## Chapter four

## Reversible and Irreversible Processes

## (A) Reversible non-flow processes:-

## 1-Constant volume process:-

For Steam:
$Q=\left(u_{2}-u_{1}\right)+W$
$W=\int_{v_{1}}^{v_{2}} p d v=0$
$Q=u_{2}-u_{1}$


For mass; $m$ of the working substance
$Q=U_{2}-U_{1}$
For Ideal gas:
$W=0$
$Q=u_{2}-u_{1}=\mathrm{c}_{v}\left(T_{2}-T_{1}\right)$
Note:- All the heat supplied in a constant volume process

goes to increasing the internal energy.

## 2-Constant pressure process:-

For Steam:
$W=\int_{v_{1}}^{v_{2}} p d v=p \int_{v_{1}}^{v_{2}} d v=p\left(v_{2}-v_{1}\right)$
$Q=\left(u_{2}-u_{1}\right)+W$
$Q=\left(u_{2}-u_{1}\right)+p\left(v_{2}-v_{1}\right)$
$Q=\left(u_{2}+p v_{2}\right)-\left(u_{1}+p v_{1}\right)$


$$
Q=h_{2}-h_{1}
$$

For mass; $m$ of a fluid ; $Q=H_{2}-H_{1}$

## For Ideal gas:

$W=p\left(v_{2}-v_{1}\right)$
$Q=h_{2}-h_{1}=c_{p}\left(T_{2}-T_{1}\right)$


## Example(4.1):

A mass of 0.05 kg of a certain fluid is healed at a constant pressure of 2 bar until the volume occupied is $0.0658 \mathrm{~m}^{3}$. Calculate the heat supplied and the work done:
(i) when the fluid is steam, initially dry saturated;
(ii) when the fluid is air, initially at $130^{\circ} \mathrm{C}$.

Solution:
$m=0.05 \mathrm{~kg}, \mathrm{p}_{1}=\mathrm{p}_{2}=2 \mathrm{bar}, V_{2}=0.0658 \mathrm{~m}^{3}, Q=?, W=$ ?

## (i):

$W=p\left(v_{2}-v_{1}\right)$
$v_{1}=v_{g}=0.8856 \mathrm{~m}^{3} / \mathrm{kg}$
$v_{2}=\frac{V}{m}=\frac{0.0658}{0.05}=1.316 \mathrm{~m}^{3} / \mathrm{kg}$
$W=2 * 10^{5}(1.316-0.8856)=86080 \mathrm{~J} / \mathrm{kg}$
Work done by the total mass $=86080 * 0.05=4304 \mathrm{~J}$
$Q=h_{2}-h_{1}$
$h_{1}=h_{\mathrm{g}}=2707 \mathrm{~kJ} / \mathrm{kg}$ at $\mathrm{p}=2 \mathrm{bar}$
$h_{2}=3072 \mathrm{~kJ} / \mathrm{kg}$ at $\mathrm{p}=2$ bar and $\mathrm{v}=1.316 \quad \mathrm{~m}^{3} / \mathrm{kg}$
$Q=3072-2707=365 \mathrm{~kJ} / \mathrm{kg}$
Heat supplied $=365^{*} 0.05=18.25 \mathrm{~kJ}$
(ii): air; $T_{1}=130^{\circ} \mathrm{C}$
$W=p\left(v_{2}-v_{1}\right)$
$p_{1} V_{1}=m R T_{1}$
$V_{1}=\frac{m R T_{1}}{p_{1}}=\frac{0.05 * 287 * 403}{2 * 10^{5}}=0.0289 \mathrm{~m}^{3}$

$v_{1}=0.578 \mathrm{~m}^{3} / \mathrm{kg}$
$v_{2}=1.316 \mathrm{~m}^{3} / \mathrm{kg}$
$W=2 * 10^{5}(1.316-0.578)=147600 \mathrm{~J} / \mathrm{kg}$
Total work $=147600 * 0.05=7380 \mathrm{~J}=7.38 \mathrm{~kJ}$
Or
$W=p\left(v_{2}-v_{1}\right)=R\left(T_{2}-T_{1}\right)$
$p_{2} V_{2}=m R T_{2} \Rightarrow T_{2}=\frac{2 * 10^{5} * 0.0658}{0.05 * 287}=917 \mathrm{~K}$
$W=m R\left(T_{2}-T_{1}\right)=0.05 * 0.287(917-403)=7.38 \mathrm{~kJ}$
$Q=m c_{p}\left(T_{2}-T_{1}\right)=0.05 * 1.005 *(917-403)=25.83 \mathrm{~kJ}$

## 3-Constant temperature process (Isothermal process):-

A process at constant temperature is called an isothermal process.

## For Steam:

$W=$ shaded area
$Q=\left(u_{2}-u_{1}\right)+W$


## For Ideal gas:

$p v=R T$
$p v=$ cons $\tan t \Rightarrow p v=c$
$p_{1} \mathrm{v}_{1}=p_{2} \mathrm{v}_{2}=p_{3} \mathrm{v}_{3}=$ $\qquad$
$W=\int_{v_{1}}^{v_{2}} p d v=\int_{v_{1}}^{v_{2}} \frac{c}{v} d v=c \int_{v_{1}}^{v_{2}} \frac{d v}{v}=p_{1} v_{1} \ln \frac{v_{2}}{v_{1}}$

$$
W=p_{1} v_{1} \ln \frac{v_{2}}{v_{1}}
$$


$W=p_{2} v_{2} \ln \frac{v_{2}}{v_{1}}$
$\frac{v_{2}}{v_{1}}=\frac{p_{1}}{p_{2}}$
$W=p_{1} v_{2} \ln \frac{p_{1}}{p_{2}}$
$p_{1} v_{1}=R T$
$W=R T \ln \frac{p_{1}}{p_{2}}$
$Q=\left(u_{2}-u_{1}\right)+W$
$u_{2}-u_{1}=c_{v}\left(T_{2}-T_{1}\right) \Rightarrow$ but $T_{2}=T_{1} \Rightarrow \therefore u_{2}=u_{1}$

$$
Q=W
$$

## Example(4.2):

Steam at 7 bar and dryness fraction 0.9 expands in a cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bar. Calculate the change of internal energy and the change of enthalpy per kg of steam, The heat supplied during the process is found to be 400 $\mathrm{kJ} / \mathrm{kg}$, calculate the work done per kilogram of steam.

## Solution:

At $p=7 \mathrm{bar}, \mathrm{T}_{\mathrm{s}}=165^{\circ} \mathrm{C}$
At $\mathrm{p}=1.5$ bar and $\mathrm{T}=165^{\circ} \mathrm{C}$ is superheated
Because at $\mathrm{p}=1.5$ bar, $\mathrm{T}_{\mathrm{s}}=111.4^{\circ} \mathrm{C}$
$u_{1}=(1-x) u_{f}+x u_{g}$
$u_{1}=(1-0.9) * 696+0.9 * 2573=2385.3 \mathrm{~kJ} / \mathrm{kg}$
Interpolation from superheated tables at 1.5 bar and $165^{\circ} \mathrm{C}$

$\frac{200-150}{2656-2580}=\frac{165-150}{u-2580}$
$u=\frac{15}{50}(2656-2580)+2580=2602.8 \mathrm{~kJ} / \mathrm{kg}$
$\therefore u_{2}=2602.8 \mathrm{~kJ} / \mathrm{kg}$
Gain in internal energy $=u_{2}-u_{1}=217.5 \mathrm{~kJ} / \mathrm{kg}$
$h_{1}=h_{f}+x h_{f g}=697+0.9 * 2067=2557.3 \mathrm{~kJ} / \mathrm{kg}$
Interpolation from superheated tables
$h=\frac{15}{50}(2873-2773)+2773=2803 \mathrm{~kJ} / \mathrm{kg}$
$\therefore h_{2}=2803 \mathrm{~kJ} / \mathrm{kg}$
$\therefore h_{2}-h_{1}=245.7 \mathrm{~kJ} / \mathrm{kg}$
$Q=\left(u_{2}-u_{1}\right)+W$
$\therefore W=400-217.5=182.5 \mathrm{~kJ} / \mathrm{kg}$

## Example(4.3):

1 kg of nitrogen (molar mass $28 \mathrm{~kg} / \mathrm{kmol}$ ) is compressed reversibly and isothermally from 1.01 bar, $20^{\circ} \mathrm{C}$ to 4.2 bar. Calculate the work done and the heat flow during the process. Assume nitrogen to be a perfect gas.

## Solution:

$m=1 \mathrm{~kg}, \mathrm{~N}_{2}, \quad \mathrm{M}=28, \mathrm{~T}=\mathrm{C}, \mathrm{p}_{1}=1.01 \mathrm{bar}, \mathrm{T}_{1}=\mathrm{T}_{2}=20^{\circ} \mathrm{C}$
$\mathrm{p}_{2}=4.2 \mathrm{bar}, W=?, Q=$ ?
$W=R T \ln \frac{p_{1}}{p_{2}}$
$R=\frac{R_{o}}{M}=\frac{8314}{28}=297 \mathrm{~J} / \mathrm{kg} \cdot \mathrm{K}$
$W=297 * 293 \ln \frac{1.01}{4.2}=-124 \mathrm{~kJ} / \mathrm{kg}$
The work input $=124 \mathrm{~kJ} / \mathrm{kg}$
$Q=W=-124 \mathrm{~kJ} / \mathrm{kg}$
The heat is rejected

## 4-Adiabatic process $(Q=0)$ :-

An adiabatic process is one in which no heat is transferred to or from the fluid during the process. Such a process can be reversible or irreversible.
For reversible adiabatic non-flow process:-
For Steam:
$Q=\left(u_{2}-u_{1}\right)+W$
$\because Q=0$
$\therefore W=u_{1}-u_{2}$
Note:- For an adiabatic process to take place, perfect thermal insulation for the system must be available.

## For Ideal gas:

$d Q=d u+d W$
For a reversible process, $d W=p d v$
$d Q=d u+p d v=0$
For a perfect gas ; $p v=R T \Rightarrow d u=c_{v} d T$
$d(p v)=d(R T)$
$p d v+v d p=R d T$
$\therefore d u+p d v=0$

$$
\begin{aligned}
& c_{v} d T+p d v=0 \\
& \frac{c_{v}}{R}[p d v+v d p]+p d v=0 \\
& c_{v}=\frac{R}{\gamma-1} \Rightarrow \gamma-1=\frac{R}{c_{v}}
\end{aligned}
$$

$$
\frac{1}{\gamma-1}[p d v+v d p]+p d v=0
$$

$$
p d v+v d p+(\gamma-1) p d v=0
$$

$$
p d v+v d p+\gamma p d v-p d v=0
$$

$$
[v d p+\gamma p d v=0] \div p v
$$

$$
\frac{d p}{p}+\gamma \frac{d v}{v}=0
$$

$\ln p+\gamma \ln v=C \Rightarrow \ln p+\ln v^{\gamma} C$
$\ln \left(p v^{\gamma}\right)=C$
$p v^{\gamma}=e^{C}=$ cons $\tan t$

$$
p v^{\gamma}=C \quad \text { Adiabatic process for perfect gas }
$$

$p_{1} v_{1}^{\gamma}=p_{2} v_{2}^{\gamma}$

$$
\frac{p_{1}}{p_{2}}=\left(\frac{v_{2}}{v_{1}}\right)^{\gamma}
$$

$p v=R T \Rightarrow p=\frac{R T}{v}$
$\frac{R T}{v} v^{\gamma}=C \Rightarrow T v^{\gamma-1}=\frac{C}{R}=$ cons $\tan t$

$$
T v^{\gamma-1}=C
$$

$$
\frac{T_{1}}{T_{2}}=\left(\frac{v_{2}}{v_{1}}\right)^{\gamma-1}
$$

$p v=R T \Rightarrow v=\frac{R T}{p}$
$p\left(\frac{R T}{p}\right)^{\gamma}=C \Rightarrow \frac{T^{\gamma}}{p^{\gamma-1}}=\frac{C}{R}=$ cons $\tan t$

$$
\frac{T_{1}}{T_{2}}=\left(\frac{p_{1}}{p_{2}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}
$$

The work done in an adiabatic process per kg of gas
$Q=\left(u_{2}-u_{1}\right)+W$
$\therefore W=u_{1}-u_{2}=c_{v}\left(T_{1}-T_{2}\right)$
$\because c_{v}=\frac{R}{\gamma-1}$

$$
\therefore W=\frac{R\left(T_{1}-T_{2}\right)}{\gamma-1}
$$


$\because p v=R T \Rightarrow p_{1} v_{1}=R T_{1}, p_{2} v_{2}=R T_{2}$

$$
\therefore W=\frac{p_{1} v_{1}-p_{2} v_{2}}{\gamma-1}
$$

**: A reversible adiabatic process for a perfect gas is shown on a p-v diagram.
$W=\int_{v_{1}}^{v_{2}} p d v=\int_{v_{1}}^{v_{2}} \frac{c}{v^{\gamma}} d v=c \int_{v_{1}}^{v_{2}} \frac{d v}{v^{\gamma}}=c\left[\frac{v^{-\gamma+1}}{-\gamma+1}\right]_{v_{1}}^{v_{2}}=c\left[\frac{v_{1}^{-\gamma+1}-v_{2}^{-\gamma+1}}{\gamma-1}\right]$
$c=p_{1} \mathbf{v}_{1}=p_{2} \mathbf{v}_{2}$
$W=\frac{p_{1} v_{1}-p_{2} v_{2}}{\gamma-1}$

## Example(4.4):

1 kg of steam at 100 bar and $375^{\circ} \mathrm{C}$ expands reversibly in a perfectly thermally insulated cylinder behind a piston until the pressure is 38 bar and the steam is then dry saturated.
Calculate the work done by the steam.
Solution:
$\mathrm{m}=1 \mathrm{~kg}, \mathrm{p}_{1}=100$ bar, $\mathrm{T}_{1}=375^{\circ} \mathrm{C}, Q=0, \mathrm{p}_{2}=38 \mathrm{bar}, W=$ ? at $\mathrm{p}=100$ bar $\mathrm{T}_{\mathrm{s}}=311^{\circ} \mathrm{C}<\mathrm{T}_{1}\left(375^{\circ} \mathrm{C}\right)$
the steam is superheated at point (1)
$W=u_{1}-u_{2}$
From superheated table we find $u_{1}$
At 100 bar and $375^{\circ} \mathrm{C}$
$v=v_{1}=0.02453 \mathrm{~m}^{3} / \mathrm{kg}, \quad h=h_{1}=3017 \mathrm{~kJ} / \mathrm{kg}$

$\therefore u=h-p v$
$u_{1}=3017-\frac{100 * 10^{5} * 0.02453}{1000}=2771.7 \mathrm{~kJ} / \mathrm{kg}$
$u_{2}=u_{g}$ atp $=38 b a r$
$\therefore u_{2}=2602 \mathrm{~kJ} / \mathrm{kg}$
$\therefore W=2771.7-2602=169.7 \mathrm{~kJ} / \mathrm{kg}$

## Example(4.5):

Air at 1.02 bar, $22^{\circ} \mathrm{C}$, initially occupying a cylinder volume of $0.015 \mathrm{~m}^{3}$, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate the final temperature, the final volume, and the work done on the mass of air in the cylinder.
Solution:
air, $\mathrm{p}_{1}=1.02$ bar, $\mathrm{T}_{1}=22^{\circ} \mathrm{C}, V_{1}=0.015 \mathrm{~m}^{3}$
$\mathrm{p}_{2}=6.8 \mathrm{bar}, \mathrm{T}_{2}=$ ?, $V_{2}=$ ?, $W=$ ?
for adiabatic process
$\frac{T_{1}}{T_{2}}=\left(\frac{p_{1}}{p_{2}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$
$T_{2}=T_{1}\left(\frac{p_{2}}{p_{1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$

$T_{2}=295\left(\frac{6.8}{1.02}\right)^{\left(\frac{0.4}{1.4}\right)}=507.3 \mathrm{~K}$
The final temperature $=507.3-273=234.3^{\circ} \mathrm{C}$
$\frac{p_{1}}{p_{2}}=\left(\frac{V_{2}}{V_{1}}\right)^{\gamma} \Rightarrow V_{2}=V_{1}\left(\frac{p_{1}}{p_{2}}\right)^{(1 / \gamma)}$
$V_{2}=0.015\left(\frac{1.02}{6.8}\right)^{(1 / 1.4)}=0.00387 m^{3}$
$W=u_{1}-u_{2}=c_{v}\left(T_{1}-T_{2}\right)=0.718(22-234.3)=-152.4 k J / k g$
$p V=m R T$
$m=\frac{p_{1} V_{1}}{R T_{1}}=\frac{1.02 * 10^{5} * 0.015}{287 * 295}=0.0181 \mathrm{~kg}$
The total work $=-152.4 * 0.0181=2.76 \mathrm{~kJ}$

## 5- Polytropic processes:-

It is found that many processes in practice approximate to a reversible law of the form $p v^{n}=C$; where $n$ is a constant. Both vapors and perfect gases obey this type of law closely in many non-flow processes. Such processes are internally reversible.

## For Steam:

For any reversible process, $W=\int p d v$
$p v^{n}=c$
Where; $n$ and $c:$ constant
$\therefore W=\int_{v_{1}}^{v_{2}} \frac{c}{v^{n}} d v=c\left[\frac{v^{-n+1}}{-n+1}\right]_{v_{1}}^{v_{2}}=c\left[\frac{v_{1}^{1-n}-v_{2}^{1-n}}{n-1}\right]$
$c=p_{1} v_{1}^{n}=p_{2} \mathrm{v}_{2}^{n}$
$\therefore W=\frac{p_{1} v_{1}^{n} v_{1}^{1-n}-p_{2} v_{2}^{n} v_{2}^{1-n}}{n-1}$

$$
W=\frac{p_{1} v_{1}-p_{2} v_{2}}{n-1}
$$

$$
\frac{p_{1}}{p_{2}}=\left(\frac{v_{2}}{v_{1}}\right)^{n}
$$

## For Ideal gas:

$p v=R T$
for polytropic process; $p v^{n}=C$
$p=\frac{R T}{v}$
$\left(\frac{R T}{v}\right) v^{n}=c \Rightarrow T v^{n-1}=\frac{c}{R}=c$
$T_{1} \mathrm{v}_{1}^{n-1}=T_{2} \mathrm{v}_{2}^{n-1}$

$$
\begin{equation*}
\frac{T_{1}}{T_{2}}=\left(\frac{v_{2}}{v_{1}}\right)^{n-1} \tag{1}
\end{equation*}
$$

$v=\frac{R T}{p}$
$p\left(\frac{R T}{p}\right)^{n}=c \Rightarrow \frac{T^{n}}{p^{n-1}}=\frac{c}{R^{n}}=$ cons $\tan t$
$\frac{T^{n}}{p^{n-1}}=c \Rightarrow \frac{T}{p^{\left(\frac{n-1}{n}\right)}}=c$
$\frac{T_{1}}{p_{1}{ }^{\frac{n-1}{n}}}=\frac{T_{2}}{p_{2}{ }^{\frac{n-1}{n}}}$

$$
\begin{equation*}
\frac{T_{1}}{T_{2}}=\left(\frac{p_{1}}{p_{2}}\right)^{\left(\frac{n-1}{n}\right)} \tag{2}
\end{equation*}
$$

Note: It can be seen that these equations are exactly similar to the equations for a reversible adiabatic process for a perfect gas. This is mean that the reversible adiabatic process for a perfect gas is a particular case of a polytropic process with the index; $n$ equal to $\gamma$.

For polytropic process, $W=\frac{p_{1} v_{1}-p_{2} v_{2}}{n-1}$
$p_{1} v_{1}=R T_{1} \Rightarrow p_{2} v_{2}=R T_{2}$
$\therefore W=\frac{R\left(T_{1}-T_{2}\right)}{n-1}$
Or for mass; $m$
$\therefore W=\frac{m R\left(T_{1}-T_{2}\right)}{n-1}$
$Q=\left(u_{2}-u_{1}\right)+W=c_{v}\left(T_{2}-T_{1}\right)+\frac{R\left(T_{1}-T_{2}\right)}{n-1}$
$Q=\frac{R}{n-1}\left(T_{1}-T_{2}\right)-\frac{R}{\gamma-1}\left(T_{1}-T_{2}\right)$
$Q=R\left(T_{1}-T_{2}\right)\left[\frac{1}{n-1}-\frac{1}{\gamma-1}\right]$
$Q=R\left(T_{1}-T_{2}\right)\left[\frac{\gamma-1-(n-1)}{(n-1)(\gamma-1)}\right]$
$Q=R\left(T_{1}-T_{2}\right)\left[\frac{\gamma-n}{(n-1)(\gamma-1)}\right]$
$Q=\left(\frac{\gamma-n}{\gamma-1}\right)\left[\frac{R\left(T_{1}-T_{2}\right)}{n-1}\right]$

$$
Q=\left(\frac{\gamma-n}{\gamma-1}\right) W
$$

## Example(4.0):

In a steam engine the steam at the beginning of the expansion process is at 7 bar, dryness fraction 0.95 , and the expansion follows the law $p v^{1.1}=$ constant , down to a pressure of 0.34bar. Calculate the work done per kg of steam during the expansion, and the heat flow per kg of steam to or from the cylinder walls during the expansion.

## Solution:

$\mathrm{p}_{1}=7$ bar, $x=0.95, p v^{1.1}=c, \mathrm{p}_{2}=0.34 \mathrm{bar}, W=?, Q=$ ?
$W=\frac{p_{1} v_{1}-p_{2} v_{2}}{n-1}$
$v_{1}=x v_{g}$
$\mathrm{v}_{g}=0.2728 \mathrm{~m}^{3} / \mathrm{kg}($ at 7 bar$)$
$\therefore v_{1}=0.95(0.2728)=0.259 \mathrm{~m}^{3} / \mathrm{kg}$
$p_{1} v_{1}^{n}=p_{2} v_{2}^{n} \Rightarrow \frac{v_{2}}{v_{1}}=\left(\frac{p_{1}}{p_{2}}\right)^{1 / n}$
$\therefore v_{2}=v_{1}\left(\frac{p_{1}}{p_{2}}\right)^{1 / 2}=0.259\left(\frac{7}{0.34}\right)^{1 / 1.1}=4.05 \mathrm{~m}^{3} / \mathrm{kg}$
$p_{2}=0.34 \mathrm{bar} \Rightarrow \mathrm{v}_{\mathrm{g}}=4.649 \mathrm{~m}^{3} / \mathrm{kg}$
$v_{g}>v_{2}$
the steam is wet at point 2
$W=\frac{7 * 10^{5} * 0.259-0.34 * 10^{5} * 4.05}{1.1-1}=436 \mathrm{~kJ} / \mathrm{kg}$
$Q=\left(u_{2}-u_{1}\right)+W$
$u_{1}=u_{f 1}+x_{1}\left(u_{g 1}-u_{f 1}\right)=696+0.95(2573-696)=2479.15 \mathrm{~kJ} / \mathrm{kg}$
$v_{2}=x_{2} v_{g} \Rightarrow x_{2}=\frac{v_{2}}{v_{g}}=\frac{4.05}{4.649}=0.871$
$u_{2}=u_{f 2}+x_{2}\left(u_{g 2}-u_{f 2}\right)=302+0.871(2472-302)=2192.1 \mathrm{~kJ} / \mathrm{kg}$
$\therefore Q=(2192.1-2479.15)+436=148.9 \mathrm{~kJ} / \mathrm{kg}$
Heat is supplied

## Example(4.7):

1 kg of a perfect gas is compressed from $1.1 \mathrm{bar}, 27^{\circ} \mathrm{C}$ according to law, $p v^{1.3}=$ constan $t$ until the pressure is 6.6 bar. Calculate the heat flow to or from the cylinder walls.
(i) When the gas is ethane (molar mass $30 \mathrm{~kg} / \mathrm{kmol}$ ), which has $\mathrm{c}_{\mathrm{p}}=1.75 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$.
(ii) When the gal is argon (molar mass $40 \mathrm{~kg} / \underline{\mathrm{kmol}}$ ), which has $\mathrm{c}_{\mathrm{p}}=0.515 \mathrm{~kJ} / \mathrm{kl} \mathrm{K}$.

Solution:
(i)/ $\mathrm{M}=30, \mathrm{c}_{\mathrm{p}}=1.75 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K}$
$\therefore Q=\left(\frac{\gamma-n}{\gamma-1}\right) W$
$\gamma=\frac{c_{p}}{c_{v}} \Rightarrow R=c_{p}-c_{v}$
$R=\frac{R_{o}}{M}=\frac{8.314}{30}=0.277 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$
$\therefore c_{v}=c_{p}-R=1.75-0.277=1.473 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K}$
$\therefore \gamma=\frac{1.75}{1.473}=1.188$
$W=\frac{R\left(T_{1}-T_{2}\right)}{n-1}$
$\frac{T}{p^{\frac{n-1}{n}}}=c \Rightarrow \frac{T_{2}}{T_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}} \Rightarrow T_{2}=T_{1}\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}$
$T_{2}=(27+273)\left(\frac{6.6}{1.1}\right)^{\frac{0.3}{1.3}}=453.6 K$
$\therefore W=\frac{0.277(300-453.6)}{1.3-1}=-141.8 \mathrm{~kJ} / \mathrm{kg}$
$\therefore Q=\left(\frac{1.188-1.3}{1.188-1}\right)(-141.8)=84.5 \mathrm{~kJ} / \mathrm{kg}$
The heat is supplied
(ii)/ use the same method for argon
$R=\frac{8.314}{40}=0.208 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$
$c_{v}=0.515-0.208=0.307 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K}$
$\gamma=\frac{0.515}{0.307}=1.678$
$W=\frac{0.208(300-453.6)}{1.3-1}=-106.5 \mathrm{~kJ} / \mathrm{kg}$
$Q=\left(\frac{1.678-1.3}{1.678-1}\right)(-106.5)=59.4 \mathrm{~kJ} / \mathrm{kg}$
Heat is supplied
Note:- The various processes considered in previous sections are cases of the polytropic process for a perfect gas.

Where;
$n=0, p v^{0}=c \Rightarrow p=$ cons $\tan t$
$n=\infty, p v^{\infty}=c \Rightarrow p^{\frac{1}{\infty}} v=c^{\frac{1}{\infty}} \Rightarrow v=$ cons $\tan t$
$n=1, p v=c \Rightarrow T=$ cons $\tan t$
$n=\gamma, p v^{\gamma}=c$

State $1 \rightarrow$ A (constant pressure cooling; $n=0$ )
State $1 \rightarrow B$ (Isothermal compression; $n=1$ )
State $1 \rightarrow C$ (Reversible adiabatic compression; $n=\gamma$ )
State $1 \rightarrow \mathrm{D}$ (Constant volume heating; $n=\infty$ )
State $1 \rightarrow$ A' (constant pressure heating; $n=0$ )
State $1 \rightarrow B^{\prime}$ (Isothermal expansion; $n=1$ )
State $1 \rightarrow C^{\prime}($ Reversible adiabatic expansion; $n=\gamma)$


State $1 \rightarrow$ D' (Constant volume cooling; $n=\infty$ )
Note:- $(\gamma)$ is always greater than unity, then process $1 \rightarrow \mathrm{C}$ must lie between processes $1 \rightarrow \mathrm{~B}$ and $1 \rightarrow$ D similarly, process $1 \rightarrow$ C' must lie between processes $1 \rightarrow B$ ' and $1 \rightarrow$ D'.

Note:- A vapor may undergo a process according to a law $p v=c$. In this case the process is not isothermal because the characteristic equation of state, $p v=R T$, does not apply to vapor. Therefore tables must be used to find the properties at the end states, making use of the fact that $p_{1} v_{1}=p_{2} v_{2}$.

## Example(4.8):

In the cylinder of a steam engine the steam expands from 5.5 bar to 0.75 bar according to a hyperbolic law. If the steam is initially dry saturated, calculate the work done per kg of steam, and the heat flow to or from the cylinder walls.

## Solution:

$\mathrm{p}_{1}=5.5$ bar, $\mathrm{p}_{2}=0.75 \mathrm{bar}, p v=c,, W=?, Q=$ ?
$\left.W=\int_{v_{1}}^{v_{2}} p d v=\int_{v_{1}}^{v_{2}} \frac{c}{v} d v=c \ln v\right]_{v_{1}}^{v_{2}}=p_{1} v_{1} \ln \frac{v_{2}}{v_{1}}$
$v_{1}=v_{g}=0.3427 \mathrm{~m}^{3} / \mathrm{kg} \Leftarrow p_{1} 5.5 \mathrm{bar}$
$p_{1} v_{1}=p_{2} v_{2} \Rightarrow \frac{v_{2}}{v_{1}}=\frac{p_{1}}{p_{2}}$
$W=p_{1} v_{1} \ln \frac{p_{1}}{p_{2}}=5.5 * 10^{5} * 0.3427 \ln \frac{5.5}{0.75}=375.5 \mathrm{~kJ} / \mathrm{kg}$
$Q=\left(u_{2}-u_{1}\right)+W$
$u_{1}=u_{g}=2565 \mathrm{~kJ} / \mathrm{kg} \Leftarrow p_{1}=5.5 \mathrm{bar}$
$v_{2}=v_{1}\left(\frac{p_{1}}{p_{2}}\right)=0.3427\left(\frac{5.5}{0.75}\right)=2.513 \mathrm{~m}^{3} / \mathrm{kg}$
$p_{2}=0.75$ bar,$v_{2}=2.513 \mathrm{~m}^{3} / \mathrm{kg} \Rightarrow \mathrm{v}_{g}=2.217 \mathrm{~m}^{3} / \mathrm{kg}$
$v_{2}>v_{g}$ the steam is superheated at point 2
$\frac{2585-2510}{2.588-2.271}=\frac{u-2510}{2.513-2.271}$
$u_{2}=2567.3 \mathrm{~kJ} / \mathrm{kg}$
$Q=(2567.3-2565)+375.5=377.8 \mathrm{~kJ} / \mathrm{kg}$

## (B) Irreversible processes:-

## 1-Unresisted, or free expansion:

$Q=\left(u_{2}-u_{1}\right)+W$
$Q=0$
$W=0$
$u_{2}-u_{1}=0$

$$
\therefore u_{1}=u_{2}
$$


**: In a free expansion process, the internal energy initially equals the internal energy finally.

## (For ideal gas

$u=c_{v} T$
$c_{v} T_{1}=c_{v} T_{2}$

$$
\therefore T_{1}=T_{2}
$$

## 2-Throttling:

A flow fluid is said to be throttling when there is some restriction to the flow, such as, partly open valve, an orifice or any sudden reduction in the cross section of the flow.
$h_{1}+\frac{C_{1}^{2}}{2}+Q=h_{2}+\frac{C_{2}^{2}}{2}+W$
$Q=0, W=0$
$h_{1}+\frac{C_{1}^{2}}{2}=h_{2}+\frac{C_{2}^{2}}{2}$


When the velocities $C_{1}$ and $C_{2}$ are small, or when $C_{1}$ is very nearly equal to $C_{2}$, then the kinetic energy can be neglected.

$$
h_{1}=h_{2}
$$

Therefore for throttling process, the enthalpy initially is equal to the enthalpy finally.
(For ideal gas
$h=c_{p} T$
$c_{p} T_{1}=c_{p} T_{2}$

$$
\therefore T_{1}=T_{2}
$$

## 3- Adiabatic mixing:

The mixing of two streams of fluid is quite common in engineering practices, and can usually be assumed to occur adiabatically.
$Q=0$
$W=0$
Neglecting change in kinetic energy.
$H_{1}+H_{2}=H_{3}$
$m_{1}^{\bullet} h_{1}+m_{2}^{\bullet} h_{2}=m_{3}^{\bullet} h_{3}$

$m_{1}^{\bullet} h_{1}+m_{2}^{\bullet} h_{2}=\left(m_{1}^{\bullet}+m_{2}^{\bullet}\right) h_{3}$
$h=c_{p} T$
$m_{1}^{\bullet} c_{p} T_{1}+m_{2}^{\bullet} c_{p} T_{2}=\left(m_{1}^{\bullet}+m_{2}^{\bullet}\right) c_{p} T_{3}$

$$
m_{1}^{\bullet} T_{1}+m_{2}^{\bullet} T_{2}=\left(m_{1}^{\bullet}+m_{2}^{\bullet}\right) T_{3}
$$

## Example(4.9):

Air at 20 bar is initially contained in vessel A as shown in figure below, the volume of this vessel is $1 \mathrm{~m}^{3}$. The valve is opened and the air expands to fill vessels A and B. Assuming that the vessels are of equal volume ,calculate the final pressure of the air.

## Solution:

Air is a perfect gas
The process is free expansion
$u_{1}=u_{2}$
$\therefore T_{1}=T_{2}$

$p_{1} V_{1}=p_{2} V_{2}$
$V_{2}=2 V_{1}=2 * 1=2 \mathrm{~m}^{3}$
$\frac{p_{2}}{p_{1}}=\frac{V_{1}}{V_{2}} \Rightarrow p_{2}=p_{1}\left(\frac{V_{1}}{V_{2}}\right)=20\left(\frac{1}{2}\right)=10 \mathrm{bar}$
Note:- The point 1 and 2 lie on isothermal line, but the process between 1 and 2 cannot be called isothermal, since the intermediate temperatures are not the same throughout the
 process.

## Example(4.10):

Steam at 19 bar is throttled to 1 bar and the temperature after throttling is found to be $150^{\circ} \mathrm{C}$.
Calculate the initial dryness fraction of the steam.

## Solution:

The process is throttling, $\therefore h_{1}=h_{2}$
Point 2 at superheated, at $\mathrm{p}=1$ bar $\mathrm{T}_{\mathrm{s}}=99.6^{\circ} \mathrm{C}$
$\mathrm{T}_{\mathrm{s}}<\mathrm{T}$

From superheated table at $\mathrm{p}=1 \mathrm{bar}$ and $\mathrm{T}=150^{\circ} \mathrm{C}$.
$h=2777 \mathrm{~kJ} / \mathrm{kg}$
$h_{1}=h_{2}=2777 \mathrm{~kJ} / \mathrm{kg}$
$h_{1}=h_{f}+x h_{f g}$
$2777=897+x(1901) \Rightarrow x=0.989$
Note:- States 1 and 2 are fixed but the intermediate states are indeterminate, therefore for a vapor, throttling can be used as a means of finding the dryness fraction of wet steam, as in example.

## Reversible Flow Processes:-

$h_{1}+\frac{C_{1}^{2}}{2}+Q=h_{2}+\frac{C_{2}^{2}}{2}+W \Rightarrow Q=0$
$W=\left(h_{1}-h_{2}\right)+\left(\frac{C_{1}^{2}-C_{2}^{2}}{2}\right)$
If the kinetic energy terms are negligible small;
$W=\left(h_{1}-h_{2}\right)$
For ideal gas: $W=c_{p}\left(T_{1}-T_{2}\right)$
Note:- The work done in a reversible adiabatic flow process between two states $W=\left(h_{1}-h_{2}\right)$ is not equal to the work done in a reversible adiabatic non-flow process between the same states $W=\left(u_{1}-u_{2}\right)$

## Example(4.11):

A gas turbine receives gases from the combustion chamber at7 bar and $650^{\circ} \mathrm{C}$, with velocity of $9 \mathrm{~m} / \mathrm{s}$. The gases leave the turbine at 1 bar with a velocity of $45 \mathrm{~m} / \mathrm{s}$. Assuming that the expansion is adiabatic and reversible in the ideal case, calculate the work done per kg of gas , For the gases take $\gamma=1.333$ and $c_{p}=1.11 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$.
Solution:
$h_{1}+\frac{C_{1}^{2}}{2}+Q=h_{2}+\frac{C_{2}^{2}}{2}+W \Rightarrow Q=0$
$W=\left(h_{1}-h_{2}\right)+\left(\frac{C_{1}^{2}-C_{2}^{2}}{2}\right)$
For gas; $h=c_{p} T$
$\therefore W=c_{p}\left(T_{1}-T_{2}\right)+\left(\frac{C_{1}^{2}-C_{2}^{2}}{2}\right)$
$\frac{T_{1}}{T_{2}}=\left(\frac{P_{1}}{p_{2}}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{2}=T_{1}\left(\frac{p_{2}}{p_{1}}\right)^{\frac{\gamma-1}{\gamma}}=(650+273)\left(\frac{1}{7}\right)^{\frac{0.333}{1.333}}=567.7 \mathrm{~K}$
$\therefore W=1.11 * 10^{3}(923-567.7)+\frac{1}{2}\left(9^{2}-45^{2}\right)=393.4 \mathrm{~kJ} / \mathrm{kg}$

