FLOW THROUGH POROUS MEDIA

Nature of soil body :

In ground water, the body of soil is considered to be of continuous interconnected particles with opening which serve as a fluid carrier.

In general, sand are considered as a porous media composed of macroscopic rounder or angular in shape particles, they drain do not swell possess insignificant capillary, potential and when dry exhibit no volume change.

On other hand clay are microscopic particles plate like in shape, they are highly impervious exhibit considerable swelling.

Ground water movement :

Ground water in its natural is invariably moving . this movement is governed by established hydraulic principles. The flow through porous media, most of which are natural soil, can be expressed by what is known as Darcy's law. Hydraulic conductivity , which is a measure of permeability of the media , is an important constant in the flow equation .
Determination of hydraulic conductivity can be made by several laboratory or field techniques.

Darcy's law (1856) :

More than a century ago henry darcy a French hydraulic engineer investigated the flow through horizontal beds of sand to be used for water filtration . The experimental verification of darcy's law can be performed with water flowing at a rate (Q) through a cylinder of cross sectional area (A) packed with sand and having piezometers a distance (L) apart , as shown in the figure (1). Total energy head , or fluid potential , above a datum plane may be expressed by the energy equation (Bernoulli's equation) :

$$HL = (\frac{P_1}{\gamma} + Z_1) - (\frac{P_2}{\gamma} + Z_2) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Therefore , the resulting head loss is defined as the potential loss within the sand cylinder , this energy being lost by frictional resistance dissipated as heat energy . it follows that the head loss is independent on the inclination of the cylinder .

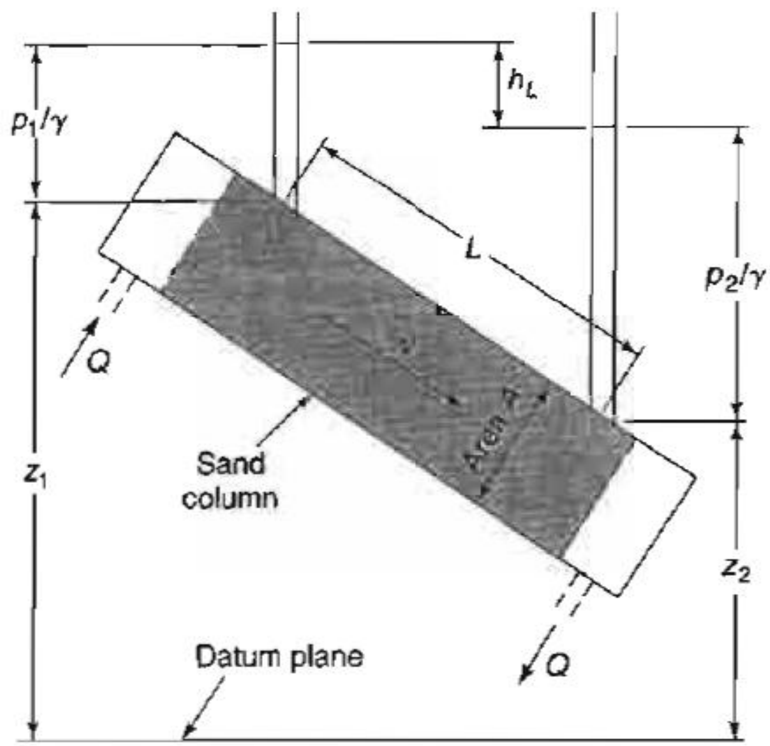


Figure (1) pressure distribution and head loss in flow through a sand column .

Note : velocity in porous media are usually low so $\frac{V^2}{2g}$ (velocity head) is neglected.

Now, darcy 's measurements showed that the proportionalities Q ~ HL and Q ~ $\frac{1}{L}$ exist .

Introducing a proportionality constant (K) leads to the equation :

$$Q = -KA\frac{HL}{L} \quad \text{………………….(2)}$$

Expressed in general term

$$Q = -KA\frac{dh}{dL} \quad \text{………………….(3)}$$

Or simply

$$V = \frac{Q}{A} = -K\frac{dh}{dL} \quad \text{……………....(4)}$$

Where (V) is darcy velocity or specific discharge , (K) is the hydraulic conductivity , a constant that serve as a measure of the permeability of porous media , and $\frac{dh}{dL}$ is the hydraulic gradient . the negative sign indicates that the flow of water is in the direction of decreasing head .

Darcy velocity :

The flow is limited only to the pore space so that the average interstitial velocity :

$$Va = \frac{Q}{\alpha A}$$

Where (α) is the effective porosity . this indicates that for a sand with a porosity of 33% , Va = 3V . to define the actual flow velocity , one must consider the microstructure of the rock material .

In water flowing through sand , for example the pore spaces vary continuously with location within the medium . this means that the actual velocity is nonuniform , involving endless accelerations , decelerations , and change in direction .

Validity of darcy's law :

In applying darcy's law it is important to know the range of validity within which it is applicable . because velocity in laminar flow , such as water flowing in capillary tube , the reynold's number has been employed to establish the limit of flows described by darcy's law :

$$N_R = \frac{\rho VD}{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where (ρ) is the fluid density , (V) the velocity , (D) the diameter of the pipe and ($\boldsymbol{\mu}$) is the (dynamic) viscosity of the fluid .

Darcy's law is valid for ($N_R < 1$) .

Example 1 : if the length and diameter of a field sample of an unconfined aquifer of a packed in a test cylinder are 50 cm and 6 cm , respectively . the field sample is tested for a period of 3 min under a constant head difference of 16.3 cm . as a result 45.2 cm^3 of water is collected at the outlet . determine the hydraulic conductivity of the sample ?

Solution :

The cross section area of the sample is

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.06)^2}{4} = 0.00283 \ m^2$$

The hydraulic gradient , $\frac{dh}{dL}$, is given by

$$\frac{dh}{dL} = \frac{-16.3}{50} = -0.326$$

And the average flowrate is

$$Q = \frac{45.2}{3} = 15.07 \ cm^3/\text{min} = 0.0217 \ m^3/\text{day}$$

Apply darcy’s law to obtain the hydraulic conductivity

$$Q = -KA \frac{dh}{dL}$$

$$K = -\frac{Q}{A\left(\frac{dh}{dL}\right)} = \frac{-0.0217}{0.00283 \times -0.326} = 23.5 \ \text{m/day}$$

Example 2: for example 1,the sample has a median grain size of 0.037 cm , and a porosity of 0.3 . the test is conducted using pure water at 20° C . determine the darcy velocity, average interstitial velocity , and assess the validity of darcy's law ?

$$V = -K\ \frac{dh}{dL} = -23.54 \times -0.326 = 7.67 \text{ m/day}$$

$$Va = \frac{Q}{\alpha\ A} = \frac{V}{\alpha} = \frac{7.67}{0.3} = 25.6 \text{ m/day}$$

$$N_R = \frac{\rho\ VD}{\mu}$$

$\boldsymbol{\mu} = 1.005 \times 10^{-3}$ N/m^2 for water at 20° C $= 1.005 \times 10^{-3}\ \frac{kg}{m.sec}$

$\rho = 998.2$ kg/m^3

so that for N_R=1

$$V_{max} = \frac{\mu}{\rho\ D} = \frac{1.005 \times 10^{-3}}{998.2 \times 0.00037} = 0.00272 \text{ m/sec} = 235 \text{ m/day}$$

Then darcy's law will be valid for darcy velocity equal to or less than 235 m/day . for this sample since V = 7.67 m/day < 235 m/day (darcy's law is valid).

Note : the porosity (α or η) = $\frac{\text{voliume of voides (Vv)}}{\text{total volume V}}$

α or η = $\frac{e}{1+e}$ whrere e = void ratio .

determination of hydraulic conductivity :

hydraulic conductivity (K) in saturated zones can be determined by a variety of techniques , including calculation from formulas , laboratory methods , tracer tests, auger hole tests , and pumping tests of wells .

- ### Formulas :

Several formulas have resulted based on analytic or experimental work . most permeability formulas have the general form :

$K = Cd^2$ ………………….(6)

Where (C) is a dimensionless coefficient , or

$K = F_S\, F_\alpha\, d^2$ …………………..(7)

Where (F_s) is a grain (or pore) shape factor , (F_α) is a porosity factor , and (d) is characteristic grain diameter .

Few formulas give reliable estimates of results because of the difficulty of including all possible variables in porous media . for an ideal medium , such as assemblage of spheres of uniform diameter , hydraulic conductivity can be evaluated accurately from known porosity and packing conditions . because of the problems inherent in formulas , other techniques for determining hydraulic conductivity are preferable .

- ### laboratory methods :

In the laboratory , hydraulic conductivity can be determined by a permeameter , in which flow is maintained through a small sample of material while measurements of flow rate and head loss are made . the constant-head and falling-head types of permeameters are simple to operate and widely employed .

Constant head method :

The constant head permeameter shown in figure (2) can measure hydraulic conductivities of consolidated or unconsolidated formations under low heads . water enters the medium cylinder from the bottom and is collected as overflow after passing upward through the material . from darcy's law it follows that the hydraulic conductivity can be obtained from :

$$K = \frac{V\,L}{A\,t\,h} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where V is the flow volume in time t , and the other dimensions , A , L , and h , are shown in figure (2) .

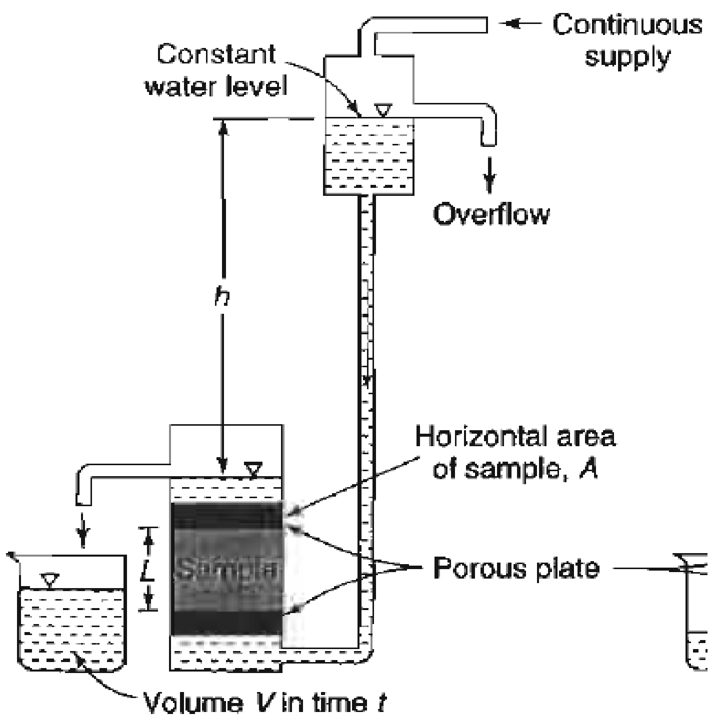


Figure (2): permeameter for measuring hydraulic conductivity at constant head .

It is important that the medium be thoroughly saturated to remove entrappted air . several different heads in a series of tests provide a reliable measurement .

Falling head method :

The water is added to the tall tube as shown in figure (3) . it flows upward through the cylindrical sample and is collected as overflow . the test consists of measuring the rate of fall of the water level in the tube . the hydraulic conductivity can be obtained by noting that the flow rate Q in the tube

$$Q = \pi r_t^2 \frac{dh}{dt} \quad \text{……………….(9)}$$

Most equal that through the sample , which by darcy's law is

$$Q = \pi r_c^2 K \frac{h}{L} \quad \text{……………(10)}$$

After equating and integrating ,

$$K = \frac{r_t^2 L}{r_c^2 t} \ln \frac{h_1}{h_2} \quad \text{…………...(11)}$$

Where L , r_t and r_c are shown in figure (3) , and t is the time interval for the water level in the tube to fall from h_1 to h_2 .

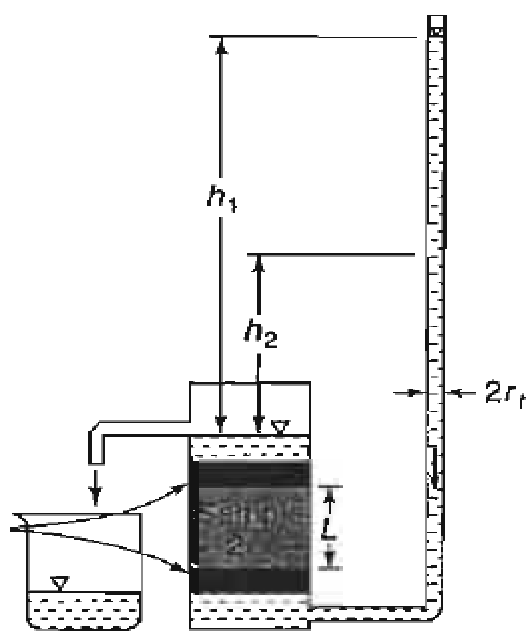


Figure (3) : permeameter for measuring hydraulic conductivity at falling head .

Permeameter results may been little relation to actual field hydraulic conductivitie . undisturbed samples of unconsolidated material are difficult to obtain . while disturbed samples experience change in porosity , packing , and grain orientation , which modify hydraulic conductivities . then two , one or even several samples from an aquifer may not represent the overall hydraulic conductivity of an aquifer variation of several orders of magnitude frequently occur for different depths and locations in an aquifer . furthermore , directional properties of hydraulic conductivity may not be recognized .

Example 3 : A field sample of medium sand with a medium grain size of 0.84 mm , will be tested to determine the hydraulic conductivity using a constant-head permeameter . the sample has a length of 30 cm and a diameter of 5 cm . for pure water at 20 ℃ , estimate the range of piezometric head difference to be used in this test if K = 12 m/day (from table 1) ?

Solution :

Assuming $N_R = 1$ for d = 0.84 mm (max allowable darcy's velocity) .

$$V_{max} = \frac{\mu}{\rho\, D} = \frac{1.005 \times 10^{-3}}{998.2 \times 0.00084} = 0.0012 \text{ m/s} = 103.6 \text{ m/day}$$

Thus , the darcy velocity in the test must be equal to or less than 103.6 m/day. so that darcy's law will be valid , so that

$$V = -K \frac{dh}{dL} \leq 103.6$$

$$|dh| \leq \frac{103.6 \times 0.3}{K}$$

K = 12 m/day

dh = 260 cm .

Table (1) : values of hydraulic conductivity .

material	Hydraulic conductivity (m/day)
Gravel , course	150
Gravel , medium	270
Gravel , fine	450
Sand , course	45
Sand , medium	12
Sand , fine	2.5
Silt	0.08
Clay	0.0002
Sand stone , medium grained	3.1
Sand stone , fine grained	0.2
Lime stone	0.94
Dolomite	0.001
Dune sand	20
Basalt	0.01
Granite , weathered	1.4

Transmissivity :

The term transmissivity T is widly employed in groundwater hydraulic . it may be defined as the rate at which water of prevailing kinematic viscosity is transmitted through a unit width of aquifer under a unit hydraulic gradient . it follows that

$$T = K\,b$$

where T in m^2/day .

and b is the saturated thickness of the aquifer .

Example 4 : the results of constant head permeability on a fine sand are as following : area of soil spacemen 180 cm^2 , length of spacemen 320 mm . constant head maintained 460 mm and flow of water through the spacemen 200 ml in 5 minutes . determine the coefficient of permeability ?

Solution :

$V = 200$ ml = 200 cm^3

$L = 320$ mm = 32 cm

$A = 180$ cm^2

$t = 5$ min

$h = 460$ mm = 46 cm

$$K = \frac{V\,L}{A\,t\,h} = \frac{200 \times 32}{180 \times 5 \times 46} = 0.155 \text{ cm/min} .$$

- field methods :

1- Tracer test :

field determination of hydraulic conductivity can be made by measuring the time interval for a water tracer to travel between two observation wells or test hole , we can apply the following equation :

$$K = \frac{\alpha\, L^2}{h\, t}$$

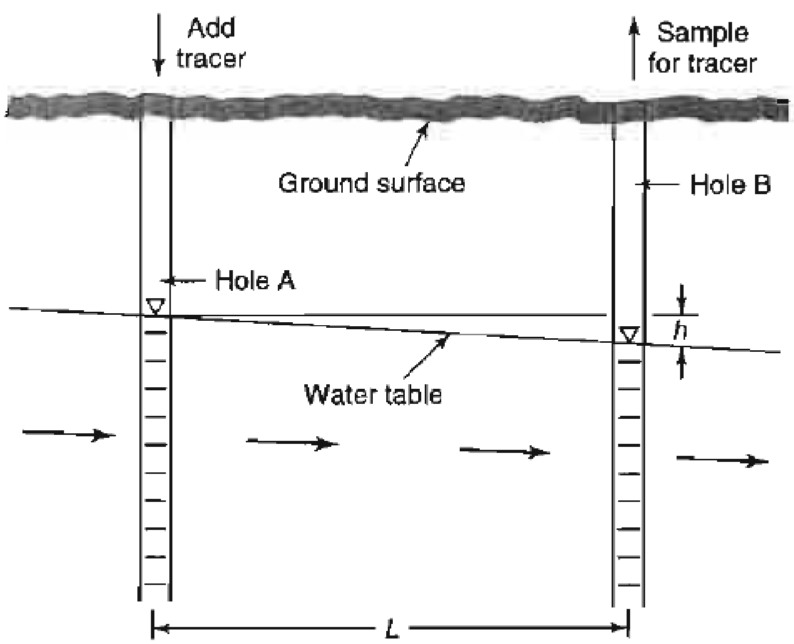
α = porosity

t = travel time interval for the tracer between the holes .

h = difference head between holes .

L = the distance between holes .

Using dye such as calcium chloride .



Example : a tracer test is conducted to determine the hydraulic conductivity of an unconfined aquifer , the water values in the two observation wells 20 m apart are 18.4 m and 17.1 m , the tracer injected in the first well arrives at the second observation well in 167 hours . compute the hydraulic conductivity of the unconfined aquifer given that the porosity of the formulation is 0.25 ?

Solution :

α = 0.25

L = 20 m

h = 18.4 – 17.1 = 1.3 m

t = 167 hr = 6.96 days

$K = \frac{\alpha\, L^2}{h\ t} = \frac{0.25 \times 20^2}{1.3 \times 6.96}$ = 11.1 m/day (sand medium) .

2- Auger hole test :

it is most adaptable to shallow water condition . the permeability K is given by

$$K = \frac{C}{864}\frac{dy}{dt} = \text{m/day} .$$

C = from the table .

$\frac{dy}{dt}$ = measured rate of the rise (cm/sec) .

K = permeability in (m/day) .

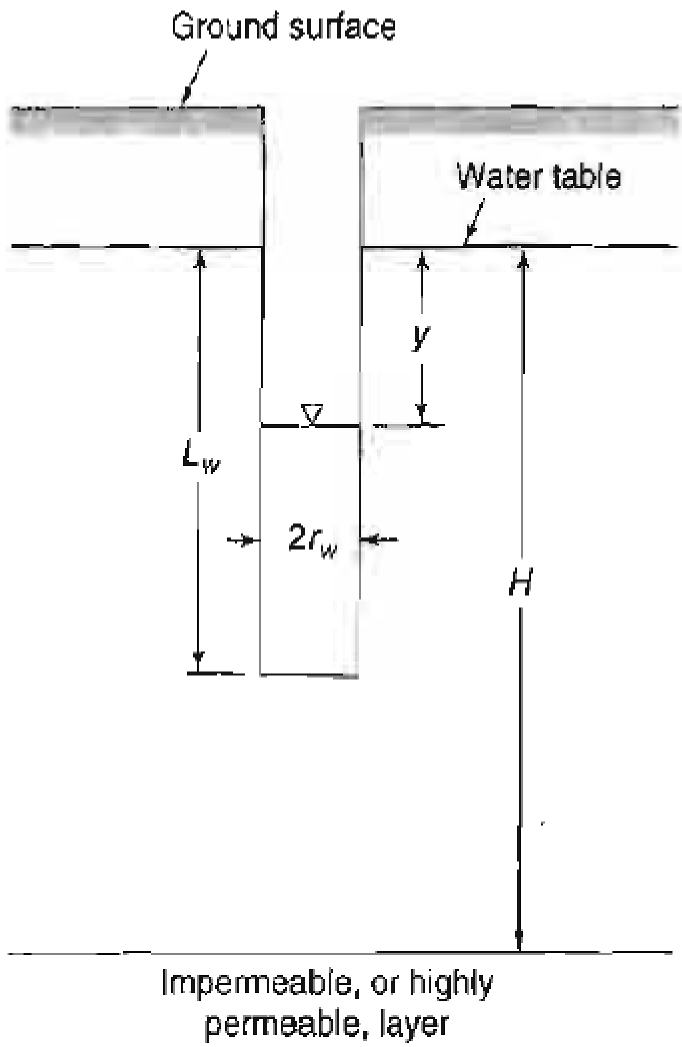


Diagram of an auger hole dimension for determining hydraulic conductivity .

3- Pumping test :

$$K = \frac{Q \times \ln\,(r_2/r_1)}{\pi\,(z_2^2 - z_1^2)}$$

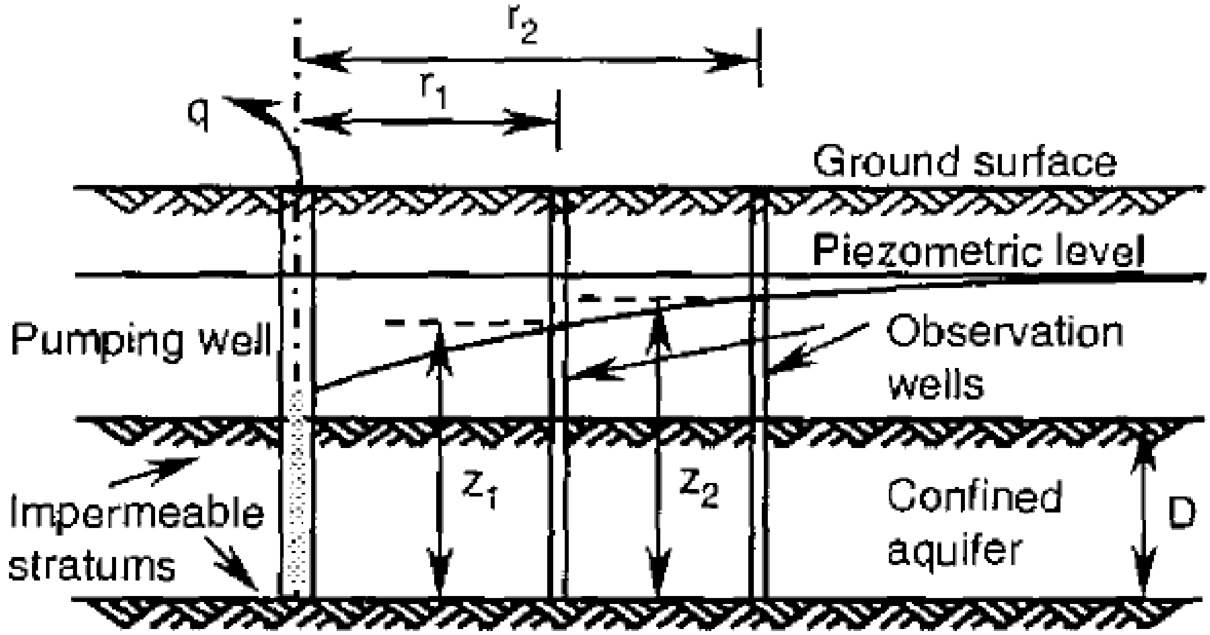
r_1 = radius of well 1 .

r_2 = radius of well 2 .

z_1 = height of water in well 1 .

z_2 = height of water in well 2 .

Q = discharge .



Pumping well in a confined aquifer

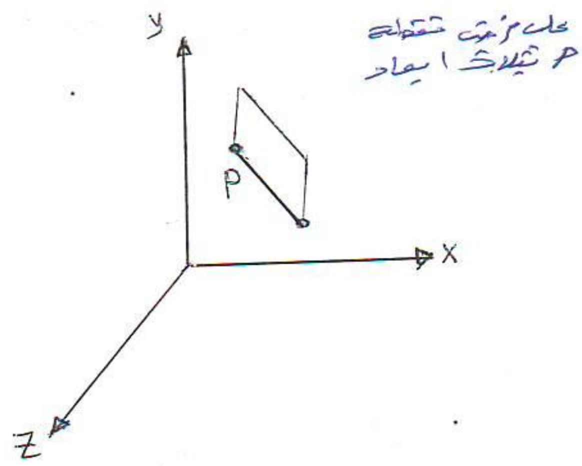
Generalization of darcy's law :

Consider a point in the flow region three dimensional (3D)

$$V_X = -K_X \frac{\partial h}{\partial x}$$

$$V_Y = -K_Y \frac{\partial h}{\partial y}$$

$$V_Z = -K_Z \frac{\partial h}{\partial z}$$



Assumption :

1. homogeneous porous media .
2. continuous saturated flow .
3. homogenous fluid .
4. constant porosity .
5. no change in voids .

consider an element in the flow region with dimension dx , dy , dz

fluid entering the element

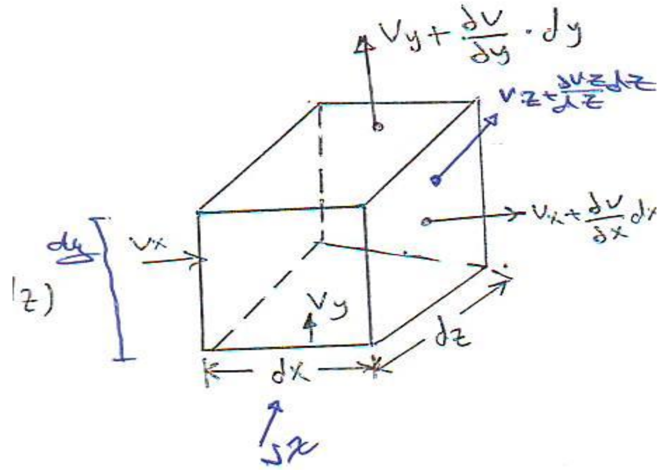
$$q_{in} = V_X .dy .dz + V_Y .dx .dz + V_Z .dx .dy$$

$$q_{out} = \left(V_X + \frac{\partial V_x}{\partial x} . dx \right) dy.dz + \left(V_Y + \frac{\partial V_y}{\partial y} . dy \right) dx.dz + \left(V_Z + \frac{\partial V_z}{\partial z} . dz \right) dx.dy$$

if continuity is valid

$$q_{in} = q_{out}$$

$$\boxed{\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0}$$



if two dimension flow is considered , then

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

$$V_X = -K_X \frac{\partial h}{\partial x}$$

$$\frac{\partial V_x}{\partial x} = -K_X \frac{\partial^2 h}{\partial x^2}$$

$$V_Y = -K_Y \frac{\partial h}{\partial y}$$

$$\frac{\partial V_y}{\partial y} = -K_Y \frac{\partial^2 h}{\partial y^2}$$

$$K_X \frac{\partial^2 h}{\partial x^2} + K_Y \frac{\partial^2 h}{\partial y^2} = 0$$

If the soil is isotropic ($K_X = K_Y = K_Z$)

$$\boxed{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0}$$

∗ Laplace equation ∗

The laplace equation governs the flow in 2D if the above mentioned assumptions are valid .

To solve this equation ; we need to determine a function in terms of (x , y) or functions that satisfy the laplace equation .

Let $\emptyset$. Function = potential function .

ψ. Function = stream function .

the potential function $\emptyset$ (x,y) is assumed so that :

$$\frac{\partial \emptyset}{\partial x} = V_X = u = -K \frac{\partial h}{\partial x}$$

$$\frac{\partial \emptyset}{\partial y} = V_Y = V = -K \frac{\partial h}{\partial y}$$

By integrating the above two equation :

$\emptyset$ (x,y) = K . h (x,y) + C

This equation physically means the following :

K and C are constant .

$\emptyset$ = K . h (x,y) + C

Then if a constant value $\emptyset_1$ is substituted in the above equation , a constant value for h say h(x,y) obtained .

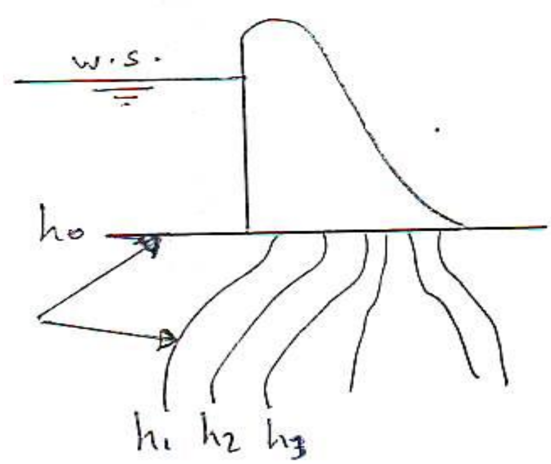
The value represent a line or curve in the flow region plane (x,y) where all points at this line in curve has a constant total head of h_1 .

If a series of $\emptyset$ values are substituted in the equation , then a family of curves are obtained in the flow region corresponding to a set of equipotential lines .

The equation that may arise at the stage :

If $\emptyset$ the only function that satisfies the laplace equation .

Equipotential line



A second function Ψ(x,y) can be considered as a (stream function) or (flow function) .

$$\frac{\partial \psi}{\partial y} = -V_X = K\frac{\partial h}{\partial x} \quad \gg \quad \frac{\partial^2 \psi}{\partial y^2} = K\frac{\partial^2 h}{\partial x \partial y}$$

$$\frac{\partial \psi}{\partial x} = V_Y = -K\frac{\partial h}{\partial y} \quad \gg \quad \frac{\partial^2 \psi}{\partial x^2} = -K\frac{\partial^2 h}{\partial y \partial x}$$

$K\frac{\partial^2 h}{\partial x \partial y} - K\frac{\partial^2 h}{\partial y \partial x} = 0$ therefore Ψ(x,y) satisfy the laplace equation .

$$d\psi = \frac{\partial \psi}{\partial x} . dx + \frac{\partial \psi}{\partial y} . dy$$

$$0 = -V_y . dx + V_X . dy$$

$\boxed{\frac{dy}{dx} = \frac{V_y}{V_x}}$ This is represent the slope of the flow line .

$$d\emptyset = \frac{\partial \emptyset}{\partial x} . dx + \frac{\partial \emptyset}{\partial y} . dy$$

$$0 = V_X . dx + V_y . dy$$

$$\boxed{\frac{dy}{dx} = \frac{-V_x}{V_y}}$$

$\frac{V_y}{V_x} . \frac{-V_x}{V_y} = -1$ ∴ the two lines intersection at 90° .

Stream lines and equipotential lines :

$$\frac{V_y}{V_x} = \tan\theta = \frac{dy}{dx}$$

$V_X \cdot dy = V_y \cdot dx$

$V_X \cdot dy - V_y \cdot dx = 0$

$U = \frac{\partial\psi}{\partial y}$ and $V = \frac{-\partial\psi}{\partial x}$

$$d\psi = \frac{\partial\psi}{\partial x} \cdot dx + \frac{\partial\psi}{\partial y} \cdot dy = 0 = d\psi$$

$\therefore\ \Psi$ = constant

Consider the curves represented by $\emptyset$ (x,0)

$$d\emptyset = \frac{\partial\emptyset}{\partial x} \cdot dx + \frac{\partial\emptyset}{\partial y} \cdot dy$$

$u = \frac{\partial\emptyset}{\partial x}$, $V = \frac{\partial\emptyset}{\partial y}$

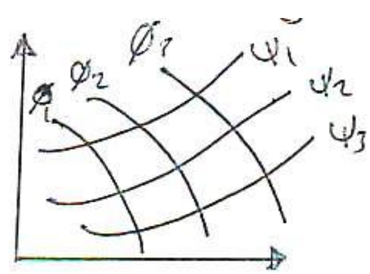
$u. dx + v.dy = 0$ $\therefore$ $\boxed{\frac{dy}{dx} = \frac{-u}{V}}$

the complete solution of the laplace equation is represented for isotropic soil by two families of curves :

the $\emptyset$ family : represents equipotential lines

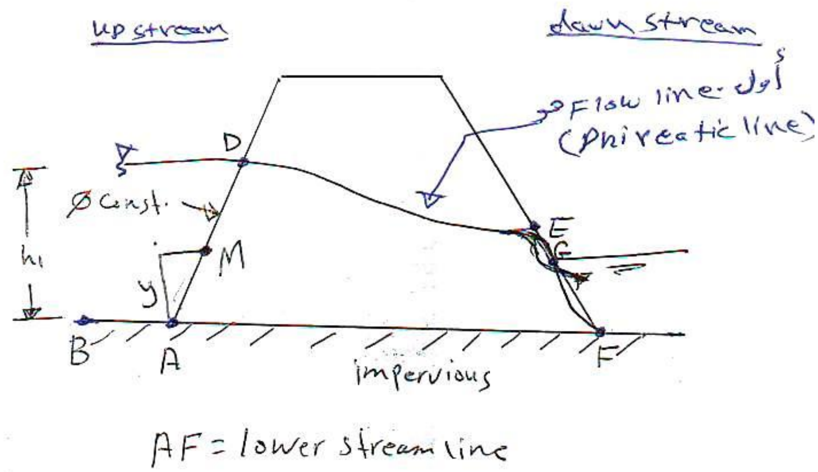
the ψ family : represent stream lines

the two families intersect each other orthogonally .



the intersection of the stream lines and equipotential lines for what is known as the flow net .

Boundary conditions :



1- impervious boundary .

n = normal

t = tangential

$$\frac{\partial \phi}{\partial n} = \frac{\partial \psi}{\partial t} = 0$$

2- Boundary of the reservoir .

At point M

$P = \gamma_w \, (h_1 - y)$

$\phi = -K \left(\frac{p}{\gamma_w} + y \right) + C$

Given $\phi = -K \, h_1 + C = \text{constant}$

∴ AB and AD are equipotential lines .

3- Consider surface of seepage EF .

Pressure = constant = atmospherical pressure = P

∴ $\phi = -K \, \frac{p}{\gamma_w} - Ky + C$

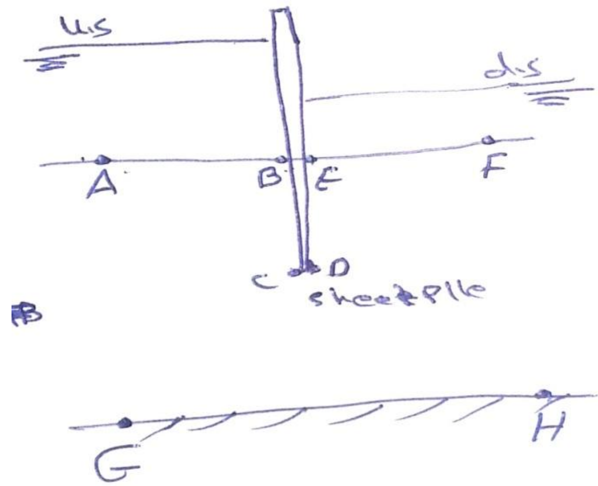
$\phi + Ky = \text{constant}$.

4- Line of seepage DE .

$\phi + Ky = \text{constant}$ flow line .

Example : find the boundary condition of the following hydraulic structure ?

Solution :



AB = equipotential line

BC = flow line

DE = flow line

EF = equipotential line

GH = flow line

∗ The general form of laplace equation is

$$K_y \frac{\partial^2 h}{\partial y^2} + K_X \frac{\partial^2 h}{\partial x^2} = \frac{1}{1+e} \frac{\partial (s.e)}{\partial t}$$

Four possible cases :

1. e and s are both constant [steady state flow] .
2. e varies and s constant [consolidation] .
3. e constant and s varies [constant volume drainage] .
4. e and s varies [compression or expansion problem] .

for all seepage problem consider in this course steady state flow .

the solution of laplace equation for isotropic porous media is a mesh with the following properties :

1. each element is a curvalinear square $\frac{b}{L} \approx 1$.
2. the potential lines intersect the flow lines at angles 90° [right angles] .

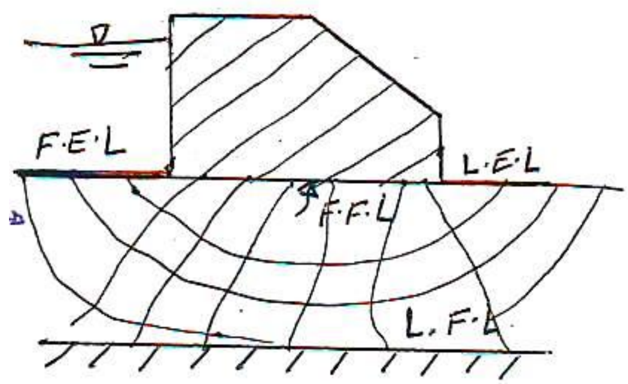
solution of the laplace equation :

1. graphical solution .
2. direct mathematical solution .
3. numerical solution .
4. electrical analogy solution .

1- graphical solution :

means plotting a flow net with a sensible scale (1m = 1 cm) , and then fix the boundary conditions , first flow line (F.F.L) , and last flow line (L.F.L) , also first equipotential line (F.E.L) and last equipotential line (L.E.L) .

* choosing the downstream as a datum is preferable .

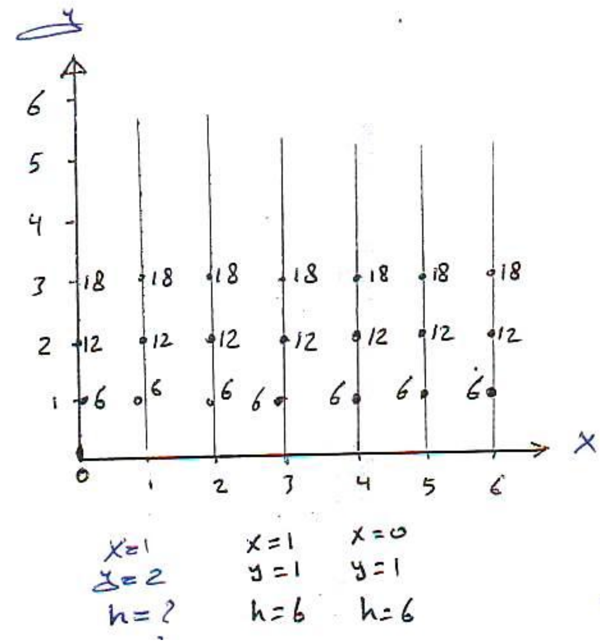


2- direct mathematical solution :

packing functions that satisfies , the laplace equation and choosing the boundary conditions , that make the solution apply to physical problems .

Example : h = 6y one dimensional flow , is this function satisfies laplace equation ? and draw the function ?

Solution :



Laplace equation is : $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$

$\frac{\partial^2 h}{\partial x^2} = 0$

$\frac{\partial h}{\partial y} = 6 \gg \frac{\partial^2 h}{\partial y^2} = 0$

∴ satisfies laplace equation .

This function one dimensional flow .

Example : A soil flow field is described by a stream function $\psi = 2y - 2x^2$. find whether the flow satisfy laplace equation ? draw the function ?

Solution :

$\frac{\partial \psi}{\partial x} = -4x \quad \gg \quad \frac{\partial^2 \psi}{\partial x^2} = -4$

$\frac{\partial \psi}{\partial y} = 2 \quad \gg \quad \frac{\partial^2 \psi}{\partial y^2} = 0$

$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ (for laplace)

$-4 + 0 = -4$

∴ laplace equation is not satisfy .

3- numerical solutions :

using finite difference (F.D.) form of the flow equation . as we have seen before that the flow equation in two dimensional form to :

$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$ assuming the flow in the x-y plane .

Before the finite difference is introduced , a brief revision is set out (maclaurian series)

Assuming F(x) can be expanded

$$y = F(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots\dots\dots + a_n x^n$$

$$\overline{y} = \overline{F}(x) = a_1 + 2a_2 x + 3a_3 x^2 + \dots\dots\dots\dots + n\, a_n x^{n-1}$$

$$\overline{\overline{y}} = \overline{\overline{F}}(x) = 2a_2 + 6a_3 x + \dots\dots\dots\dots + a_n\, n\,(n-1)\, x^{n-2}$$

$$\overline{\overline{\overline{y}}} = \overline{\overline{\overline{F}}}(x) = 6a_3 + \dots\dots\dots + a_n\, n\,(n-1)\,(n-2)\, x^{n-3}$$

If we put X = 0

$$a_0 = F(0) \quad , \quad a_1 = \overline{F}(0) \quad , \quad a_2 = \frac{\overline{\overline{F}}(0)}{2!} \quad , \quad a_3 = \frac{\overline{\overline{\overline{F}}}(0)}{3!}$$

In general

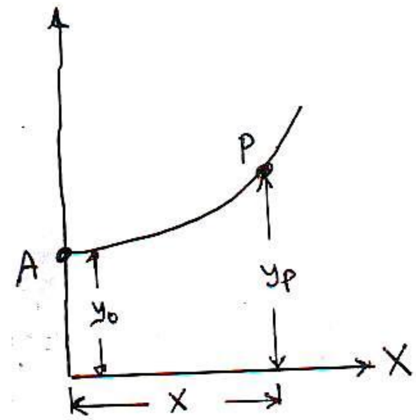
$$\boxed{a_n = \frac{F^n(0)}{n!}}$$

Substitute back into the equation

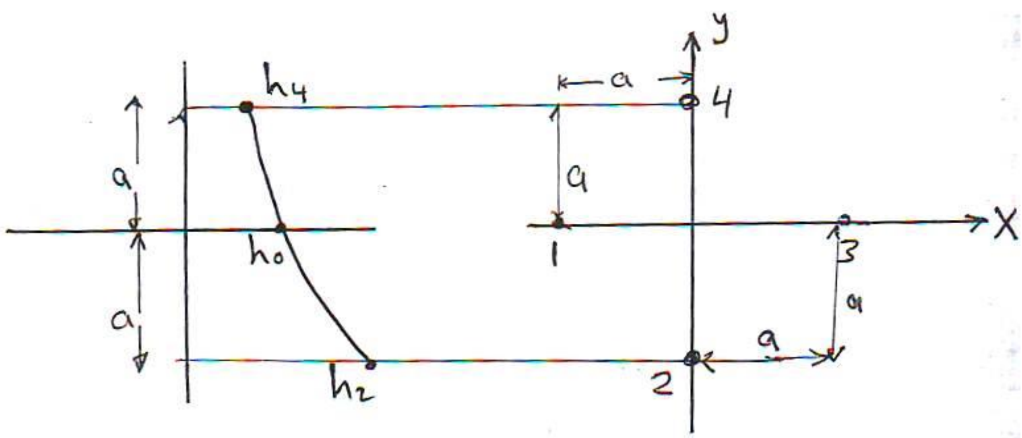
$$F(x) = F(0) + X\,\overline{F}(0) + X^2\,\frac{\overline{\overline{F}}(0)}{2!} + X^3\,\frac{\overline{\overline{\overline{F}}}(0)}{3!} + \dots\dots + X^n\,\frac{F^n(0)}{n!}$$

At point (p) in the figure below

$$F(x) = Y_p = Y_o + X\,\overline{Y_o} + X^2\,\frac{\overline{\overline{Y}}_o}{2!} + X^3\,\frac{\overline{\overline{\overline{Y}}}_o}{3!} + \ldots\ldots$$



in flow problems consider the figure



$$h_2 = h_o + a\,\overline{h_o} + a^2\,\frac{\overline{\overline{h}}_o}{2!} + a^3\,\frac{\overline{\overline{\overline{h}}}_o}{3!} + \ldots\ldots\ldots\ldots$$

$$h_4 = h_o - a\,\overline{h_o} + a^2\,\frac{\overline{\overline{h}}_o}{2!} - a^3\,\frac{\overline{\overline{\overline{h}}}_o}{3!} + \ldots\ldots\ldots\ldots$$

a = negative in the direction of flow .

$$h_2 + h_4 = 2h_o + a^2\,\overline{\overline{h}}_o$$

$$\overline{\overline{h}}_o = \frac{h_2 + h_4 - 2h_o}{a^2} = \frac{\partial^2 h}{\partial y^2}$$

Similar in the X-direction

$$\bar{\bar{h}}_o = \frac{h_1 + h_3 - 2h_o}{a^2} = \frac{\partial^2 h}{\partial x^2}$$

Laplace equation is : $\dfrac{\partial^2 h}{\partial x^2} + \dfrac{\partial^2 h}{\partial y^2} = 0$

$$\frac{h_1 + h_2 + h_3 + h_4 - 4h_o}{a^2} = 0$$

$$\boxed{h_o = \frac{1}{4}\,(h_1 + h_2 + h_3 + h_4)}$$

We can proceed in two ways :

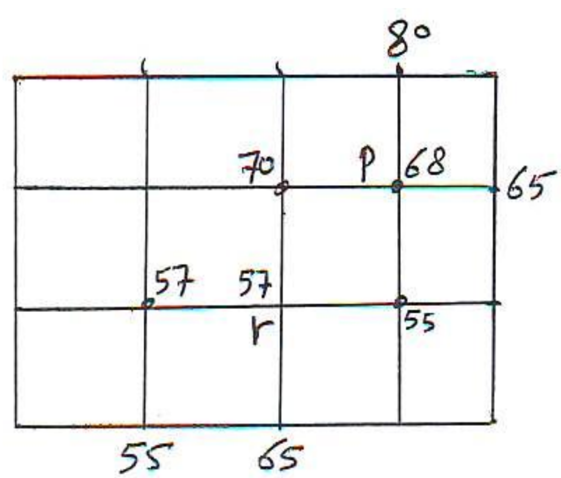
1- direct solution :

set up a matrix to solve for head at all points in a square grid (with the aid of a computer) .

2- relaxation method (solution) :

Adopting values of head given by an initial flow net and progressing correcting the numerical values of the head .

Assume a trial flow net :



Interpolate initial values to give nodal values when the values are substituted in the finite difference equation :

$$h_1 + h_2 + h_3 + h_4 - 4h_o = R$$

R is the termed the residual , we increase the value of (h_o) by (R/4).

This correction means that equation is now satisfied at the interior , but in the process , the residual at the four adjacent nodes has been increased by (R/4) .

From the figure :

At point (p) the residual is :

$65 + 80 + 70 + 55 - (4 \times 68) = -2$ negligible because is very small .

At point (r) the residual is :

$55 + 70 + 57 + 65 - (4 \times 57) = 19$

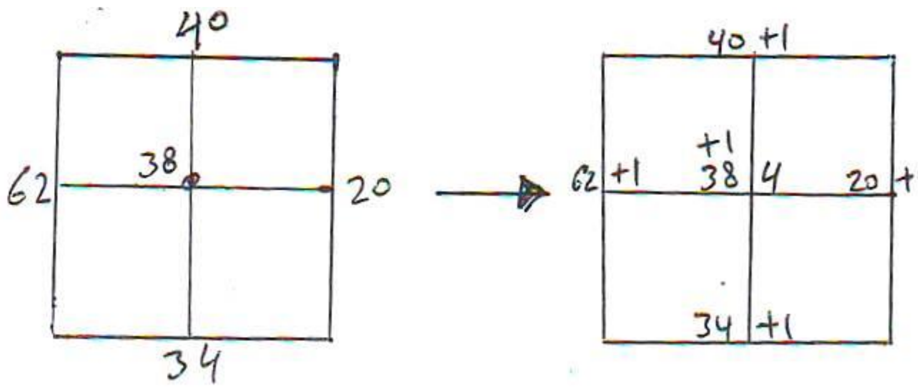
$$\frac{19}{4} \approx 5$$

The corrected head = 57 + 5 = 62

[+5 is added to <u>each residual</u> at each adjacent nodes]

Example :

Determine a more accurate potential for the central point :



Solution :

$h_1 + h_2 + h_3 + h_4 - 4h_o = R$

$40 + 62 + 34 + 20 - (4 \times 38) = 4 = R$

$\frac{R}{4} = \frac{4}{4} = +1$

This residual is eliminated by adding (+1) to the value of the potential at the center point and (+1) to the residuals at points 1 , 2 , 3 and 4 .

Method of fragments :

To determine the seepage discharge for confined flow problems pavlovsky 1956 introduced the method and was brought to attention of the western world by harr 1962 .

Basic assumptions :

Equipotential lines at selected critical points in the flow net into fragments . table (6–2) in harr page (156 – 157) gives summary of fragments type and form factors . with the aid of tables (6 –2) and appendix B –1 . harr page 286.

We can compute the discharge in the m-th fragments as :

$$\boxed{Q = \frac{K\,h_m}{\emptyset_m}} \quad , \; m = 1,2,3,\ldots.$$

h_m = head loss through fragment

$\emptyset_m$ = dimensionless form factor

Discharge the same between the fragments

$$Q_1 = \frac{K\,h_1}{\emptyset_1} \quad , \quad Q_2 = \frac{K\,h_2}{\emptyset_2} \quad , \quad Q_3 = \frac{K\,h_3}{\emptyset_3}$$

$$Q = \frac{K \sum h_m}{\sum \emptyset_m} = K \frac{h_{\,total}}{\sum_{m=1}^{n} \emptyset_m}$$

Head loss in the m-th fragment (h_m)

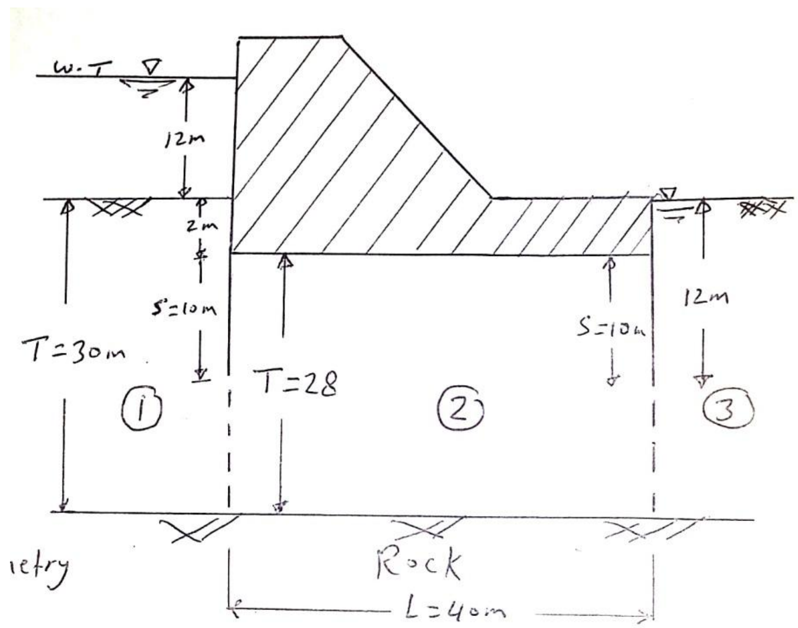
$$\boxed{h_m = \frac{h_{\,total} \times \emptyset_m}{\sum \emptyset}}$$

Next step , define types of fragments and determine the value of the form , ∅ factor for each fragment .

Example :

the dam with the data shown in figure below ; determine

1. quantity of seepage loss under dam if $K = 20 \times 10^{-4}$ cm/sec .
2. exit gradient .
3. pressure distribution on the base of the dam .



solution :

1)

1. divide problem into fragments to match the particular boundary conditions of the problem .
2. in this example head loss (hL = 12 cm) = $\sum h_m$ = sum of head loss in each fragment .
3. from table (6 –2) and the geometry of the problem :

fragment 1 and 3 type (||)

fragment 2 type (V)

4. calculate the form factor $\emptyset$ for two types of fragments

$\emptyset = \frac{K}{\overline{K}}$ ≫ both K and $\overline{K}$ are function of m

$$\boxed{m = \sin\frac{\pi\, S}{2T}}$$

S = depth of sheet pile = 12 m

T = thickness of the soil layer

$m = \sin\frac{\pi \times 12}{2 \times 30} = 0.588$

$m^2 = 0.345$

$\frac{K}{\overline{K}} = 0.865$

From table appendix B , $\emptyset = 0.865$

Fragment 1 and 3 same form factor .

<u>Fragment 2</u> :

S = 10 , 2S = 20

L = 40 , then L > 2S

$$\emptyset_2 = 2\ln\left(1+\frac{S}{a}\right) + \left(\frac{L-2S}{T}\right) = 2\ln\left(1+\frac{10}{18}\right) + \frac{40-(2\times 10)}{28} = 1.598$$

$$Q = \frac{K.h}{\sum\emptyset} = \frac{20\times 10^{-4}\ cm/\sec \times \frac{1}{100}\times 12}{0.865+1.598+0.865} = 7.21\times 10^{-5}\ m^3/\sec \text{ per m}$$

2) exit gradient (i exit) :

From table (6 –2) , type (||)

i exit = $\frac{h\ \pi}{2\ K\ T m}$ ≫ * h in this section (third fragment) .

m = 0.588

h = head loss in third fragment = h_3

$h_3 = \frac{h_t\ .\ \emptyset_3}{\sum \emptyset} = \frac{0.865 \times 12}{3.328}$ = 3.12 m

m^2 = 0.345 , K = 1.741 m^3/sec . per m. (from table)

i exit = $\frac{3.12 \times \pi}{2 \times 1.74 \times 30 \times 0.588}$ = 0.16

3) pressure distribution under dam :

assumption that the pressure or head loss varies linearly from fragment 1 to 3

steps :

1. compute head loss per fragment :

$\sum \emptyset = \emptyset_1 + \emptyset_2 + \emptyset_3$ = 0.865 + 1.598 + 0.865 = 3.328 m

2. head loss for fragment :

$h_1 = \frac{\emptyset_1\ .\ h_{total}}{\sum \emptyset} = \frac{0.865 \times 12}{3.328}$ = 3.12 m

$h_2 = \frac{\emptyset_2\ .\ h_{total}}{\sum \emptyset} = \frac{1.598 \times 12}{3.328}$ = 5.76 m

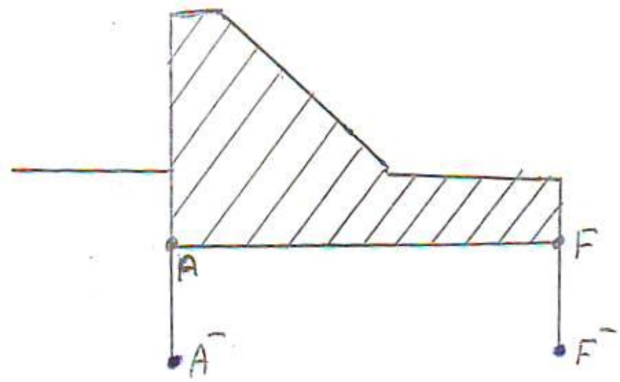
$h_3 = h_1$ = due to symmetry = 3.12 m

<u>at point $\bar{A}$</u> :

h = 12 – 3.12 = 8.88 m

<u>at point $\bar{F}$</u> :

h = 12 – 3.12 – 5.76 = 3.12 m



assuming linearly of the head loss from points $\bar{A}$, A , F , $\bar{F}$

total distance = 10 + 40 + 10 = 60 m

head loss / m = $\frac{h_2}{60} = \frac{5.76}{60}$ = 0.096 m/m

head loss at point A = $h_{\bar{A}}$ – (10 × 0.096) = 8.88 – (10 × 0.096) = 7.92 m

this is correct out for other points pressure head = $h_t - h_2$ = 7.92 – (–2) = 9.92 m .

Head at point F = 7.92 – (0.096 × 40) = 4.08 m

<u>Or</u> pressure at F + (10 × 0.096) = 3.12 + (10 × 0.096) = 4.08 m .

Seepage through earth dams on impervious base :

Several solutions to determine discharge free surface through homogenous earth dams :

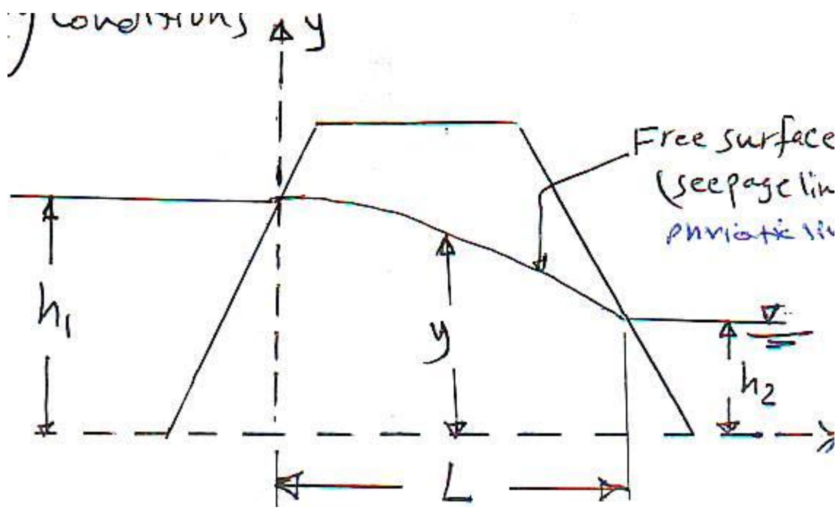
- ### dupuits solution

discharge per unit width through any vertical section :

$$Q = -K \cdot y \cdot \frac{dy}{dx}$$

Integrating and applying boundary conditions

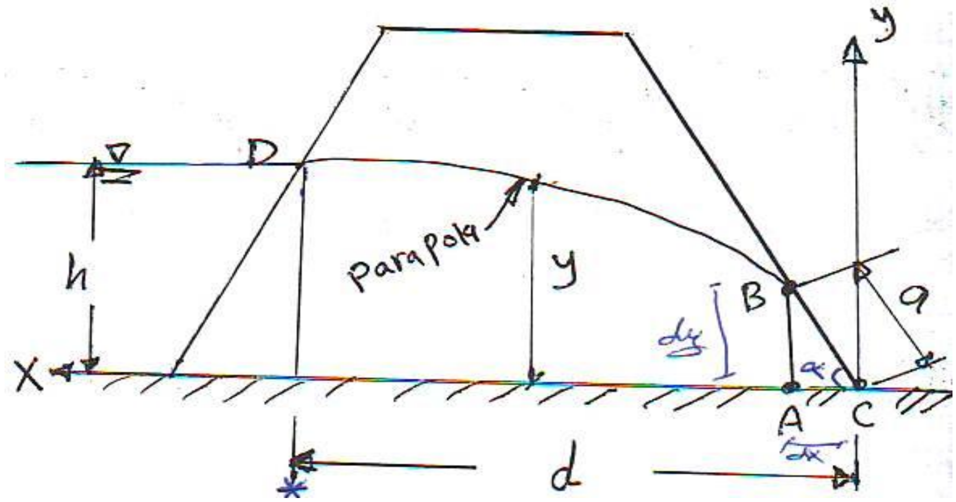
$$\boxed{Q = \frac{K\,(h_1^2 - h_2^2)}{2L}}$$



Lamations :

1- no recognition to the entrance and exit condition .
2- if $h_2 = 0$, free surface intersects the impervious base .
3- free surface and discharge are independent of the slope of the dam .

- schaffornak and van iterson solution :



consider the triangle C A B .

Q = K . y . $\frac{dy}{dx}$ = K (a sin α) (tan α)

a = length of the surface of the seepage .

y . $\frac{dy}{dx}$ = a sin α tan α

$$\int_{a \sin\alpha}^{h} y \, . dy = \int_{a \cos\alpha}^{d} (a \sin\alpha \tan\alpha) \, dx$$

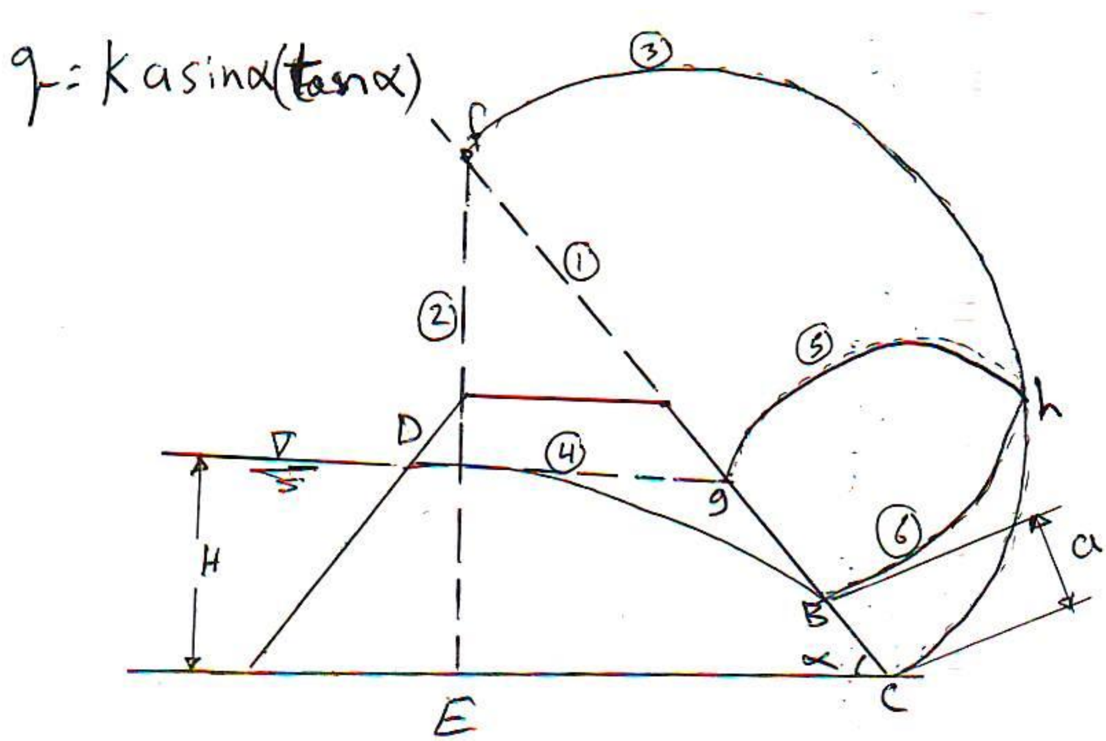
$$\boxed{a = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{(\cos\alpha)^2} - \frac{h^2}{(\sin\alpha)^2}}}$$

- Schaffornak suggested a graphical procedure to determine the value of (a) by the following :

1- extend the downstream slope line CB upward .
2- draw a vertical line DE through the point D . this will intersect the projection of line CB at F .
3- with FC diameter draw a semicircle FhC .
4- draw a horizontal line Dg .
5- with C as a center and Cg as a radius , draw an arc of a circle , hg .
6- with F at the center Fh as the radius , draw an arc of a circle , hB .
7- measure BC = a .

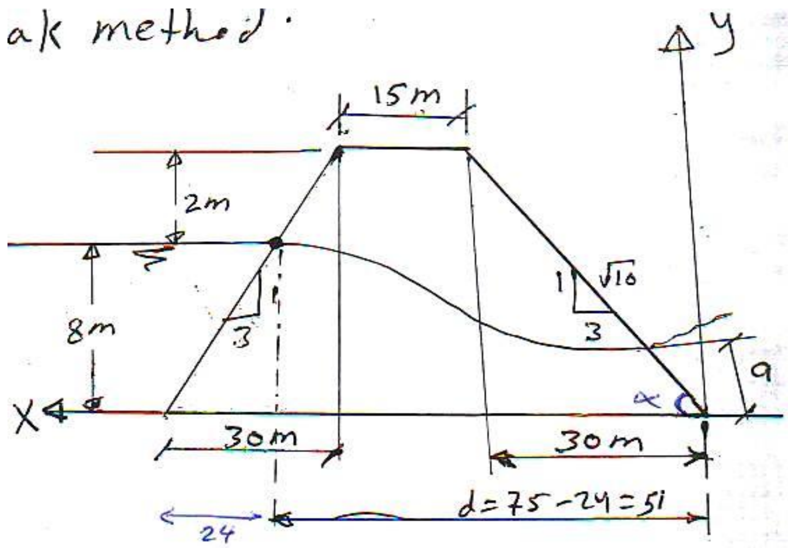
then

$$\boxed{Q = K\,(a \sin\alpha)\,(\tan\alpha)}$$



Example : determine the rate of seepage through the dam shown in figure below using schaffornak method ?

Given K = 3×10^{-3} m/sec



Solution :

Q = K (a $\sin\alpha$) ($\tan\alpha$)

$$a = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{(\cos\alpha)^2} - \frac{h^2}{(\sin\alpha)^2}}$$

$$\cos\alpha = \frac{3}{\sqrt{10}} \quad ; \quad \sin\alpha = \frac{1}{\sqrt{10}}$$

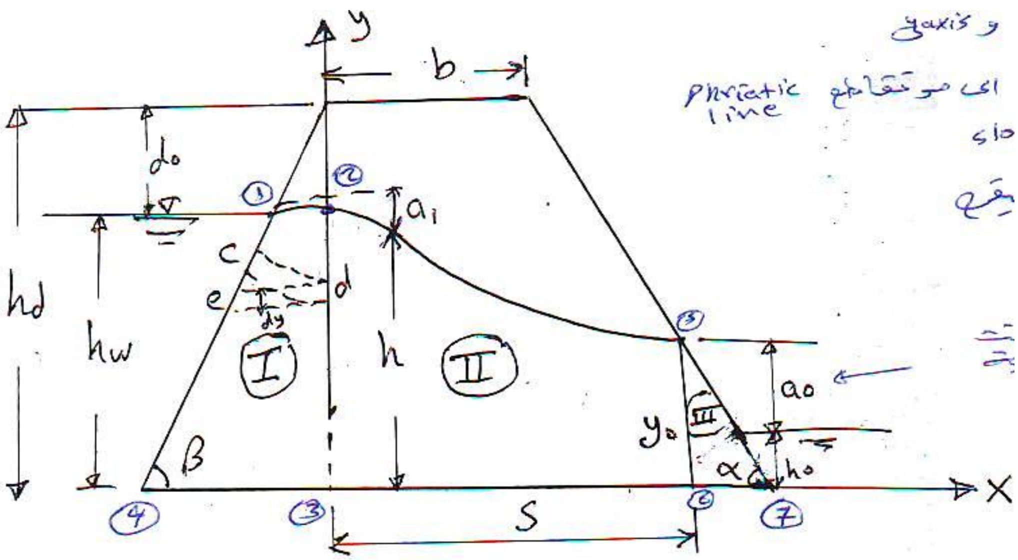
d = 75 – 24 = 51 m

$$\therefore \ a = \frac{51}{3/\sqrt{10}} - \sqrt{\frac{51^2}{(3/\sqrt{10})^2} - \frac{8^2}{(1/\sqrt{10})^2}} = 6.3246 \text{ m}$$

$$Q = 3 \times 10^{-3} \times 6.3246 \times \frac{1}{\sqrt{10}} \times \frac{1}{3} = 2 \times 10^{-3} \ \text{m}^3\text{/sec/m}$$

Also can you calculate the value of (a) using drawing method .

- pavlovsky's solution :



Assumptions :

Zone (|) : bounded by the upstream slope and the y-axis .

Zone (‖) : between the y-axis and the discharge point of the free surface .

Zone (III) : by the discharge point and the downstream slope .

In zone (|) : stream lines are curvilinear may be replaced by horizontal stream lines .

Cd may be replaced by ed .

$$dq = K \cdot \frac{a_1}{m\,(hd-y)}\, dy$$

m = cot β .

$\frac{a_1}{m\,(hd-y)}$ is the hydraulic gradient .

$$\boxed{\frac{q}{K} = \frac{h_w-h}{m} \ln \frac{hd}{hd-h}} \qquad \dots\dots\dots\dots\dots\dots (1)$$

Zone (Ⅱ) : pavlovsky's used dupuit's formula for this zone

$$\boxed{\frac{q}{K} = \frac{h^2 - (\,a_o + h_o\,)^2}{2\,S}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (\,2\,)$$

Zone (III)

A) no tail water $h_o = 0$, assuming horizontal flow .

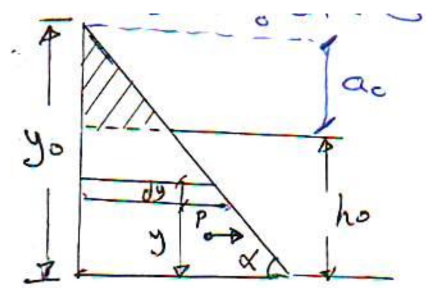
$$\int_0^q dq = \text{K}\ \frac{1}{m_1} \int_0^{a_o} dy \qquad , \quad m_1 = \cot\alpha$$

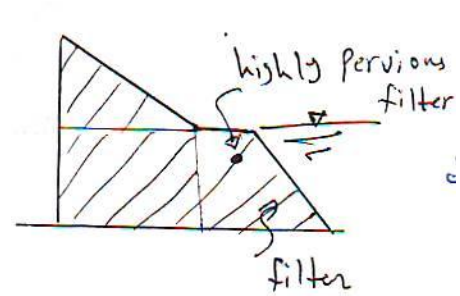
$$\boxed{\frac{q}{K} = \frac{a_o}{m_1}}$$

B) considering tail water $h_o > 0$

$$\frac{q}{K} = \int_{h_o}^{y_o} \frac{dy}{m_1} + \int_0^{h_o} \frac{(\,y_o - h_o\,)}{m_1\,(\,y_o - y\,)}\,\text{dy}$$

$$\boxed{\frac{q}{K} = \frac{a_o}{m_1}\,[\,1 + \ln(\,\frac{a_o + h_o}{a_o}\,)]}$$





From the geometry of the figure

$S = b + m_1 [hd - (a_o + h_o)]$ ………….. (4)

Four unknowns , four equations h , a_o , s , q

If tail water can neglected then

$a_o = \frac{b}{m_1} + hd - \sqrt{(\frac{b}{m_1} + hd)^2 - h^2}$ ………………(a)

$\frac{a_o . m}{m_1} = (h_W - h) \ln(\frac{hd}{hd-h})$ ……………. (b)

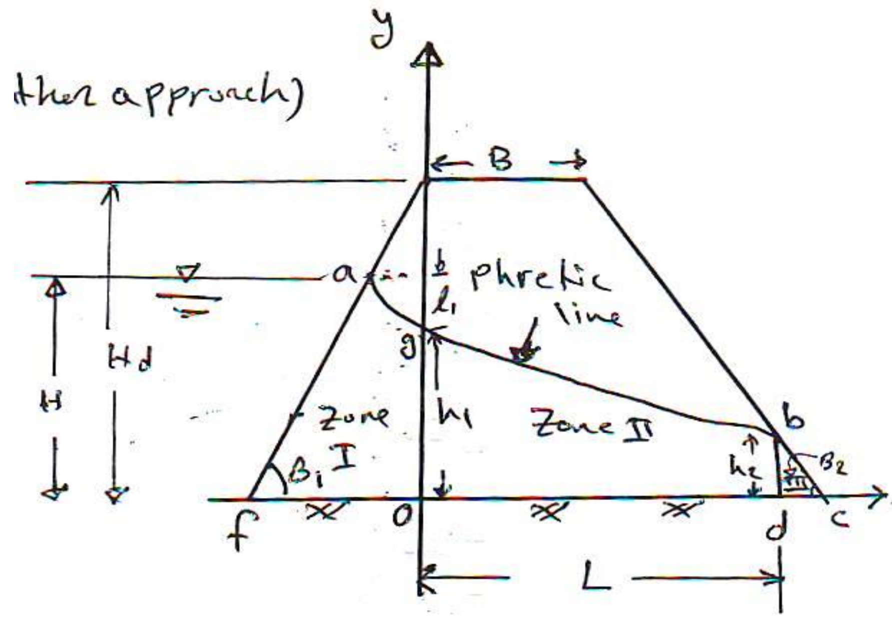
Two equations may be solved to obtain a_o and h .

- ## Pavlovsky's solution (another approach) :

From the figure and divide the dam into three zones , and the rate of seepage can be calculated as :

Zone (|) : area of (agof)

$$q = \frac{K\,(H-h_1)}{\cot\beta_1} \ln \frac{Hd}{Hd-h_1}$$



zone (∥) : area of (ogbd)

$$q = \frac{K}{2L}\,(h_1^2 - h_2^2)$$

where

$L = B + (Hd - h_2)\cot\beta_2$

zone (III) : area of (bed)

$$q = \frac{K\,h_2}{\cot\beta_2}$$

using the following equations to find two unknowns (h_1) and (h_2) which can be solved graphically , often that apply any of above equations to find the rate of seepage per unit length of dam .

$$h_2 = \frac{B}{\cot\beta_2} + Hd - \sqrt{\left(\frac{B}{\cot\beta_2} + Hd\right)^2 - h_1^2} \quad \text{.................... (1)}$$

$$\frac{(H-h_1)}{\cot\beta_1} \ln \frac{Hd}{Hd-h_1} = \frac{h_2}{\cot\beta_2} \quad \text{........................... (2)}$$

• L – Casagrand's method :

Assumptions :

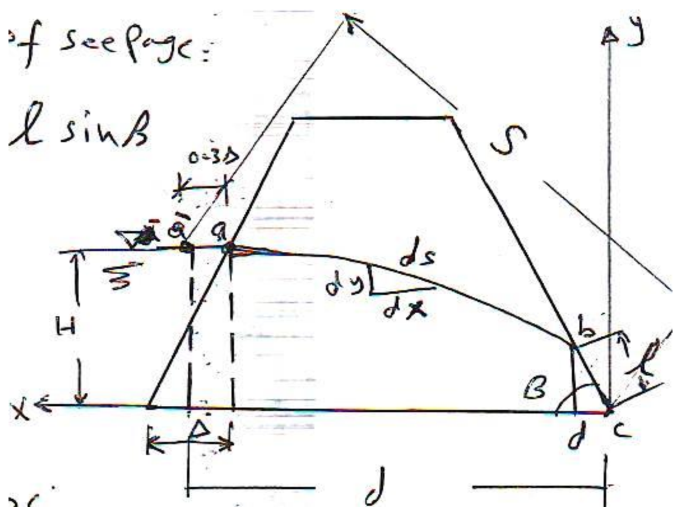
* $\frac{dy}{ds}$ is used instead of $\frac{dy}{dx}$.

* S = is the long of free surface as shown in figure .

* casagrand shown experimentally that the free surface parabola should actually start from (0.3Δ) .

* for a downstream slope $\beta > 30°$, the deviations from dupuit's assumption become more noticeable .

Consider the triangle (bcd) , the rate of seepage



$i = \frac{dy}{ds} = \sin\beta$

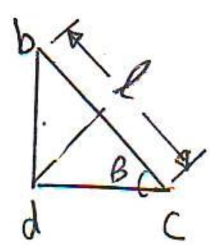
A = (bd) × 1 = $\ell \sin\beta$

$q = K\,y\,\frac{dy}{ds} = K\,\ell\,(\sin\beta)^2$

$\int_{l \sin\beta}^{H} y\,dy = \int_{l}^{S} [\, l\,(\sin\beta)^2\,]\,ds$

Where S is the length of curve $\bar{a}$ b c

$\frac{1}{2}\left(H^2 - \ell^2 (\sin\beta)^2\right) = \ell\,(\sin\beta)^2\,(S - \ell)$



$H^2 - \ell^2(\sin\beta)^2 = 2\ell\,S\,(\sin\beta)^2 - 2\ell^2(\sin\beta)^2$

$\ell^2 - 2\ell\,S + \frac{H^2}{(\sin\beta)^2} = 0$ (solve the equation)

$$\boxed{\ell = S - \sqrt{S^2 - \frac{H^2}{(\sin\beta)^2}}}$$

with about (4 – 5) % error , we can approximate (S) the length of a straight line ($\bar{a}$c) so

$S = \sqrt{d^2 + H^2}$ then

$$\boxed{\ell = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2 \, (\cot\beta)^2}}$$

Once ℓ is known , the rate of seepage calculate :

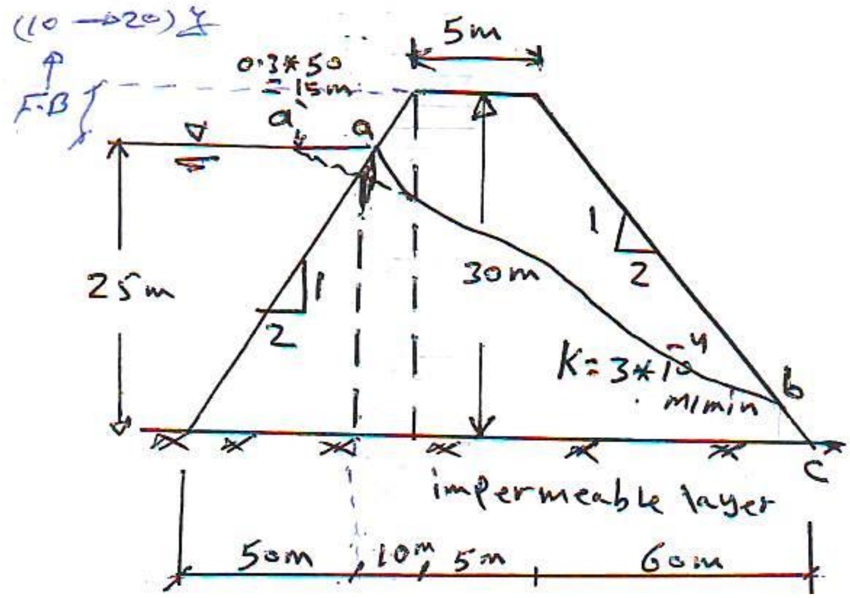
$$\boxed{q = K\ell \, (\sin\beta)^2}$$

Note : the symbol (a) in pavlovsky = the symbol (ℓ) in casagrand .

General example :

The cross section of an earth dam shown in figure below . calculate the rate of seepage through the dam [q in $m^3/(\text{min.m})$] by :

a) Dupuit's method
b) Schaffornak's method
c) L.casagrande's method
d) Pavlovsky's method



Solution :

a) Dupuit's method

$$q = \frac{K\,(h_1^2 - h_2^2)}{2L}$$

$h_1 = 25$ m , $h_2 = 0$

L = horizontal distance

Between points (a) and (C)

L = 60 + 5 + 10 = 75 m

$$q = \frac{3 \times 10^{-4}}{2 \times 75} \times 25^2 = 12.5 \times 10^{-4}\ m^3/(\text{min.m})\ .$$

b) Schaffornak's method

$q = K\,(a \sin\alpha)\,(\tan\alpha)$

$$a = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{(\cos\alpha)^2} - \frac{h^2}{(\sin\alpha)^2}}$$

$\alpha = \beta$, $a = \ell$

Using casagrande's correction , d (the horizontal distance between $\bar{a}$ and C)

Is equal to 60 + 5 + 10 + 15 = 90 m .

Also $\alpha = \tan^{-1} \frac{1}{2} = 26.57°$, h = 25 m

$$a = \frac{90}{\cos 26.57} - \sqrt{\left(\frac{90}{\cos 26.57}\right)^2 - \left(\frac{25}{\sin 26.57}\right)^2}$$

$= 100.63 - \sqrt{100.63^2 - 55.89^2} = 16.95$ m

$q = 3 \times 10^{-4} \times 16.95 \times \sin 26.57 \times \tan 26.57 = 11.37 \times 10^{-4}$ m^3/(min.m) .

a) L.casagrande's method

$q = K \ell \, (\sin\beta)^2$

$\ell = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2 \, (\cot\beta)^2}$

$\beta = 26.57°$

d = 90

H = 25

$\therefore \ell = 19$ m

$q = 3 \times 10^{-4} \times 19 \times \sin 26.57 = 11.4 \times 10^{-4}$ m^3/ (min.m) .

a) Pavlovsky's method

$$h_2 = \frac{B}{\cot\beta_2} + Hd - \sqrt{\left(\frac{B}{\cot\beta_2} + Hd\right)^2 - h_1^2}$$

$$h_2 = \frac{5}{2} + 30 - \sqrt{\left(\frac{5}{2} + 30\right)^2 - h_1^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{(H - h_1)}{\cot\beta_1} \ln\frac{Hd}{Hd - h_1} = \frac{h_2}{\cot\beta_2}$$

$$\frac{(25 - h_1)}{2} \ln\frac{30}{30 - h_1} = \frac{h_2}{2}$$

$$h_2 = (25 - h_1) \ln\frac{30}{30 - h_1} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

equations (1) and (2) must be solved by trial and error .

h_1 (m)	h_2 from equ. (1) , (m)	h_2 from equ. (2) , (m)
2	0.062	1.587
4	0.247	3.005
6	0.559	4.24
8	1	5.273
10	1.577	6.082
12		
14		
16		
18	5.4	6.414
20	6.882	5.493

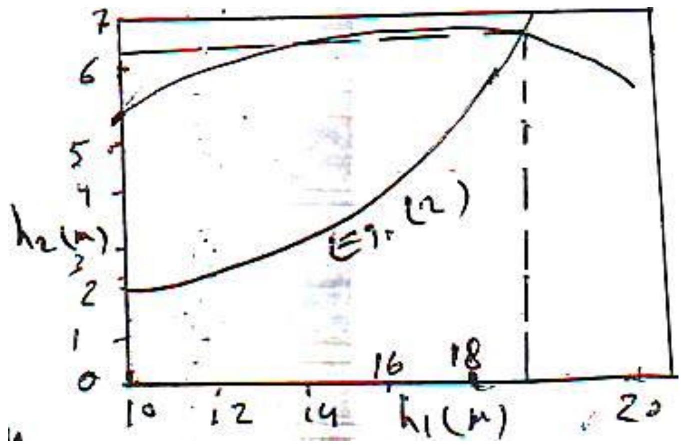
Plot of h_2 against h_1 using the table and from this graph when $h_1 = 18.9$ m

And $h_2 = 6.06$ m the curves will intersect .

$$q = \frac{K\, h_2}{\cot \beta_2}$$

$$= \frac{3 \times 10^{-4} \times 6.06}{2}$$

$$= 9.09 \times 10^{-4}\ m^3/(\text{min.m}) .$$



∗ choose the correct answer :

In earth dam a phreatic line is :

a) Line below which there are positive hydrostatic pressure in the dam .
b) Imaginary line passing through centers of gravity of the sections .
c) An equipotential line .
d) A flow line .

∗ put true or false :

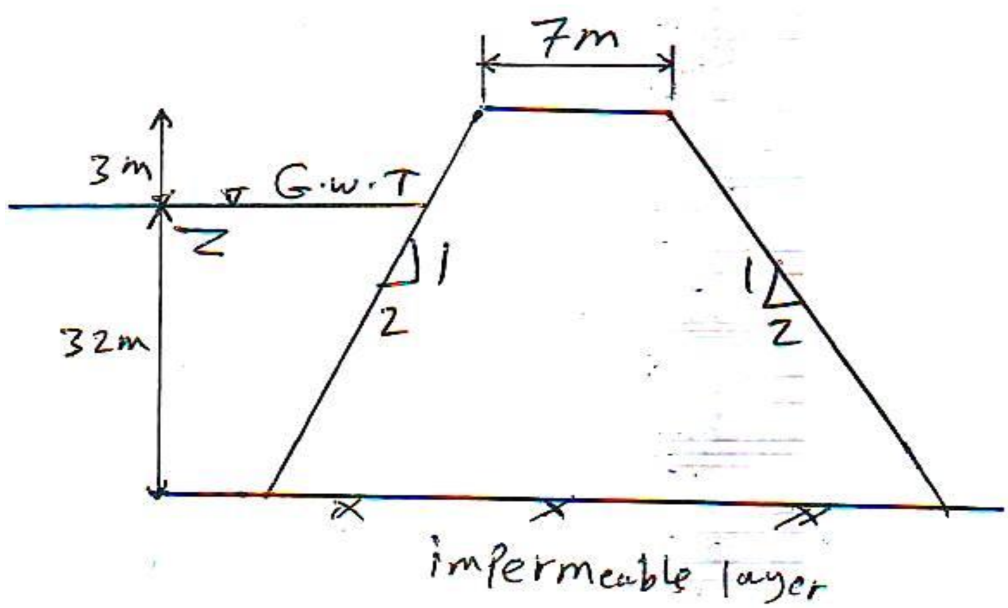
1- Sheet pile walls are used as a seepage preventing devices .
2- Maximum failures of earth dams are due to erosion of soil due to flowing water .

H.W :

An earth dam section shown in figure , determine the rate of seepage through the earth dam using :

a) dupuit's method
b) Schaffornak's method (using two ways)
c) L.casagrande's method
d) Pavlovsky's method
Using K = 10^{-5} m/min

Also using Pavlovsky's solution using K = 4 × 10^{-5} mm/sec.
Discuss the results by make a table ?



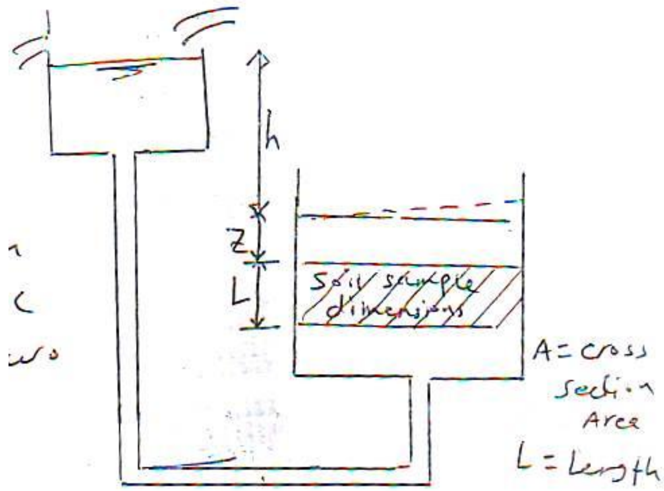
Seepage force :

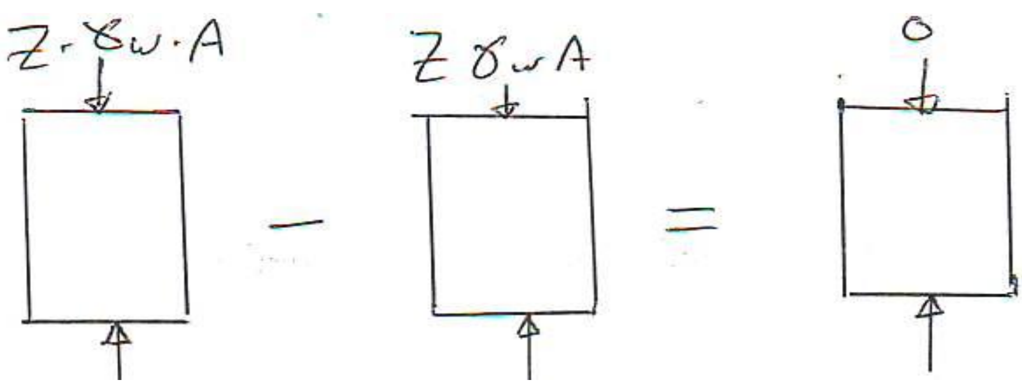
As water flows through the pores of the soil , it exerts some sort of a drag force , this force can be determined according to the simple figure below :

to determine the seepage force ,

first determine the forces on the boundaries , then determine the forces on boundaries under static condition .

the difference between the two cases will provide the seepage force .





$(h + Z + L)\ \gamma_W \cdot A$ $\qquad$ $(Z + L)\ \gamma_W \cdot A$ $\qquad$ $h \cdot \gamma_W \cdot A$

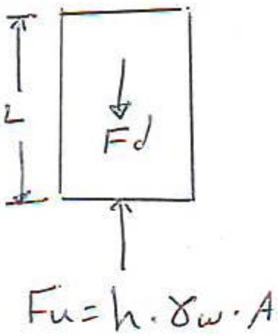
Boundary forces $\qquad$ static condition $\qquad$ seepage force

A better presentation of seepage force is the term " j "

$$j = \frac{\text{seepage frce}}{\text{volume of element}}$$

$$j = \frac{h \cdot \gamma_W \cdot A}{A \cdot L} = \gamma_W \cdot i \qquad \gg \qquad \boxed{j = \gamma_W \cdot i}$$

the seepage force become hazardous when it tends to lift the soil up in the upward flow condition .



$F_d = \bar{\gamma}_{soil} \times L \times A$

When F_U in the large enough , that becomes equal to the downward force (F_d),

the condition of boiling or piping or quick sand condition occurs .

$F_d = F_u$

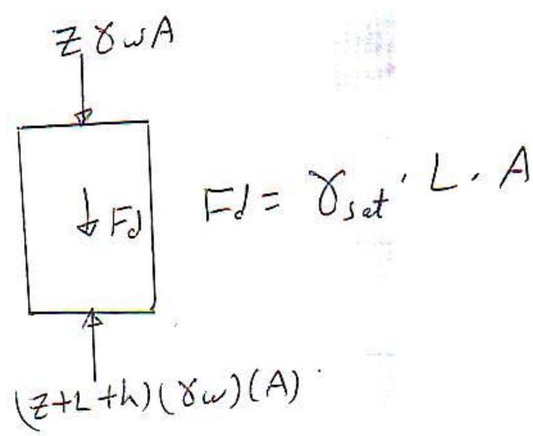
$L \,.\, A \,.\, \bar{\gamma} = h \,.\, \gamma_W \,.\, A$

$L \,.\, \bar{\gamma} = h \,.\, \gamma_W$

$\frac{h}{L} = \frac{\bar{\gamma}}{\gamma_W} = i_{critical}$

$i_{critical} \approx 1 = \frac{\gamma_{sat} - \gamma_W}{\gamma_W} = \frac{20-10}{10} = 1$

if the piping condition or boiling condition is to be checked from the boundary forces , total unit weight must be used as shown below :



in a similar manner , if (F_U) is large enough to lift the soil element .

$F_U = (Z + L + h)\, \gamma_W \,.\, A = Z \,.\, \gamma_W \,.\, A + \gamma_{sat} \,.\, L \,.\, A$

$(L + h)\, \gamma_W \,.\, A = \gamma_{sat} \,.\, L \,.\, A$

$L \,.\, \gamma_W \,.\, A + h \,.\, \gamma_W \,.\, A = \gamma_{sat} \,.\, L \,.\, A$

$h \,.\, \gamma_W \,.\, A = (\gamma_{sat} - \gamma_W)\, L \,.\, A$

$$\frac{h}{L} = i_{critical} = \frac{\gamma_{sat} - \gamma_W}{\gamma_W}$$

the factor of safety against piping can be defined as

$F.S = \frac{i_{\,critical}}{i_{\,exit}}$ the range between (3 – 4) .

Safty of hydraulic structures against piping :

When upward seepage occurs and hydraulic gradient (i) is equal to (i_{cr}) piping or heaving originates in the soil mass .

$$\boxed{i_{cr} = \frac{\gamma_b}{\gamma_W}}$$

If $i_{exit} > i_{cr}$ the piping will occur .

$$\gamma_b = \gamma_{sat} - \gamma_W = \frac{G_S \cdot \gamma_W + e \cdot \gamma_W}{1+e} - \gamma_W = \frac{(G_S - 1)\,\gamma_W}{1+e}$$

$$\boxed{i_{cr} = \frac{(G_S - 1)}{1+e}}$$

For the combinations of G_S and e generally encountered in soils , i_{cr} varies within a range of about (0.85 – 1.1) .

The factor of safety against piping (F.S) can be defined as :

$$\boxed{F.S = \frac{i_{critical}}{i_{exit}}}$$

Harza define

(i_{exit}) can be determined from flow net or from the following equation for single sheet pile in the d/s of the hydraulic structure :

$$\boxed{i_{exit} = \frac{1}{\pi} \frac{\max hydralic\ head}{depth\ of\ pentration\ of\ sheet\ pile}}$$

Lane's method :

Lane method (weighted creep line method) suggested an empirical approach to the safety of dam against piping , which is determined from the shortest flow path :

$$\boxed{L_W = \frac{\sum L_h}{3} + \sum L_V}$$

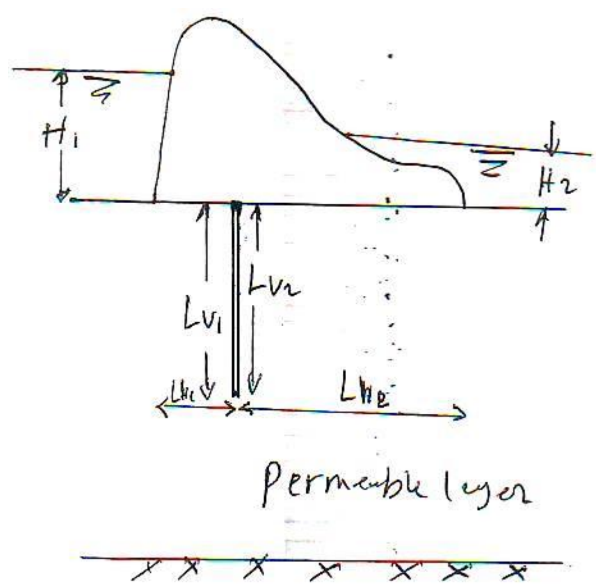
Where : L_W = weighted creep length

L_h = sum of horizontal distance

L_V = sum of vertical distance

Once the weighted creep length has been calculated , the weighted creep ratio can be determined as :

$$\boxed{\text{Weighted creep ratio} = \frac{L_W}{H_1 - H_2}}$$



For a structure to be safe against piping , lane suggested that the weighted creep ratio should be equal to or greater than the safe values shown in table below .

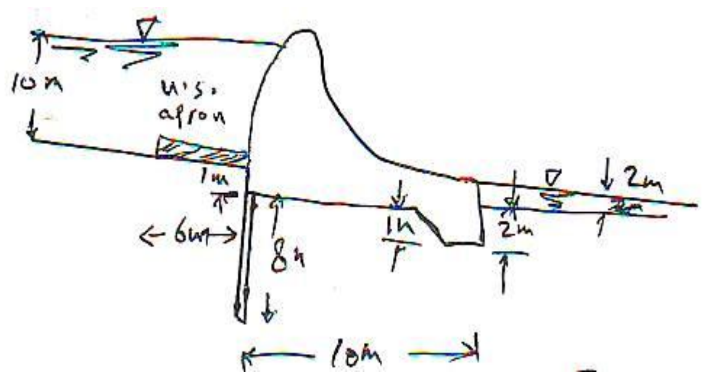
Material	Safe weighted Creep ratio
Very fine sand or silt	8.5
Fine sand	7
Medium sand	6
Coarse sand	5
Fine gravel	4
Coarse gravel	3
Soft to medium clay	2 – 3
Hard clay	1.8
Hard pan	1.6

Safe values for the weighted creep ratio

if the cross section has slope steeper than 45° , it should consider as a horizontal path .

example :

A dam section shown in the figure , the subsoil is fine sand . using lane's method , determine whether the structure is safe against piping ?



Solution :

$L_W = \frac{\sum L_h}{3} + \sum L_V$

$\sum L_h$ = 6 + 10 = 16 m

$\sum L_V$ = 1 + 8 + 8 + 1 + 2 = 20 m

$L_W = \frac{16}{3}$ + 20 = 25.33 m

Weighted creep ratio = $\frac{L_W}{H_1 - H_2} = \frac{25.33}{10-2}$ = 3.17

From table the safe weighted creep ratio for fine sand is about 7 .

Since calculated weighted creep ratio is 3.17 , the structure is unsafe .

Terzaghi's method :

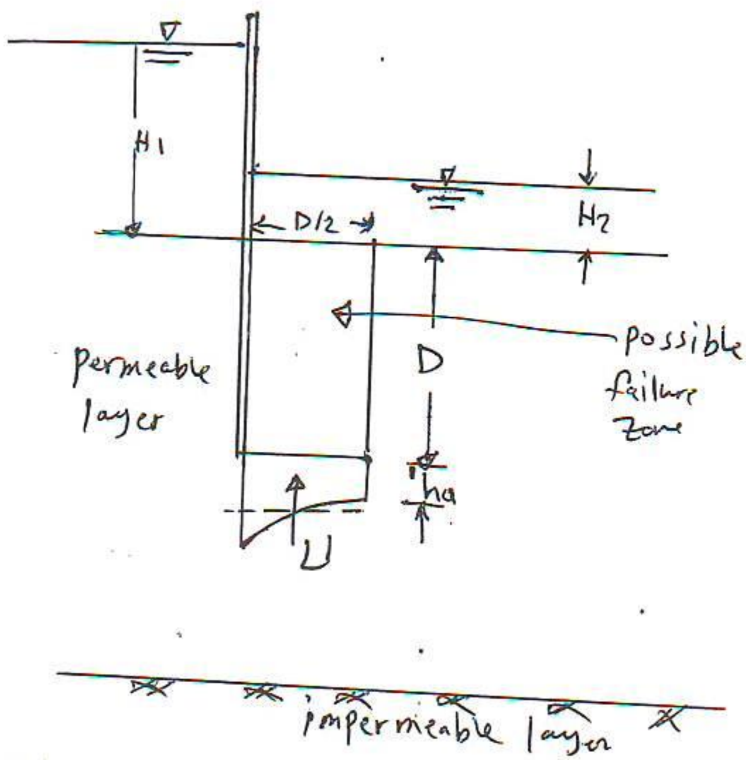
Terzaghi conducted some model tests with a single row of sheet piles as shown in the figure .

He found that the failure due to piping take place within a distance of D/2 from the sheet pile (D is the depth of penetration of the sheet pile) . therefore the stability of this type of structure can be determined by considering a soil prism on the downstream side of unit thickness and of section (D × D/2) .

Using the flow net , the hydraulic uplifting pressure can be determined as :

$$U = \frac{1}{2}\ \gamma_W \cdot D \cdot h_a$$

Where (h_a) is the average hydraulic head at the base of the soil prism .



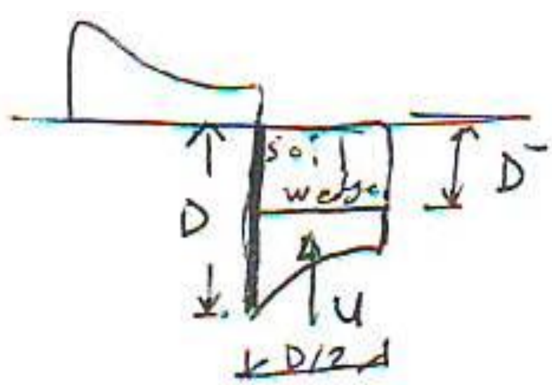
The submerged weight of the soil prism acting vertically downward can be given by :

$$\boxed{\bar{W} = \frac{1}{2}\,\gamma_b\ .\ D^2}$$

Hence the factor of safety against piping (heaving) is :

$$\text{F.S} = \frac{\bar{W}}{\text{U}} = \frac{\frac{1}{2}\,\gamma_b\ .D^2}{\frac{1}{2}\ \gamma_W\ .\text{D}\ .h_a} = \frac{\gamma_b.\ \text{D}}{\gamma_W\ .\ h_a}$$

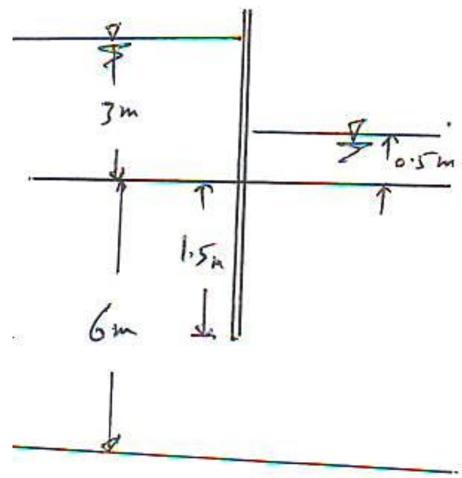
A factor of safety of about (4) is generally considered adequate for a structure other than a single row of sheet piles , such that shown in the figure above , terzaghi recommended that the stability of several soil prisms of size (D/2 × $\bar{D}$ × 1) be investigated to find minimum factor of safety . note that ($0 < \bar{D} \leq$ D) . however , harr suggested that the factor of safety of (4 – 5) with ($\bar{D}$ = D) should be sufficient for safe performance of the structure .



Example :

If the last flow element in flow net is 0.82 m at the figure below determine :

a) The factor of safety against piping by harza's method .
b) The factor of safety against piping by terzaghi's method .
   Assume γ_b = 10.2 KN/m^3
   Take Nd = 6
   Nd = number of potential drops .



Solution :

a) $i_{exit} = \frac{\Delta h}{L}$

h = 3 – 0.5 = 2.5 m

L = 6 m

$\Delta h = \frac{2.5}{6}$ = 0.417 m

$i_{exit} = \frac{0.417}{0.82}$ = 0.509 for the last element .

We can check with the theoretical equation

$$i_{exit} = \frac{1}{\pi}\,\frac{\text{max hydraulic gradient}}{\text{depth of sheet pile}} = \frac{1}{\pi}\,\frac{3-0.5}{1.5} = 0.503$$

Which is close to the value obtained above .

$$i_{critical} = \frac{\gamma_b}{\gamma_W} = \frac{10.2\ KN/m^3}{9.81\ KN/m^3} = 1.04$$

$$F.S = \frac{i_{critical}}{i_{exit}} = \frac{1.04}{0.509} = 2.04$$

b) A soil prism of cross section (D × D/2) , where D = 1.5 m , on the downstream side adjacent to the sheet pile is plotted in figure below (a) . the approximate hydraulics heads at the bottom of prism can be evaluated (Nd = 6) for flow net .

$h_a = \frac{3}{6}$ (3 – 0.5) = 1.25 m

$h_B = \frac{2}{6}$ (3 – 0.5) = 0.833 m

$h_C = \frac{1.8}{6}$ (3 – 0.5) = 0.75 m

$h_a = \frac{0.375}{0.75} \left(\frac{1.25+0.75}{2} + 0.833 \right) = 0.917$ m

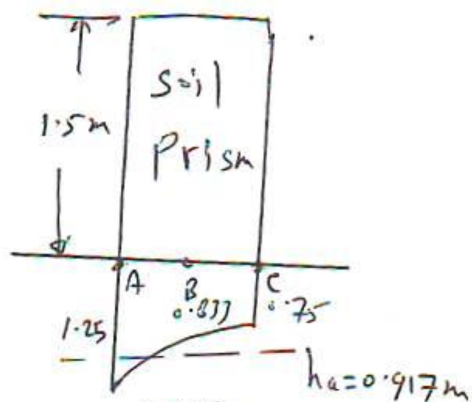


Fig (a)

$$F.S = \frac{\gamma_b .\ D}{\gamma_W .\ h_a} = \frac{1.5 \times 10.2}{0.917 \times 9.81} = 1.7$$

The factor of safety calculated here is rather low .

However , it can be increased by placing some filter material on the downstream side above the ground surface . this will increase the weight of the soil prism (figure b) .

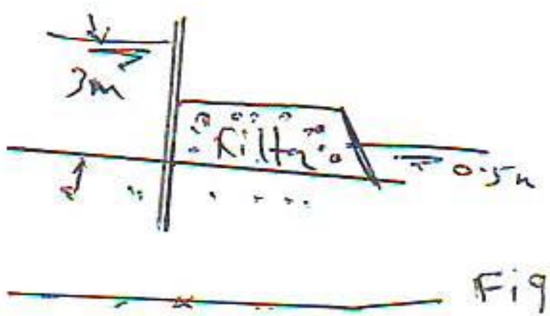


Fig (b)

Anisotropy :

If (K) is independent on the direction of the velocity , the soil is said to be " isotropic " .

Soil with the same coefficient of permeability at all points within the flow region is said to be " homogeneous and isotropic " .

Homogenous and Anisotropic :

If (K) is dependent on the direction of the velocity and if this directional dependence is the same at all points of the flow region , the soil is said " Homogenous and Anisotropic " .

Most soils are anisotropic to some degree . generally in homogenous natural deposits $k_X >>> k_y$.

In general the flow velocity in any direction n :

$V_n = - k_n$. grad n . h

From two dimensional flow in the X and y directions

$V_x = - k_x$. grad x . h

Where :

$$V_x = - k_x \cdot \frac{\partial h}{\partial x}$$

$V_y = - k_y$. grad y . h

$$V_y = - k_y \cdot \frac{\partial h}{\partial y}$$

Before going through the anisotropic conditions , let us consider the flow in stratified soil in both directions :

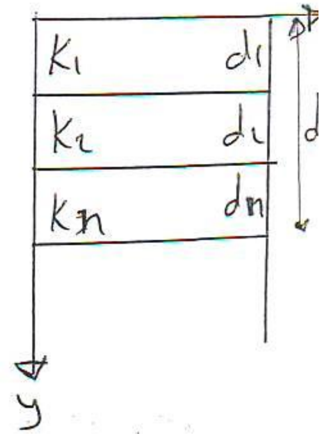
Stratified layers :

A-For purely horizontal flow :

$q = \sum q = k_x . d . i = k_{1x} . d_1 . i + k_{2x} . d_2 . i + \ldots\ldots\ldots + k_{nx} . d_n . i$

equivalent $k_x = \dfrac{k_{x1} . d_1 + k_{x2} . d_2 + \ldots\ldots + k_{xn} . d_n}{d}$

$$\boxed{\overline{K}_X = \sum_{m=1}^{m=n} \frac{k_{xm} . d_m}{d}}$$



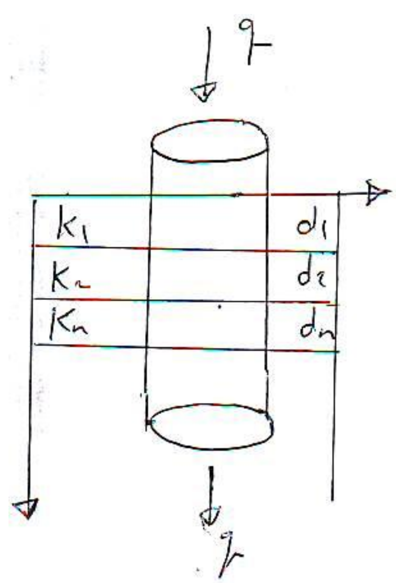
B-For purely vertical flow :

$V = V_1 = V_2 = V_3$

$id = i_1 . d_1 + i_2 . d_2 + i_3 . d_3 + \ldots\ldots + i_n . d_n$

$\dfrac{V}{\overline{K}_y} . d = \dfrac{V}{K_{1y}} . d_1 + \dfrac{V}{K_{2y}} . d_2 + \ldots\ldots + \dfrac{V}{K_{ny}} . d_n$

$$\boxed{\overline{\overline{K}}_y = \frac{d}{\sum_{m=1}^{m=n} \frac{d_m}{K_m}}}$$

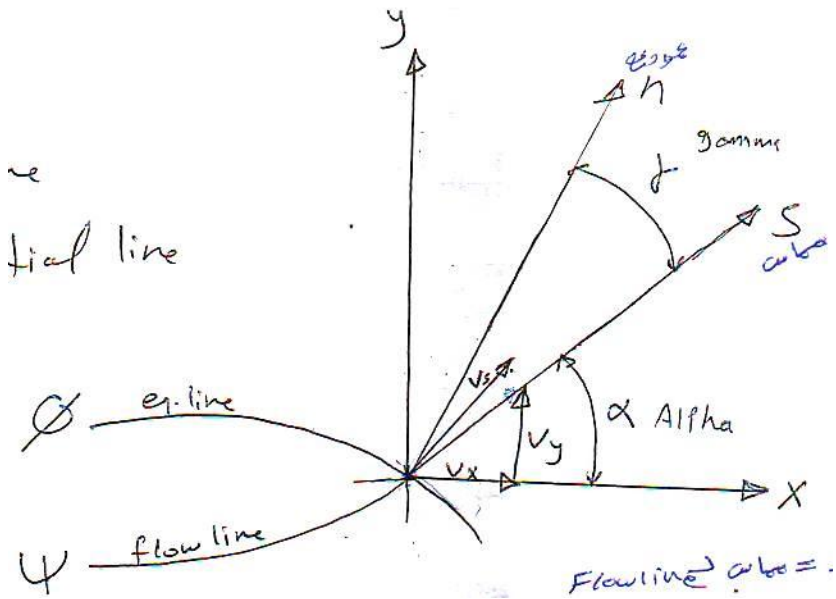


In anisotropic soil :

In the figure

S : tangent to the stream line

n : normal to the equipotential line



in anisotropic soil flow , the direction of stream line will not in general coincide with the direction of the normal to the equipotential line .

$$\therefore v_s = -K \frac{\partial h}{\partial s}$$

$$v_x = -k_x \frac{\partial h}{\partial x} = v_s \cos\alpha$$

$$v_y = -k_y \frac{\partial h}{\partial y} = v_s \sin\alpha$$

$$\frac{\partial h}{\partial s} = \frac{\partial h}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial h}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{1}{K_s} = \frac{(\cos\alpha)^2}{K_x} + \frac{(\sin\alpha)^2}{K_y}$$

$$K_s = \frac{K_X \, K_y}{K_x\,(\sin\alpha)^2 + K_y\,(\cos\alpha)^2}$$

This equation can converted into rectangular coordinates

X = r cos α , y = r sin α

$$\frac{r^2}{K_s} = \frac{X^2}{K_x} + \frac{y^2}{K_y}$$

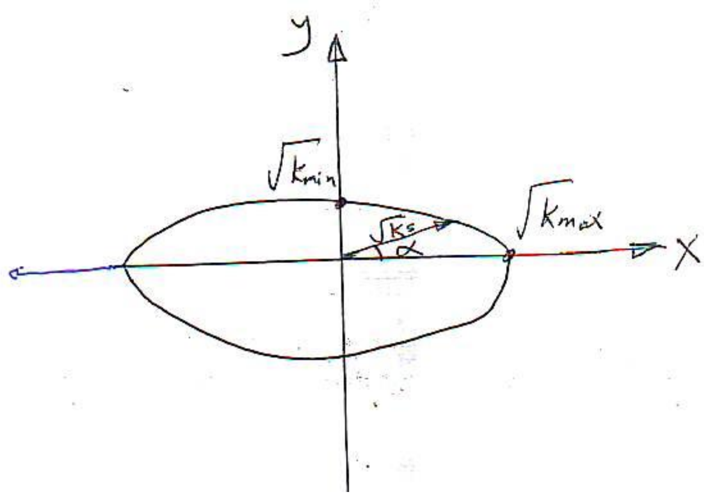
This is an equation of an ellipse :

Major axis = $\sqrt{K_X}$

Minor axis = $\sqrt{K_y}$

$K_X = K_{max}$

$K_y = K_{min}$



Anisotropic soil to isotropic soil :

Common conditions $K_X > K_y$
a simple transformation is used to change

$$K_X \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} = 0$$

into

$$\frac{\partial^2 h}{\partial x_1^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

by introducing $x_1 = X \sqrt{\frac{K_y}{K_x}}$

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial x_1} \frac{\partial x_1}{\partial x} = \frac{\partial h}{\partial x_1} \sqrt{\frac{K_y}{K_x}}$$

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial^2 h}{\partial x_1^2} \sqrt{\frac{K_y}{K_x}}$$

$$\frac{\partial^2 h}{\partial x_1^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Using the same technique for isotropic conditions , after the flow net is plotted in the y - x_1 plan . the plot is transformed back to the x – y plane . this distortion destroys the orthogonality of the flow lines .

x_1 = X . a where a = $\sqrt{\frac{K_y}{K_x}}$

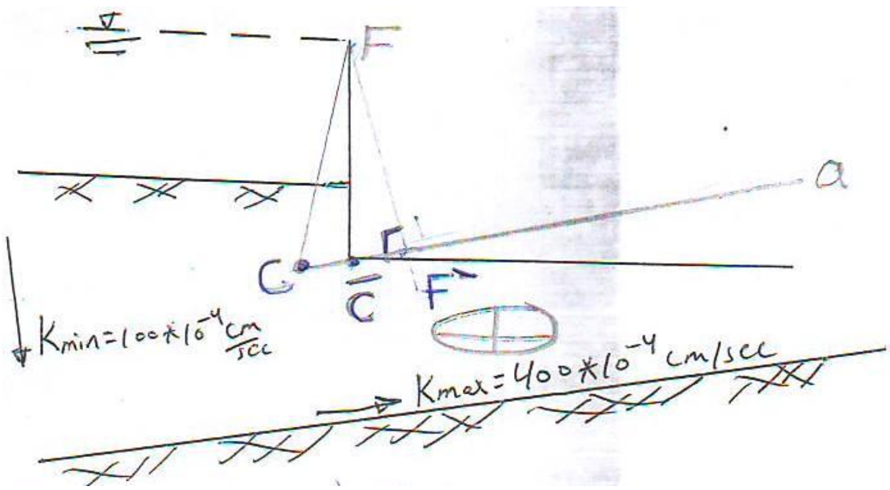
in determining the amount of flow

$$q = \bar{k} \cdot hL \cdot \frac{M}{N}$$

$\bar{k}$ is determined by comparing the flow in two elements , one in the original flow net and the corresponding transformed element .

Example :

In the figure below , obtain the transformed section ?

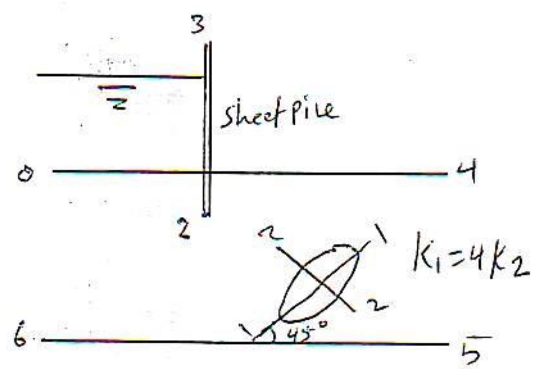


Solution :

The transformed section is obtained :

1- Chose point F .
2- $\sqrt{\frac{K\ min}{K\ max}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$
3- The location of C is obtained by plotting Ca parallel to K_{max} direction plot $\mathrm{F}\bar{\mathrm{F}} \perp$ Ca .
4- Measure $\mathrm{C}\bar{\mathrm{F}}$ and multiply by $\sqrt{\frac{K\ min}{K\ max}} = \frac{1}{2}$ to locate $\bar{\mathrm{C}}$ (from the original point) .
5- Carry on for the next points .

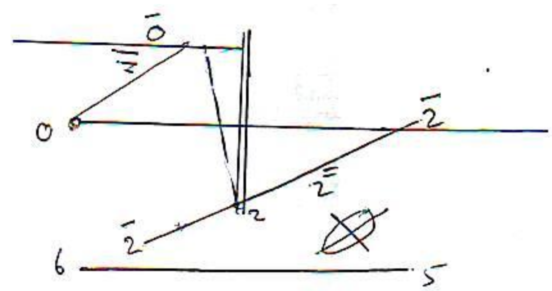
Example : draw the flow line of the figure below ?



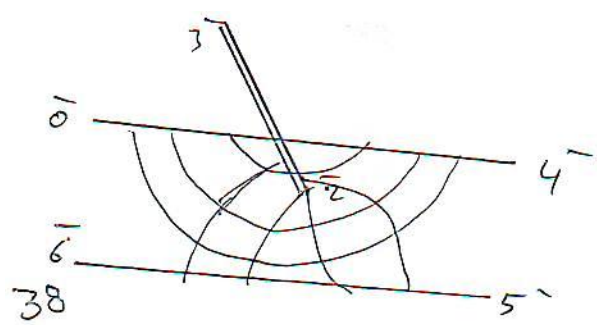
Solution :

Find the new location of each point to transform the figure from anisotropic to isotropic .

(a)

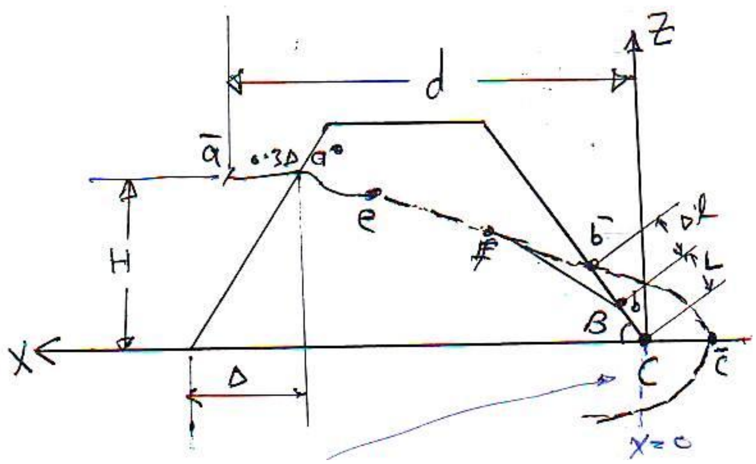


(b)



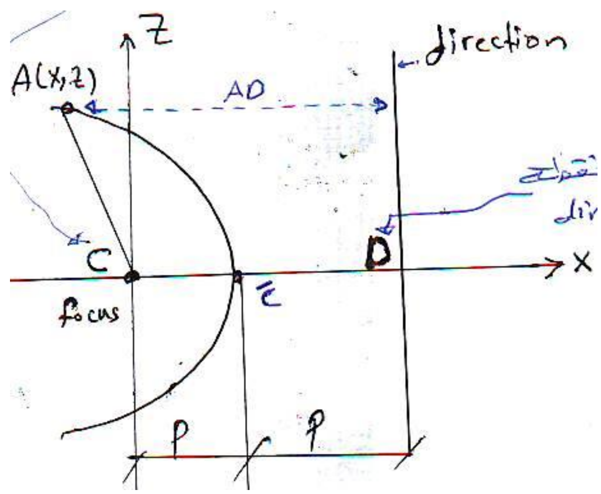
Plotting phreatic line for seepage through dams :

(figure 1)



aefb is the actual phreatic line .

the curve $\bar{a}$ e f $\bar{b}\,\bar{c}$ is a parabola with a focus at C .
the phreatic line consider with this parabola , but some deviation at the upstream and dawn stream occurs .



At point a , the phreatic line starts at angle 90° to the upstream face of the dam and $a\bar{a}$ = 0.3 Δ .

1- Let $C\bar{C}$ = P , in figure 2

$AC = \sqrt{x^2 + z^2}$
$AD = 2P + x$
$\sqrt{x^2 + z^2} = 2P + X$

At X = d , Z = H in figure (1)

$$\boxed{P = \frac{1}{2}\left(\sqrt{d^2 + H^2} - d\right)}$$

Since d and H are known , P can be calculated .

$\sqrt{x^2 + z^2} = 2P + X$

$X^2 + Z^2 = 4P^2 + 4PX + X^2$

$4PX = Z^2 - 4P^2$

$$\boxed{X = \frac{Z^2 - 4P^2}{4P}}$$

2- With P known , the values of Z can be determine for different X values and the parabola line can be constructed .
to complete the phreatic line , ae is drawn by hand .

* when $\beta < 30°$, the values of L can be calculated as :

$$\boxed{\ell = \frac{d}{\cos\beta} - \sqrt{\frac{d^2}{(\cos\beta)^2} - \frac{H^2}{(\sin\beta)^2}}}$$

* when $\beta > 30°$, casagrande proposed the figure no (3 – 5) page 66 in Harr .

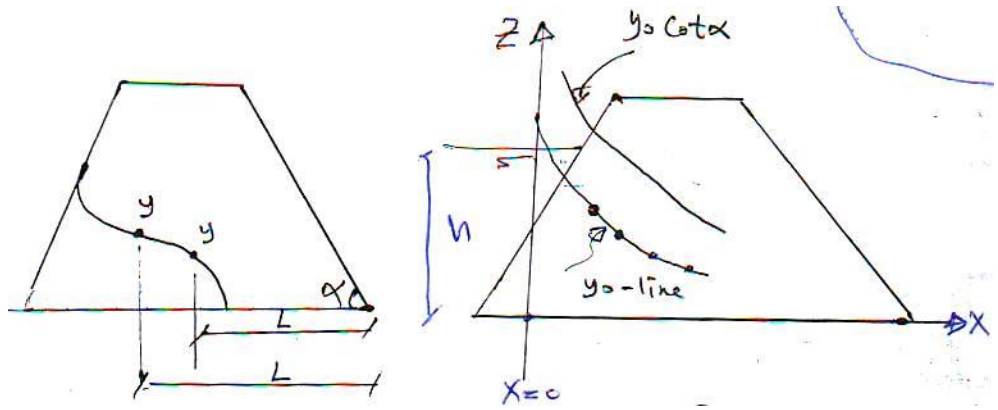
$\Delta \ell = b\bar{b}$, $bc = \ell$

After locating the point (b) on the dawn stream face (fb) can approximated by hand .

Minimum length of filter (L_{min}) :

1- Draw (y∘) line for different values of X (y∘ is the intersect of the basic parabola)

$$y_\circ = \sqrt{d^2 + h^2} - d$$



2- Draw $y_\circ \cot\alpha$

3- Locate 2-line for A a line with 2α , from A vertical intersect the 2α line horizontally at point 2 .

4- Several points will locate the 2-line . the desired d distance is plotted parallel to the dawn stream level intersect the 2-line at point 3 , from point 3 , line 2α intersects the X- axis at A_1 fixing the length L .

$$K = C\ D_{15}^2$$

$$K_{filter} = 25\ K_{soil}$$

$$D_{15\ drain} \geq 5\ D_{15\ soil}$$