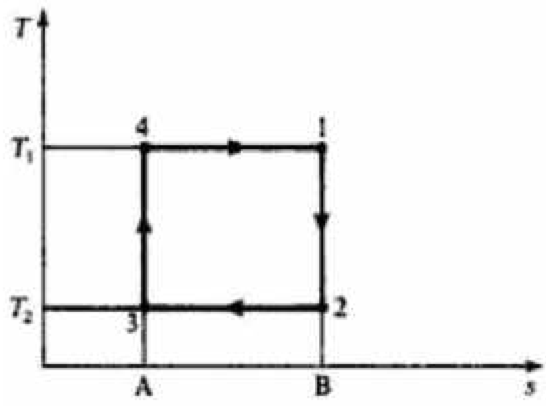
Chapter Six

The Heat Engine Cycle

The Carnot Cycle:-

Carnot, a French engineer, showed in a paper written in 1824 that the most efficient possible cycle is one in which all the heat supplied is supplied at one fixed temperature, and all the heat rejected is rejected at a lower fixed temperature.

The cycle therefore consists of two isothermal processes joined by two adiabatic processes, since all processes are reversible, then the adiabatic processes in the cycle are also isentropic. The cycle is represented on *T-s* diagram as shown in figure below.



$(1 \rightarrow 2)$ isentropic expansion from T_1 to T_2.

$(2 \rightarrow 3)$ isothermal heat rejection.

$(3 \rightarrow 4)$ isentropic compression from T_2 to T_1.

$(4 \rightarrow 1)$ isothermal heat supply.

The thermal efficiency of heat engine;

$$\boxed{\eta_{th.} = 1 - \frac{Q_2}{Q_1}}$$

: In the Carnot cycle, the heat supplied Q_1 represent the area 41BA4.

Q_1=area 41BA4=$T_1\,(s_B\text{-}s_A)$

Q_2=area 23AB2=$T_2\,(s_B\text{-}s_A)$

The thermal efficiency of Carnot cycle $= 1 - \dfrac{T_2(s_B - s_A)}{T_1(s_B - s_A)}$

$$\boxed{\therefore \eta_{th.} = 1 - \frac{T_2}{T_1}}$$

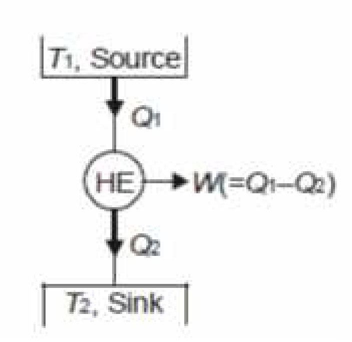
: When the ratio (T_2/T_1) decreases, the thermal efficiency increases.

Note:- The maximum possible thermal efficiency between any two temperatures is that of the Carnot cycle. Such that the highest thermal efficiency possible for a heat engine in practice is only about half that of the ideal theoretical Carnot cycle, between the same temperature limits. This is due to irreversibility in the actual cycle, and to deviations from the ideal cycle, which are made for various practical reasons.

****:** The work output of the Carnot cycle can be found from the *T*-*s* diagram.

From the first law $\sum Q = \sum W$

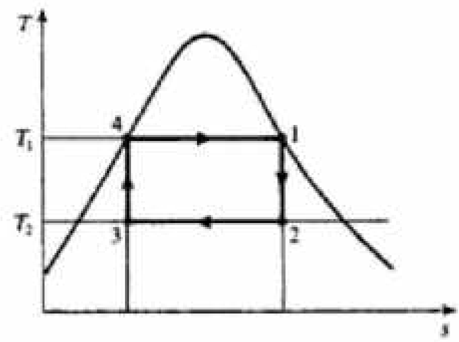
$$W = Q_1 - Q_2$$



$$\boxed{W_{Carnot} = (T_1 - T_2)(s_B - s_A)}$$

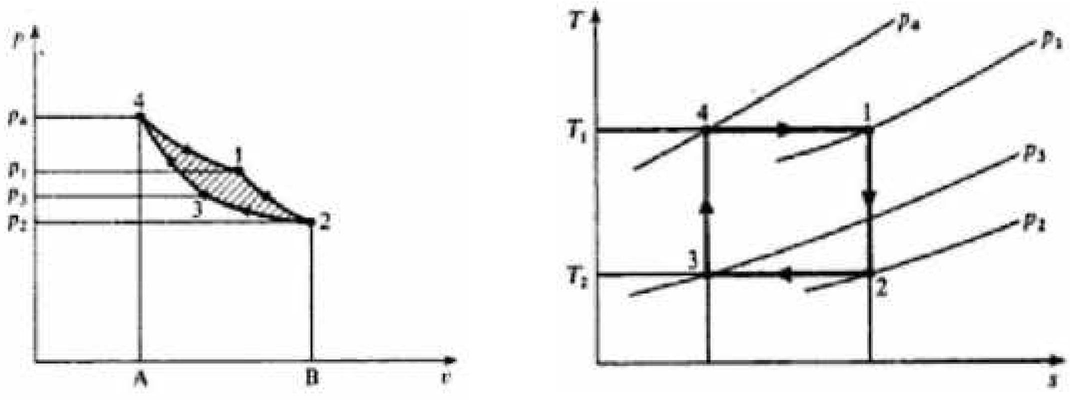
(A) The Carnot Cycle of Steam:-

The *T*-*s* diagram of the Carnot cycle for steam is shown below.



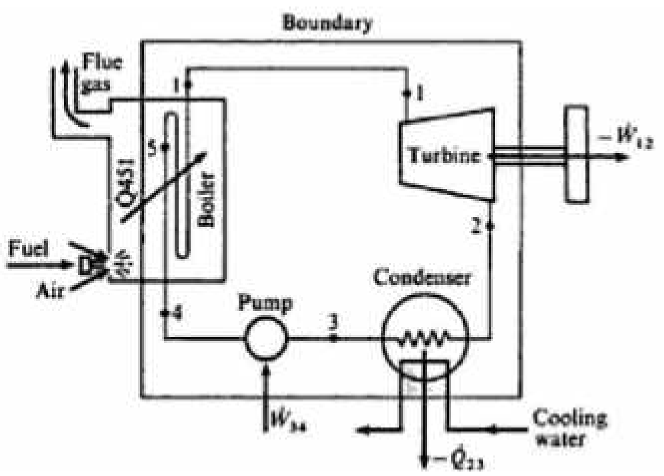
(B) The Carnot Cycle for a Perfect Gas:-

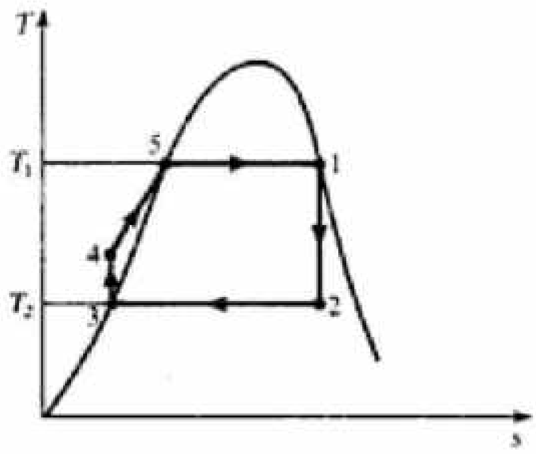
A Carnot cycle for a perfect gas is shown on a *T*-*s* diagram as shown in figure below.



(C) Modified Carnot Cycle (Rankine Cycle) for Steam:-

The Rankine cycle and the power cycle for the steam are shown below.





(1) Boiler: The heat supplied in the boiler.

$$h_4 + Q_{451} = h_1 + W \Rightarrow W = 0$$

$$\boxed{\therefore Q_{451} = h_1 - h_4}$$

(2) Turbine: The work done through the turbine.

(the expansion is adiabatic; Q=0 and isentropic; s_1=s_2)

$$h_1 + Q_{1\to 2} = h_2 + W_{1\to 2} \Rightarrow Q_{1\to 2} = 0$$

$$\boxed{\therefore W_{1\to 2} = h_1 - h_2}$$

(3) Condenser: The heat rejected through the condenser.

$$h_2 + Q_{2\to 3} = h_3 + W_{2\to 3} \Rightarrow W_{2\to 3} = 0$$

$$\boxed{Q_{2\to 3} = h_3 - h_2 = -(h_2 - h_3)}$$

(4) Pump: (The compression is adiabatic, Q=0 and isentropic, s_3=s_4)

$$h_3 + Q_{3\to 4} = h_4 + W_{3\to 4} \Rightarrow Q_{3\to 4} = 0$$

$$\boxed{W_{3\to 4} = h_3 - h_4 = -(h_4 - h_3)}$$

Net work done in the cycle;

$$W_{net} = W_{1\to 2} + W_{3\to 4}$$
$$W_{net} = (h_1 - h_2) - (h_4 - h_3)$$

If the feed pump work is neglected,

$$W_{net} = h_1 - h_2$$

$$Thermal\ efficiency = \frac{Net\ work}{Heat\ supplied}$$

$$\boxed{\eta_R = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}}$$

$$\eta_R = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_3) - (h_4 - h_3)}$$

If the feed pump term, $(h_4 - h_3)$ is neglected;

$$\boxed{\eta_R = \frac{(h_1 - h_2)}{(h_1 - h_3)}}$$

Pump work $= W_{3\to 4} = h_4 - h_3$

For a liquid, which is assumed to be incompressible (υ=constant) the increase in enthalpy for isentropic compression is given by;

$$h_4 - h_3 = \upsilon (p_4 - p_3)$$

proof :

$$dQ = du + p d\upsilon$$
$$h = u + p\upsilon$$
$$dh = du + p d\upsilon + \upsilon dp$$
$$du = dh - p d\upsilon - \upsilon dp$$
$$dQ = dh - p d\upsilon - \upsilon dp + p d\upsilon$$
$$dQ = dh - \upsilon dp$$

For isentropic process; *dQ=0*

$$\therefore dh = \upsilon dp$$
$$\int_3^4 dh = \int_3^4 \upsilon dp \Rightarrow \upsilon = constant(liquid)$$

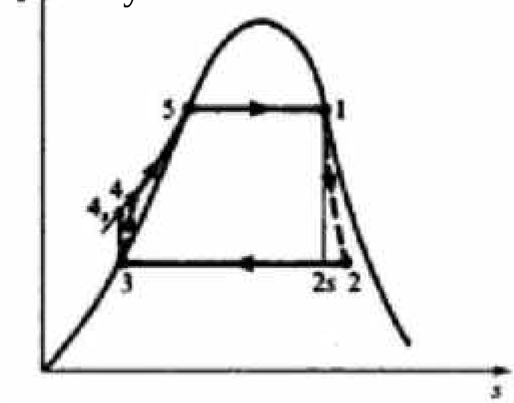
The pump work input;

$$h_4 - h_3 = \upsilon(p_4 - p_3)$$

Where;

(υ): taken from tables for water ($\upsilon = \upsilon_f$) at $p = p_3$

The efficiency ratio:- Is the ratio of the actual efficiency to the ideal efficiency. In vapor cycles the efficiency ratio compares the actual cycle efficiency to the Rankine cycle efficiency.



$$\text{Efficiency ratio} = \frac{\text{Cycle efficiency}}{\text{Rankine efficiency}}$$

: The actual expansion and compression processes $(1 \rightarrow 2')$, $(3 \rightarrow 4')$ are irreversible.

$$\text{Isentropic efficiency} = \frac{\text{Actual work}}{\text{Isentropic work}}$$ (for an expansion process)

$$\eta_{Is.} = \frac{W_{1\rightarrow 2'}}{W_{1\rightarrow 2}} = \frac{h_1 - h_{2'}}{h_1 - h_2}$$

$$\text{Isentropic efficiency} = \frac{\text{Isentropic work input}}{\text{Actual work input}}$$ (for a compression process)

Work ratio:- Is the ratio of net work to the gross work.

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Gross work}}$$

:- Both efficiency and work ratio are criteria of performance as well as another criterion in steam plant is the specific steam consumption.

The specific steam consumption(s.s.c.):- Is the steam flow in kg/h required to develop $1kW$.

$$s.s.c. = \frac{3600}{W} = \frac{3600}{h_1 - h_2}$$ (If neglected the pump work)

Example (6.1):-

What is the highest possible theoretical efficiency of a heal engine operating with a hot reservoir of furnace gases at 2000°C, when the cooling water available is at 10°C?

Solution:

$\eta_{Carnot}=?$, T_1=2000°C, T_2=10°C

$$\eta_{Carnot}=1-\frac{T_2}{T_1}=1-\frac{283}{2273}=0.8755=87.55\%$$

Example (6.2):-

A steam power plant operates between a boiler pressure of 42 bar and a condenser pressure of 0.035 bar. Calculate for these limit, the cycle efficiency, the work ratio, and the specific steam consumption:

(i) for a Carnot cycle using wet steam;

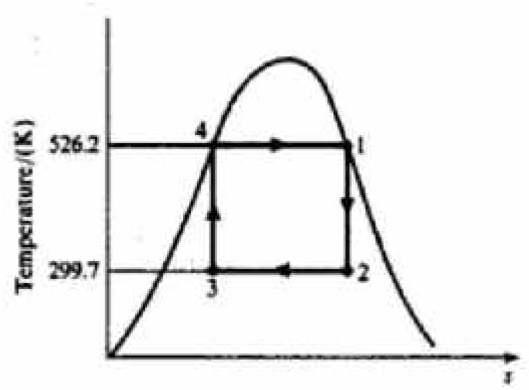
(ii) for a Rankine cycle with dry saturated steam at entry to the turbine;

(iii) for the Rankine cycle of (ii), when the expansion process has an isentropic efficiency of 80 %.

Solution:

$\eta_{th.}=?$, $W.r.=?$, $s.s.c.=?$

(i)/ for Carnot cycle



$$T_1=T_4=T_s=253.2°C \Rightarrow at, p=42bar$$

$$T_2=T_3=T_s=26.7°C \Rightarrow at, p=0.035bar$$

$$\eta_{Carnot}=1-\frac{T_2}{T_1}=1-\frac{299.7}{526.2}=43\%$$

$$Work\ ratio=\frac{Net\ work}{Gross\ work}$$

$$\eta_{th.}=\frac{W}{Q}$$

$$Q_{1\to4}=h_1-h_4=h_g-h_f=h_{fg}=1698kJ/kg$$

$$\therefore 0.43=\frac{W}{1698}\Rightarrow W=730.14kJ/kg$$

$$or)W=(T_1-T_2)(s_1-s_4)=(T_1-T_2)(s_g-s_f)=(T_1-T_2)s_{fg}=(253.2-26.7)(3.226)=730.7kJ/kg$$

$$W_{1\to2}=h_1-h_2$$

$$h_1=h_g=2800kJ/kg\Rightarrow s_1=s_2=s_g=6.049kJ/kg.K$$

$$s_2=s_{f2}+x_2s_{fg2}$$

$$6.049=0.391+x_2(8.13)\Rightarrow x_2=0.696$$

$$h_2=h_{f2}+x_2h_{fg2}=112+0.696(2438)=1808.85kJ/kg$$

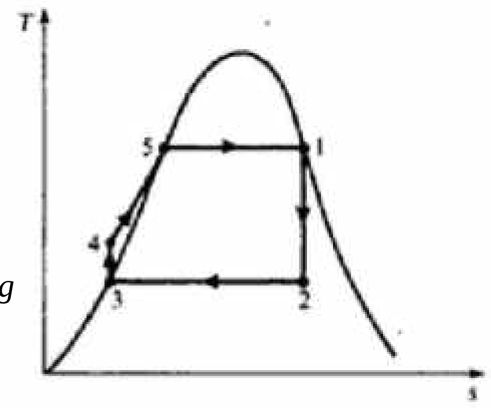
$$\therefore W_{1\to2}=2800-1808.85=991.15kJ/kg$$

$$\therefore work-ratio=\frac{W}{W_{1\to2}}=\frac{730.14}{991.15}=0.737=73.7\%$$

$$s.s.c.=\frac{3600}{W}=\frac{3600}{730.14}=4.93kg/kW.h$$

(ii)/ for Rankine cycle

$$\eta_R = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_3) - (h_4 - h_3)}$$



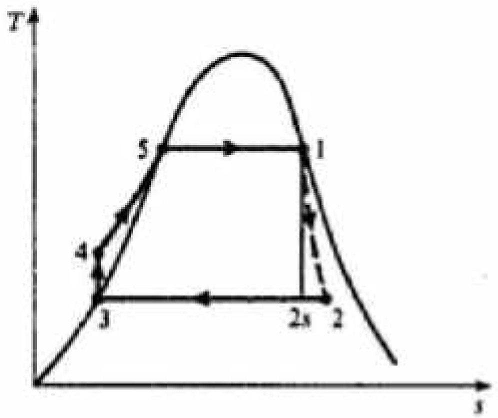
$$h_1 = h_g = 2800 kJ/kg$$
$$h_2 = 1808.85 kJ/kg$$
$$h_3 = h_f = 112 kJ/kg$$
$$(h_4 - h_3) = v_f (p_4 - p_3) = 0.001003(42 - 0.035) * 10^2 = 4.21 kJ/kg$$
$$\therefore \eta_R = \frac{(2800 - 1808.85) - 4.21}{(2800 - 112) - 4.21} = 0.368 = 36.8\%$$

$$Work\ ratio = \frac{Net\ work}{Gross\ work}$$

$$Network = W_{1\to2} + W_{3\to4} = (h_1 - h_2) - (h_4 - h_3) = 986.94 kJ/kg$$
$$Grosswork = W_{1\to2} = h_1 - h_2 = 991.15 kJ/kg$$
$$\therefore Work - ratio = \frac{986.94}{991.15} = 0.996 = 99.6\%$$
$$s.s.c. = \frac{3600}{W} = \frac{3600}{(h_1 - h_2) - (h_4 - h_3)} = \frac{3600}{986.94} = 3.65 kg/kW.h$$

(iii)/ isentropic efficiency of 80 %.

$$Isentropic\ efficiency = \frac{Actual\ work}{Isentropic\ work}$$



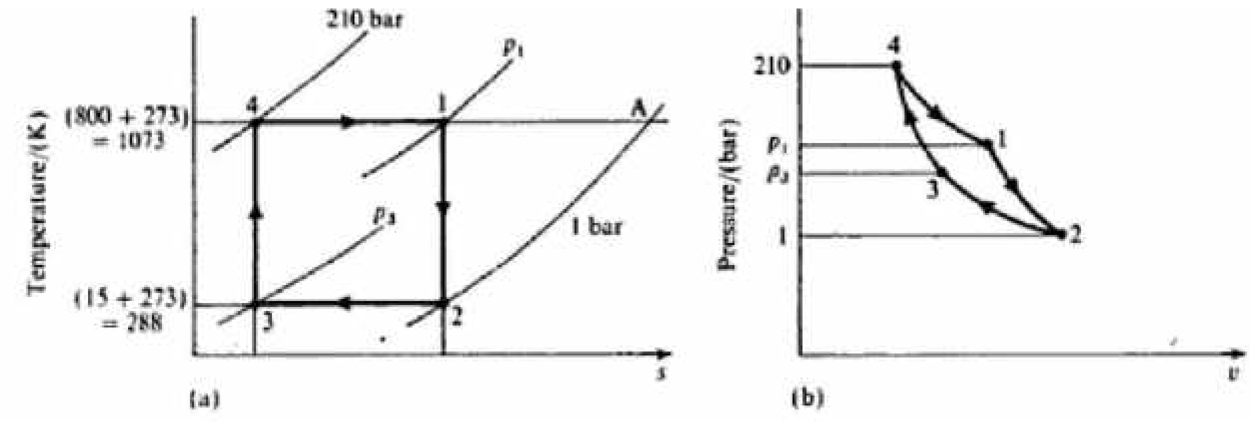
$$\eta_{Is.} = \frac{W_{1\to2'}}{W_{1\to2}} = \frac{h_1 - h_{2'}}{h_1 - h_2} = 0.8$$
$$W_{1\to2'} = W_{1\to2} * 0.8 = (h_1 - h_2) * 0.8$$
$$\therefore W_{1\to2'} = (2800 - 1808.85) * 0.8 = 792.92 kJ/kg$$
$$\eta_{th.} = \frac{W_{1\to2'}}{Q_1} = \frac{W_{1\to2'}}{h_1 - h_4} = \frac{W_{1\to2'}}{(h_1 - h_3) - (h_4 - h_3)}$$
$$\therefore \eta_{th.} = \frac{792.92}{(2800 - 112) - 4.21} = 0.295 = 29.5\%$$
$$Work - ratio = \frac{W_{1\to2'} - W_{3\to4}}{W_{1\to2'}} = \frac{(h_1 - h_{2'}) - (h_4 - h_3)}{(h_1 - h_{2'})} = \frac{792.92 - 4.21}{792.92} = 0.995$$
$$s.s.c. = \frac{3600}{(h_1 - h_{2'}) - (h_4 - h_3)} = \frac{3600}{792.92 - 4.21} = 4.56 kg/kW.h$$

Example (6.3):-

A hat reservoir at 800°C and a cold reservoir at 15°C are available. Calculate the thermal efficiency and the work ratio of a Carnot cycle using air as the working fluid, if the maximum and minimum pressures in the cycle are 210 bar and 1 bar.

Solution:

T_1=800°C, T_2=15°C, η_{Carnot}=?, *Work ratio*=?, air, p_4=210 bar, p_2=1 bar



$$\eta_{Carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{288}{1073} = 0.732 = 73.2$$

$$Work-ratio = \frac{Network}{Grosswork} = \frac{area(41234)}{area(412BA4)}$$

$$Network = Q_1 - Q_2 = T_1(s_1 - s_4) - T_2(s_1 - s_4) = (T_1 - T_2)(s_1 - s_4)$$

$$(s_1 - s_4) = (s_A - s_4) - (s_A - s_2) \Rightarrow s_1 = s_2$$

$$(s_A - s_4) = R \ln \frac{p_4}{p_A} = R \ln \frac{p_4}{p_2} = 0.287 \ln \frac{210}{1} = 1.535 kJ/kg.K$$

$$(s_A - s_2) = c_p \ln \frac{T_A}{T_2} = c_p \ln \frac{T_1}{T_2} = 1.005 \ln \frac{1073}{288} = 1.322 kJ/kg.K$$

$$\therefore (s_1 - s_4) = 1.535 - 1.322 = 0.213 kJ/kg.K$$

$$\therefore Network = (800 - 15)(0.213) = 167.21 kJ/kg$$

$$Grosswork = W_{4\to1} + W_{1\to2}$$

$$W_{4\to1} = Q_{4\to1}$$

W=*Q* (for isothermal process for ideal gas)

$$W_{4\to1} = T_1(s_1 - s_4) = 1073(0.213) = 228.55 kJ/kg$$

$$W_{1\to2} = u_1 - u_2 = c_v(T_1 - T_2) \Rightarrow Q_{1\to2} = 0, (isentropic-process)$$

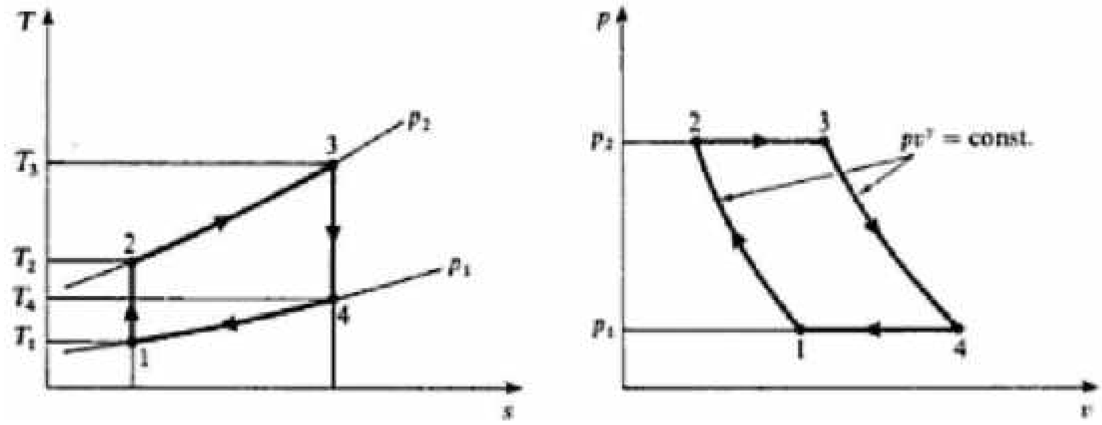
$$W_{1\to2} = 0.718(800 - 15) = 563.63 kJ/kg$$

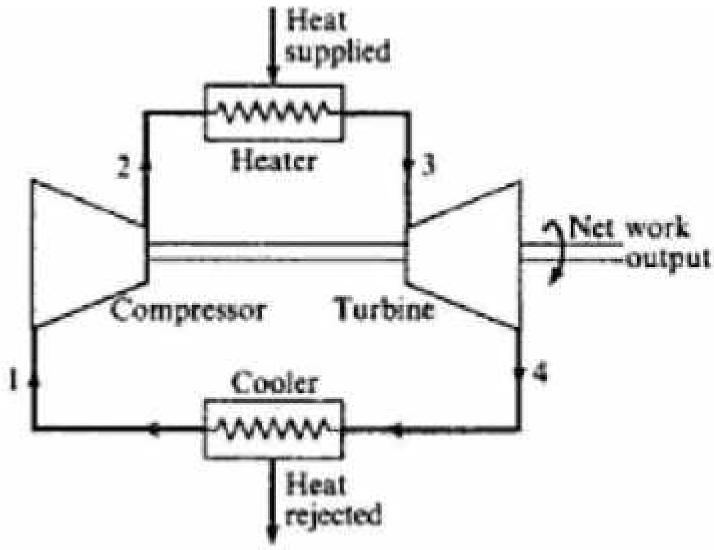
$$\therefore Grosswork = 228.55 + 563.63 = 792.2 kJ/kg$$

$$\therefore Workratio = \frac{167.21}{792.2} = 0.211$$

<u>The Constant Pressure Cycle:-</u>

In this cycle the heat supply and heat rejection processes occur reversibly at constant pressure. The expansion and compression processes are isentropic. The cycle is shown on a *T-s* and *p-υ* diagrams in figures below.





Work input to compressor, $W_{1\to 2} = (h_2 - h_1) = c_p(T_2 - T_1)$

Work output from turbine, $W_{3\to 4} = (h_3 - h_4) = c_p(T_3 - T_4)$

Heat supplied in heater, $Q_1 = (h_3 - h_2) = c_p(T_3 - T_2)$

Heat rejected in cooler, $Q_2 = (h_4 - h_1) = c_p(T_4 - T_1)$

$$\boxed{\eta_{th.} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_4 - T_1}{T_3 - T_2}}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \Rightarrow \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$\boxed{\frac{T_2}{T_1} = r_p^{\left(\frac{\gamma-1}{\gamma}\right)} \Rightarrow \frac{T_3}{T_4} = r_p^{\left(\frac{\gamma-1}{\gamma}\right)}}$$

Where;

r_p : is the pressure ratio (p_2/p_1)

$$T_2 = T_1 r_p^{(\frac{\gamma-1}{\gamma})} \Rightarrow T_3 = T_4 r_p^{(\frac{\gamma-1}{\gamma})}$$

$$\therefore \eta_{th.} = 1 - \frac{T_4 - T_1}{(T_4 - T_1) r_p^{(\frac{\gamma-1}{\gamma})}}$$

$$\boxed{\eta_{th.} = 1 - \frac{1}{r_p^{(\frac{\gamma-1}{\gamma})}}}$$

****:** for the constant pressure cycle, the thermal efficiency depends only on the pressure ratio.

$$work\ ratio = \frac{Net\ work}{Gross\ work} = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_4)}$$

$$\boxed{\therefore Workratio = 1 - \frac{T_2 - T_1}{T_3 - T_4}}$$

$$workratio = 1 - \frac{T_1(r_p^{(\frac{\gamma-1}{\gamma})} - 1)}{T_3(1 - \frac{1}{r_p^{(\frac{\gamma-1}{\gamma})}})} = 1 - \frac{T_1(r_p^{(\frac{\gamma-1}{\gamma})} - 1)}{T_3(r_p^{(\frac{\gamma-1}{\gamma})} - 1)} r_p^{(\frac{\gamma-1}{\gamma})}$$

$$\boxed{\therefore Workratio = 1 - \frac{T_1}{T_3} r_p^{(\frac{\gamma-1}{\gamma})}}$$

****:** It can be seen from this equation that the work ratio depends, not only on the pressure ratio but also on the ratio of minimum and maximum temperatures.

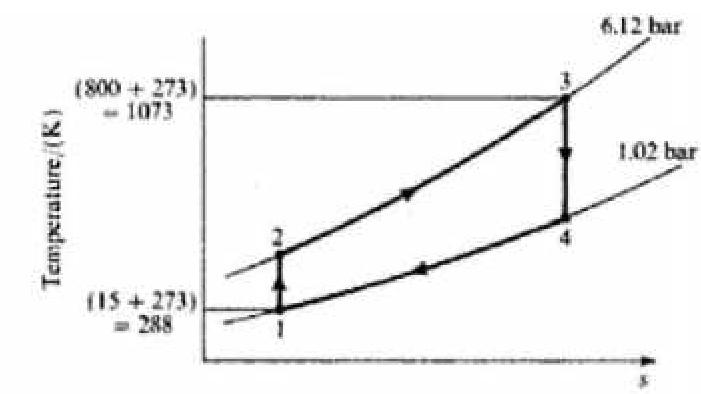
Example (6.4):-

In a gas turbine unit air is drawn at 1.02 bar and 15°C, and is compressed to 6.12 bar. Calculate the thermal efficiency and the work ratio of the ideal constant pressure cycle, when the maximum cycle temperature is limited to 800°C.

Solution:

Air, p_1=1.02 bar , T_1=15°C, p_2=6.12 bar, $\eta_{th.}=?$, *Work ratio*=?, T_3=800°C

$$\eta_{th.} = 1 - \frac{1}{r_p^{(\frac{\gamma-1}{\gamma})}}$$



$$r_p = \frac{p_2}{p_1} = \frac{6.12}{1.02} = 6$$

$$\therefore \eta_{th.} = 1 - \frac{1}{(6)^{\frac{0.4}{1.4}}} = 0.401 = 40.1\%$$

$$\therefore Workratio = 1 - \frac{T_1}{T_3} r_p^{(\frac{\gamma-1}{\gamma})} = 1 - \frac{288}{1073}(6)^{\frac{0.4}{1.4}} = 0.552 = 55.2\%$$

Or) $work\ ratio = \dfrac{Net\ work}{Gross\ work}$

$$Network = c_p(T_3 - T_4) - c_p(T_2 - T_1)$$

$$T_2 = T_1 r_p^{(\frac{\gamma-1}{\gamma})} = 288(6)^{\frac{0.4}{1.4}} = 480.5K$$

$$T_4 = \frac{T_3}{r_p^{(\frac{\gamma-1}{\gamma})}} = \frac{1073}{(6)^{\frac{0.4}{1.4}}} = 643.1K$$

$$\therefore Network = 1.005[1073 - 643.1 - 480.5 + 288] = 238.6 kJ/kg$$

$$Grosswork = c_p(T_3 - T_4) = 1.005(1073 - 643.1) = 432.05 kJ/kg$$

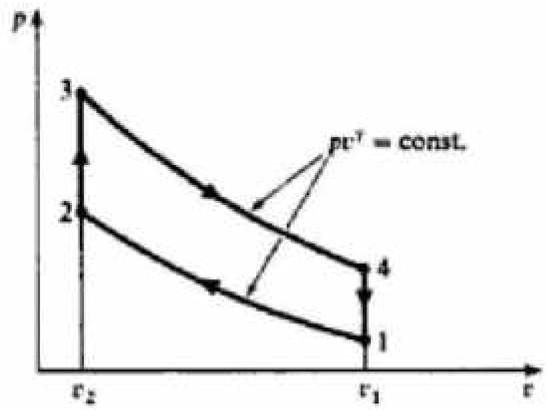
$$\therefore workratio = \frac{238.6}{432.05} = 0.552 = 55.2\%$$

The air standard cycle:-

(A) The Otto Cycle:

The Otto cycle is the ideal air standard cycle for the petrol engine, the gas engine, and the high-speed oil engine.

The cycle is shown in the figure below on a *p-υ* diagram.



($1 \rightarrow 2$): Isentropic compression.

($2 \rightarrow 3$): Constant volume heating.

($3 \rightarrow 4$): Isentropic expansion.

($4 \rightarrow 1$): Constant volume cooling.

$$\eta_{th.} = 1 - \frac{Q_2}{Q_1}$$

$$Q_1 = c_\upsilon (T_3 - T_2)$$

$$Q_2 = c_\upsilon (T_4 - T_1)$$

$$\boxed{\therefore \eta_{th.} = 1 - \frac{T_4 - T_1}{T_3 - T_2}}$$

Since processes; $1 \rightarrow 2$ and $3 \rightarrow 4$ are isentropic, then

$$\frac{T_2}{T_1} = \left(\frac{\upsilon_1}{\upsilon_2}\right)^{\gamma - 1} \Rightarrow \frac{T_3}{T_4} = \left(\frac{\upsilon_4}{\upsilon_3}\right)^{\gamma - 1} = \left(\frac{\upsilon_1}{\upsilon_2}\right)^{\gamma - 1}$$

$$\boxed{\frac{T_2}{T_1} = r_\upsilon^{\ \gamma - 1}} \qquad \boxed{\frac{T_3}{T_4} = r_\upsilon^{\ \gamma - 1}}$$

r_υ : is the compression ratio (υ_1/υ_2)

$T_2 = T_1 r_\upsilon^{\ \gamma - 1}$ and $T_3 = T_4 r_\upsilon^{\ \gamma - 1}$

$$\boxed{\therefore \eta_{th.} = 1 - \frac{1}{r_\upsilon^{\gamma - 1}}}$$

$$\therefore \eta_{th.} = 1 - \frac{T_4 - T_1}{(T_4 - T_1) r_\upsilon^{\gamma - 1}}$$

****:** It can be seen from this equation that the thermal efficiency of Otto cycle depend only on the compression ratio; r_υ.

Example (6.5):-

Calculate the ideal air standard cycle efficiency based On the Otto cycle for a petrol engine with a cylinder bore of 50 mm, and a stroke of 75 mm, and a clearance volume of 21.3cm^3.

Solution:

$$\eta_{th.} = 1 - \frac{1}{r_\upsilon^{\gamma - 1}}$$

$$r_\vartheta = \frac{swept\ volume + clearance\ volume}{clearance\ volume} = \frac{\vartheta_1}{\vartheta_2}$$

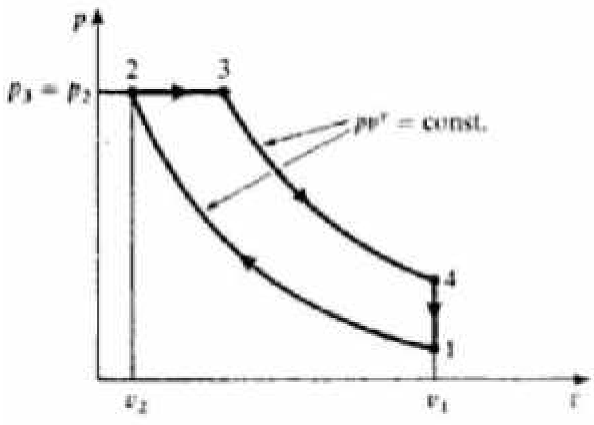
Swept volume=(π/4)(50)^2*75=147.3cm^3

$r_\upsilon = \upsilon_1/\upsilon_2$=(147.3+21.3)/21.3=7.92

$$\therefore \eta_{th.} = 1 - \frac{1}{(7.92)^{0.4}} = 0.563 = 56.3\%$$

(B) The Diesel Cycle:

The engine in use today which are called diesel engines. The ideal air standard diesel cycle as shown below.



$(1 \rightarrow 2)$: Isentropic compression.

$(2 \rightarrow 3)$: Constant pressure heating.

$(3 \rightarrow 4)$: Isentropic expansion.

$(4 \rightarrow 1)$: Constant volume cooling.

$$\eta_{th.} = 1 - \frac{Q_2}{Q_1}$$

$$Q_1 = c_p (T_3 - T_2) \Rightarrow Q_2 = c_v (T_4 - T_1)$$

$$\boxed{\eta_{th.} = 1 - \frac{\beta^{\gamma} - 1}{\gamma(\beta - 1) r_v^{\gamma - 1}}}$$

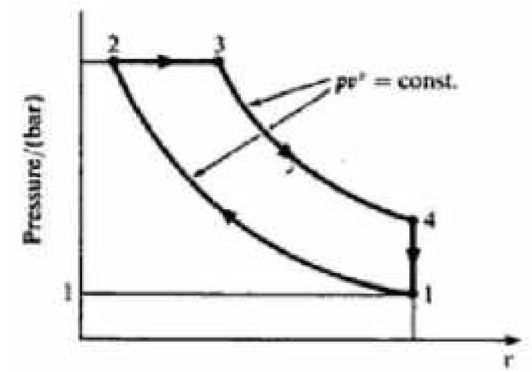
$\beta = (\upsilon_3/\upsilon_2)$: cut-off ratio

Example (6.6):-

A diesel engine has an inlet temperature and pressure of 15°C and 1 bar respectively. The compression ratio is 12/1 and the maximum cycle temperature is 1100°C. Calculate the air standard thermal efficiency based on the diesel cycle.

Solution:

T_1=15°C, p_1=1 bar , r_υ=12, T_3=1100°C, $\eta_{th.}=?$



$$\eta_{th.} = 1 - \frac{\beta^{\gamma} - 1}{\gamma(\beta - 1) r_v^{\gamma - 1}}$$

$$\beta = \frac{\upsilon_3}{\upsilon_2}$$

$(2 \rightarrow 3)$ constant pressure process; $\frac{\upsilon}{T} = c$

$$\frac{3_2}{T_2}=\frac{3_3}{T_3}\Rightarrow\frac{3_3}{3_2}=\frac{T_3}{T_2}$$

$$\frac{T_2}{T_1}=\left(\frac{3_1}{3_2}\right)^{¥-1}\Rightarrow\therefore T_2=288(12)^{0.4}=778.2K$$

$$\therefore\quad=\frac{3_3}{3_2}=\frac{T_3}{T_2}=\frac{1373}{778.2}=1.764$$

$$\therefore ¦_{th.}=1-\frac{(1.764)^{1.4}-1}{0.764(12)^{0.4}*1.4}=0.58=58\%$$

$$or)¦_{th.}=1-\frac{Q_2}{Q_1}$$

$$Q_1=c_p(T_3-T_2)=1.005(1373-778.2)=597.8kJ/kg$$

$$Q_2=c_3(T_4-T_1)$$

$$\frac{T_4}{T_3}=\left(\frac{3_3}{3_4}\right)^{¥-1}$$

$$\frac{3_3}{3_4}=\frac{3_3}{3_1}\Rightarrow\frac{3_2}{T_2}=\frac{3_3}{T_3}\Rightarrow\therefore\frac{3_3}{3_2}=\frac{T_3}{T_2}=1.764$$

$$\frac{3_3}{3_4}=\frac{3_3}{3_1}=\frac{3_2}{3_1}*\frac{3_3}{3_2}=\frac{1}{12}*1.764=0.147$$

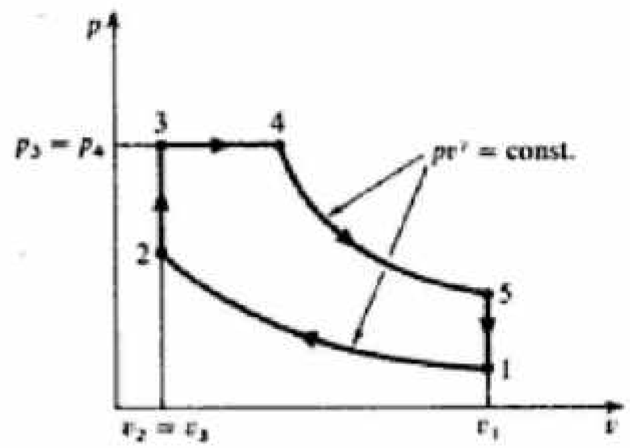
$$T_4=T_3\left(\frac{3_3}{3_4}\right)^{¥-1}=1373(0.147)^{0.4}=637.7K$$

$$\therefore Q_2=0.718(637.7-288)=251.1kJ/kg$$

$$\therefore ¦_{th.}=1-\frac{251.1}{597.8}=0.58=58\%$$

(C) The Dual Combustion Cycle:

The dual combustion cycle or the mixed cycle represented on a *p-υ* diagram as shown in figure below.



$(1\rightarrow2)$: Isentropic compression.

$(2\rightarrow3)$: Constant volume heating.

$(3\rightarrow4)$: Constant pressure heating.

$(4\rightarrow5)$: Isentropic expansion.

$(5\rightarrow1)$: Constant volume cooling

**: The heat is supplied in two parts; the first part at constant volume and the remainder at constant pressure.

$$\eta_{th.} = 1 - \frac{Q_2}{Q_1}$$

$$Q_1 = c_\upsilon (T_3 - T_2) + c_p (T_4 - T_3)$$

$$Q_2 = c_\upsilon (T_5 - T_1)$$

$$\boxed{\eta_{th.} = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)] r_\upsilon^{\gamma - 1}}}$$

Where;

(k): the ratio of pressure; $k=p_3/p_2$

(β): the ratio of volume; $\beta=\upsilon_4/\upsilon_3$

(r_υ): the compression ratio; $r_\upsilon=\upsilon_1/\upsilon_2$

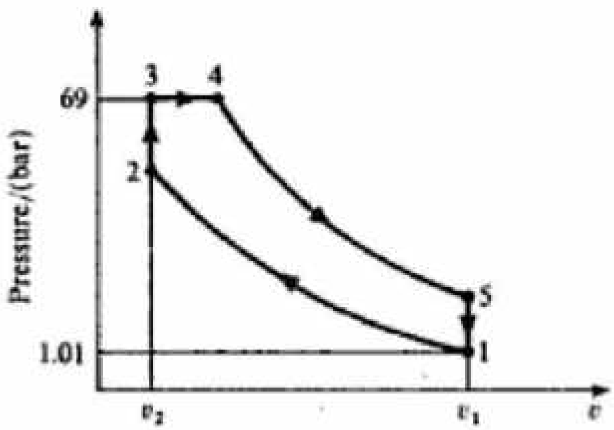
Example (6.7):-

An oil engine takes in air at 1.01 bar, 20°C and the maximum cycle pressure is 69 bar. The compressor ratio is 18/1. Calculate the air standard thermal efficiency based on the dual-combustion cycle, Assume that the heat added at constant volume is equal to the heat added at constant pressure.

Solution:

p_1=1.01 bar, T_1=20°C, p_3=69 bar, r_υ=18/1, $\eta_{th.}=?$

$$\eta_{th.} = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)] r_\upsilon^{\gamma - 1}}$$



$$r_\upsilon = \frac{\upsilon_1}{\upsilon_2} = 18 \Rightarrow \beta = \frac{\upsilon_4}{\upsilon_3}$$

$$\frac{\upsilon_4}{T_4} = \frac{\upsilon_3}{T_3} \Rightarrow \frac{\upsilon_4}{\upsilon_3} = \frac{T_4}{T_3}$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2} \Rightarrow \frac{p_3}{p_2} = \frac{T_3}{T_2}$$

$$\frac{T_2}{T_1} = \left(\frac{\upsilon_1}{\upsilon_2}\right)^{\gamma - 1} \Rightarrow \therefore T_2 = 293(18)^{0.4} = 931K$$

$$p_1\upsilon_1^{\gamma} = p_2\upsilon_2^{\gamma} \Rightarrow \therefore p_2 = 1.01(18)^{1.4} = 57.8bar$$

$$\therefore T_3 = T_2\left(\frac{p_3}{p_2}\right) = 931\left(\frac{69}{57.8}\right) = 1111.4K$$

Heat supplied at constant volume=Heat supplied at constant pressure

$$c_\upsilon (T_3 - T_2) = c_p (T_4 - T_3)$$

$$0.718(1111.4 - 931) = 1.005(T_4 - 1111.4) \Rightarrow \therefore T_4 = 1240.3K$$

$$\therefore \quad = \frac{\upsilon_4}{\upsilon_3} = \frac{T_4}{T_3} = \frac{1240.3}{1111.4} = 1.116$$

$$k = \frac{p_3}{p_2} = \frac{69}{57.8} = 1.194$$

$$\therefore \eta_{th.} = 1 - \frac{1.194(1.116)^{1.4} - 1}{[0.194 + 1.4 * 1.194 * 0.116](18)^{0.4}} = 1 - \frac{0.3923}{1.2326} = 68.2\%$$

$$or)\eta_{th.} = 1 - \frac{Q_2}{Q_1}$$

$$Q_1 = c_v(T_3 - T_2) + c_p(T_4 - T_3) = 2c_v(T_3 - T_2)$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2} \Rightarrow \frac{T_3}{T_2} = \frac{p_3}{p_2}$$

$$T_2 = T_1\left(\frac{\upsilon_1}{\upsilon_2}\right)^{\gamma - 1} = 931K$$

$$p_2 = p_1\left(\frac{\upsilon_1}{\upsilon_2}\right)^{\gamma} = 57.8bar$$

$$\therefore T_3 = T_2\left(\frac{p_3}{p_2}\right) = 1111.4K$$

$$\therefore Q_1 = 2 * 0.718(1111.4 - 931) = 259.05kJ/kg$$

$$Q_2 = c_v(T_5 - T_1)$$

$$\frac{T_5}{T_4} = \left(\frac{\upsilon_4}{\upsilon_5}\right)^{\gamma - 1} = \left(\frac{\upsilon_4/\upsilon_5}{\upsilon_1/\upsilon_2}\right)^{\gamma - 1} = \left(\frac{1.116}{18}\right)^{0.4} = 0.3288$$

$$\therefore T_5 = 0.3288 * 1240.3 = 407.8K$$

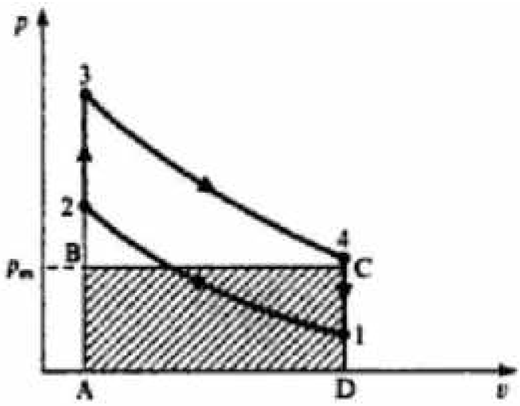
$$\therefore Q_2 = 0.718(407.8 - 293) = 82.4kJ/kg$$

$$\therefore \eta_{th.} = 1 - \frac{82.4}{259.05} = 68.2\%$$

Mean Effective Pressure:

The mean effective pressure is defined as the height of a rectangle having the same length and area as the cycle plotted on a *p-υ* diagram. This is illustrated for an Otto cycle in figure below. The rectangle ABCDA is equal to area 12341. Then the mean effective pressure, p_m, is the height AB of the rectangle. The work done per kg of air can therefore be written as;

$$\boxed{W = area(ABCDA) = p_m(\upsilon_1 - \upsilon_2)}$$



Example (6.8):-

Calculate the mean effective pressure for the cycle of Example 6.7.In Example 6.7 the heat supplied, Q_1 , and the cycle efficiency were found to be 260 kJ/kg and 68.2% respectively.

Solution:

$$\eta_{th.} = \frac{W}{Q_1}$$

$$W = \eta Q_1 = 0.682 * 260 = 177 kJ/kg$$

$$W = p_m(\upsilon_1 - \upsilon_2)$$

$$r_\upsilon = \frac{\upsilon_1}{\upsilon_2} = 18$$

$$(\upsilon_1 - \upsilon_2) = (\upsilon_1 - \frac{\upsilon_1}{18}) = \frac{17}{18}\upsilon_1$$

$$\because p_1\upsilon_1 = RT_1 \Rightarrow \therefore \upsilon_1 = \frac{RT_1}{p_1} = \frac{287*293}{1.01*10^5} = 0.8326 m^3/kg$$

$$\therefore (\upsilon_1 - \upsilon_2) = 0.786 m^3/kg$$

$$\therefore p_m = \frac{177*1000}{o.786} = 225190.8 N/m^2 = 2.25 bar$$