

The number of protons and neutrons are roughly for stable lighter nuclei, however the number of neutrons is substantially greater than the number of protons for stable heavy nuclei. For light nuclei, the energy required to remove a proton or a neutron from the nucleus is roughly the same. The energy required to remove a proton or a neutron from a stable nucleus (\mathbf{Z}, \mathbf{A}) is

$$S_p = B(Z, A) - B(Z - 1, A - 1); \quad S_n = B(Z, A) - B(Z, A - 1)$$

The binding energy

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p \delta A^{-1/2}$$

Hence

$$S_p - S_n = -a_c(2Z - 1)(A - 1)^{-1/3} + a_a(A - 2Z)(A - 1)^{-1}$$

where $a_c = 0.714$ MeV; $a_a = 23.2$ MeV for stable nuclei.

$$Z = \frac{A}{2 + \frac{2a_c}{a_a} A^{2/3}} \approx \frac{A}{2} \left(1 - \frac{a_c}{a_a} A^{2/3}\right)$$

$$S_p - S_n = \frac{a_c}{A - 1} [A^{5/3} - (A - 1)^{5/3} + \frac{a_c}{a_a} A^{5/3} (A - 1)^{2/3}]$$

For heavy nuclei $A \gg \gg 1$ and $S_p - S_n = 5.5 \times 10^{-3} A^{4/3}$ hence $S_p - S_n$ increase with A . i.e to dissociate a proton from a heavy nucleus needs more energy then to dissociate a neutron.

For isobar nuclei (same A and different Z), one can derive a relationship between A and Z , the stable nuclei should satisfy

$$\frac{\partial(B.E)}{\partial Z} = -2A^{-1/3} a_c Z + 4a_a A^{-1} (A - 2Z) = 0$$

given

$$Z = \frac{A}{2 + \frac{a_c}{2a_a} A^{2/3}}$$

hence

$$Z = \frac{A}{2 + 0.0154 A^{2/3}}$$

Q) The greatest binding energy per nucleon occurs near ^{56}Fe and is much less for ^{238}U . Explain this in terms of the semi-empirical theory (you need not specify the values of the various coefficients).