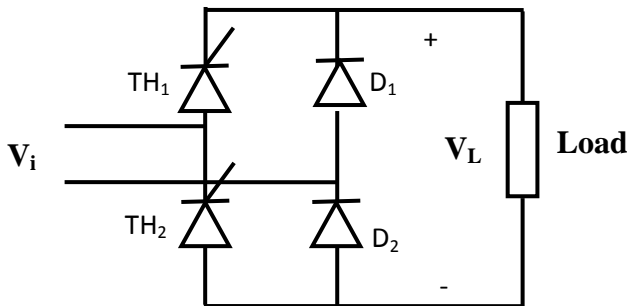
Single-phase half controlled converter



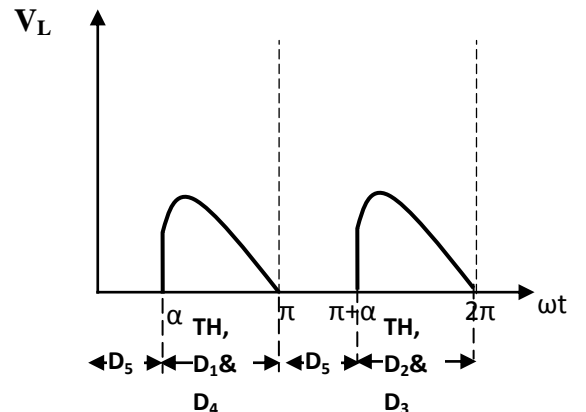
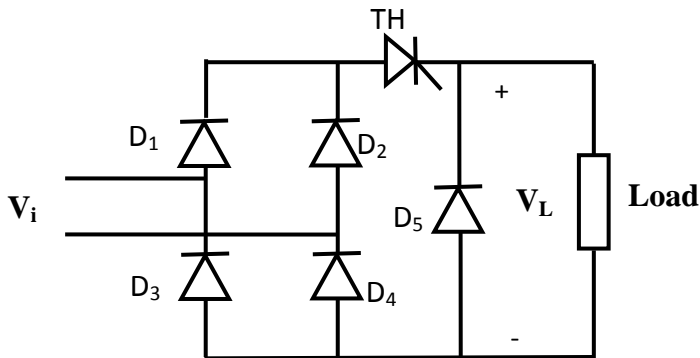
b-

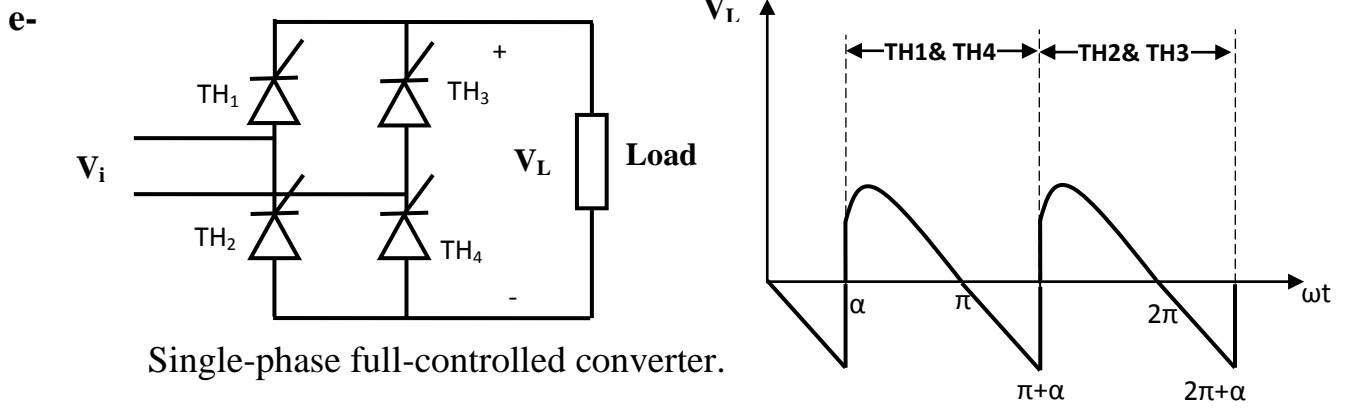


c-



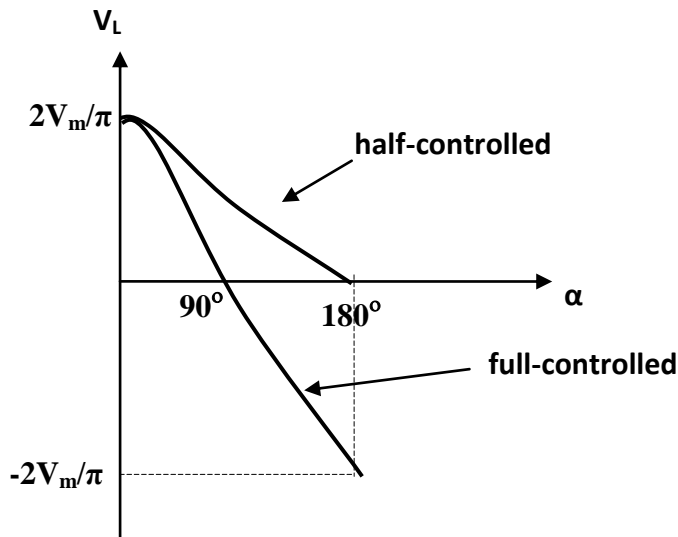
d-





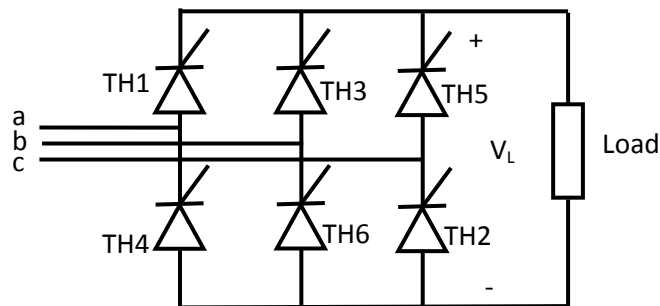
$$V_L(av) = \frac{V_m}{\pi} (1 + \cos \alpha) \quad \text{For half-controlled converter}$$

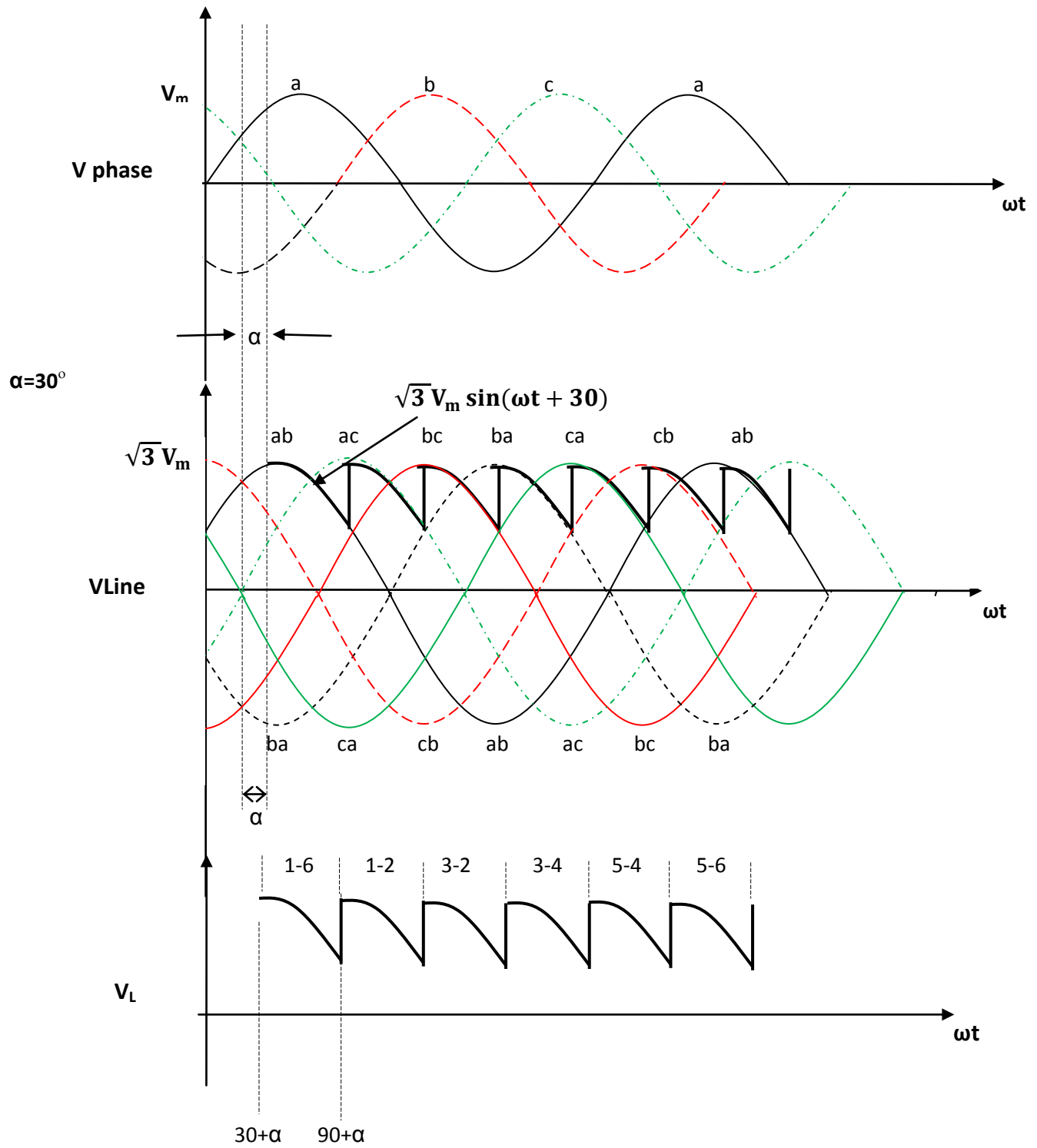
$$V_L(av) = \frac{2V_m}{\pi} \cos \alpha \quad \text{For full-controlled converter}$$



### 3. Three-phase bridge converter:

#### a-Three-phase full-controlled converter:



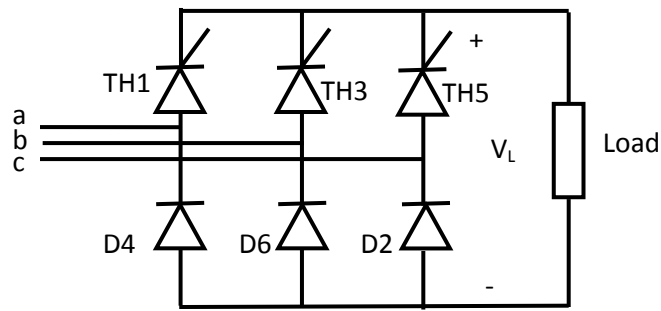


Ripple frequency=6f

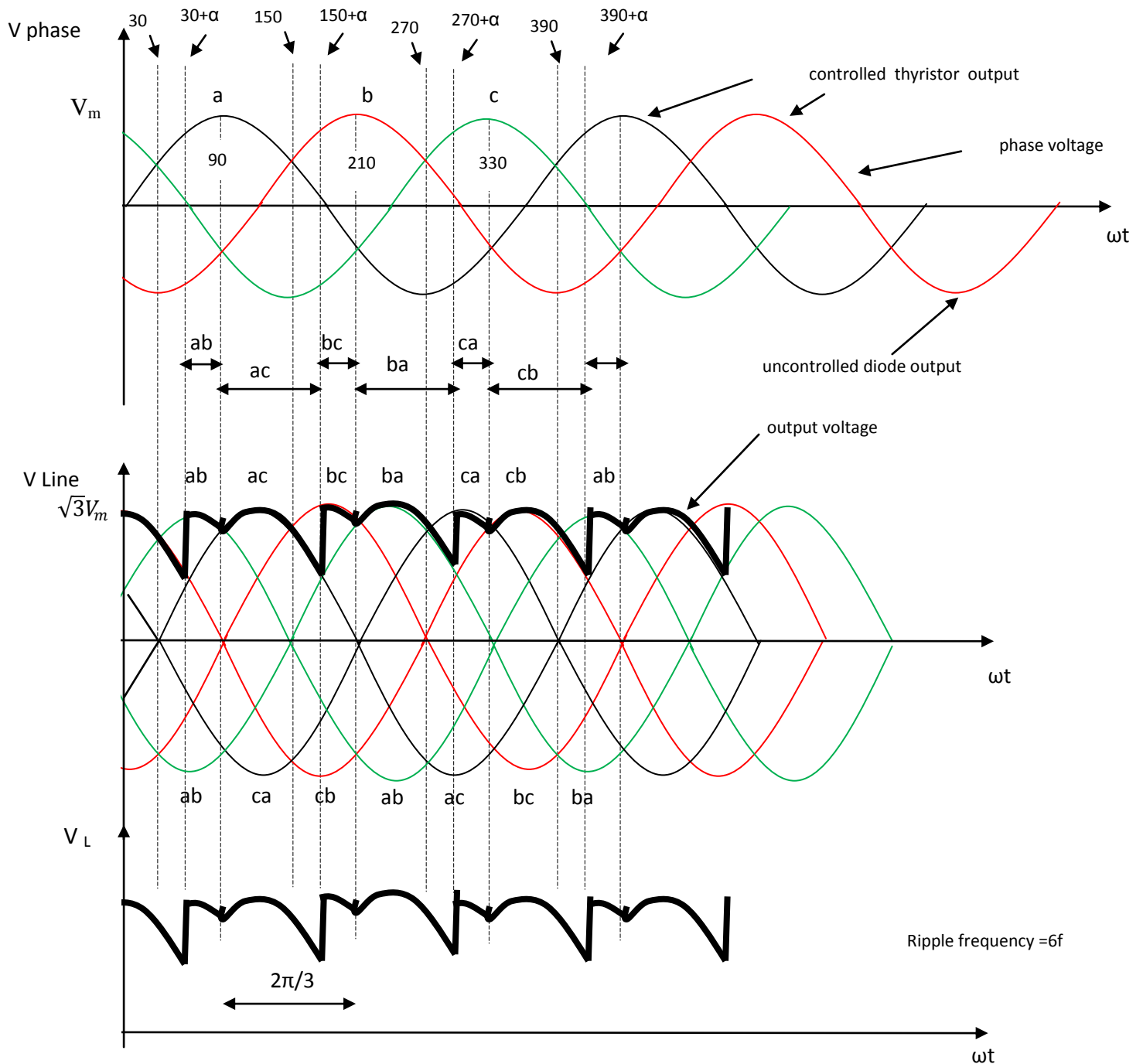
$$V_{L(av)} = \frac{1}{\pi/3} \int_{\alpha+30}^{\alpha+90} \sqrt{3} V_m \sin(\omega t + 30) d\omega t = \frac{3\sqrt{3}}{\pi} V_m \cos \alpha$$



## b-Three-phase half-controlled converter:



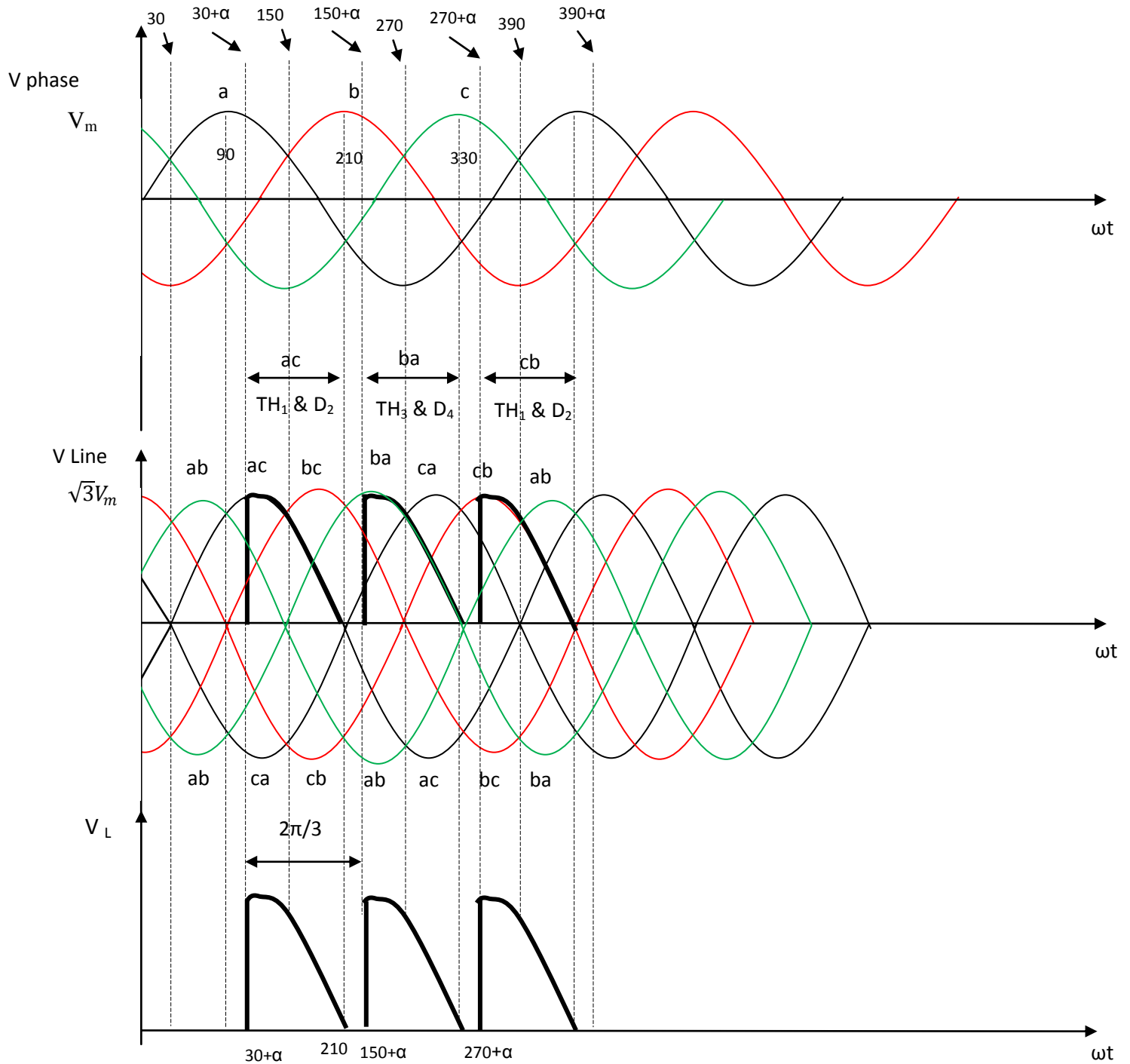
for  $\alpha < 60^\circ$



$$V_{L(av)} = \frac{1}{\frac{2\pi}{3}} \left[ \int_{30+\alpha}^{90} \sqrt{3} V_m \sin(\omega t + 30) d\omega t + \int_{90}^{150+\alpha} \sqrt{3} V_m \sin(\omega t - 30) d\omega t \right]$$

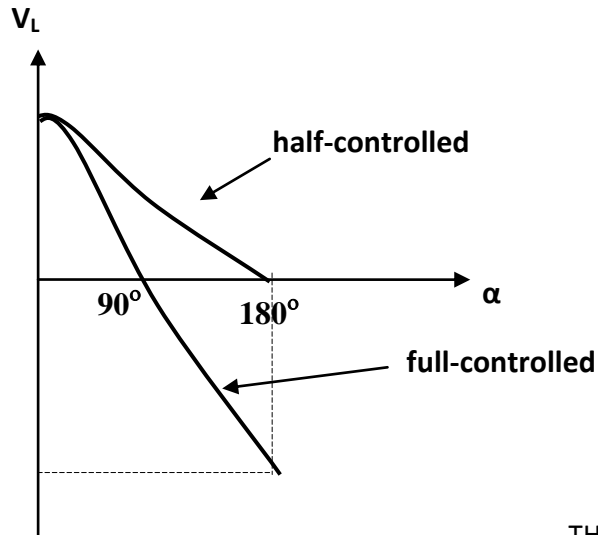
$$= \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos \alpha)$$

for  $\alpha > 60^\circ$

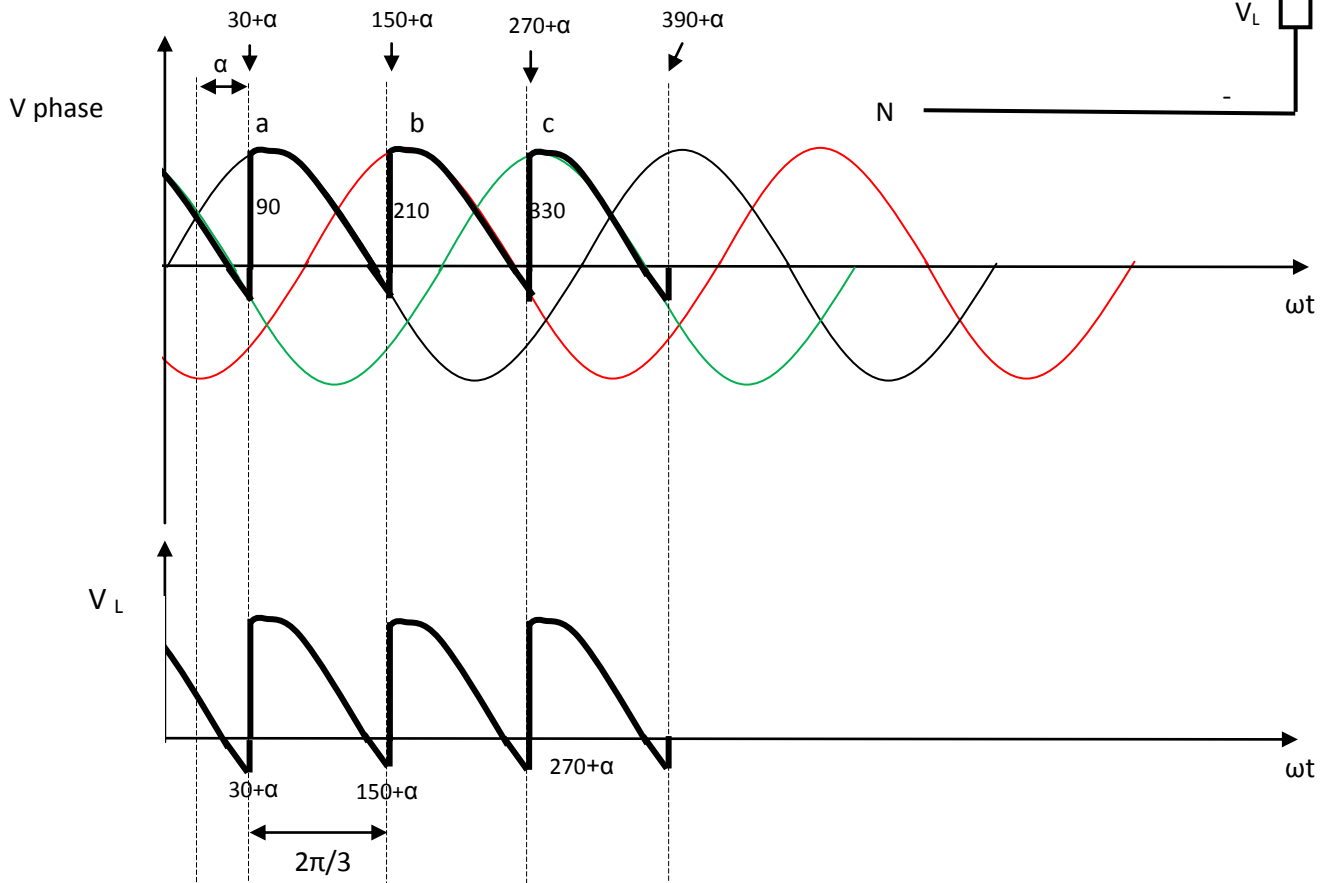
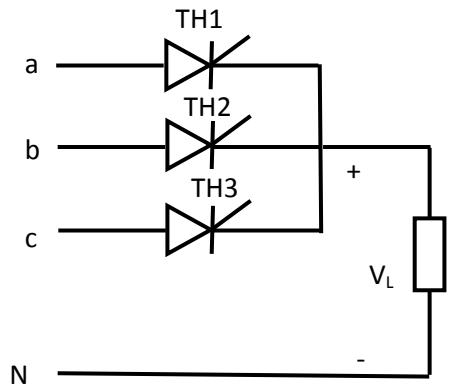


$$V_{L(av)} = \frac{1}{\frac{2\pi}{3}} \left[ \int_{30+\alpha}^{210} \sqrt{3} V_m \sin(\omega t - 30) d\omega t \right]$$

$$= \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos \alpha)$$



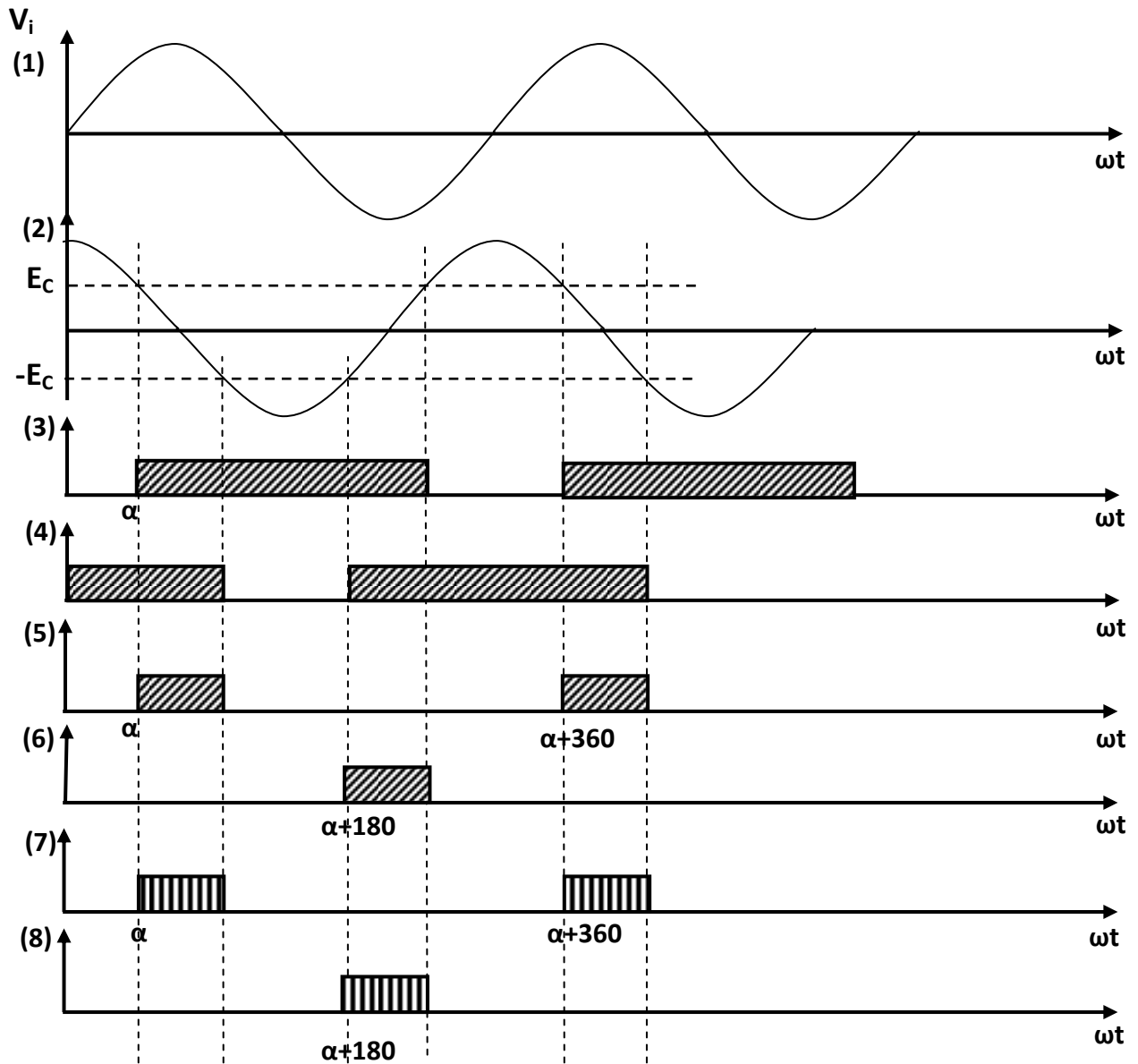
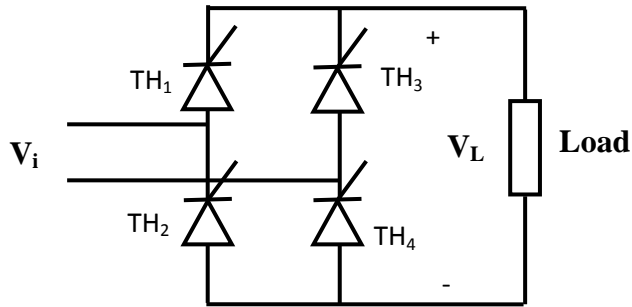
**c-Three-phase half-controlled converter:**

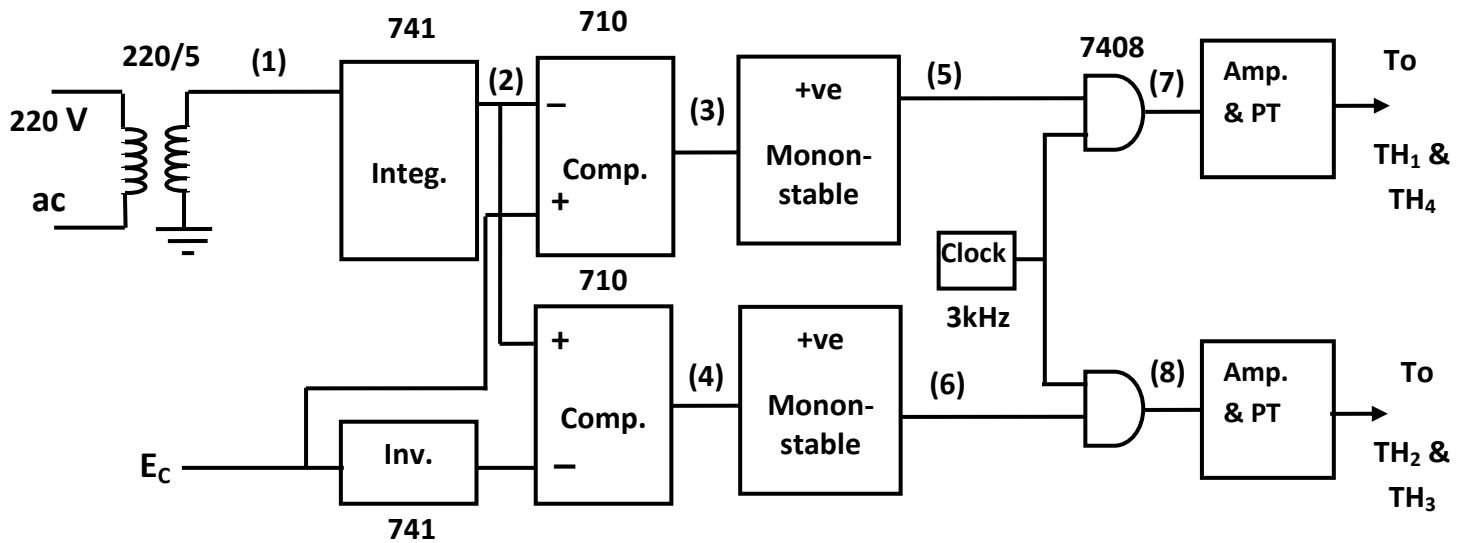


$$V_{L(av)} = \frac{1}{\frac{2\pi}{3}} \left[ \int_{30+\alpha}^{150+\alpha} V_m \sin(\omega t - 30) d\omega t \right]$$

$$= \frac{3\sqrt{3}}{2\pi} V_m (\cos \alpha)$$

**Single-phase controlled converter control circuit:**





### Performance parameters of converters:

There are different types of AC/DC converters and the performances of a converter are normally evaluated in terms of the following parameters.

The average value of the output load voltage,  $V_{dc}$

The average value of the output load current,  $I_{dc}$

The output dc power,  $P_{dc} = V_{dc} I_{dc}$

The rms value of the output voltage,  $V_{rms}$

The rms value of the output current,  $I_{rms}$

The output ac power =  $V_{rms} I_{rms}$

The efficiency (or rectification factor) of a rectifier is defined as:

$$\zeta = P_{dc} / P_{ac}$$

The output voltage is composed of two components: (1) the dc voltage, and (2) the ac component or ripple.

The effective (rms) value of the ac component of output voltage is:

$$V_{ac} = \sqrt{V_{ms}^2 - V_{dc}^2}$$

The form factor, which is a measure of the shape of output voltage, is:

$$FF = \frac{V_{rms}}{V_{dc}}$$

The ripple factor, which is a measure of the ripple content, is defined as:

$$RF = \frac{V_{ac}}{V_{dc}}$$

$$\therefore RF = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \sqrt{FF^2 - 1}$$

The transformer utilization factor is defined as:

$$TUF = \frac{P_{dc}}{V_s I_s}$$

where  $V_s$  and  $I_s$  are the rms voltage and rms current of the transformer secondary respectively.

If  $\varphi$  is the angle between the fundamental components of the input current and voltage,  $\varphi$  is called the displacement angle. The displacement factor is defined as:

$$DF = \cos(\varphi)$$

The harmonic factor of the input current is defined as:

$$HF = \left(\frac{I_s^2 - I_1^2}{I_1^2}\right)^{1/2} = \left[\left(\frac{I_s}{I_1}\right)^2 - 1\right]^{1/2}$$

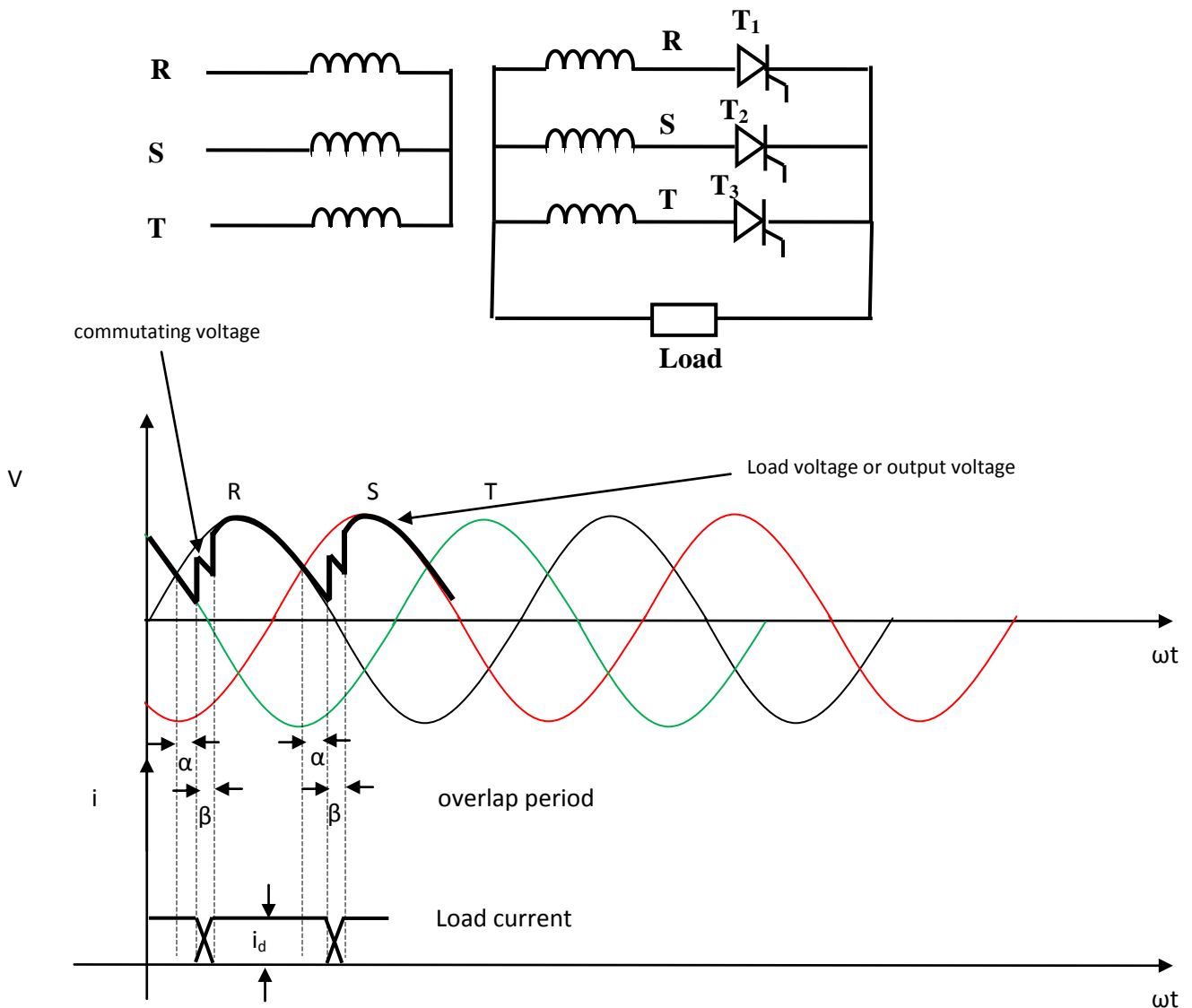
where  $I_1$  is the fundamental rms component of the input current.

The input power factor is defined as:

$$PF = \frac{V_s I_1}{V_s I_s} \cos(\varphi) = \frac{I_1}{I_s} \cos(\varphi)$$

If the input current is purely sinusoidal,  $I_1=I_s$  and the power factor PF=displacement factor, DF. An ideal rectifier should have:  $\zeta=100\%$ ,  $V_{ac}=0$ ,  $FF=1$ ,  $RF=0$ ,  $TUF=1$ ,  $HF=0$ , and  $PF=1$ .

**Overlap in AC/DC converters:**



Due to the leakage reactance of the transformer windings, the commutation cannot occur instantaneously. The outgoing thyristor continues for some time along with the incoming thyristor. This period is called overlap period. During this period two phases of the transformer get short circuited.

When the conduction of the incoming thyristor starts, the conduction of outgoing thyristor continues during the overlap period. The difference in the anode voltages of the two thyristors is called commutating voltage. This voltage circulates a short circuit current in

the transformer windings and through the thyristors. This current is called commutating current. If the firing angle  $\alpha$  is increased then the commutating voltage available will be larger. This increases the commutating current. This current opposes the current in the outgoing thyristor and aids the current in the incoming thyristor. This current will quickly bring the current in the outgoing thyristor to zero and to its full value ( $I_d$ ) in the incoming thyristor. This reduces the overlap period  $\beta$ . Due to overlap period the dc average output voltage is reduced.



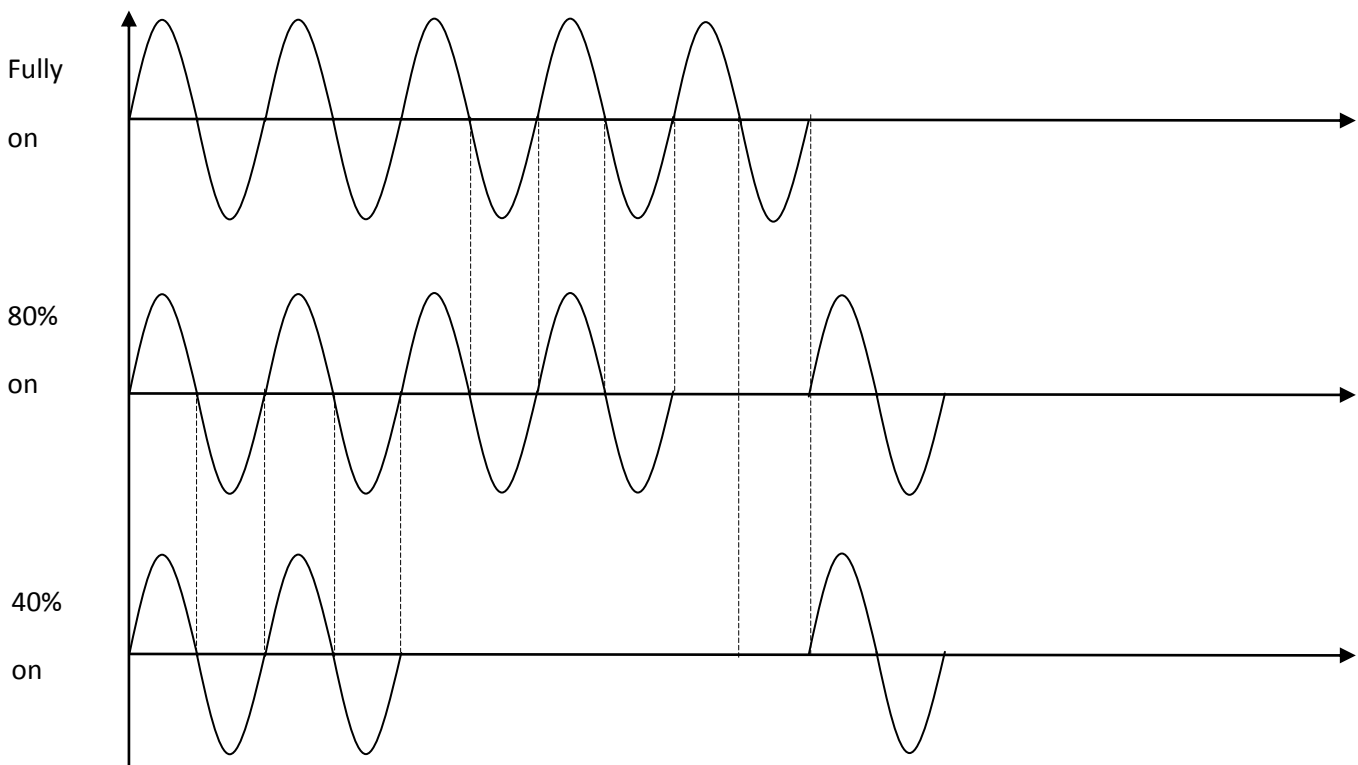
## AC voltage control:

AC voltage controllers are employed to vary the rms value of the alternating voltage applied to a load circuit by introducing thyristors between it and a constant-voltage ac source. There are two methods of control:

1- On-off control.

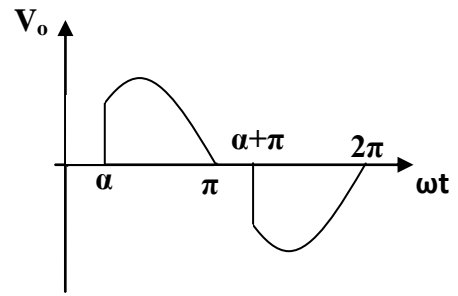
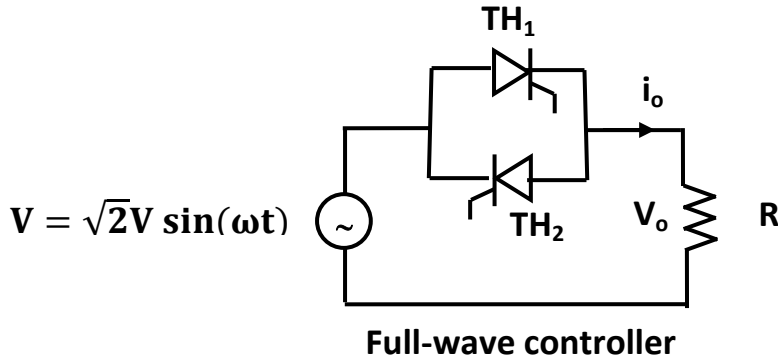
2- phase control.

In on-off control, the thyristors are employed as switches to connect the load circuit to the source for a few cycles of the source voltage and then to disconnect it for a comparable period. The thyristor thus acts as a high-speed contactor.

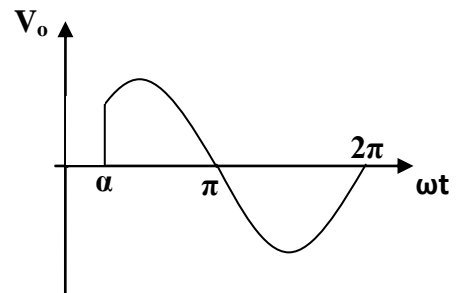
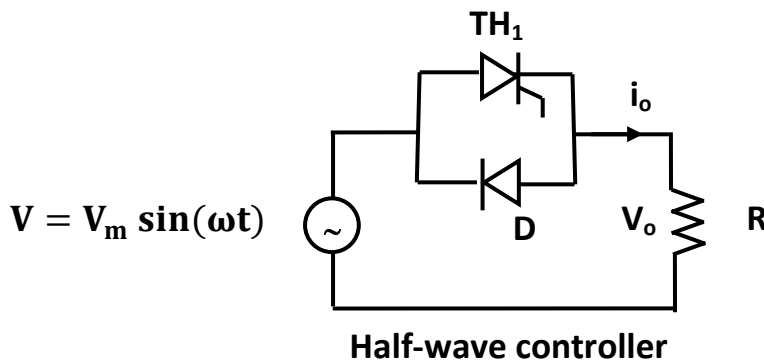


In phase control, the thyristors are employed as switches to connect the load circuit to the source for a chosen portion of each cycle of the source voltage.

## A.C. phase control:



$$V_m = \sqrt{2}V$$



For full-wave controller (resistive load):

$$V_{o(\text{rms})} = \left[ \frac{1}{\pi} \int_{\alpha}^{\pi} (V_m \sin(\omega t))^2 d\omega t \right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi} \right]^{\frac{1}{2}}$$

$$I_{o(\text{rms})} = \frac{V_o}{R} = \frac{V_m}{\sqrt{2}R} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi} \right]^{\frac{1}{2}}$$

For R-L load:

$$L \frac{di}{dt} + Ri = V_m \sin(\omega t)$$

$$i = \frac{V_m}{Z} \left[ \sin(\omega t - \phi) - \sin(\alpha - \phi) e^{-\frac{R}{L}(\omega t - \alpha)} \right]$$

$$Z = \sqrt{R^2 + \omega^2 L^2} \quad \Omega$$

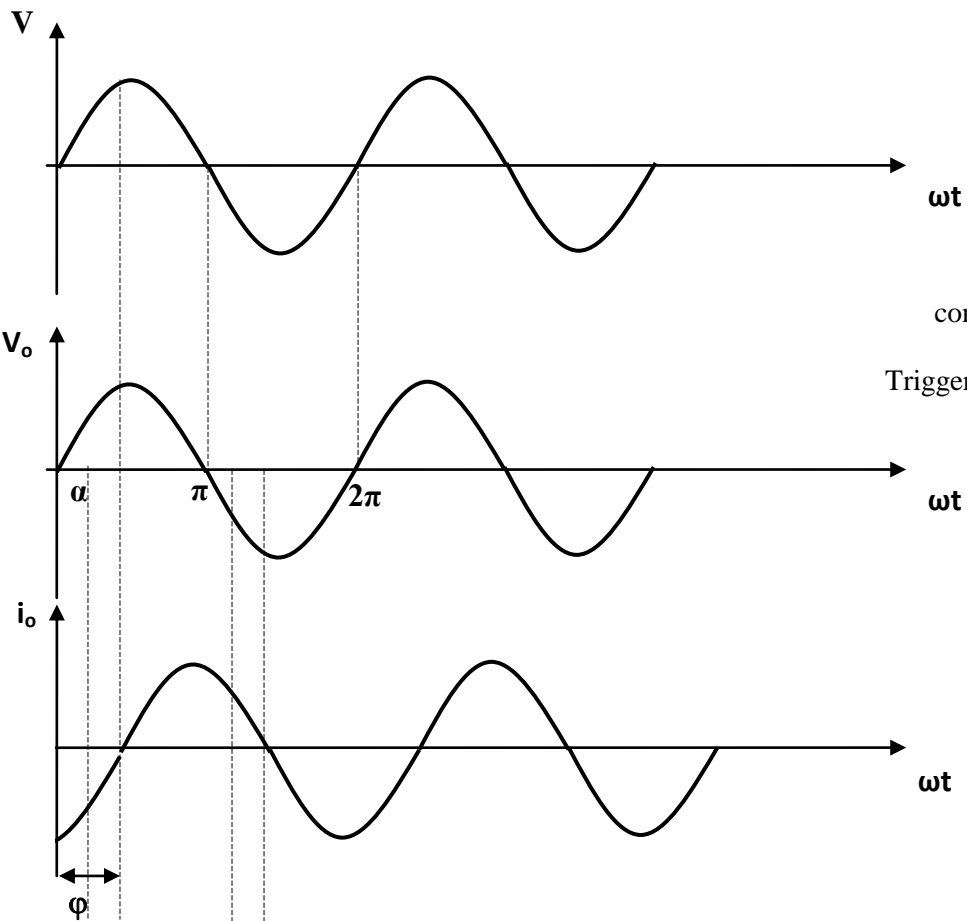
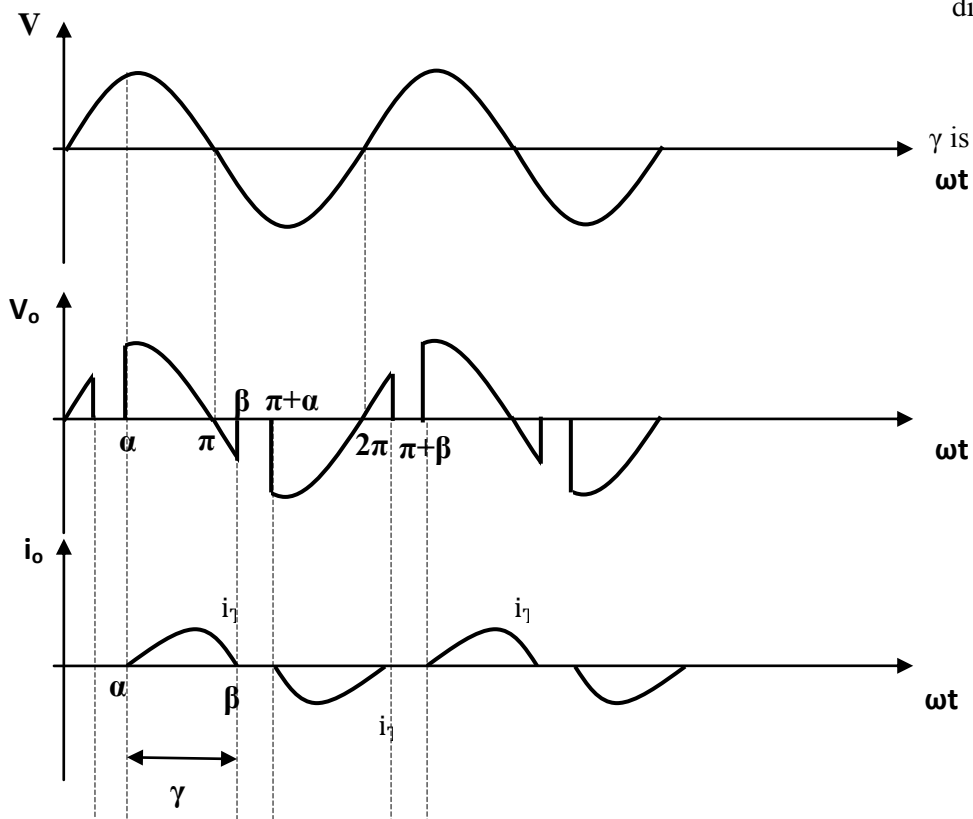
$$\varphi = \tan^{-1} \frac{\omega L}{R}$$

$$\alpha > \varphi$$

discontinuous current

$$\gamma = \beta - \alpha$$

$\gamma$  is the conduction angle.  
 $\omega t$



$$\alpha \leq \varphi$$

continuous current

Trigger pulse width  $> (\varphi - \alpha)$

For  $\alpha \leq \phi$ , trigger pulse of TH<sub>1</sub> must be wider, because at the time of firing TH<sub>1</sub>,  $i_o$  is negative and the thyristor TH<sub>1</sub> will not turn on until the thyristor TH<sub>2</sub> turns off.

From these it can be seen that as  $\alpha$  is reduced until  $\gamma=180^\circ$  the waveforms of  $i_o$  and  $V_o$  approach the sinusoidal form for which  $\alpha=\phi$ .

The relationships between  $\alpha$ ,  $\gamma$ ,  $\phi$ , and  $\beta$  can be obtained by substituting for  $\omega t=\beta$  (i.e.  $i_o=0$ ) in the current equation:

$$i = 0 = \frac{V_m}{Z} \left[ \sin(\beta - \phi) - \sin(\alpha - \phi) e^{\frac{R}{L} \left( \frac{\alpha - \beta}{\omega} \right)} \right]$$

$$\sin(\beta - \phi) = \sin(\alpha - \phi) e^{\frac{R}{L} \left( \frac{\alpha - \beta}{\omega} \right)} \text{-----(1)}$$

$$\gamma = \beta - \alpha \text{-----(2)}$$

If  $\alpha = \phi$ , then from equation (1)

$$\sin(\beta - \alpha) = \sin(\alpha - \alpha) e^{\frac{R}{L} \left( \frac{\alpha - \beta}{\omega} \right)} = 0$$

$$\beta - \alpha = \gamma = 180^\circ$$

### **Harmonic Analysis:**

The load or line-current waveform may be described by the Fourier series

$$i_o = \sum_{n=1}^{\infty} a_n \sin(n\omega t) + \sum_{n=1}^{\infty} b_n \cos(n\omega t) + \frac{a_o}{2} \quad A$$

where:

$$a_n = \frac{1}{\pi} \int_0^{2\pi} i_o \sin(n\omega t) d\omega t$$

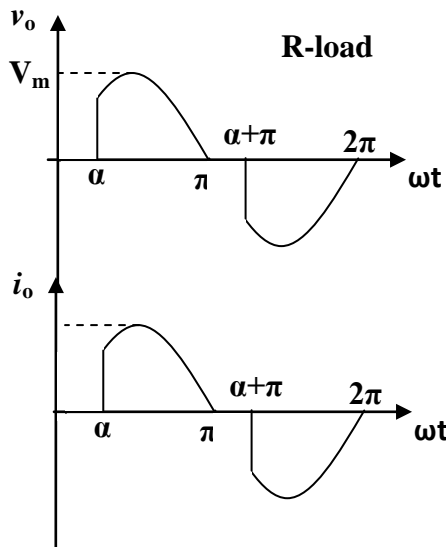
$$b_n = \frac{1}{\pi} \int_0^{2\pi} i_o \cos(n\omega t) d\omega t$$

$$\frac{a_o}{2} = \frac{1}{2\pi} \int_0^{2\pi} i_o d\omega t$$

**a-For R-load:**

$$\frac{a_o}{2} = \frac{1}{2\pi} \left[ \int_{\alpha}^{\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) d\omega t + \int_{\alpha+\pi}^{2\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) d\omega t \right] = 0$$

$$i_o = \frac{\sqrt{2}V}{R} \sin(\omega t) \quad \alpha \leq \omega t \leq \pi \quad \text{for resistive load}$$



$$a_n = \frac{1}{\pi} \left[ \int_{\alpha}^{\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) \sin(n\omega t) d\omega t + \int_{\alpha+\pi}^{2\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) \sin(n\omega t) d\omega t \right] \text{ -----(1)}$$

$$= \frac{\sqrt{2}V}{2\pi R} \left[ \int_{\alpha}^{\pi} (\cos(n-1)(\omega t) - \cos(n+1)(\omega t)) d\omega t + \int_{\alpha+\pi}^{2\pi} (\cos(n-1)(\omega t) - \cos(n+1)(\omega t)) d\omega t \right]$$

$$= \frac{\sqrt{2}V}{2\pi R} \left[ \frac{1}{n-1} (\sin(n-1)\pi - \sin(n-1)\alpha + \sin(n-1)2\pi - \sin(n-1)(\pi + \alpha)) - \frac{1}{n+1} (\sin(n+1)\pi - \sin(n+1)\alpha + \sin(n+1)2\pi - \sin(n+1)(\pi + \alpha)) \right]$$

$$a_n = \frac{\sqrt{2}V}{2\pi R} \left[ \frac{1}{n-1} (-\sin(n-1)\alpha - \sin(n-1)(\pi + \alpha)) - \frac{1}{n+1} (-\sin(n+1)\alpha - \sin(n+1)(\pi + \alpha)) \right] \quad n \neq 1$$

For even n:

$$a_2 = a_4 = a_6 = \dots = 0$$

For odd n:

$$a_n = \frac{\sqrt{2}V}{\pi R} \left[ \frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1} \right]$$

For n=1 in equation (1):

$$\begin{aligned} a_1 &= \frac{1}{\pi} \left[ \int_{\alpha}^{\pi} \frac{\sqrt{2}V}{R} \sin^2(\omega t) d\omega t + \int_{\alpha+\pi}^{2\pi} \frac{\sqrt{2}V}{R} \sin^2(\omega t) d\omega t \right] \\ &= \frac{\sqrt{2}V}{\pi R} \left[ \pi - \alpha + \frac{1}{2} \sin(2\alpha) \right] \end{aligned}$$

Similarly for b<sub>n</sub>:

$$b_n = \frac{1}{\pi} \left[ \int_{\alpha}^{\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) \cos(n\omega t) d\omega t + \int_{\alpha+\pi}^{2\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) \cos(n\omega t) d\omega t \right] \quad \text{-----(2)}$$

$$\begin{aligned} b_n &= \frac{\sqrt{2}V}{2\pi R} \left[ \int_{\alpha}^{\pi} (\sin(n+1)(\omega t) - \sin(n-1)(\omega t)) d\omega t \right. \\ &\quad \left. + \int_{\alpha+\pi}^{2\pi} (\sin(n+1)(\omega t) - \sin(n-1)(\omega t)) d\omega t \right] \end{aligned}$$

$$\begin{aligned} &= \frac{\sqrt{2}V}{2\pi R} \left[ \frac{1}{n+1} (\cos(n+1)\alpha - \cos(n+1)\pi + \cos(n+1)(\pi + \alpha) - \cos(n+1)2\pi) \right. \\ &\quad - \frac{1}{n-1} (\cos(n-1)\alpha - \cos(n-1)\pi + \cos(n-1)(\pi + \alpha) - \cos(n-1)2\pi) \left. \right] \quad n \neq 1 \end{aligned}$$

For even n:

$$b_2 = b_4 = b_6 = \dots = 0$$

For odd n:

$$b_n = \frac{\sqrt{2}V}{\pi R} \left[ \frac{\cos(n+1)\alpha - \cos[(n+1)\pi]}{n+1} - \frac{\cos(n-1)\alpha - \cos[(n-1)\pi]}{n-1} \right]$$

For n=1 in equation (2):

$$b_1 = \frac{1}{\pi} \left[ \int_{\alpha}^{\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) \cos(\omega t) d\omega t + \int_{\alpha+\pi}^{2\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) \cos(\omega t) d\omega t \right]$$

$$b_1 = -\frac{\sqrt{2}v}{\pi R} \sin^2(\alpha)$$

The rms value of the nth harmonic is then given by:

$$I_n = \sqrt{\frac{a_n^2 + b_n^2}{2}}$$

The rms value of the nth harmonic of the output voltage is given by:

$$V_n = I_n * [R^2 + (n\omega L)^2]^{\frac{1}{2}} \text{ V}$$

The rms value of the load current is:

$$i_{load (rms)} = \sqrt{i_0^2 + i_1^2 + i_2^2 + i_3^2 + \dots}$$

where  $i_0 = 0, i_2 = 0, i_4 = 0$

The rms value of the load voltage is:

$$V_{load (rms)} = \sqrt{V_0^2 + V_1^2 + V_2^2 + V_3^2 + \dots}$$

where  $V_0 = 0, V_2 = 0, V_4 = 0$

The normalized harmonic rms values is:

$$H_{n\alpha} = \frac{\text{rms value of } n\text{th harmonic at angle } \alpha}{\text{rms value of line current at } \alpha = 0}$$

$$\text{Harmonic factor} = \frac{\sum_{n=2}^{\infty} I_n}{I_1}$$

**b-For R-L load:**

Similar analysis can be repeated but with integration boundaries from  $\alpha$  to  $\beta$  and from  $\pi$  to  $\pi+\beta$

$$\frac{a_o}{2} = \frac{1}{2\pi} \left[ \int_{\alpha}^{\beta} \frac{\sqrt{2}V}{z_o} \sin(\omega t) d\omega t + \int_{\alpha+\pi}^{\pi+\beta} \frac{\sqrt{2}V}{R} \sin(\omega t) d\omega t \right] = 0$$

$$a_n = \frac{1}{\pi} \left[ \int_{\alpha}^{\beta} \frac{\sqrt{2}V}{z_n} \sin(\omega t) \sin(n\omega t) d\omega t + \int_{\alpha+\pi}^{\pi+\beta} \frac{\sqrt{2}V}{z_n} \sin(\omega t) \sin(n\omega t) d\omega t \right] \text{ -----(1)}$$

$$= \frac{\sqrt{2}V}{2\pi z_n} \left[ \frac{1}{n-1} (\sin(n-1)\beta - \sin(n-1)\alpha + \sin(n-1)(\pi+\beta) - \sin(n-1)(\pi+\alpha)) \right. \\ \left. - \frac{1}{n+1} (\sin(n+1)\beta - \sin(n+1)\alpha + \sin(n+1)(\pi+\beta) - \sin(n+1)(\pi+\alpha)) \right] \quad n \neq 1$$

For even n:

$$a_2 = a_4 = a_6 = \dots = 0$$

For odd n:

$$a_n = \frac{\sqrt{2}V}{\pi z_n} \left[ \frac{1}{n-1} (\sin(n-1)\beta - \sin(n-1)\alpha) \right. \\ \left. - \frac{1}{n+1} (\sin(n+1)\beta - \sin(n+1)\alpha) \right]$$



For n=1 in (1)

$$a_1 = \frac{\sqrt{2}V}{\pi z_1} \left[ \int_{\alpha}^{\beta} \sin^2(\omega t) d\omega t + \int_{\alpha+\pi}^{\pi+\beta} \sin^2(\omega t) d\omega t \right]$$

$$= \frac{\sqrt{2}V}{\pi z_1} \left[ \beta - \alpha + \frac{1}{2} \sin(2\alpha) - \frac{1}{2} \sin(2\beta) \right]$$

Similarly for  $b_n$ :

$$b_n = \frac{1}{\pi} \left[ \int_{\alpha}^{\beta} \frac{\sqrt{2}V}{z_n} \sin(\omega t) \cos(n\omega t) d\omega t + \int_{\alpha+\pi}^{\pi+\beta} \frac{\sqrt{2}V}{z_n} \sin(\omega t) \cos(n\omega t) d\omega t \right] \text{ -----(2)}$$

$$= \frac{\sqrt{2}V}{2\pi z_n} \left[ \frac{1}{n+1} (\cos(n+1)\alpha - \cos(n+1)\beta + \cos(n+1)(\pi + \alpha) - \cos(n+1)(\pi + \beta)) \right.$$

$$\left. - \frac{1}{n-1} (\cos(n-1)\alpha - \cos(n-1)\beta + \cos(n-1)(\pi + \alpha) - \cos(n-1)(\pi + \beta)) \right] \quad n \neq 1$$

For even n:

$$b_2 = b_4 = b_6 = \dots = 0$$

For odd n:

$$b_n = \frac{\sqrt{2}V}{\pi z_n} \left[ \frac{\cos(n+1)\alpha - \cos(n+1)\beta}{n+1} - \frac{\cos(n-1)\alpha - \cos(n-1)\beta}{n-1} \right]$$

For n=1 in equation (2):

$$b_1 = \frac{1}{\pi} \left[ \int_{\alpha}^{\beta} \frac{\sqrt{2}V}{z_1} \sin(\omega t) \cos(\omega t) d\omega t + \int_{\alpha+\pi}^{\pi+\beta} \frac{\sqrt{2}V}{z_1} \sin(\omega t) \cos(\omega t) d\omega t \right]$$

$$b_1 = -\frac{\sqrt{2}V}{2\pi z_1} [\cos(2\beta) - \cos(2\alpha)]$$

**Example-1:** A single-phase full-wave controller is used to control the power from a 2300 V ac source into a resistive load that can vary from 1.15 to 2.3  $\Omega$ . The maximum output power desired is 2300kW. calculate the maximum value of thyristor voltage, the rms thyristor current, the average thyristor current , and the maximum rms value of third-harmonic current in the line for any operating condition of this system.

**Solution:** a) For R=2.3  $\Omega$ . When maximum power is to be delivered to the maximum load-circuit resistance, then  $\alpha=0$ :

$$P_o = R I^2$$

$$2300 * 10^3 = 2.3 I^2$$

$$I = 1000 \text{ A}$$

For a resistive load,  $\beta = \pi$

$$i_o = \frac{\sqrt{2}}{R} V \sin(\omega t) \quad \alpha < \omega t < \pi$$

The average thyristor current is:

$$\begin{aligned} I_{qav} &= \frac{1}{2\pi} \left[ \int_{\alpha}^{\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) d\omega t \right] = \frac{\sqrt{2}V}{2\pi R} (\cos(\alpha) + 1) \\ &= \frac{2300 * \sqrt{2}}{2 * \pi * 2.3} (\cos(0) + 1) = 450.158 \text{ A} \end{aligned}$$

The rms thyristor current is:

$$\begin{aligned} I_{qrms} &= \left[ \frac{1}{2\pi} \left[ \int_{\alpha}^{\pi} \left( \frac{\sqrt{2}V}{R} \sin(\omega t) \right)^2 d\omega t \right] \right]^{\frac{1}{2}} = \frac{V}{\sqrt{2}R} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi} \right]^{\frac{1}{2}} \\ &= \frac{V}{\sqrt{2}R} = \frac{2300}{\sqrt{2} * 2.3} = 707.106 \text{ A} \end{aligned}$$

b) For  $R=1.15 \Omega$ . When maximum power is delivered to the maximum load-circuit resistance, then  $\alpha > 0$ :

$$2300 * 10^3 = 1.15 I^2$$

$I=1414.214 \text{ A}$  r.m.s. load current.

$$= \frac{V_m}{\sqrt{2R}} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi} \right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2} * 2300}{\sqrt{2} * 1.15} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi} \right]^{\frac{1}{2}}$$

$$\sin(2\alpha) = 2\alpha - \pi$$

$$\therefore \alpha = \frac{\pi}{2} = 90^\circ$$

The average thyristor current is:

$$I_{qav} = \frac{\sqrt{2}V}{2\pi R} (\cos(\alpha) + 1) = \frac{2300 * \sqrt{2}}{2 * \pi * 1.15} (\cos(90^\circ) + 1) = 450.158 \text{ A}$$

The rms thyristor current is:

$$I_{qrms} = \frac{V}{\sqrt{2R}} \left[ 1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi} \right]^{\frac{1}{2}}$$

$$= \frac{2300}{\sqrt{2} * 1.15} \left[ 1 - \frac{\pi/2}{\pi} + \frac{\sin(\pi)}{2\pi} \right]^{\frac{1}{2}} = 1000 \text{ A}$$

The peak forward and reverse voltages applied to the thyristor are  $2300 * \sqrt{2} = 3252.691 \text{ V}$

$$a_3 = \frac{\sqrt{2}V}{\pi R} \left[ \frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1} \right]$$

mean 3rd harmonic is at  $\alpha = \frac{\pi}{2}$

$$b_3 = \frac{\sqrt{2}V}{\pi R} \left[ \frac{\cos(n+1)\alpha - \cos(n+1)\pi}{n+1} - \frac{\cos(n-1)\alpha - \cos(n-1)\pi}{n-1} \right]$$

$$I_{3rms} = \sqrt{\frac{a_3^2 + b_3^2}{2}}$$

$$\frac{dI_{3rms}}{d\alpha}$$

$$\therefore \alpha = \frac{\pi}{2}$$

$$a_3 = \frac{\sqrt{2} * 2300}{\pi * 1.15} \left[ \frac{\sin(3+1) * \frac{\pi}{2}}{3+1} - \frac{\sin(3-1) * \frac{\pi}{2}}{3-1} \right] = 0$$

$$b_3 = \frac{\sqrt{2} * 2300}{\pi * 1.15} \left[ \frac{\cos(3+1) * \frac{\pi}{2} - \cos(3+1)\pi}{3+1} - \frac{\cos(3-1) * \frac{\pi}{2} - \cos(3-1)\pi}{3-1} \right]$$

$$= \frac{\sqrt{2} * 2300}{\pi * 1.15} \left[ \frac{\cos 2\pi - \cos 4\pi}{4} - \frac{\cos \pi - \cos 2\pi}{2} \right] = 900.316 \text{ A}$$

$$I_{3rms} = \sqrt{\frac{a_3^2 + b_3^2}{2}} = \sqrt{\frac{0 + 900}{2}} = 636.62 \text{ A}$$

**Example-2:** A single-phase full-wave controller is used to control the power from a 2300 V ac source into a load circuit of 2.3  $\Omega$  resistance and 2.3  $\Omega$  inductive reactance. Determine:

a-The control range (i.e., the range of variation of  $\alpha$  necessary to vary the current from zero to the maximum possible value).

b-The maximum rms current.

c-The maximum power and power factor.

d-The rms thyristor current, the condition angle and the power factor at the source for  $\alpha=\pi/2$ .

**Solution:**

(a)  $v = 2300\sqrt{2} \sin(\omega t)$

at  $p=0$   $\alpha = \alpha_{\max} = \pi$

at  $p=p_{\max}$   $\alpha = \varphi = \tan^{-1} \frac{\omega L}{R} = \frac{\pi}{4} = \alpha_{\min}$  (pure ac)

Thus the control range is  $\pi/4 \leq \alpha \leq \pi$ .

(b) Maximum current is when  $\alpha = \alpha_{\min}$

$$I = \frac{V}{Z} = \frac{2300}{\sqrt{2.3^2 + 2.3^2}} = 707.107 \text{ A}$$

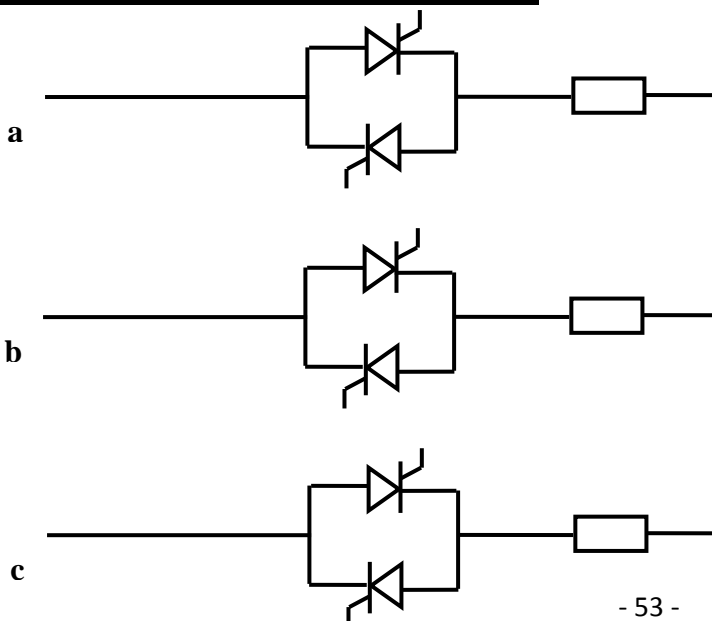
(c)  $p_{\max} = R I^2 = 2.3(707.107)^2 = 1150 \text{ kW}$

$$\text{power factor} = \frac{R I^2}{V I} = \frac{1150 * 10^3}{2300 * 707.107} = 0.7071$$

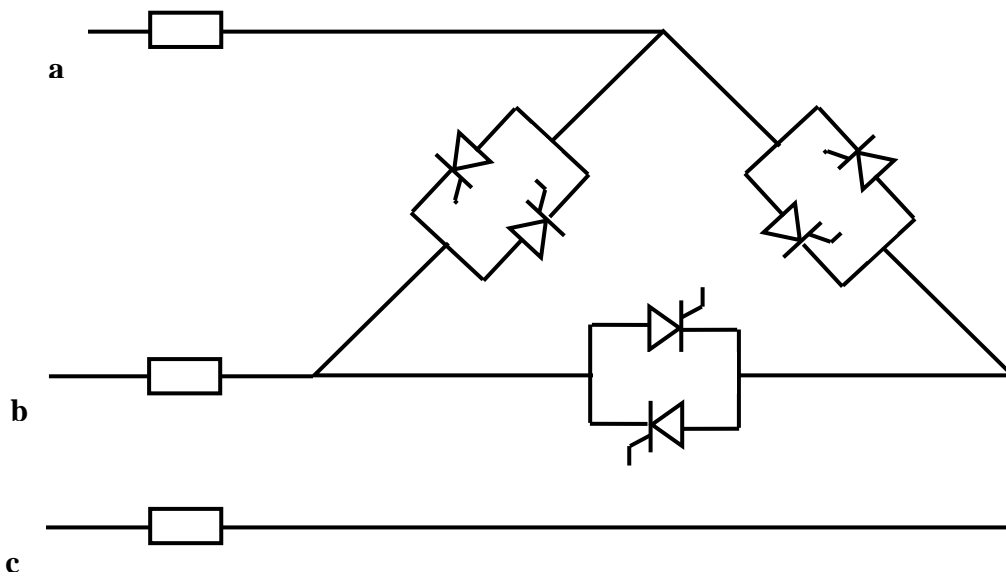
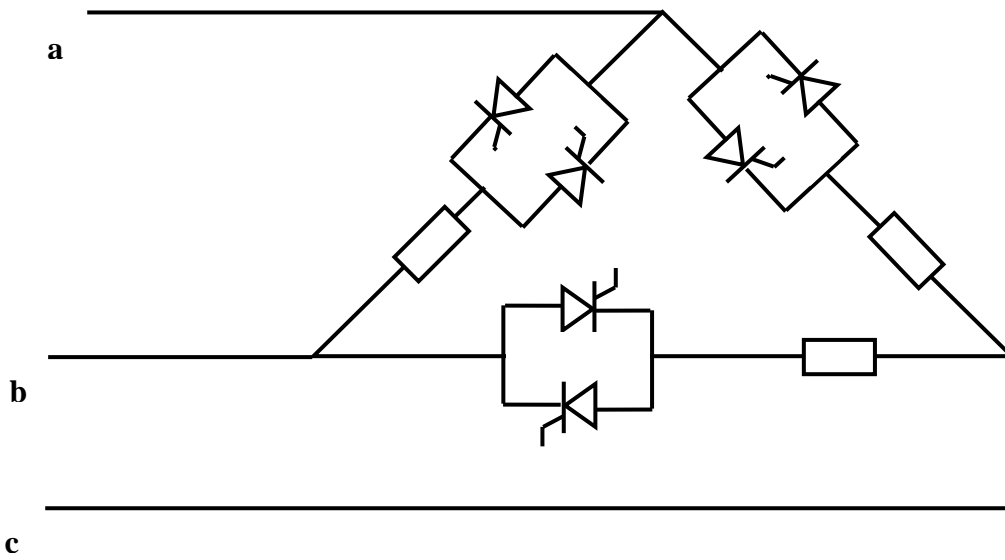
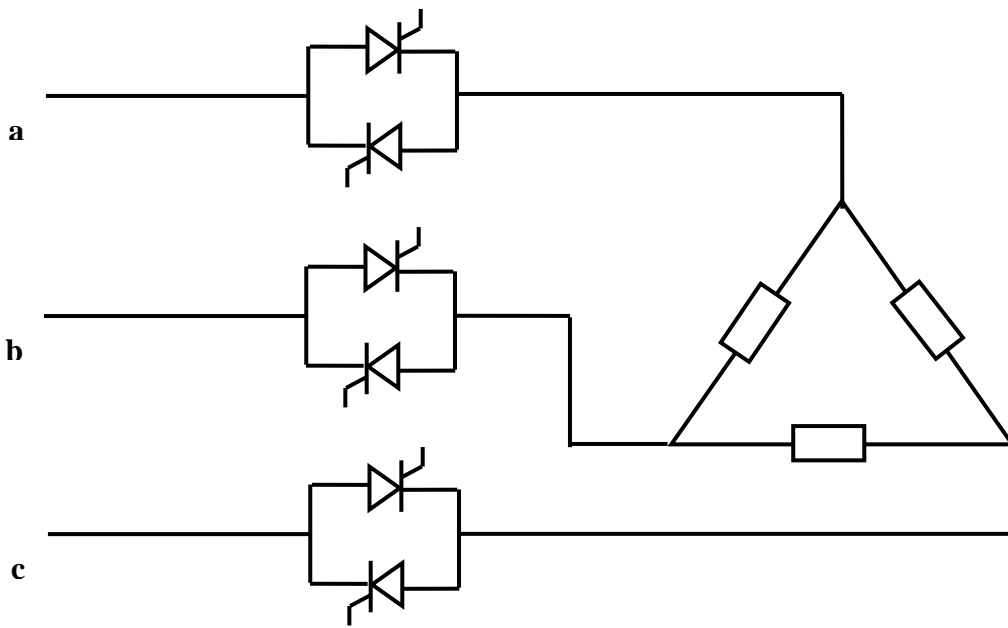
From this is clear that the power factor is  $\cos\varphi$ , but this is not true for the nonsinusoidal condition of operation when  $\alpha > \alpha_{\min}$ .

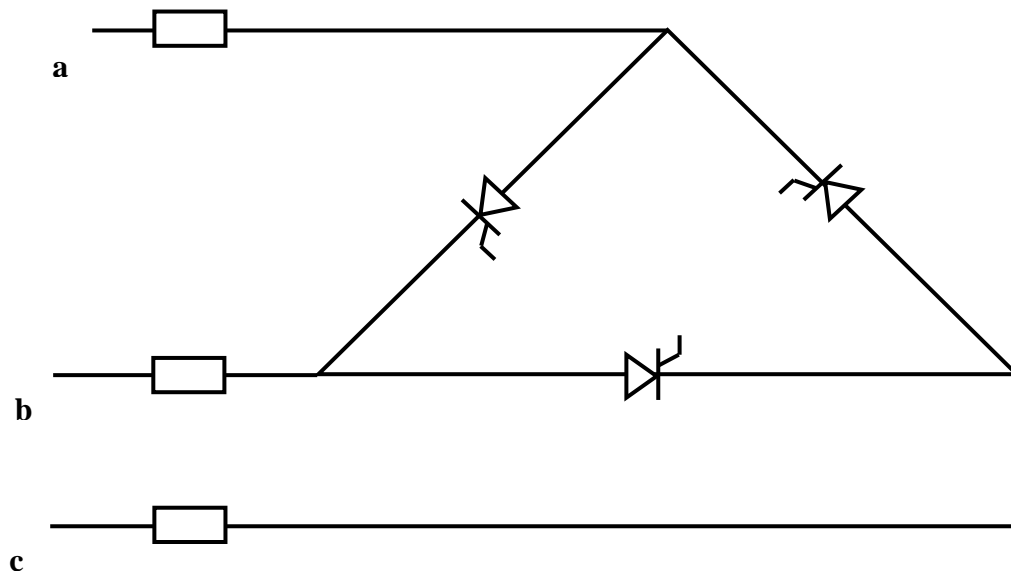
(d) Left to you (H.W.).

**Three-phase a.c. phase control:**

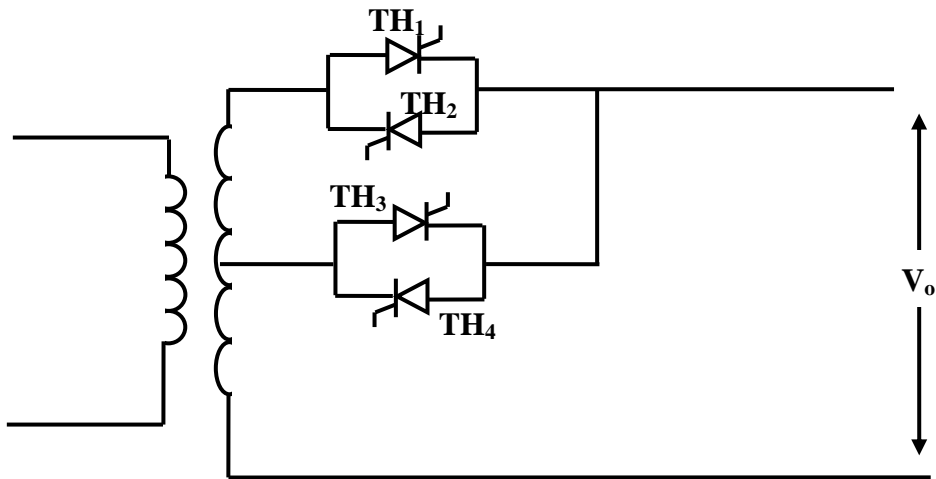


or





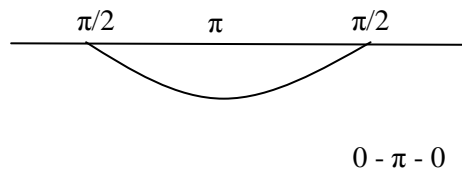
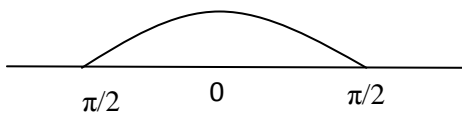
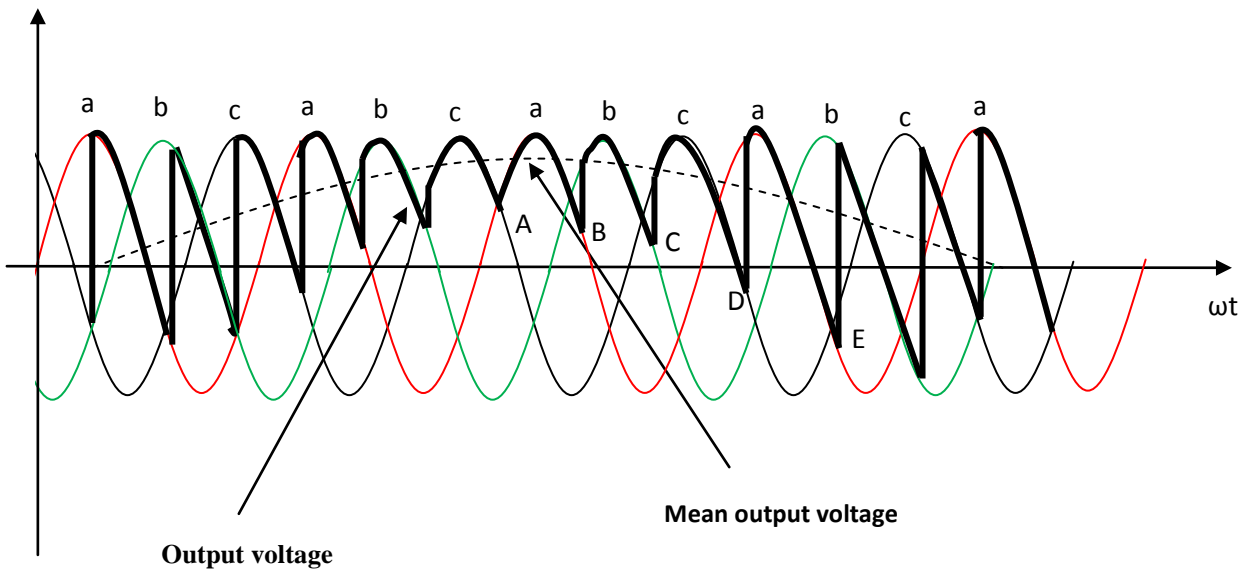
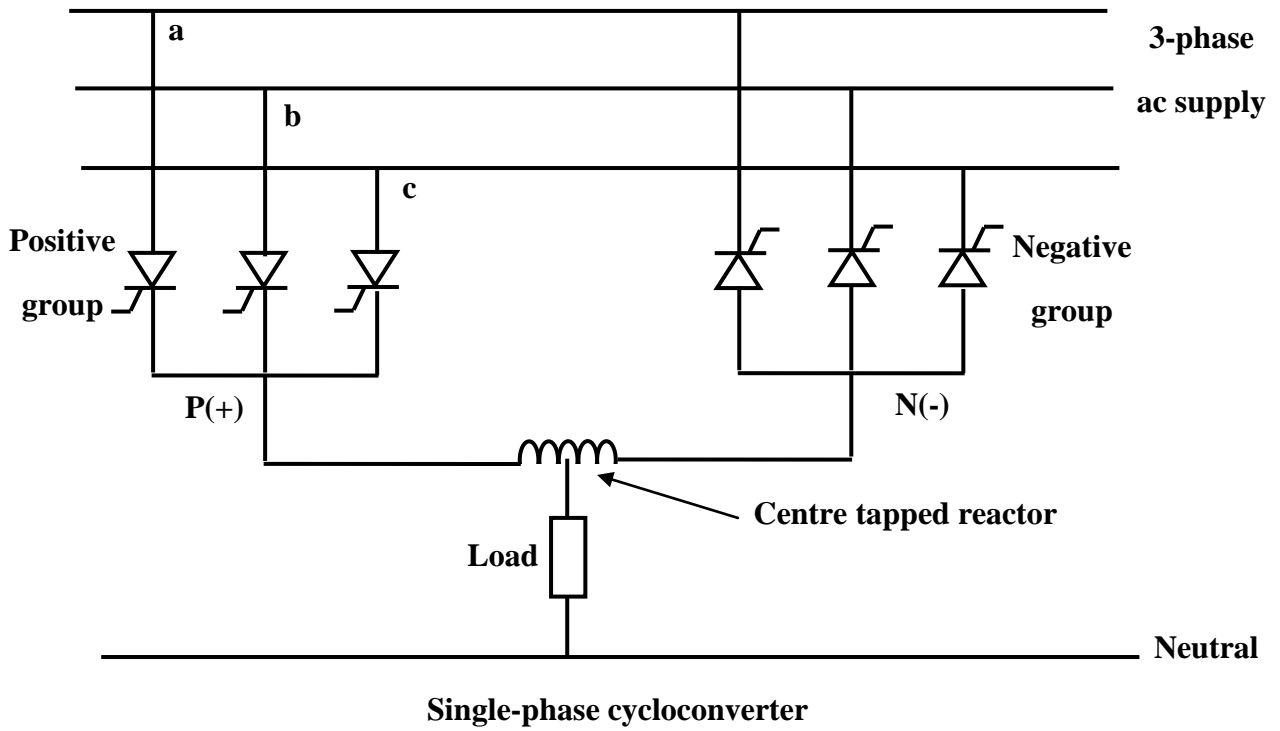
**synchronous tap changer:**



If  $TH_1$  &  $TH_2$  are on, full voltage is applied to the load.

If  $TH_3$  &  $TH_4$  are on, part of the transformer secondary is applied to the load.

# Cycloconverter:





In cycloconverter, output ac voltage wave is fabricated from the segment of the ac input voltage wave. We know that the average d.c. output of a rectifier is given by  $V_{dc} = V_{dm} \cos(\alpha)$ . This average value can be controlled by controlling the firing angle  $\alpha$ . If the firing angle  $\alpha$  is suitably varied from  $\pi/2$  to 0 and back to  $\pi/2$  an ac half wave can be superimposed on the dc output voltage similarly if  $\alpha$  is varied from  $\pi/2$  to  $\pi$  and back to  $\pi/2$  a negative half wave can be superimposed on the dc output voltage. Thus by varying  $\alpha$  from 0 to  $\pi$  and back to zero complete ac voltage wave can be fabricated.

Since the thyristors conduct current only in one direction, it is necessary to connect two groups of thyristors in inverse parallel as shown in the figure. Here in the figure are shown two half wave rectifiers, but any rectifier configuration can be used. The average output voltages of the two groups must be equal and opposite, otherwise large circulating current will flow at output frequency. This can be achieved by making  $\alpha_P = \pi - \alpha_N$ . However the instantaneous output voltages of the two groups are quite different and hence harmonic currents will circulate. These harmonic currents can be limited by using a centre tapped reactor.

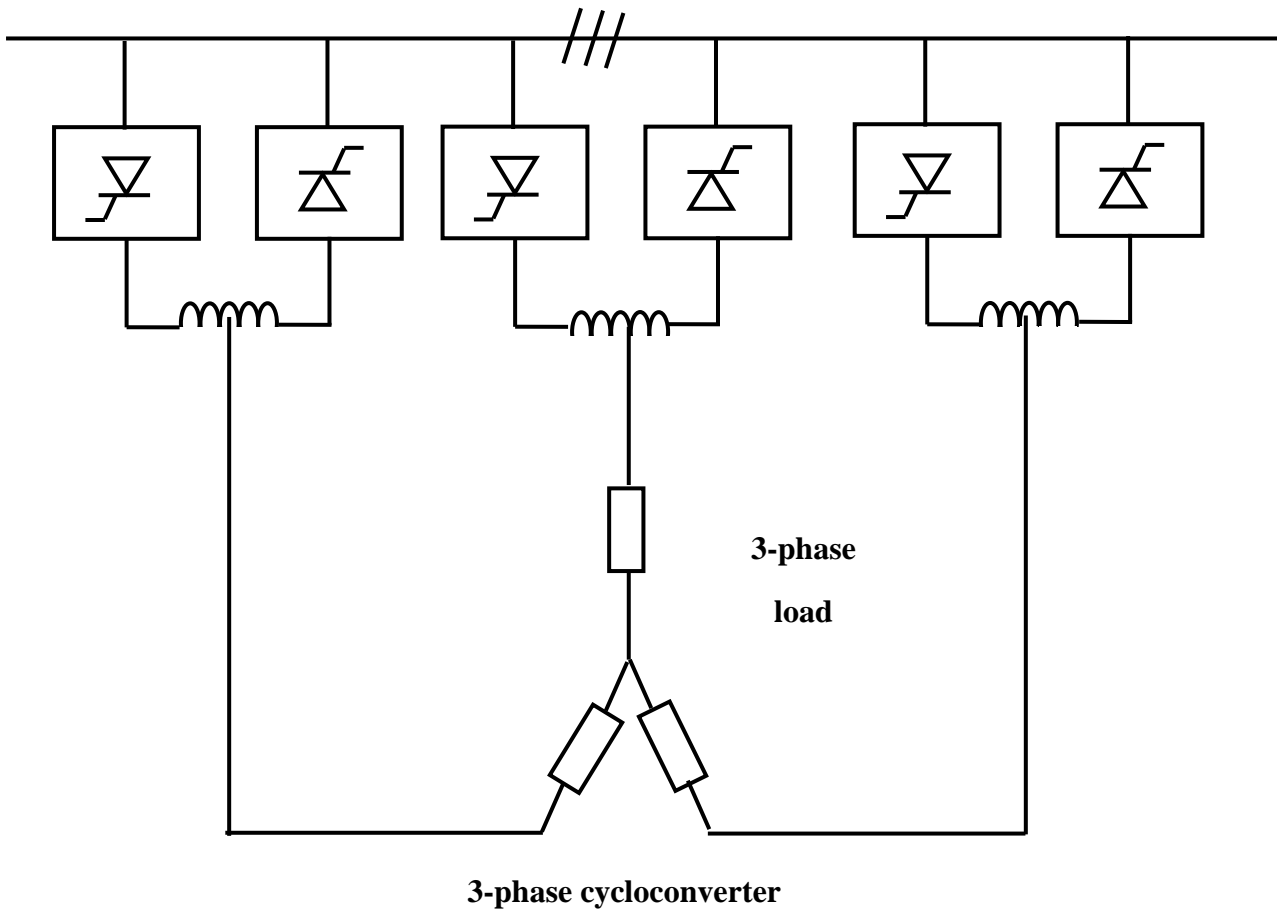
These harmonic currents can be suppressed all together by removing the gating signals for the half cycle for the group which is not conducting current.

The output frequency is independent of the input frequency it depends upon the rate of variation of  $\alpha$ .

The output frequency is usually kept lower than the input frequency. It is however possible to get output frequency larger than the input frequency but the losses in the thyristor will increase considerably.

The harmonics in the output wave can be reduced by decreasing the ratio of output to input frequency and also by increasing the number of phases.

A three phases cycloconverter consists of three single phase cycloconverters. The outputs of which are displaced by  $120^\circ$ . If the load is balanced then the neutral connection is not required. If 3-phase half wave rectifiers are used, then eighteen thyristor will be required for a 3-phase cycloconverter.



**Voltage equaton:**

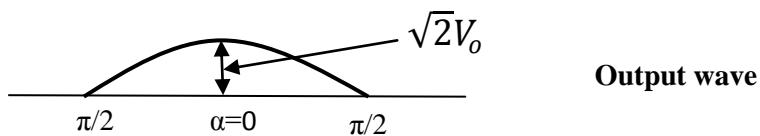
The average dc output voltage of 3-phase half-wave converter is:

$$V_{dc} = \frac{3\sqrt{3}}{2\pi} V_m \cos(\alpha) = V_{dm} \cos(\alpha)$$

where  $V_{dm}$  is the output dc voltage when  $\alpha=0$ .

Let the rms value of the a.c. output wave be  $V_o$ .

$\therefore$  The maximum of the a.c. output wave =  $\sqrt{2}V_o$



$$\therefore \sqrt{2}V_o = V_{dm} = \frac{3\sqrt{3}}{2\pi} V_m$$

$$\therefore V_o = \frac{3\sqrt{3}}{2\sqrt{2}\pi} V_m$$

## Sheet No.1

1- A d.c. source of 100 V supplies a purely inductive load of 0.1 H. The controller is a thyristor in series with the source and load. From the response of this simple circuit find the minimum width of the gating pulse to ensure thyristor turn-on. The specification for the thyristor gives the latching current to be 4 mA.

---

2- An a.c. supply of  $v_i=100 \sin(377t)$  is connected in series with a resistance  $10 \Omega$ , a thyristor and a 50 V battery, whose anode is connected to the thyristor cathode. Compute the average value of current in the circuit, if the thyristor is fired by a continuous d.c. signal.

---

3- A voltage source  $v=100 \sin(377t)$  supplies a resistive load of  $100 \Omega$  through a thyristor, which performs half-wave controlled rectification. Calculate the power supplied to the load when the thyristor firing angle is fixed at  $45^\circ$  with respect to the supply voltage waveform.

---

4- A single-phase full controlled converter is used to supply a load of  $R=2 \Omega$  and  $L= 5\text{mH}$ . If the input voltage is 220 V at 50 Hz calculate:

a- Range of  $\alpha$  to change the current from zero to maximum allowable value.

b- Maximum rms value of load current.

c- Maximum power and power factor.

---

5- A two-SCR rectifier has zero internal drop and is connected to auto-transformer that supplies 240 V, 60 Hz per leg. The load resistance equals its inductive reactance. When the SCR's fires with zero delay, a dc ammeter reads 5.4 A in the load:

a- Find the ammeter reading when the firing angle  $\alpha=75^\circ$ .

b- Find the henrys of the load.

c- Find the largest value of  $\alpha$  at which the load current does not become zero momentarily.

d- When the ammeter reads 0.81 A, find  $\alpha$ .

---

**6-** A single-phase full-controlled converter is connected to RL load ( $R=10\ \Omega$  &  $L=11.6\ \text{mH}$ ) and 240 V, 50 Hz supply.

a- Find the average current in the load when  $\alpha=30^\circ$ .

b- Find the firing angle  $\alpha$  when the average load current is 10 A.

c- For a half-wave controlled rectifier, calculate the average load voltage when the load is RL load with  $\alpha=30^\circ$ .

---

**7-** A half-wave controlled converter is connected to a single-phase transformer. The load is purely resistive load of R and the delay angle is  $\alpha=\pi/2$ . Determine the:

a-rectification efficiency; b- form factor, FF; c- ripple factor, RF; d- transformer utilization factor, TUF; and e- Peak inverse voltage, PIV, of the thyristor.

---

**8-** A single-phase full-wave bridge converter is connected to a 120 V 60Hz supply. The load current,  $I_o$ , can be assumed to be continuous and its ripple content is negligible. The turns ratio of the transformer is unity.

a- Express the input current in a Fourier series; determine the harmonic factor of input current, HF; displacement factor, DF, and input power factor, PF.

b- If delay angle is  $\alpha=\pi/3$ , calculate  $V_{dc}$ ,  $V_n$ ,  $V_{rms}$ , HF, DF, and PF.

---

**9-** For a delay angle of  $\alpha=\pi/2$ , repeat example-8 for the single-phase full-wave half-controlled bridge converter.

---

**10-** A three-phase half-wave converter is operated from a three-phase Y-connected 208 V 60Hz transformer supply and the load resistance is  $R=10\ \Omega$ . If it is required to obtain an average output voltage of 50% of the maximum possible output voltage, calculate the:

a- delay angle  $\alpha$ ; b- rms and average output currents; c- average and rms thyristor currents; d- rectification efficiency; e-transformer utilization factor, TUF; and f- input power factor, PF.

---

**11-** Repeat example-10 for the three-phase half-controlled bridge converter.

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**12-** Repeat example-10 for the three-phase full-controlled bridge converter.

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**13-** The load current of a three-phase full converter is continuous with a negligible ripple current. a-Express the input current in Fourier series, and determine the harmonic factor of the input current, HF; displacement factor, DF; and the input power factor, PF; b- If the delay angle  $\alpha=\pi/3$  and the peak input phase voltage,  $V_m=169.83$  V calculate  $V_n$ , HF, DF, and PF.