## ✓ Entropy

Entropy *is defined as the average number of bits needed for storage or communication*. In other words, it is the sum of the expected values of self-information in the communication system and has the symbol "H". Entropy calculated according to the following formula:

$$H(Xi) = \sum_{i=1}^{n} P(xi) * I(Xi)$$

***Example 1*:** Find the amount of entropy for two events which have the following Probabilities: first event P = ¼, the second event P = ¾.

$$H = \sum_{i=0}^{n} P(xi) * \log2(1/P(xi))$$

$= 0.25 * \log_2(4) + 0.75 * \log_2 (4/3)$

$= 0.25 * 3.322 * \log_{10} (4) + 0.75 * 3.322 * \log_{10} (4/3)$

$= 0.25 * 3.322 * 0.602 + 0.75 * 3.322 * 0.125$

$= 0.5 + 0.311$

$= 0.811$ Bits

***Example 2*:** Suppose that we have a horse race with eight horses. The probabilities of winning for the eight horses are $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64})$. We can calculate the entropy of the horse race as:

$H(X) = 1/2 \log_2(2) + 1/4 \log_2(4) +1/8 \log_2(8) + 1/16 \log_2(16) + 4*(1/64 \log_2(64))$

$= 1/2 * 3.322*\log_{10}(2) + 1/4 * 3.322*\log_{10}(4) +1/8 * 3.322*\log_{10}(8)$

$+ 1/16* 3.322*\log_{10}(16) + 4*(1/64 * 3.322*\log_{10}(64))$

$= 2$ bits

***Example3*:** Let A with P (1/2), B with P (1/4), C with P (1/8) and D with P (1/8). Find the entropy of X.

$H (X) = 1/2*3.322*\log_{10}(2) + 1/4*3.322*\log_{10}(4) +2*(1/8*3.322*\log_{10}(8))$

$= 1.75$ bits

## ✓ Joint entropy

The **joint entropy** of two discrete random variables X and Y is merely the entropy of their pairing (X, Y). This implies if X and Y are independent, then their joint entropy is the sum of their individual entropies.

For example, if (X, Y) represents the position of a chess piece. X represents the row and Y the column, then the joint entropy of the row of the piece and the column of the piece will be the entropy of the position of the piece.

$$H(X, Y) = \sum_X \sum_Y P(x, y) \log(\frac{1}{p(x,y)})$$ unit of information

OR

$$H(X, Y) = H(X) + H(Y)$$ unit of information

***Example1*:** Find the joint entropy for tossing a coin X and rolling a dice Y.

*Note:* The chance of getting " tail " or " head " is equal, as well, the chance of getting "1", "2", "3", "4", "5" and "6" is equal.

$P(x_{=Head}) = P(_{X=Tail}) = 1/2$

$P(y=1) = P(_{Y=2}) = P(_{Y=3}) = P(_{Y=4}) = P(_{Y=5}) = P(_{Y=6}) = 1/6$

$P(x,y) = P(_X) * P(_Y) = 1/2 * 1/6 = 1/12$

$H(x,y) = \sum_{i=1}^{2} \sum_{j=1}^{6} P(x_i,y_j) \log_2(1/P(x_i,y_j))$

$= 2*(6*(1/12*3.322*\log_{10}(12)))$

$= 12 * (0.083 * 3.322 * 1.079)$

$= 3.584$ bits

OR

$H(x) = \sum_{i=1}^{2} P(x_i) \log_2(1/P(x_i))$

$= 2 * (1/2 * 3.322 * \log_{10}(2))$

$= 2 * (0.5 * 3.322 * 0.301)$

$= 1$bits

$H(y) = \sum_{j=1}^{6} P(y_j) \log_2(1/P(y_j))$

$= 6 * (1/6 * 3.322 * \log_{10}(6))$

$= 6 * (0.167 * 3.322 * 0.778)$

$= 2.585$ bits

$H(x,y) = H(x) + H(y)$

$= 1 + 2.585$

$= 3.585$ bits