I. ELECTROMAGNETIC MULTIPOLE MOMENTS AND TRANSITIONS

The electromagnetic decay processes of nuclei are described as resulting from the interaction of the nucleus with an external electromagnetic field. The complete field consists of the electric field **E** and magnetic field **B**.

The electromagnetic radiation field can be expanded in multipoles containing spherical harmonics. The field is quantized in terms of photons. Creation and annihilation of photons is described in occupation number representation. Consider the electromagnetic decay of an excited nucleus to its ground state. The unperturbed initial state of the system is the excited nuclear state and the electromagnetic field in its ground state, i.e. no photons. The final state is the nuclear ground state and the electromagnetic field with one photon created into it.

A. Transition Probability and Half-Life

Consider the transition probability per unit time, usually called just transition probability, of gamma decay from an initial nuclear state **i** to a final nuclear state **f**, denoted $\mathbf{T_{fi}}$. The lifetime of the transition is $\mathbf{1/T_{fi}}$ and its half-life is

$$t_{1/2} = \frac{ln2}{T_{fi}} - - - - - - - - - - - - - -(1)$$

Gamma transitions proceed by multipole components $\lambda\mu$ of the radiation field (electric $\sigma = E$ or magnetic $\sigma = M$ type). The $\sigma\lambda\mu$ transition probability, calculated by the '**golden rule**' of time-dependent perturbation theory, is

$$T_{fi}^{(\sigma\lambda\mu)} = \frac{2}{\epsilon_0\hbar}\frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2}(\frac{E_\gamma}{\hbar c})^{(2\lambda+1)}|\langle\xi_f J_f m_f|\mathcal{M}_{\sigma\lambda\mu}|\xi_i J_i m_i\rangle|^2 - - - - - - - - (2)$$

E_γ is the energy of the transition and $\mathcal{M}_{\sigma\lambda\mu}$ is the nuclear operator associated with the multipole radiation field $\sigma\lambda\mu$. Magnetic substates are normally not observed separately. Averaging (2) over the initial substates and summing over the final substates and all yields the transition probability.

$$T_{fi}^{(\sigma\lambda)} = \frac{1}{2J_i+1}\sum_{m_i\mu m_f} T_{fi}^{(\sigma\lambda\mu)} = \frac{2}{\epsilon_0\hbar}\frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2}(\frac{E_\gamma}{\hbar c})^{(2\lambda+1)}B(\sigma\lambda;\xi_f J_f \to \xi_i J_i) - - - -(3)$$

where we have the reduced transition probability

$$B(\sigma\lambda;\xi_i J_i \rightarrow \xi_f J_f) - - - - (3) \equiv \frac{1}{2J_i+1}|\langle \xi_f J_f|\mathcal{M}_{\sigma\lambda}|\xi_i J_i\rangle|^2 - - - - - - -(4)$$

The usual notation is $\mathcal{M}_{E\lambda} = Q_\lambda$ and $\mathcal{M}_{M\lambda} = M_\lambda - - -$ (5)