Absolute Value Properties

1.	$\vert -a \vert = \vert a \vert$	A number and its additive inverse or negative have the same absolute value.
2.	$\vert ab \vert = \vert a \vert \vert b \vert$	The absolute value of a product is the product of the absolute values.
3.	$\left\vert \frac{a}{b} \right\vert = \frac{\vert a \vert}{\vert b \vert}$	The absolute value of a quotient is the quotient of the absolute values.
4.	$\vert a + b \vert \leq \vert a \vert + \vert b \vert$	The **triangle inequality**. The absolute value of the sum of two numbers is less than or equal to the sum of their absolute values.

Absolute Values and Intervals

If a is any positive number, then

5.	$\vert x \vert = a$	if and only if	$x = \pm a$
6.	$\vert x \vert < a$	if and only if	$-a < x < a$
7.	$\vert x \vert > a$	if and only if	$x > a$ or $x < -a$
8.	$\vert x \vert \leq a$	if and only if	$-a \leq x \leq a$
9.	$\vert x \vert \geq a$	if and only if	$x \geq a$ or $x \leq -a$

Examples:

1. Solve the equation $|2x-3|=1$

<u>Sol</u>: $2x - 3 = \pm 1 \quad \Rightarrow \quad$ either $2x - 3 = 1 \Rightarrow 2x = 4 \Rightarrow x = 2$
or $\quad 2x - 3 = -1 \Rightarrow 2x = 2 \Rightarrow x = 1$

$\therefore$ The solutions are $x = 1$ and $x = 2$.

2. Solve the inequality $|2x-3| \leq 1$

<u>Sol</u>: $|2x-3| \leq 1 \Rightarrow -1 \leq 2x-3 \leq 1 \Rightarrow 2 \leq 2x \leq 4 \Rightarrow 1 \leq x \leq 2$

$\therefore$ The solution set is the closed interval [1,2].

3. Solve the inequality $|2x-3| \geq 1$

<u>Sol</u>: $|2x-3| \geq 1 \Rightarrow$ either $2x-3 \geq 1 \Rightarrow 2x \geq 4 \Rightarrow x \geq 2$
or $\quad 2x-3 \leq -1 \Rightarrow 2x \leq 2 \Rightarrow x \leq 1$

$\therefore$ The solution set is $(-\infty,1] \cup [2, \infty)$.

4. Solve the inequality $\left|5-\frac{2}{x}\right| < 1$

<u>Sol</u>: $\left|5-\frac{2}{x}\right| < 1 \Rightarrow -1 < 5-\frac{2}{x} < 1 \Rightarrow -6 < -\frac{2}{x} < -4$

$\Rightarrow 3 > \frac{1}{x} > 2 \Rightarrow 1/3 < x < 1/2$

$\therefore$ The solution set is the open interval (1/3,1/2).

5. Solve the inequality $|x-3|+|x+2|<11$.

Sol. Recall the definition of absolute value:

$$y=|x|=\sqrt{x^2}=\begin{cases} x & \text{if } x>0 \\ -x & \text{if } x<0 \end{cases}$$

It follows that: $|x-3|=\begin{cases} x-3 & \text{if } x-3>0 \\ -(x-3) & \text{if } x-3<0 \end{cases} = \begin{cases} x-3 & \text{if } x\geq 3 \\ -x+3 & \text{if } x<3 \end{cases}$

Similarly, $|x+2|=\begin{cases} x+2 & \text{if } x+2\geq 0 \\ -(x+2) & \text{if } x+2<0 \end{cases} = \begin{cases} x+2 & \text{if } x\geq -2 \\ -x-2 & \text{if } x<-2 \end{cases}$

These expressions show that we must consider three cases:

$x<-2$, $-2\leq x<3$ and $x\geq 3$

Case I: if $x<-2$ we have
$$|x-3|+|x+2|<11$$
$$-x+3-x-2<11$$
$$-2x<10$$
$$x>-5$$

Case II: if $-2\leq x<3$ we have
$$|x-3|+|x+2|<11$$
$$-x+3+x+2<11$$
$$5<11 \quad \text{(always true)}$$

Case III: if $x\geq 3$ we have
$$|x-3|+|x+2|<11$$
$$x-3+x+2<11$$
$$x<6$$

Combining cases I, II, and III, we see that the inequality is satisfied when

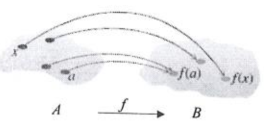
$$-5<x<6.$$

So the solution is the interval (-5, 6).

Functions

Definition:
A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

- The set A is called the **domain** of the function f.
- The number $f(x)$ or y is the value of f at x.
- The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain.
- x & y are variables.
- x is called the independent variable.
- y is called the dependent variable.



<u>Domain</u> (D_f): is the set of all possible inputs (x-values).
<u>Range</u> (R_f): is the set of all possible outputs (y-values).
To find Domain (D_f) and Range (R_f) the following points must be noticed:

1. The denominator in a function must not equal zero (never divide by zero).
2. The values under even roots must be positive.

Examples: Find the domain (D_f) and range (R_f) of the following functions:

1. $y = f(x) = \frac{1}{x}$

<u>Sol</u>: denominator must not equal zero $\Rightarrow x \neq 0 \Rightarrow D_f = \{x: x \neq 0\}$.
To find R_f: convert the function from $y = f(x)$ into $x = f(y)$.
$\therefore x = \frac{1}{y} \Rightarrow R_f = \{y: y \neq 0\}$, or $R_f = \mathrm{R} \setminus \{0\}$, or $R_f = (-\infty, 0) \cup (0, \infty)$.

2. $y = f(x) = \sqrt{4-x}$

<u>Sol</u>: The values under even roots must be positive
$\Rightarrow 4 - x \geq 0 \Rightarrow 4 \geq x$
$\therefore \quad D_f = \{x: x \leq 4\}$.
To find R_f: convert the function from $y = f(x)$ into $x = f(y)$.
$y = \sqrt{4-x} \Rightarrow y^2 = 4 - x$
$\Rightarrow x = 4 - y^2 \Rightarrow$
$\therefore \quad R_f = R$.
But the values of y must be always positive, we must exclude negative values,
$\Rightarrow \quad R_f = \{y: y \geq 0\}$, or $R_f = [0, \infty)$.

3. $y = f(x) = \sqrt{1-x^2}$

<u>Sol</u>: The values under even roots must be positive
$\Rightarrow 1 - x^2 \geq 0 \Rightarrow -x^2 \geq -1 \Rightarrow x^2 \leq 1 \Rightarrow$ either $x \leq 1$, or $-x \leq 1 \Rightarrow x \geq -1$
$\therefore \quad D_f = \{x: -1 \leq x \leq 1\}$.
To find R_f: convert the function from $y = f(x)$ into $x = f(y)$.

$y=\sqrt{1-x^2} \Rightarrow y^2=1-x^2$

$\Rightarrow x^2=1-y^2 \Rightarrow x=\pm\sqrt{1-y^2}$

So the values under even roots must be positive

$1 - y^2 \geq 0 \Rightarrow -y^2 \geq -1 \Rightarrow y^2 \leq 1 \Rightarrow$ either $y \leq 1$, or $-y \leq 1 \Rightarrow y \geq -1$

$\therefore \quad R = \{y: -1 \leq y \leq 1\}$.

But the values of y must be always positive, we must exclude negative values,

$\Rightarrow \quad R_f = \{y: 0 \leq y \leq 1\}$.

4. $y=f(x)=\dfrac{1}{\sqrt{9-x^2}}$

Sol: The values under even roots must be positive and the denominator must not equal zero, so:

$9 - x^2 > 0 \Rightarrow -x^2 > -9 \Rightarrow x^2 < 9 \Rightarrow$ either $x < 3$, or $-x < 3 \Rightarrow x > -3$

$\therefore \quad D_f=\{x: -3 < x < 3\}$.

To find R_f: convert the function from $y = f(x)$ into $x = f(y)$.

$$y=\frac{1}{\sqrt{9-x^2}} \Rightarrow y^2=\frac{1}{9-x^2} \Rightarrow 9-x^2=\frac{1}{y^2} \Rightarrow x^2=9-\frac{1}{y^2}$$

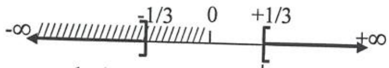
$$\Rightarrow x^2=\frac{9y^2-1}{y^2} \Rightarrow x=\pm\sqrt{\frac{9y^2-1}{y^2}} \Rightarrow x=\pm\frac{\sqrt{9y^2-1}}{y}$$

The values under even roots must be positive, so:

$9y^2 - 1 \geq 0 \Rightarrow y^2 \geq 1/9 \Rightarrow$ either $y \geq 1/3$, or $-y \geq 1/3 \Rightarrow y \leq -1/3$

$\Rightarrow \quad R_f= (-\infty, -1/3] \cup [1/3, \infty)$.

The denominator must not equal zero $\Rightarrow y \neq 0$



But the values of y must be always positive; we must exclude negative values,

$\Rightarrow \quad R_f= [1/3, \infty)$, or $R_f=\{y: 1/3 \leq y \leq \infty\}$.

5. $y=f(x)=\dfrac{1}{x^2-9}$

Sol: denominator must not equal zero $\Rightarrow x^2-9 \neq 0 \Rightarrow x \neq \pm 3 \Rightarrow D_f = R \setminus \{-3, 3\}$.

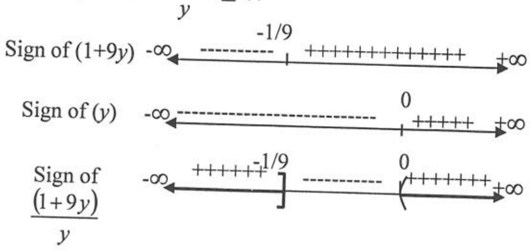
To find R_f: convert the function from $y = f(x)$ into $x = f(y)$.

$$y(x^2-9)=1 \Rightarrow yx^2-9y=1 \Rightarrow x=\pm\sqrt{\frac{1+9y}{y}}.$$

The values under even roots must be positive, so $\Rightarrow \dfrac{1+9y}{y} \geq 0$.

$\therefore R_f= (-\infty, -1/9] \cup (0, \infty)$.

Or $R_f = R \setminus (-1/9, 0]$.



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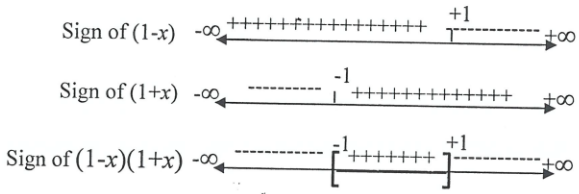
6. $y = f(x) = -\sqrt{1-x^2}$

Sol: The values under even roots must be positive:

$1 - x^2 \geq 0 \quad \Rightarrow \quad (1 - x)(1 + x) \geq 0$

$\therefore\ D_f = [-1,+1]$

To find R_f : convert the function from $y = f(x)$ into $x = f(y)$.



$y = -\sqrt{1-x^2} \Rightarrow y^2 = 1 - x^2$

$\Rightarrow x^2 = 1 - y^2 \Rightarrow x = \mp\sqrt{1-y^2}$

The values under even roots must be positive:

$1 - y^2 \geq 0 \quad \Rightarrow \quad (1 - y)(1 + y) \geq 0$

$\Rightarrow R_f = [-1,+1]$

But the values of y must be always negative; we must exclude positive values,

$\Rightarrow \quad R_f = [-1,0]\ \therefore$

Chapter One

Graph of Functions (Graph of Curves):

To graph the curve of a function, we can follow the following steps:

1. Find the domain and range of the function.
2. Check the symmetry of the function
3. Find (if any found) points of intersection with x-axis and y-axis.
4. Choose some another points on the curve.
5. Draw s smooth line through the above points.

Symmetry Tests for Graphs:

If $f(x,y) = 0$ is any function then:

1. Symmetry about ***x*-axis**: If $f(x,-y)=f(x,y)$
2. Symmetry about ***y*-axis**: If $f(-x,y)=f(x,y)$ It is called an *even* function.
3. Symmetry about **the origin**: If $f(-x,-y)=f(x,y)$ It is called an *odd* function

Examples 1: Check the symmetry of the graph of the following curves:

1. $y = f(x)=x^2$

sol.: $f(x,y) = x^2 - y = 0$

$\therefore$ (*i*) $f(x,-y)= x^2 - (-y) = 0$ $\Rightarrow$ $.f(x,-y)= x^2 + y = 0 \neq f(x,y)$ not o.k.

(*ii*) $f(-x,y)= (-x)^2 - y = 0$ $\Rightarrow$ $.f(-x,y)= x^2 - y = 0 = f(x,y)$ o.k.

(*iii*) $f(-x,-y)= (-x)^2 - (-y) = 0$ $\Rightarrow$ $.f(-x,-y)= x^2 + y = 0 \neq f(x,y)$ not o.k.

So the function has symmetry only about y-axis. It is called an even function.

2. $y= f(x)=x^3$

Sol. $f(x,y) = x^3 - y = 0$

$\therefore$ (*i*) $f(x,-y)= x^3 - (-y) = 0$ $\Rightarrow$ $.f(x,-y)= x^3 + y = 0 \neq f(x,y)$ not o.k.

(*ii*) $f(-x,y)= (-x)^3 - y = 0$ $\Rightarrow$ $.f(-x,y)= -x^3 - y = 0 \neq f(x,y)$ not o.k.

(*iii*) $f(-x,-y)= (-x)^3 - (-y) = 0$ $\Rightarrow$ $.f(-x,-y)= -x^3 + y = 0$ (multiply by -1)

$\Rightarrow$ $f(-x,-y)= x^3 - y = 0 = f(x,y)$ o.k.

So the function has symmetry only about *the origin*. It is called an odd function.

3. $x^2 = y^2 + 4$

Sol. $f(x,y) = y^2 - x^2 + 4 = 0$

$\therefore$ (*i*) $f(x,-y) = (-y)^2 - x^2 + 4 = 0$ $\Rightarrow$ $.f(x,-y) = y^2 - x^2 + 4 = 0 = f(x,y)$ o.k.

(*ii*) $f(-x,y) = y^2 - (-x)^2 + 4 = 0$ $\Rightarrow$ $.f(-x,y) = y^2 - x^2 + 4 = 0 = f(x,y)$ o.k.

(*iii*) $f(-x,-y) = (-y)^2 - (-x)^2 + 4 = 0$ $\Rightarrow$ $.f(-x,-y) = y^2 - x^2 + 4 = 0 = f(x,y)$ o.k.

So the function has symmetry about x-axis, y-axis and *the origin*.

DEFINITIONS **Even Function, Odd Function**

A function $y = f(x)$ is an

even function of x if $f(-x) = f(x)$ symmetry about y-axis

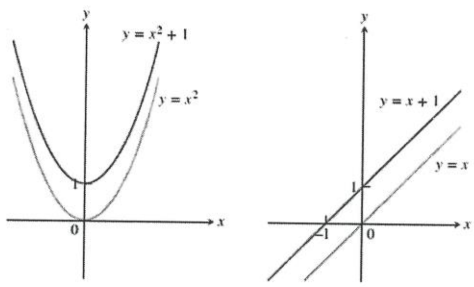
odd function of x if $f(-x) = -f(x)$ symmetry about origin

for every x in the function's domain.

Examples 2: Recognizing Even and Odd functions

- $f(x) = x^2$ Even function: $(-x)^2 = x^2$ for all x; symmetry about y-axis.
- $f(x) = x^2 + 1$ Even function: $(-x)^2 + 1 = x^2 + 1$ for all x; symmetry about y-axis.
- $f(x) = x$ Odd function: $(-x) = x$ for all x; symmetry about *the origin*.
- $f(x) = x + 1$ Not odd: $f(-x) = -x + 1$, but $-f(x) = -x - 1$. The two are not equal.

  Not even: $f(-x) = -x + 1$, but $f(x) = x + 1$. for all $x \neq 0$.



(a) (b)

Example 3: Sketch the graph of the curve $y = f(x) = x^2 - 1$

Sol.: Step 1: Find D_f, R_f of the function?

$D_f=(-\infty,\infty)$;

To find R_f: we must convert the function from $y=f(x)$ into $x=f(y)$.

$y=x^2-1 \Rightarrow \quad x^2=y+1 \qquad x=\mp\sqrt{y+1}$

So $\quad y+1\geq 0 \quad \Rightarrow \quad y\geq -1 \quad \Rightarrow \quad R=[-1,\infty)$

Step 2: Find x and y intercept?

To find x-intercept put $y=0 \Rightarrow \quad x^2-1=0 \Rightarrow \quad x^2=\pm 1$

So x-intercept are $(-1,0)$ and $(+1,0)$.

To find y-intercept put $x=0 \Rightarrow \quad y=0-1 \Rightarrow \quad y=-1$

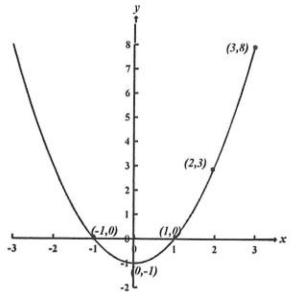
So y-intercept is $(0,-1)$.

Step 3: check the symmetry:

$f(-x)=(-x)^2-1=x^2-1=f(x)$

$-f(x)=-(x^2-1)=-x^2+1\neq f(x)$

So it is an even function (it is symmetric about y-axis).



Step 4: Choose some another point on the curve.

x	y
2	3
3	8

Step 5: Draw smooth line through the above points.

Homework: Draw the following functions:

1. $y=f(x)=3x^2+2$
2. $x^2+y^2=1$
3. $y^2=4x-1$
4. $x=y^3$
5. $y=[x]$; for $-3\leq x\leq 3$
6. $y=x-[x]$; for $-2\leq x\leq 2$
7. $y=\sqrt{4-x}$

Chapter One

Shifting, Shrinking and Stretching:

Shift formulas: (for c > 0)

Vertical shifts

$y=f(x)+c$	or	$y-c=f(x)$	shifts the graph of f up by c units.
$y=f(x)-c$	or	$y+c=f(x)$	shifts the graph of f down by c units.

Horizontal shifts

$y=f(x+c)$	shifts the graph of f left by c units.
$y=f(x-c)$	shifts the graph of f right by c units.

Shrinking, Stretching and Reflecting Formulas:

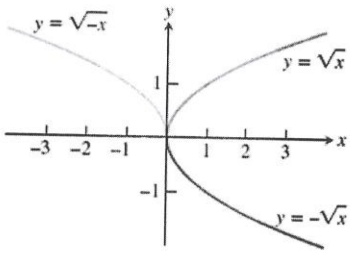
(for c > 1)

$y=c\,f(x)$	Stretches the graph of f c units along y-axis.
$y=\frac{1}{c}f(x)$	Shrinks the graph of f c units along y-axis.
$y=f(cx)$	Shrinks the graph of f c units along x-axis.
$y=f(\frac{x}{c})$	Stretches the graph of f c units along x-axis.

(for c = -1)

$y=-f(x)$	Reflects the graph of f across the x-axis.
$y=f(-x)$	Reflects the graph of f across the y-axis.

Example 1: The graph of $y=-\sqrt{x}$ is a reflection of $y=\sqrt{x}$ across the x-axis, and $y=\sqrt{-x}$ is a reflection across the y-axis.



Example 2: Shift the graph of the function

$f(x)=x^2$; if $D_f=\{x: -2 \le x \le 3\}$ and $R_g=\{y: 0 \le y \le 9\}$.

(a) one unit right. (b) two units left.

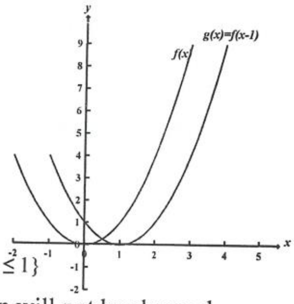
(c) one unit up. (d) two units down.

Sol.: (a) Shifting the function $f(x)$ one unit right:

$g(x)=f(x-1)=(x-1)^2$ and $D_g=\{x: -2 \le x-1 \le 3\}=\{x: -1 \le x \le 4\}$

Note: In case of horizontal shifts, the range of the function will not be changed.

x	$y=f(x)=x^2$	x-1	$y=g(x)=(x-1)^2$
-2	4	-	-
-1	1	-2	**4**
0	0	-1	**1**
1	1	0	**0**
2	4	1	**1**
3	9	2	**4**
4	-	3	**9**

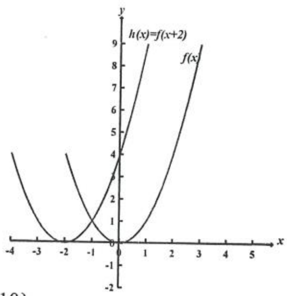


(b) Shifting the function $f(x)$ two units left:

$h(x) = f(x+2) = (x+2)^2$ and $D_h=\{x: -2 \le x+2 \le 3\}=\{x: -4 \le x \le 1\}$

Note: In case of horizontal shifts, the range of the function will not be changed.

X	$y=f(x)=x^2$	x+2	$y=h(x)=(x+2)^2$
-4	-	-2	**4**
-3	-	-1	**1**
-2	4	0	**0**
-1	1	1	**1**
0	0	2	**4**
1	1	3	**9**
2	4	-	-
3	9	-	

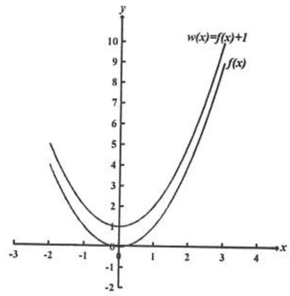


(c) Shifting the function $f(x)$ one unit up:

$w(x) = f(x)+1 = x^2 +1$ and $R_w=\{y: 0 \le y-1 \le 9\}=\{y: 1 \le y \le 10\}$

Note: In case of vertical shifts, the domain of the function will not be changed.

X	$y=f(x)=x^2$	$y=w(x)= x^2 +1$
-2	4	**5**
-1	1	**2**
0	0	**1**
1	1	**2**
2	4	**5**
3	9	**10**

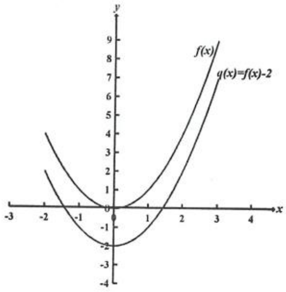


(d) Shifting the function $f(x)$ two units down:

$q(x) = f(x)-2 = x^2 -2$ and $R_q=\{y: 0 \le y+2 \le 9\}=\{y: -2 \le y \le 7\}$

Note: In case of vertical shifts, the domain of the function will not be changed.

X	$y=f(x)=x^2$	$y=q(x)= x^2 -2$
-2	4	2
-1	1	-1
0	0	-2
1	1	-1
2	4	2
3	9	7



Example 3: Sketch the graph of the curve $y= f(x) = |x|$

Sol.: Step1: Find D_f, R_f of the function?

$$y = f(x) = |x| = \begin{cases} x & if \quad x \ge 0 \\ -x & if \quad x < 0 \end{cases}$$

$\Rightarrow \quad D_f=(-\infty,\infty)$ and $R_f= [0,\infty)$;

Step2: Find x and y intercept?

To find x-intercept put $y=0 \Rightarrow \quad x=0$

To find y-intercept put $x=0 \Rightarrow \quad y=0$

So x- and y-intercept is (0,0).

Step 3: check the symmetry:

$f(-x) = |-x|=|x|= f(x)$

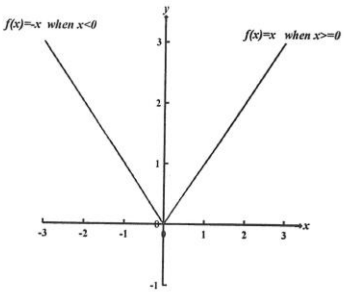
$-f(x) =-|x| \neq f(x)$

So it is an even function (it is symmetric about y-axis).

Step 4: Choose some another point on the curve.

x	y
1	1
2	2

Step 5: Draw smooth line through the above points.



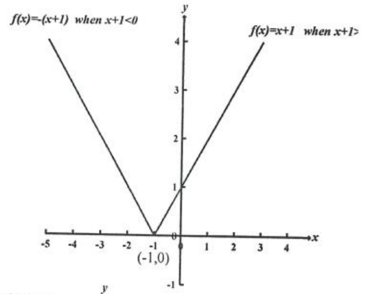
Example 4: Use graph of the function $y=|x|$ to sketch the graph of the following functions, then show their domains and range

(a) $y=|x+1|$

Sol.

$$y=|x+1|=\begin{cases}(x+1) & if \quad (x+1)\geq 0\\ -(x+1) & if \quad (x+1)<0\end{cases}$$

$$=\begin{cases}(x+1) & if \quad x\geq -1\\ -x-1 & if \quad x<-1\end{cases}$$

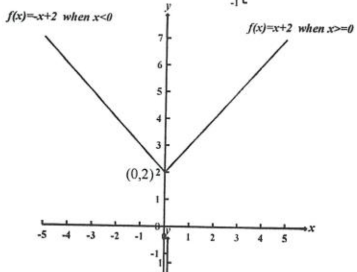


Shifting the function $y=|x|$ one unit left.

$D_f=(-\infty,\infty)$ and $R_f=[0,\infty)$

(b) $y=|x|+2$

Sol. $y==|x|+2=\begin{cases}(x)+2 & if \quad (x)\geq 0\\ (-x)+2 & if \quad (x)<0\end{cases}$

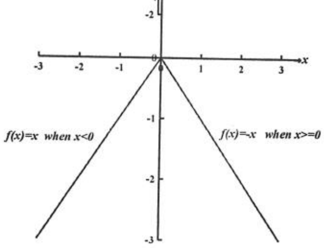


Shifting the function $y=|x|$ two up.

$D_f=(-\infty,\infty)$ and $R_f=[2,\infty)$

(c) $y=-|x|$

Sol. $y=f(x)=-|x|=\begin{cases}-(x)=-x & if \quad (x)\geq 0\\ -(-x)=x & if \quad (x)<0\end{cases}$



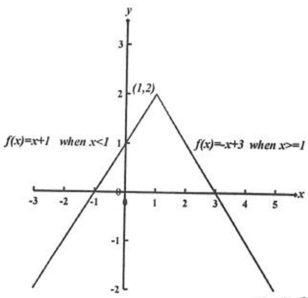
Reflecting the graph of the function $y=|x|$ across x-axis.

$D_f=(-\infty,\infty)$ and $R_f=(-\infty,0]$

(d) $y=2-|1-x|$

Sol. $y=2-|1-x|=-|1-x|+2|=-|x-1|+2$

$$=\begin{cases}-(x-1)+2 & if \quad (x-1)\geq 0\\ -(-(x-1))+2 & if \quad x-1<0\end{cases}$$



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$$= \begin{cases} -x+3 & \text{if} \quad x \ge 1 \\ x+1 & \text{if} \quad x < 1 \end{cases}$$

Reflecting the graph of the function $y=|x|$ across x-axis, then shifting it one unit right and two units up.

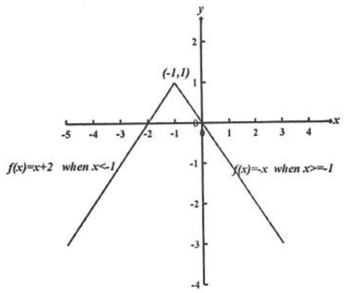
$D_f=(-\infty,\infty)$ and $R_f=(-\infty,2]$

(e) $y=1-|x+1|$

Sol. $y=1-|x+1|=-|x+1|+1$

$$= \begin{cases} -(x+1)+1 & \text{if} \quad (x+1) \ge 0 \\ -(-(x+1))+1 & \text{if} \quad (x+1) < 0 \end{cases}$$

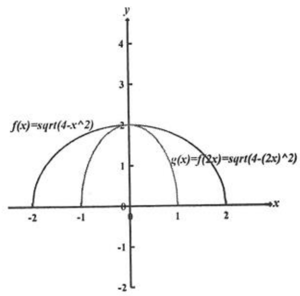
$$= \begin{cases} -x & \text{if} \quad x \ge -1 \\ x+2 & \text{if} \quad x < -1 \end{cases}$$



Reflecting the graph of the function $y=|x|$ across x-axis, then shifting it one unit left and one unit up.

$D_f=(-\infty,\infty)$ and $R_f=(-\infty,1]$

Example 5: If $f(x)=\sqrt{4-x^2}$ which has $D_f=[-2,2]$ and $R_f=[0,2]$, shrink and stretch it horizontally by two units and then sketch the original and resulting functions



Sol.: (a) shrinking:

$$g(x)=f(c.x)=\sqrt{4-(2x)^2}=\sqrt{4-4x}=2\sqrt{1-x^2}$$

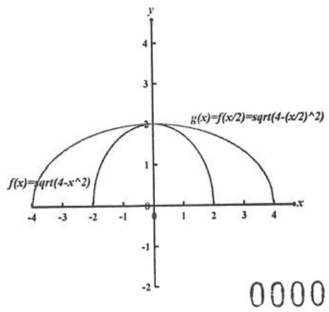
$D_g=\{x: -2 \le 2x \le 2\}=\{x: -1 \le x \le 1\}$

Note: In case of horizontal shrinks, the range of the function will not be changed.

(b) stretching:

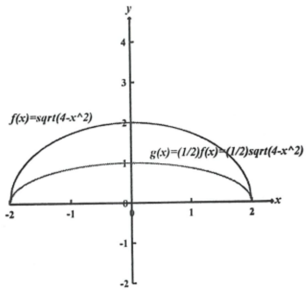
$$g(x)=f(\frac{x}{c})=\sqrt{4-(\frac{x}{2})^2}=\sqrt{4-\frac{x^2}{4}}=\sqrt{\frac{16-x^2}{4}}=\frac{1}{2}\sqrt{16-x^2}$$

$D_g=\{x: -2 \le x/2 \le 2\}=\{x: -4 \le x \le 4\}$



Note: In case of horizontal stretches, the range of the function will not be changed.

Example 6: Repeat the above example but here shrink and stretch the function vertically.

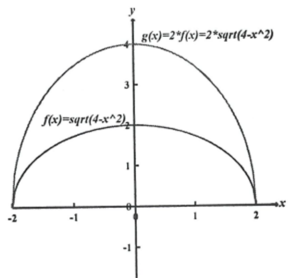


Sol.: (a) shrinking:

$$g(x)=\frac{1}{c}f(x)=\frac{1}{2}\sqrt{4-x^2}$$

$R_g=\{y: 0\le 2y\le 2\}=\{y: 0\le y\le 1\}$

Note: In case of vertical shrinks, the domain of the function will not be changed.



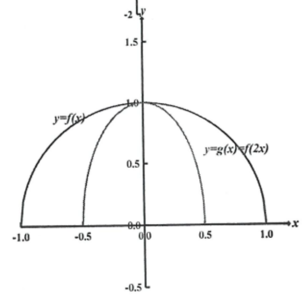
(b) stretching:

$$g(x)=cf(x)=2\sqrt{4-x^2}$$

$R_g=\{y: 0\le y/2\le 2\}=\{y: 0\le y\le 4\}$

Note: In case of vertical stretches, the domain of the function will not be changed.

Example 7: Use the graph of the function $y=f(x)=\sqrt{1-x^2}$ to sketch the graph of the following functions:



1. $y=g(x)=\sqrt{1-4x^2}$

Sol.: $y=\sqrt{1-4x^2}=\sqrt{1-(2x)^2}$

This function may be obtained by shrinking the function $f(x)=\sqrt{1-x^2}$ by two units horizontally ($g(x)=f(2x)$).

2. $y=h(x)=\sqrt{1-\dfrac{x^2}{9}}$