***Example2*:** If you know that the message sent within the system were as follows:
X = AAAABBBBCCCC. While the received message is as follows:
Y = AABCBAACCCCA.
Find the entropy for each of X and Y.
What is the measurement of the transmission of information?
What is the amount of joint entropy and conditional entropy?

X	A	A	A	B	B	B	C	C
Y	A	B	C	A	B	C	A	C
P(x)	4/12	4/12	4/12	4/12	4/12	4/12	4/12	4/12
P(y)	5/12	2/12	5/12	5/12	2/12	5/12	5/12	5/12
P(x)P(y)	.1389	.0556	.1389	.1389	.0556	.1389	.1389	.1389
P(x, y)	2/12	1/12	1/12	2/12	1/12	1/12	1/12	3/12
P(x| y)	2/5	½	1/5	2/5	1/2	1/5	1/5	3/5
P(y| x)	2/4	¼	¼	2/4	1/4	¼	1/4	¾

<u>Entropy of X</u>

$H(X) = \sum_{i=1}^{3} P_i \log_2(1/P_i)$

$= P_A * \log_2 (1/P_A) + P_B * \log_2 (1/P_B) + P_c * \log_2 (1/P_c)$

$= 3 * (4/12 * 3.322 * \log_2 (3))$

$= 3 * 0.527$

$= 1.581$ Bits

<u>Entropy of Y</u>

$H(y) = P_A * \log_2 (1/P_A) + P_B * \log_2 (1/P_B) + P_c * \log_2 (1/P_c)$

$= 2 * (5/12 * 3.322 * \log_2 (12/5)) + 2/12 * 3.322 * \log_2 (6)$

$= 2 * (0.4167 * 3.322 * 0.3802) + 0.1667 * 3.322 * 0.778$

$= 1.053 + 0.431$

$= 1.484$ Bits

<u>Joint Entropy</u>

$H(X,Y) = \sum_Y \sum_X P(x,y) \log_2(1/P(x,y))$

$= 2/12 * 3.322 * \log_{10}(6) + 1/12 * 3.322 * \log_{10}(12) + 1/12 * 3.322 * \log_{10}(12)$
$+ 2/12 * 3.322 * \log_{10}(6) + 1/12 * 3.322 * \log_{10} (12) + 1/12 * 3.322 * \log_{10}(12)$
$+ 1/12 * 3.322 * \log_{10} (12) + 3/12 * 3.322 * \log_{10} (4)$

$= 0.432 + 0.298 + 0.298 + 0.432 + 0.298 + 0.298 + 0.298 + 0.5$

$= 2.854$ Bits

Conditional Entropy

$H(Y|X) = \sum_{x,y} P(x, y) \log_2(1/P(y|x))$

$= P(_{A, A}) * \log_2(1/P(_{A|A})) + P(_{A, B}) * \log_2(1/P(_{B|A})) +$
$+ P(_{A, C}) * \log_2(1/P(_{C|A})) + P(_{B, A}) * \log_2(1/P(_{A|B}))$
$+ P(_{B, B}) * \log_2(1/P(_{B|B})) + P(_{B, C}) * \log_2(1/P(_{C|B}))$
$+ P(_{C, A}) * \log_2(1/P(_{A|C})) + P(_{C, C}) * \log_2(1/P(_{C|C}))$
$= 2/12 * 3.322 * \log_{10}(2) + 1/12 * 3.322 * \log_{10}(4) +$
$1/12 * 3.322 * \log_{10}(4) + 2/12 * 3.322 * \log_{10}(2) +$
$1/12 * 3.322 * \log_{10}(4) + 1/12 * 3.322 * \log_{10}(4) +$
$1/12 * 3.322 * \log_{10}(4) + 3/12 * 3.322 * \log_{10}(4/3)$
$= 2 * (0.1667) + 2 * (0.1667) + 3 * (0.1667) + 0.1038$
$= 1.2707$ Bits

$H(X|Y) = \sum_{x,y} P(x, y) \log_2(1/P(x|y))$

Use Table information

Transmission Information Rate

$I(X,Y) = \sum_{x,y} P(x,y) \log_2\left(\frac{P(x,y)}{P(x) * P(y)}\right)$

$= 2/12 * 3.322 * \log_{10}(0.1667/0.1389) + 1/12 * 3.322 * \log_{10}(0.0833/0.0556) +$
$1/12 * 3.322 * \log_{10}(0.0833/0.1389) + 2/12 * 3.322 * \log_{10}(0.1667/0.1389) +$
$1/12 * 3.322 * \log_{10}(0.0833/0.0556) + 1/12 * 3.322 * \log_{10}(0.0833/0.1389) +$
$1/12 * 3.322 * \log_{10}(0.0833/0.1389) + 3/12 * 3.322 * \log_{10}(0.25/0.1389)$
$= 2 * 0.0439 + 2 * 0.0486 + 3 * 0.0614 + 0.2119$
$= 0.5811$ Bits

- **Transmission Efficiency**

The transmitter efficiency η is the ratio between the actual transmission rate and the highest rate can be send (Channel Capacity C)

$$\eta = I / C$$