

I. QUADRUPOLE MOMENT

If the nucleus has \mathbf{J} and magnetic quantum number \mathbf{M} , then Quadrupole Moment (Q.M.) will depend on M because it depends on the shape and hence the orientation of the charge distribution. The quadrupole moment is then defined as the value of (Q.M.)operator for which \mathbf{M} has its maximum value projected along the z-axis. Using spherical coordinates

$$Q = 3z^2 - r^2 = r^2 \left(\frac{16\pi}{5}\right)^{1/2} Y_{20}(\Theta, \Phi)$$

For extreme single-particle shell model only valence particle contribution to the (Q.M.), and without proof we state the resulting prediction that for odd-A, odd-Z nuclei with a single proton having a total angular momentum j outside closed sub-shells, the value of (Q.M.) is given by

$$Q.M. = \left[\begin{matrix} en \\ e_p \end{matrix} \right] \int \left(\frac{16\pi}{5}\right)^{1/2} \langle r^2 \rangle_{Nl} \langle l \frac{1}{2} jj | Y_{20} | l \frac{1}{2} jj \rangle$$

$$Q.M. = - \left[\begin{matrix} en \\ e_p \end{matrix} \right] \frac{2j-1}{2j+2} \langle r^2 \rangle_{Nl}$$

where e_n for neutron = 0 , and e_p for proton =1

Thus, $\mathbf{Q.M} = \mathbf{0}$ for $\mathbf{j} = \frac{1}{2}$. For odd-A, odd-N nuclei with a single neutron outside closed sub-shells Q.M. is predicted to be zero because the neutron has zero electric charge, as will all even-Z, odd-N nuclei because of the pairing effect.

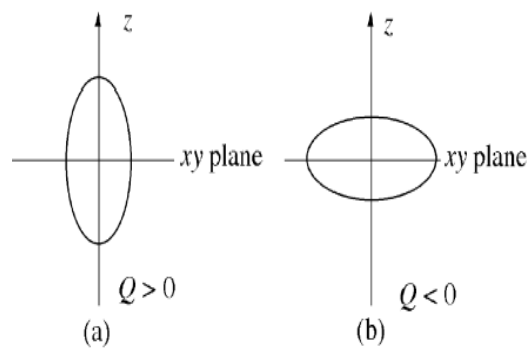
$$Q.M.(particle) = -Q.M.(hole)$$

To estimate the magnitude of Q.M., we assume that the orbital of the valence nucleon lies near the surface of the nucleus. This leads to

$$|Q.M.| \approx \langle r^2 \rangle \approx R^2 = 1.2^2 A^{2/3} = 1.44 A^{2/3} \times 10^{-2} \text{ barn}$$

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

For Comparison, the Experimental data for ^{17}F nucleus (orbital $d_{5/2}$) equal to -0.1 barn, while for ^{209}Bi nucleus (orbital $h_{9/2}$) equal to -0.46 barn.



Shapes of nuclei leading to (a) $Q > 0$ (prolate), and (b) $Q < 0$ (oblate)

FIG. 1: