# *Simultaneous Linear Equations*

***System of Equations***

*After reading this chapter, you should be able to:*

*setup simultaneous linear equations in matrix form and vice-versa.*

***Matrix algebra is used for solving systems of equations. Can you illustrate this concept?***

*Matrix algebra is used to solve a system of simultaneous linear equations. In fact, for many mathematical procedures such as the solution to a set of nonlinear equations, interpolation, integration, and differential equations, the solutions reduce to a set of simultaneous linear equations. Let us illustrate with an example for interpolation.*

***Example 1***

*The upward velocity of a rocket is given at three different times on the following table.*

***Table 5.1.*** *Velocity vs. time data for a rocket*

| *Time, t* | *Velocity, v* |
|---|---|
| *(s)* | *(m/s)* |
| *5* | *106.8* |
| *8* | *177.2* |
| *12* | *279.2* |

*The velocity data is approximated by a polynomial as*

$$v(t) = at^2 + bt + c, \quad 5 \le t \le 12.$$

*Set up the equations in matrix form to find the coefficients* $a, b, c$ *of the velocity profile.*

***Solution***

*The polynomial is going through three data points* $(t_1, v_1), (t_2, v_2), \text{and} (t_3, v_3)$ *where from table 5.1.*

$$t_1 = 5, v_1 = 106.8$$

$$t_2 = 8, v_2 = 177.2$$

$$t_3 = 12, v_3 = 279.2$$

*Requiring that* $v(t) = at^2 + bt + c$ *passes through the three data points gives*

$$v(t_1) = v_1 = at_1^2 + bt_1 + c$$

$$v(t_2) = v_2 = at_2^2 + bt_2 + c$$

$$v(t_3) = v_3 = at_3^2 + bt_3 + c$$

*Substituting the data* $(t_1, v_1), (t_2, v_2), \text{ and } (t_3, v_3)$ *gives*

$$a(5^2) + b(5) + c = 106.8$$

$$a(8^2) + b(8) + c = 177.2$$

$$a(12^2) + b(12) + c = 279.2$$

*or*

$$25a + 5b + c = 106.8$$

$$64a + 8b + c = 177.2$$

$$144a + 12b + c = 279.2$$

*This set of equations can be rewritten in the matrix form as*

$$\begin{bmatrix} 25a + & 5b + & c \\ 64a + & 8b + & c \\ 144a + & 12b + & c \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

*The above equation can be written as a linear combination as follows*

$$a\begin{bmatrix} 25 \\ 64 \\ 144 \end{bmatrix} + b\begin{bmatrix} 5 \\ 8 \\ 12 \end{bmatrix} + c\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

*and further using matrix multiplication gives*

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

*The above is an illustration of why matrix algebra is needed. The complete solution to the set of equations is given later in this chapter.*

***A general set of $m$ linear equations and $n$ unknowns,***

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n = c_1 \\ &a_{21}x_1 + a_{22}x_2 + \cdots\cdots + a_{2n}x_n = c_2 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ &a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots.. + a_{mn}x_n = c_m \end{aligned}$$

*can be rewritten in the matrix form as*

$$\begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & . & . & a_{mn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ c_m \end{bmatrix}$$

*Denoting the matrices by $[A]$, $[X]$, and $[C]$, the system of equation is*

*$[A][X] = [C]$, where $[A]$ is called the coefficient matrix, $[C]$ is called the right hand side vector and $[X]$ is called the solution vector.*

*Sometimes $[A][X] = [C]$ systems of equations are written in the augmented form. That is*

$$[A \vdots C] = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} & \vdots c_1 \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} & \vdots c_2 \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} & \vdots c_n \end{bmatrix}$$