## **Chapter three**

#### Working fluids

## (A) Ideal gas (Perfect gas):

An ideal gas is a gas which obeys the Boyle's law and Charles's law.

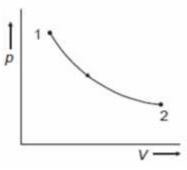
**Boyle's law:-** If a fixed mass of a gas is kept at constant temperature its volume varies inversely to its pressure.

$$V \propto \frac{1}{p}$$
, (T=constant)

$$V = \frac{C}{p}, \quad pV = C$$

$$p_1V_1 = p_2V_2$$

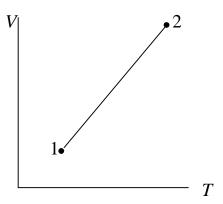
$$\frac{p_1}{p_2} = \frac{V_1}{V_2}$$



## Charles's law:-

**Statement No.(1):-** If a fixed mass of a gas receives or reject heat at constant pressure, the volume of the gas directly proportional to its absolute temperature.

$$V \propto T$$
, (p=constant)  
 $V = CT$ ,  $\frac{V}{T} = C$ ,  $\frac{V_1}{T_1} = \frac{V_2}{T_2}$   
 $\frac{V_1}{V_2} = \frac{T_1}{T_2}$ 



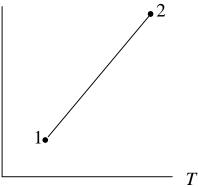
Statement No.(2):- If a fixed mass of gas receives or rejects heat at a constant volume, the pressure of the gas is directly proportional to it's absolute temperature.

p

 $p \propto T$ 

 $V_2$ 

$$p = CT$$
,  $\frac{p}{T} = C$ ,  $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ 



## Equation of state for ideal gases:

Let 1kg of ideal gas follows Boyle's law from state 1 to state *i* and then change to state 2 at constant volume.

(a) According to Boyle's law (process 1 i) pV=C $p_1 V_1 = p_i V_i$ .....(1)  $T_1 = T_i$ (b) According to Charles's law (process i = 2)  $\frac{p}{T} = C$  $\frac{p_i}{T_i} = \frac{p_2}{T_2}$ ....(2)  $V_i = V_2$ From eq.(1)  $p_i = \frac{p_1 V_1}{V_i} = \frac{p_1 V_1}{V_2}$ Substituting  $p_i$  in eq.(2):  $\frac{p_1V_1}{V_2T_i} = \frac{p_2}{T_2}$  $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$  $\frac{p^{T}}{T} = cons \tan t = R$  $p^{-} = RT$  , pV = mRT

R: the gas constant

Another form of the characteristic equation can be derived using the kilogram-mole as a unit.

**The kilogram- mole:-** Is defined as a quality of a gas equivalent to M kg of the gas, where M is the molecular weight of the gas.

Example : (the molecular weight of oxygen is 32, then 1kg mole of oxygen is equivalent to 32kg of oxygen)

$$m = nM$$

Where;

*n* : is the number of mole

M: is the molecular weight of the gas. (kg/mole)

pV=nMRT

# **Example (3.1):**

Calculate *R* for air if the air at atmosphere pressure 1.01325 bar and 0°C has the density  $1.293 \text{kg/m}^3$ .

$$p^{\hat{}} = RT$$
  
 $R = \frac{p^{\hat{}}}{T} = \frac{p}{...T} = \frac{1.01325 \times 10^5}{1.293 \times 273} = 287J / kg.K = 0.287kJ / kg.K$ 

**Avogadro's hypothesis:-** State that "equal volumes of different gases at the same pressure and temperature, contain equal number of molecules".

For gas (a):

$$p_a V_a = m_a R_a T_a = n_a M_a R_a T_a$$

For gas (b):

$$p_{b}V_{b} = m_{b}R_{b}T_{b} = n_{b}M_{b}R_{b}T_{b}$$

$$p_{a}V_{a} = p_{b}V_{b}$$

$$n_{a}M_{a}R_{a}T_{a} = n_{b}M_{b}R_{b}T_{b}$$

$$M_{a}R_{a} = M_{b}R_{b}$$

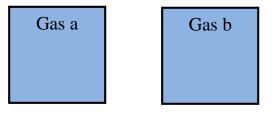
$$MR = cons \tan t = R_{o}$$

$$R = \frac{R_{o}}{M}$$

Where;

 $R_o$ : the universal gas constant

 $MR = R_o$ 



# $pV=nR_oT$

**\*\*:** Experiment has shown that the volume of 1 mole of any perfect gas at 1 bar and 0°C is approximately 22.71  $m^3$ . Therefore;

$$R_o = \frac{pV}{nT} = \frac{10^5 * 22.71}{1 * 273} = 8314 J / mole.K$$

#### **Example (3.2):**

A vessel of volume  $0.2 \text{ m}^3$  contains nitrogen at 1.013 bar and 15°C. If 0.2 kg of nitrogen is now pumped into the vessel, calculate the new pressure when the vessel has returned to its initial temperature. The molecular weight of nitrogen is 28, and it may be assumed to be a perfect gas.

#### Solution:

 $\overline{V=0.2 \ m^3, \ p_1=1.013 \ \text{bar}, \ T_1=15^{\circ}\text{C}, \ p_2=?, \ T_2=T_1, \ M=28}$   $p_1V_1 = m_1RT_1$   $R = \frac{R_o}{M} = \frac{8314}{28} = 296.9 \ J \ / \ kg.K$   $m_1 = \frac{p_1V_1}{RT_1} = \frac{1.013^{*}10^5 * 0.2}{296.9 * 288} = 0.237 \ kg$   $p_2V_2 = m_2RT_2$   $m_2 = m_1 + 0.2$   $p_2 = \frac{m_2RT_2}{V_2} = \frac{0.437 * 296.9 * 288}{0.2} = 1.87 \ bar$ 

#### **Example (3.3):**

A certain perfect gas of mass 0.01 kg occupies a volume of 0.003 m<sup>3</sup> at a pressure of 7 bar and a temperature of 131 °C. The gas is allowed to expand until the pressure is 1 bar and the final volume is 0.02 m<sup>3</sup>. Calculate: (i) the molar mass of the gas; (ii) the final temperature. **Solution:** m=0.01kg,  $V_1=0.003 m^3$ ,  $p_1=7$  bar,  $T_1=131$ °C,  $p_2=1$ bar,  $V_2=0.02 m^3$ , M=?,  $T_2=?$  $p_1V_1 = mRT_1$ 

$$R = \frac{p_1 V_1}{mT_1} = \frac{7 * 10^5 * 0.003}{0.01 * 404} = 519.8 J / kg.K$$

$$R = \frac{R_o}{M}$$
  $\Rightarrow M = \frac{R_o}{R} = \frac{8314}{519.8} = 16 kg / mole$ 

 $p_2V_2 = mRT_2$ 

$$T_2 = \frac{p_2 V_2}{mR} = \frac{1*10^5 * 0.02}{0.01*519.8} = 384.8K = 111.8'C$$

#### Specific heats:-

The specific heat of a solid or liquid is usually defined as the heat required to raise unite mass through one degree temperature rise.

In symbols for small quantities: dQ = mcdT

Where;

*m* : is the mass. (kg)

dT: is the increase in temperature. (K)

c: is the specific heat. (J/kg.K)

<u>\*\*:</u> Two specific heats for gases are defined, the specific heat at constant volume (c), and the specific heat at constant pressure, ( $c_p$ ).

$$c_p = \frac{1}{m} \frac{dQ}{dT}$$

$$c_{\uparrow} = \frac{1}{m} \frac{dQ}{dT}$$

 $Q=mc_p dT$  (For a reversible non-flow process at constant pressure)

 $dQ=mc \ dT$  (For a reversible non-flow process at constant volume)

<u>Note:-</u> For a perfect gas the values of  $c_p$  and  $c_p$  are constant for any one gas at all pressure and temperature while for real gases,  $c_p$  and  $c_p$  vary with temperature.

Heat flow in a reversible constant pressure process, Q

 $Q = mc_p (T_2 - T_1)$ 

Heat flow in a reversible constant volume process, Q

 $Q = mc_{\text{c}} \left(T_2 - T_1\right)$ 

**Joule's law:-** state's that the internal energy of a perfect gas is a function of the absolute temperature only.

$$u = F(T)$$

Let 1kg of a perfect gas be heated at constant volume.

dQ = du + dW (non-flow energy equation)

dQ=du+pdV (dW=0 because volume remain constant)

For a perfect gas, dQ = c dT

du=c dT,

$$c_{\uparrow} = \frac{du}{dT}$$

$$u=c T+K$$

where, (K) : constant

when T=0, u=0 K=0

u=c T (for a perfect gas)

$$\frac{U}{m} = c \cdot T \Longrightarrow U = mc \cdot T$$

In any process for a perfect gas, between states 1 and 2;

Gain in internal energy;

 $U_2 - U_1 = mc_1 (T_2 - T_1)$ 

#### **Relationship between the specific heats:-**

Let a perfect gas be heated at constant pressure from  $T_1$  to  $T_2$ .

 $Q = (U_2 - U_1) + W \text{ (non-flow equation)}$ For a perfect gas;  $U_2 - U_1 = mc_2 (T_2 - T_1)$  $Q = mc_2 (T_2 - T_1) + W$  $W = p(V_2 - V_1) \text{ (constant pressure process)}$ 

$$pV_{1} = mRT_{1} \Rightarrow V_{1} = \frac{mRT_{1}}{p}$$

$$pV_{2} = mRT_{2} \Rightarrow V_{2} = \frac{mRT_{2}}{p}$$

$$W = p\left[\frac{mRT_{2}}{p} - \frac{mRT_{1}}{p}\right] = mR(T_{2} - T_{1})$$

$$Q = mc_{-}(T_{2} - T_{1}) + mR(T_{2} - T_{1}) = m(c_{-} + R)(T_{2} - T_{1})$$
But for constant pressure process;  $Q = mc_{p}(T_{2} - T_{1})$ 

$$c_{p} = c + R$$

$$R = c_{p} - c$$
Enthalpy of a perfect gas:-

h = u + p

for perfect gas; p = RT

from Joule's law; u=c T

h = c T + RT = (c + R)T

 $h=c_pT$ 

(for perfect gas)

 $H=mc_pT$ 

# The ratio of specific heats:-

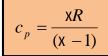
The ratio of specific heat at constant pressure to the specific heat at constant volume called the sensible heat ratio ().

$$X = \frac{C_p}{C_{\uparrow}}$$

 $R = c_p - c$ 

$$\frac{c_p}{c_r} - 1 = \frac{R}{c_r} \Longrightarrow X - 1 = \frac{R}{c_r}$$

$c_{-} = \frac{R}{X - 1}$
$c_p = Xc_{\uparrow}$



#### **Example (3.4):**

A certain perfect gas has a specific heat capacities as follows:  $c_p = 0.846$  kJ/kg K and c = 0.657 kJ/kg K Calculate the gas constant and the molar mass of the gas.  $R=c_p-c$ 

=0.846-0.657=0.189*kJ/kg.K*   $R = \frac{R_o}{M} \Rightarrow M = \frac{R_o}{R} = \frac{8314}{189} = 44kg / mole$ **Example (3.5):** 

A perfect gas has a molar rna ss of 26 kg/kmol and a value of = 1.26. Calculate the heat rejected:

(i) when the gas is contained in a rigid vessel at 3 bar and 315°C, and is then cooled until the pressure falls to1.5 bar.

(ii) when the gas enters a pipeline at 280°C, and flows steadily to the end of the pipe where the temperature is 20°C. Neglect changes in velocity of the gas in the pipeline. *Solution:-*

(i)/

Heat rejected per kg of gas;  $Q = c_{-}(T_2 - T_1)$ 

$$R = \frac{R_o}{M} = \frac{8314}{26} = 319.8J / kg.K$$

$$c_{-} = \frac{R}{x - 1} = \frac{319.8}{0.26} = 1.229kJ / kg.K$$

$$p_1 V_1 = mRT_1$$

$$p_2 V_2 = mRT_2 \Longrightarrow V_1 = V_2 \Longrightarrow \frac{p_1}{p_2} = \frac{T_1}{T_2}$$

$$T_2 = T_1(\frac{p_2}{p_1}) = 588(\frac{1.5}{3}) = 294K$$

Q = 1.229(294 - 588) = -361.3kJ / kg

(ii)/

$$h_1 + Q = h_2 + W \Longrightarrow Q = h_2 - h_2$$

For perfect gas,  $h=c_pT$  $Q=mc_p(T_2-T_1)$ 

 $R = c_p - c_2 \implies c_p = R + c_2 = 0.3198 + 1.229 = 1.548 kJ / kg.K$ 

Q = 1.548(20 - 280) = -402.5kJ / kg

**The working fluid:-** Is the matter contained within the boundaries of a system. When two independent properties of the fluid are known then the thermodynamic state of the fluid is defined.

In thermodynamic systems the working fluid can be in the liquid, vapor or gaseous phase but the solid phase is not important in engineering thermodynamics.

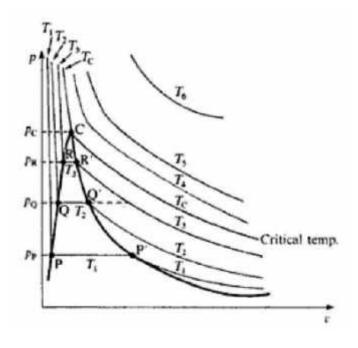
# (Liquid, Vapour and gas):-

(P, Q, R): boiling points of a liquid.

P<sub>C</sub>: critical pressure.

T<sub>C</sub>: critical temperature.

C: critical point.



<u>Saturation state:-</u> Is defined as state at which a change of phase may occur without change of pressure or temperature.

<u>The latent heat of vaporization:-</u> Is the amount of heat required to changes the phase of the substance from liquid to vapor, during this change of phase the pressure and temperature remain constant.

<u>**Critical pressure:-**</u> Is the pressure at which the turning point occurs between the saturated liquid line and the saturated vapor line.

Saturated liquid line:- Is a series of boiling points joined to form a line.

**Saturated vapor line:-** Is a series of point at which vaporization is complete joined to form line.

**Isothermals line:-** Is the line of constant temperature.

*Note:-* when a dry saturated vapor is heated at constant pressure its temperature rises and it becomes superheated.

**Degree of superheated:-** Is the difference between the actual temperature of the superheated vapor and the saturation temperature at the pressure of the vapor.

For example the vapor at point *S* is superheated at  $P_Q$  and  $T_3$  and the degree of superheated is  $(T_3-T_1)$ .

# <u> Region (1):-</u>

**Sub cool liquid (compressed liquid) :-** A liquid at a pressure higher than the saturation pressure corresponding to its temperature. (or, a liquid at a temperature lower than the saturation temperature corresponding to its pressure).

$$T \prec T_{sat}$$

# Example:-

$P=1bar$ , $T=50^{\circ}C$
----------------------------

P=1bar, T=100°C

# <u>Region (2):-</u>

Wet vapor:- Is a mixture of liquid and dry vapor

$$T=T_{sat.}$$
,  $p=p_{sat.}$ 

# <u>Region (3):-</u>

**Superheated vapor:-** It is a vapor at a temperature higher than the saturation temperature corresponding to its pressure.

$$T \succ T_{sat.}$$

# Saturation state properties:-

P	$\frac{t}{(^{\circ}C)}$	$\frac{r_s}{(m^3/kg)}$	u <sub>t</sub>	u <sub>s</sub>	hç	his	h,	s <sub>t</sub>	Sfg	s <sub>s</sub>
(bar)			(kJ/kg)		(kJ/kg)			(kJ/kg K)		
0.34	72	4.649	302	2472	302	2328	2630	0.980	6.745	7.725

## Note:-

g : is used to denote the dry saturate state.

f : is used to denote the dry saturate liquid.

When saturated water is changed to dry saturated

Vapor,

$$Q = (u_2 - u_1) + W$$

W: represented by the area under the horizontal

Line on the (p-) diagram.

$$W = (\hat{f}_g - \hat{f}_f)p$$

$$Q = (u_g - u_f) + p(\hat{f}_g - \hat{f}_f)$$

$$Q = (u_g + p\hat{f}_g) - (u_f + p\hat{f}_g)$$

$$Q = h_g - h_f = h_{fg}$$

# Ling)/Junssel 4 0.34 0.01802 4.649

## Properties of wet vapor:-

The condition or quality of a wet vapor is most frequently defined by its dryness fraction,(x).

(mixture=saturated liquid+ saturated vapor)

**Dryness fraction:**(x):- is the mass of dry vapor in 1kg of the mixture.

 $x = \frac{mas \text{ of } dry \text{ saturated steam}}{Total \text{ mass of wet steam}}$  $x = \frac{m_v}{m_t} = \frac{m_v}{m_l + m_v}$ Where; x : quality of vapor in the mixture.

 $m_v$ : mass of saturated steam.

 $m_l$ : mass of saturated liquid.

Wetness fraction=1-x

Wetness fraction:- is the mass of liquid in 1kg of the mixture.

\*\*: For a dry saturated vapor;  $x=1 \implies x_2 = \frac{m_v}{0+m_v} = 1$ 

**\*\*:** For a saturated liquid; x=0  $\Rightarrow x_1 = \frac{0}{m_1 + 0} = 0$ 

## Note:-

**\*\*:** The dryness fraction does not used in the superheated region.

**\*\*:** Variation of dryness fraction,  $0 \le x \le 1$ 

# The properties of wet vapor:

# 1-Specific volume (v):

 $\vartheta = \frac{volume \ of \ liquid + volume \ of \ dry \ vapor}{total \ mass \ of \ wet \ vapor}$ 

 $\hat{} = \hat{}_f (1-x) + \hat{}_g x$ 

\*\*: The volume of liquid is small compared to the volume of dry saturated vapor therefore is usually negligibly.

$$\hat{} = \hat{}_g x$$

## 2-The enthalpy:

Is given by the sum of the enthalpy of the liquid plus the enthalpy of the dry vapor.

$$h = (1 - x)h_f + xh_g$$

$$h = h_f + x h_{fg}$$

$$h_{fg} = h_g - h_f$$

# 3-The internal energy:

Is given by the sum of the internal energy of the liquid plus the internal energy of the dry vapor.

$$u = (1 - x)u_f + xu_g$$

$$u = u_f + x(u_g - u_f)$$

#### Properties of superheated vapor:

For steam in the superheated region temperature and pressure are independent properties. When temperature and pressure are given for superheated steam then the state is defined and all the other properties can be found. For example; steam at 2bar and 300°C.

 $T > T_{sat.}$ 

	Temperature/(°C)									
	250	300	350	400	450	500	600			
v/(m <sup>3</sup> /kg)	0.1115	0.1255	0.1386	0.1511	0.1634	0.1756	0.1995			
u/(kJ/kg)	2681	2774	2861	2946	3030	3116	3291			
h/(kJ/kg)	2904	3025	3138	3248	3357	3467	3690			
s/(kJ/kgK)	6.547	6.768	6.957	7.126	7.283	7.431	7.701			

<u>\*\*:</u> For pressure above 70 bar the internal energy can be found from  $u = h - p^{-1}$  because it is not tabulated in table.

#### **Example (3.6):**

Calculate the specific volume, specific enthalpy and specific internal energy of wet steam at 18 bar, dryness fraction 0.9.

#### <u>Solution:</u>

$$\hat{} = \hat{}_{g} x$$

At p=18bar,  $_{g}=0.1104m^{3}$ /kg,  $h_{f}=885$ kJ/kg,  $h_{fg}=1912$ kJ/kg,  $u_{f}=883$ kJ/kg,  $u_{g}=2598$ kJ/kg  $\hat{}=0.9*0.1104=0.0994m^{3}$  / kg

$$h = h_f + xh_{fg} = 885 + 0.9 * 1912 = 2605.8 kJ / kg$$

$$u = u_f + x(u_g - u_f) = 883 + 0.9(2598 - 883) = 2426.5kJ / kg$$

## **Example (3.7):**

Find the dryness fraction, specific volume and specific internal energy of steam at 7 bar and specific enthalpy 2600 kJ /kg.

#### Solution:

At p=7bar,  $h_f=697$ kJ/kg,  $h_{fg}=2067$ kJ/kg,  $u_f=696$ kJ/kg,  $u_g=2573$ kJ/kg,  $_g=0.2728m^3$ /kg

$$h = h_f + xh_{fg}$$
  

$$2600 = 697 + x(2067) \Longrightarrow x = 0.921$$
  

$$\hat{\ }_g x = 0.921(0.2728) = 0.251m^3 / kg$$

 $u = u_f + x(u_g - u_f) = 696 + 0.921(2573 - 696) = 2424.7kJ / kg$ 

#### **Example (3.8):**

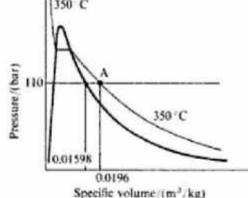
Steam at 110 bar has a specific volume of  $0.0196 \text{ m}^3/\text{kg}$ , calculate the temperature, the specific enthalpy, and the specific internal energy.

Solution:

At 
$$p=110$$
bar,  $_{g}=0.01598 m^{3}/kg$ 

$$\succ \hat{g}$$

The steam is superheated



From superheated tables at 110 bar and specific volume is 0.0196 m<sup>3</sup>/kg, the temperature; T=350°C, and h=2889 kJ/kg.

$$u = h - p^{2} = 2889 - 110 \times 10^{2} \times 0.0196 = 2673.4 kJ / kg$$

The degree of superheated =  $T_{act}$  -  $T_{sat}$  = 350 - 318 = 32' C

# **Example (3.9):**

Steam at 150 bar has a specific enthalpy of 3309 kJ/kg. Calculate the tempera lure, the specific volume and the specific internal energy.

Solution:

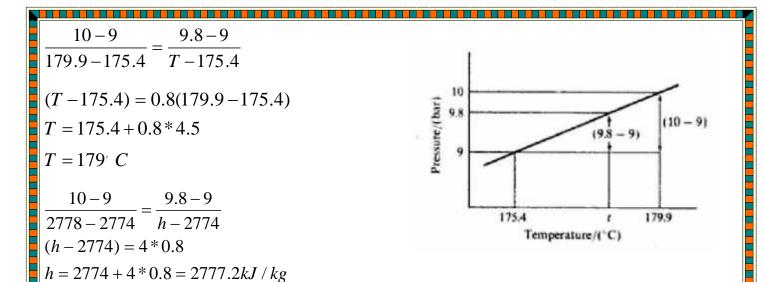
At p=150 bar,  $h_g=2611$ kJ/kg h(3309) the steam is superheated At p=150 bar and h=3309 kJ/kg T=500°C =0.02078 m<sup>3</sup>/kg  $u = h - p^{2} = 3309 - \frac{150 \times 10^{5} \times 0.02078}{10^{3}} = 2997.3$ kJ / kg

## Interpolation:-

For properties which are not tabulated exactly in the tables it is necessary to interpolate between the values tabulated.

## **Example (3.10):**

Find the temperature, specific volume, specific internal energy and specific enthalpy of dry saturated steam at 9.8 bar.



#### **Example (3.11):**

Steam at 5 bar and 320°C. Find the specific volume and enthalpy.

 $\frac{350 - 300}{0.5701 - 0.5226} = \frac{320 - 300}{-0.5226}$  $50(^{\circ} - 0.5226) = 0.0475(20)$  $^{\circ} = \frac{0.0475 * 20}{50} + 0.5226 = 0.5416m^{3} / kg$ 

*Note:-* In some cases a double interpolation is necessary for example to find the enthalpy of superheated steam at 18.5 bar and 432°C.

 $\frac{450 - 400}{3364 - 3256} = \frac{432 - 400}{h - 3256}$   $\frac{450 - 400}{432 - 400} = \frac{3364 - 3256}{h - 3256}$   $h = \frac{432 - 400}{450 - 400} (3364 - 3256) + 3256 = 3325.1 kJ / kg$  h = 3248 + 0.64 (3357 - 3248) h = 3317.8 kJ / kg  $\frac{20 - 15}{3317.8 - 3325.1} = \frac{18.5 - 15}{h - 3325.1}$   $h = (\frac{18.5 - 15}{20 - 15})(3317.8 - 3325.1) + 3325.1$  h = 0.7 (-7.3) + 3325.1 = 3320 kJ / kg

#### **Example (3.12):**

Sketch a pressure-volume diagram for steam and mark on it the following points. Labeling clearly the pressure, specific volume and temperature of each point.

#### solution:

point(a): at p=20 bar,  $T_s=212.4$ °C < T(250°C) superheated p=20 bar, =0.1115 $m^3$ /kg, T=250°C point(b): at  $T=212.4^{\circ}$ C,  $_{g}=0.0995m^{3}/kg=$ dry saturated p=20 bar,  $=_g=0.0995m^3/kg$ , T=212.4°C

#### point(c):

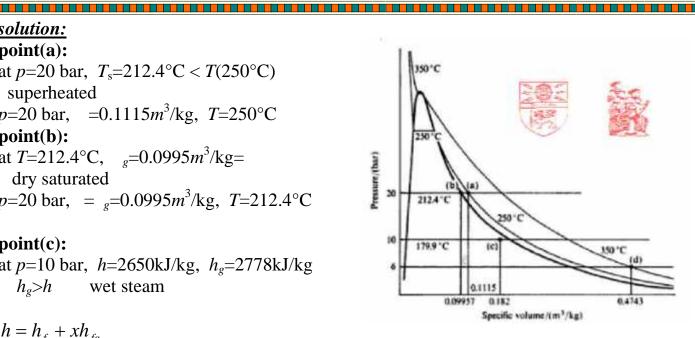
at p=10 bar, h=2650kJ/kg,  $h_g=2778$ kJ/kg  $h_{g} > h$ wet steam

$$h = h_f + xh_{fg}$$
  

$$2650 = 763 + x(2015) \Longrightarrow x = 0.936$$
  

$$\hat{\ } = x\hat{\ }_g = 0.936 * 0.1944 = 0.182 m^3 / kg$$
  

$$p = 10 \text{ bar}, = 0.182 m^3 / \text{kg}, T = T_s = 179.9^{\circ}\text{C}$$



point(d): at p=6 bar, h=3166 kJ/kg,  $h_g=2757$  kJ/kg  $h > h_{o}$ superheated at p=6 bar and h=3166 kJ/kg,  $T=350^{\circ}$ C and  $=0.4743 m^{3}/kg$