Subject :	**Mathematics I**	**Units :**	**3**
Code	**E111**		
Level	**1st level**	**Theoretical :**	**3** Hr/wk
Pre-requisite:	None	**Practical :**	

Algebraic Preliminaries

Numbers, Sets, Inequalities, Absolute value.

Functions

Domain, Range, graphs, Symmetry, Asymptotes.

Limits

Definition of Limit, Theorems, Continuity, One-Sided Limits, Limits at Infinity, L Hopital's rule.

Derivatives

Definition, Power and Sum Rules, Product and Quotient Rules, Chain rule, High-Order derivatives, Implicit differentiation.

Applications of Derivative

Maximum and minimum, mean value theorem, Increasing and Decreasing Functions, Concavity and Points of inflection, Second Derivative Test.

Definite Integration

Definition, Integral Theorems, Length of a Curve, Areas, Volume of Solids, Surface Area, Indefinite Integrals.

Transcendental Functions

Trigonometric Functions, Graphs, Derivatives of trigonometric functions, Inverse trigonometric functions, Graphs, Derivatives of Inverse trigonometric functions, Natural Logarithm Functions, Exponential Functions, Functions a^u and $\log_a u$.

PRELIMINARIES

Sets and Intervals

Definitions:

<u>Set</u>: is a collection of objects, and these objects are the **elements** of the set. If S is a set, the notation $a \in S$ means that a is an element of S, and $a \notin S$ means that a is not an element of S. If S and T are sets, then $S \cup T$ is their **union** and consists of all elements belonging either to S or T (or to both S and T). The **intersection** $S \cap T$ consists of all elements belonging to both S and T. The **empty set (null set)** ϕ is the set that contains no elements.

Some sets can be described by listing their elements in braces. For instance, the set A consisting of positive integers less than 6 can be expressed as:

$A = \{1, 2, 3, 4, 5\}$.

Another way to describe a set is to enclose in braces a rule that generates all the elements of the set. For instance, the set

$A = \{x: x \text{ is an integer and } 0 < x < 6\}$,

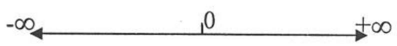
is the set of positive integers less than 6.

<u>Subset</u>: if each element of a set A is a member of a set B, then A is a subset of B and we write $A \subseteq B$. If the set B contains at least one element that is not a member of A, then A is a proper subset of B and this situation is indicated by $A \subset B$.

<u>Real numbers (R)</u>: are numbers that can be expressed as decimals, such as

$-3/4 = -0.75000....$, $1/3 = 0.3333....$, $\sqrt{2} = 1.4142....$

The dots .... in each case indicate that the sequence of decimal digits goes on forever. The real numbers can be represented geometrically as points on a number line called the **real number line**. $R = \{-\infty, +\infty\}$



Real number line

These are four special subsets of real numbers:

1. The **natural numbers (N)**: consist of zero and positive integer numbers only. $N = \{0,1,2,3,----,+\infty\}$.
2. The **integer numbers (I)**: consist of positive and negative integer numbers only. $I = \{-\infty,----,-3,-2,-1,0,1,2,3,----,+\infty\}$.
3. The **rational numbers**, namely the numbers that can be expressed in the form of a fraction m/n, where m and n are integers and $n \neq 0$. Examples are:

   1/3, -4/9, 200/13, and 57=57/1.

   The rational numbers are precisely the real numbers with decimal expansions that are either

(a) terminating (ending in an infinite string of zeros), for example,

$$3/4 = 0.75000\ldots \qquad \text{or}$$

(b) eventually repeating (ending with a block of digits that repeats over and over), for example,

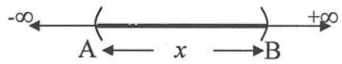
$$23/11 = 2.090909\ldots. = 2.\overline{09}.$$

4. The **irrational numbers**, they are characterized by having nonterminating and nonrepeating decimal expansions. Examples are:

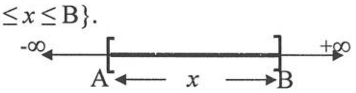
$$\pi, \sqrt{2}, \sqrt[3]{5}, \log_{10} 3, \sin 41°, 2^{\sqrt{3}}.$$

<u>**Interval:**</u> is a set of all real numbers between two points on the real number line (it is a subset of real numbers).

1. Open interval: is a set of all real numbers between A&B excluded (A&B are not elements in the set). (A, B) = {x: A < x < B}.

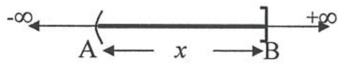


2. Closed interval: is a set of all real numbers between A&B included (A&B are elements in the set). [A, B] = {x: A ≤ x ≤ B}.



3. Half-open interval: is a set of all real numbers between A&B with one of the end-points as an element in the set.

a) (A, B] = {x: A < x ≤ B}



b) [A, B) = {x: A ≤ x < B}

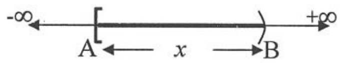


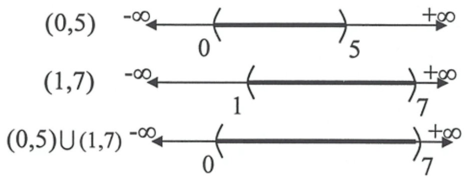
TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \le x \le b\}$	Closed	
	$[a, b)$	$\{x \mid a \le x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \le b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \ge a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \le b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	

Examples: Solve for x the following

1. $\{x: 0 < x < 5\} \cup \{x: 1 < x < 7\}$

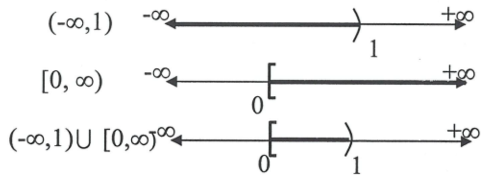
Sol: From the number line:



$\therefore$ The interval is $\{0 < x < 7\}$ or $(0,5) \cup (1,7) = (0,7)$.

2. $\{x: x < 1\} \cap \{x: x \geq 0\}$

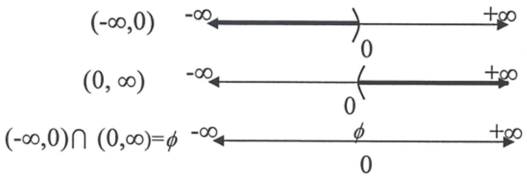
Sol: From the number line:



$\therefore$ The interval is $\{x: 0 \leq x < 1\}$ or $[0,1)$.

3. $\{x: x < 0\} \cap \{x: x > 0\}$

Sol: From the number line:



Inequalities: At some times it is not sufficient to say that two numbers a and b are unequal. We can say something about unequal such as a is greater than b ($a > b$) if and only $(a - b)$ is positive and a is less than b ($a < b$) if and only $(a - b)$ is negative.

Rules for inequalities: If a, b, and c are real numbers, then:

1. $a < b \Rightarrow a + c < b + c$
2. $a < b \Rightarrow a - c < b - c$
3. $a < b$ *and* $c > o \Rightarrow ac < bc$
4. $a < b$ *and* $c < o \Rightarrow ac > bc$

   Special case $a < b \Rightarrow -a > -b$

5. $a > 0 \Rightarrow \frac{1}{a} > 0$

6. If a and b are both positive or both negative, then $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

Solving inequalities: The process of finding the interval or intervals of numbers that satisfy an inequality in x is called **solving** the inequality.

Examples: Solve the following inequalities and show their solution set

1. $3 + 7x \le 2x - 9$

Sol: $3 + 7x - 3 \le 2x - 9 - 3 \Rightarrow 7x \le 2x - 12 \Rightarrow 7x - 2x \le 2x - 12 - 2x$
$\Rightarrow 5x \le -12 \Rightarrow x \le -12/5$

The solution set is $(-\infty, -12/5]$.

2. $7 \le 2 - 5x < 9$

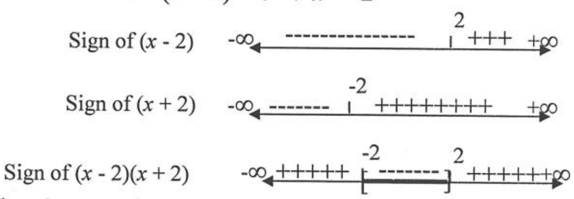
Sol: $7 - 2 \le 2 - 5x - 2 < 9 - 2 \Rightarrow 5 \le -5x < 7 \Rightarrow 5/(-5) \le -5x/(-5) < 7/(-5)$
$\Rightarrow -1 \ge x > -7/5 \Rightarrow -1 \le x \le -7/5$

The solution set is $(-7/5, -1]$.

3. $x^2 \le 4$

Sol: $x^2 \le 4 \Rightarrow x^2 - 4 \le 0 \Rightarrow (x - 2)(x + 2) \le 0$

Let $(x - 2)(x + 2) = 0 \Rightarrow$ either $(x - 2) = 0 \Rightarrow x = 2$
or $(x + 2) = 0 \Rightarrow x = -2$



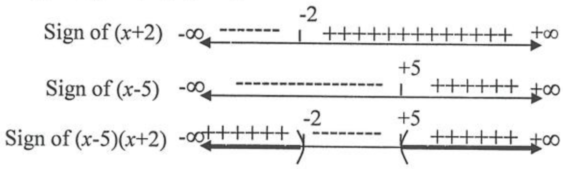
∴ The solution is $\{x: -2 \le x \le 2\}$ or $[-2, 2]$.

4. $x^2 - 3x > 10$

sol: $x^2 - 3x - 10 > 0 \Rightarrow (x - 5)(x + 2) > 0$

Let $(x - 5)(x + 2) = 0 \Rightarrow$ either $(x - 5) = 0 \Rightarrow x = 5$
or $(x + 2) = 0 \Rightarrow x = -2$

The solution set is $(-\infty, -2) \cup (5, \infty)$.



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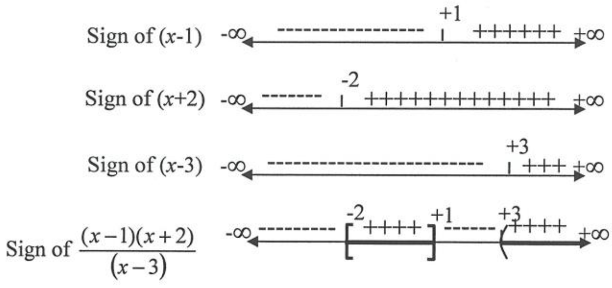
5. $\frac{(x-1)(x+2)}{x-3} \geq 0$

Sol:



$\therefore$ The solution set is $\{x: -2 \leq x \leq +1\} \cup \{x: x > +3\}$
or $[-2,+1] \cup (+3, \infty)$.

6. $\frac{6}{x-1} \geq 5$

Sol:

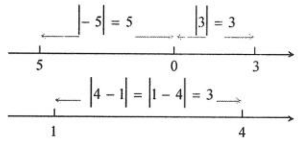
This inequality can hold only if $x > 1$, because otherwise $6/(x-1)$ is undefined or negative.

$6 \geq 5x - 5 \Rightarrow 11 \geq 5x \Rightarrow 11/5 \geq x$ or $x \leq 11/5$

$\therefore$ The solution set is the half-open interval $(1, 11/5]$.

Absolute value: The absolute value of a number x, denoted by $|x|$, is defined by the formula:

$$|x| = \begin{cases} x & if\ x > 0 \\ -x & if\ x < 0 \end{cases}$$



Example: Finding absolute values.

$|3| = 3$, $|0| = 0$, $|-5| = -(-5) = 5$.

Geometrically, the absolute value of x is the distance from x to 0 on the real number line. Since distances are always positive or 0, we see that $|x| \geq 0$ for every real number x, and $|x| = 0$ if and only if $x = 0$. Also $|x - y|$ is the distance between x and y on the real line.