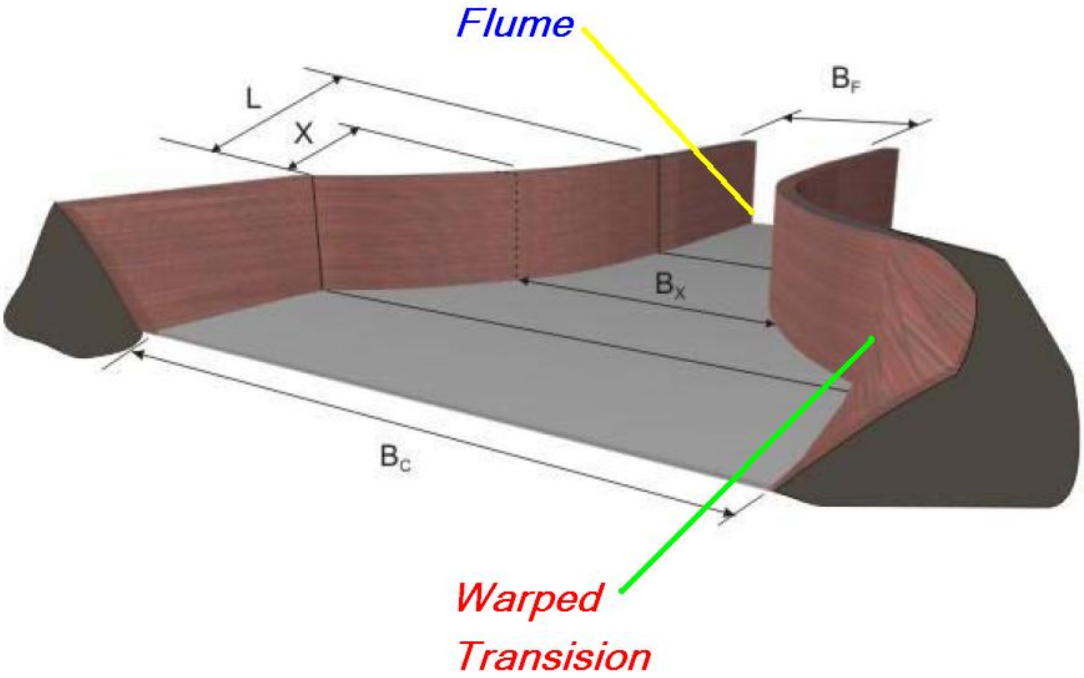
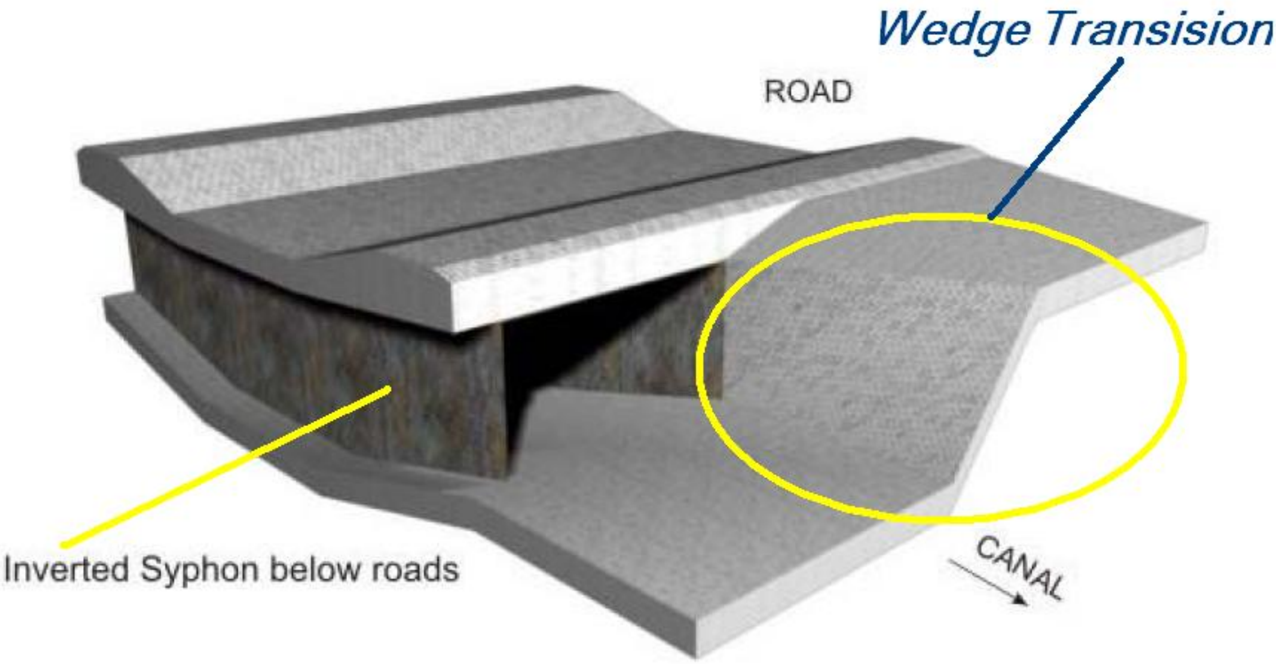
Lecture One :

Design Of Transition





Design Of Transition

Introduction

The transition in channel : is a structure design to change the shapes or cross sectional area of the flow, the function is to avoid excessive of energy losses to element cross waves and other turbulence and to provide safety for structure & water way.

The common types of transitions:

1-Inlet & outlet transition between canal & flume.

2-Inlet & outlet transition between canal & tunnel.

3-Inlet & outlet transition between canal & siphon.

Rules should be considered in design of transition:

1- The optimum max. Angle between the channel axis & a line connecting the channel sides between entrance and exit section is (12.5°).

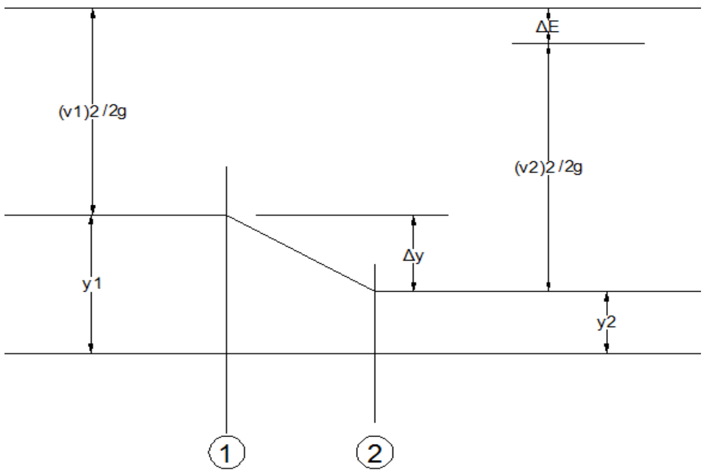
2- Losses: the energy losses in a transition consist of:

a)-friction losses which may be estimated by mean of any uniform flow formula, such as manning formula:

$$s = \frac{n^2 * v^2}{R^{4/3}}$$

b)-conversion loss is generally expressed in terms of the changes in velocity head between entrance & exit section.

Inlet structures



The entrance velocity is less than exit velocity

$\Delta y =$ drop in water

Bern. Between (1) & (2)

$y_1 + v_1^2/ 2g = y_2 + v_2^2/ 2g + \Delta E$

$y_1 - y_2 = v_2^2/ 2g - v_1^2/ 2g + k_1 * (v_2^2 - v_1^2) /2g$

$\Delta y_i = (v_2^2/ 2g - v_1^2/ 2g) (1+k_1)$

K_1= coefficient of inlet losses

Outlet structures

Bern. Between (3) & (4)

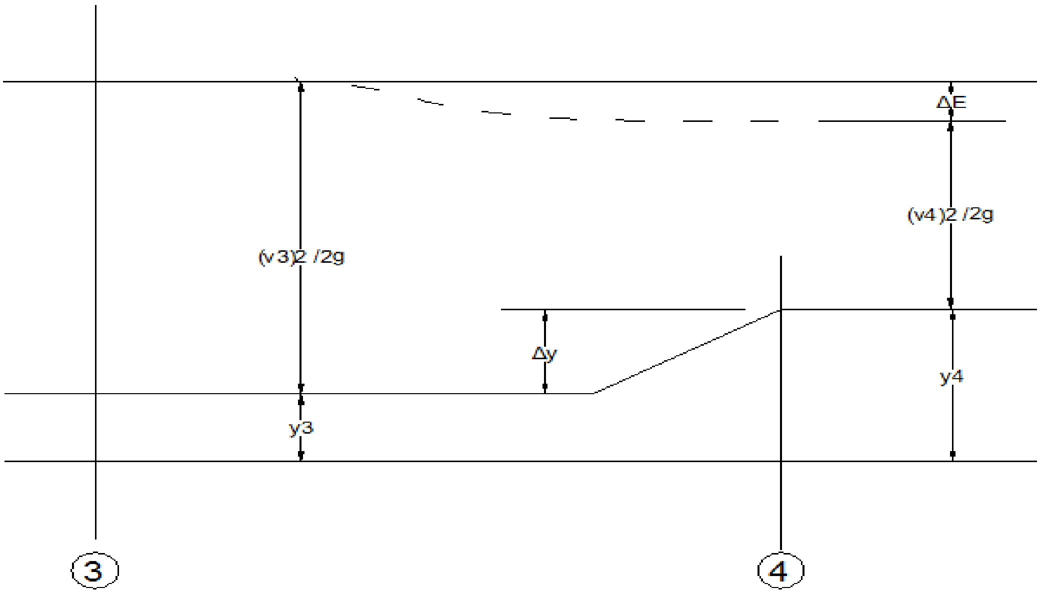
$y_3 + v_3^2/ 2g = y_4 + v_4^2/ 2g + \Delta E$

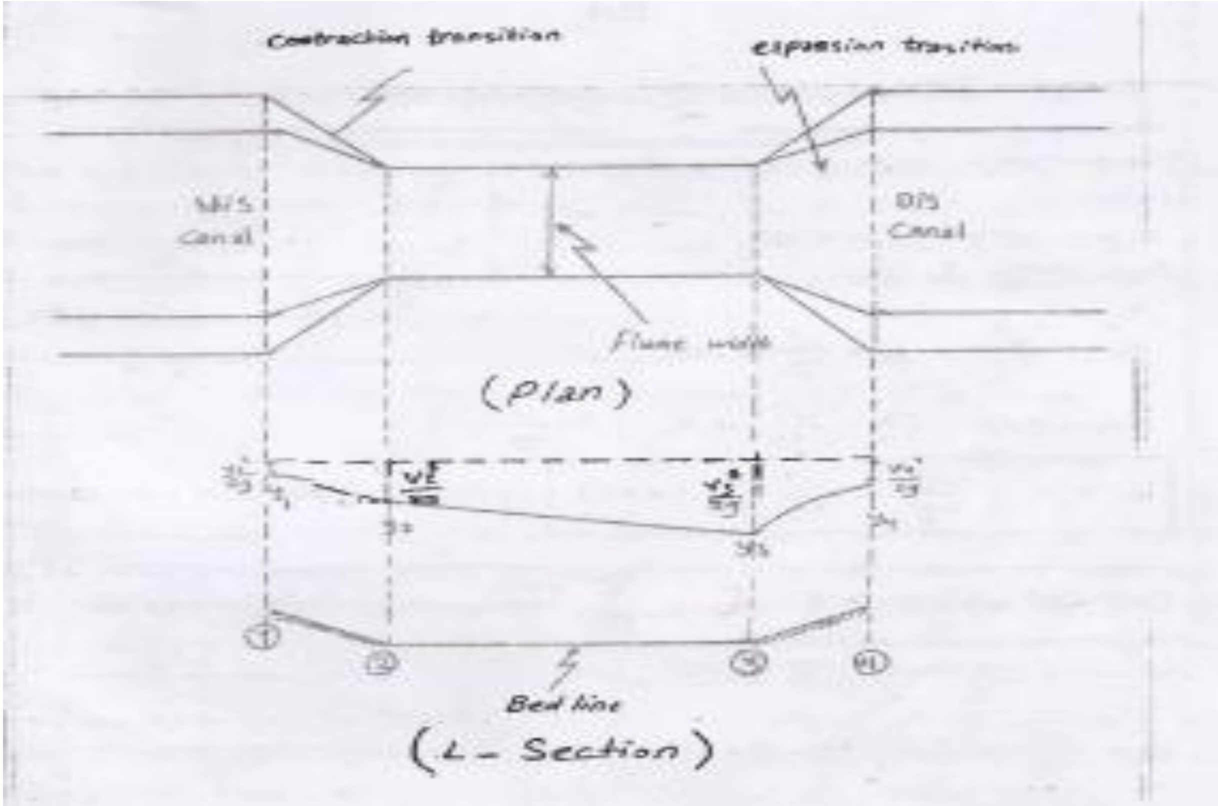
$y_4 - y_3 = v_3^2/ 2g - v_4^2/ 2g - \Delta E$

$\Delta y_o = (v_3^2/ 2g - v_4^2/ 2g) - k_2 * (v_2^2 - v_1^2) /2g$

$\Delta y_o = (v_3^2/ 2g - v_4^2/ 2g) (1-k_2)$

K2=coefficient of inlet losses





Procedure for design of transition

1- from the fig. the contraction transition start from sect. (1-1) & finish in sect. (2-2). The flume section continues from sect.(2-2) to sect. (3-3) .the expansion transition starts at sect. (3-3) and finishes at sect. (4-4) & from sec(4-4) on wards the channel flows in its normal cross section & the conditions at this section are completely known .

2-full supply level (F.S.L) at section (4-4) = B.L of sec (4-4) +y_4 (known)

T.E.L of sec (4-4) =F.S.L at sec (4-4) + $\frac{v4^2}{2g}$(known)

3-between sec (3-3) &sec (4-4), there is an energy loss in the expansion which is equal to

$$k2\left(\frac{v3^2 - v4^2}{2g}\right)$$

T.E.L at sec (3-3) =T.E.L at sec (4-4) + $k2\left(\frac{v3^2 - v4^2}{2g}\right)$

The trough dimension at sec (3-3) known and v3 is also known

T.E.L can be computed.

F.S.W.L=T.E.L at sec (3-3) - $\frac{v3^2}{2g}$

B.L at sec (3-3) =F.S.W.L-y3

4-between sec (2-2) &sec (3-3), the channel flow in trough of constant cross section .the loss in trough (h_l) is the friction loss which can be computed with manning formula:

$$s = \frac{n^2 * v^2}{R^{4/3}}$$

T.E.L at sec (2-2) =T.E.L at sec (3-3) +H_l

F.S.W.L=T.E.L- $\frac{V2^2}{2g}$

B.L=water level-y2

5-between sec (1-1) & (2-2)

Energy loss between sec (1-1) &sec (2-2) =$k1\left(\frac{v2^2-v1^2}{2g}\right)$

T.E.L of sec (1-1)=T.E.L of sec(2-2)+$k1\left(\frac{v2^2-v1^2}{2g}\right)$

W.L=T.E.L (1-1)$-\frac{V1^2}{2g}$

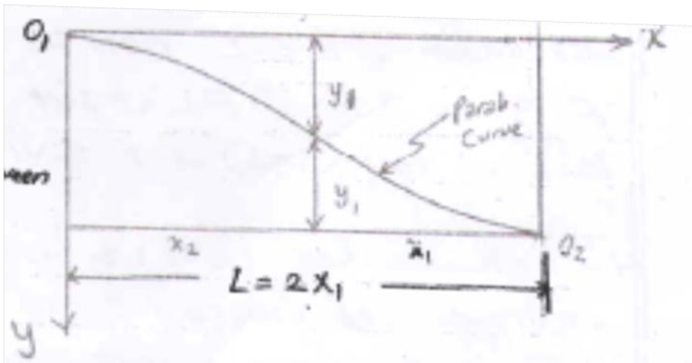
B.L=W.L-y1

6-draw T.E.L assume it is straight line between adjusted sections.

7- B-line also drew assuming straight line.

8-water surface in transition:

in contraction transition between sec(1-1)and (2-2)there will be drop in water surface due to drop in energy line & due to increase in velocity head at sec(2-2).this drop in surface has smooth curve of two parabolic
meeting tangentially at the center .



L=length of transition.

L=2x1

2y1=total difference in water level

Betweensec (1-1) & (2-2)

$y = cx^2 = y1 \quad , x = x1$

$c = \frac{y1}{x1^2} \qquad y = \frac{y1}{x1^2} * x^2$

9- Plot the water surface for expansion & contraction.

10- velocity head= vertical distance between T.E.L& water surface at any point.

11- find velocity= v = $\sqrt[2]{2ghv}$

12- the cross section= Q/v

13-in trapezoidal channel of water depth y, the bed width B &side slope Z /1

$$A = By + Zy^2$$

14-side slope brought to vertical from initial slope (Z : 1)

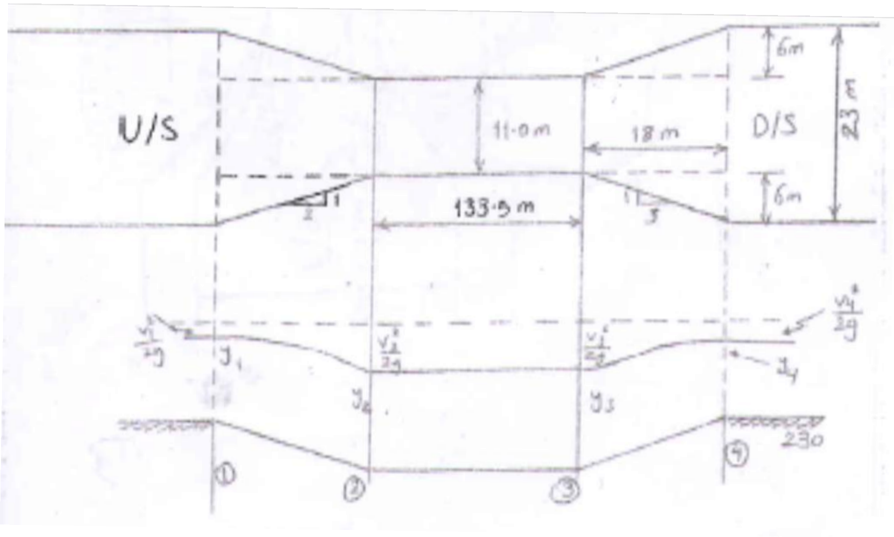
Example

Design transition (expansion &contraction) with following data:

Canal discharge=30.0 m^3/s

Bed width canal=23.0m
Depth of water=1.7m
Bed level=230.0
Side slop of canal=1.5:1
Warped transition
k1=0.2
k2=0.3

Flume:
Bed width =11.0m
Contraction=2:1
Expansion =3:1
Manning coefficient =0.016
Length of flume=133.5m



Solution:

At section (4-4):

Area of section (4-4) = (23+1.7*1.5)*1.7=43.3m^2

$v4 = \frac{Q}{A} = \frac{30}{43.3}$=0.69$m/s$

Velocity head =$\frac{v4^2}{2g}$

$\frac{0.69^2}{2 * 9.8}$

B.L=230m

W.level=230+1.7=231.7m

E.line=231.7+0.0248=231.7248m

At section (3-3):

Area of flume=11.0*1.5)*1.7=18.7m^2

$v3 = \frac{Q}{A} = \frac{30}{18.7}$=1.61 m/s

Velocity head=$\frac{v3^2}{2g}$

$\frac{1.61^2}{2 * 9.8}$

T.E.L at sec (3-3) =T.E.L at sec (4-4) + $k2\left(\frac{v3^2 - v4^2}{2g}\right)$

=231.7248+0.3*(0.138-0.0248) =231.7572m

W.level at sec (3-3) =231.7572-velocity head

=231.7572-0.138=231.6244m

B.L=231.7572-1.7=230.0572M

Another method to find W.L at 3&4

$\Delta yo =$

$=$

$\Delta yo =$

$\Delta yo =$

=231.7-231.6244=0.0756m

From sec (2—2) to sec (3—3)

The area and velocity are constant.

From manning $Q=\frac{1}{n}\ A * R^{\frac{2}{3}} S^{\frac{1}{2}}$

A=11*1.7=18.7m2 P=1.7*2+11=14.4m

$R = \frac{18.7}{14.4}$ =1.3

$S = \frac{V^2 n^2}{R^{\frac{4}{3}}} = \frac{1.61^2 * 0.016^2}{1.3^{\frac{4}{3}}}$

S=0.00045%

Friction loss in trugh =0.000457*133.5=0.061m

T.E.L at sec (2-2) =T.E.L at sec (3) + loss of head from sec (3)to sec (2)

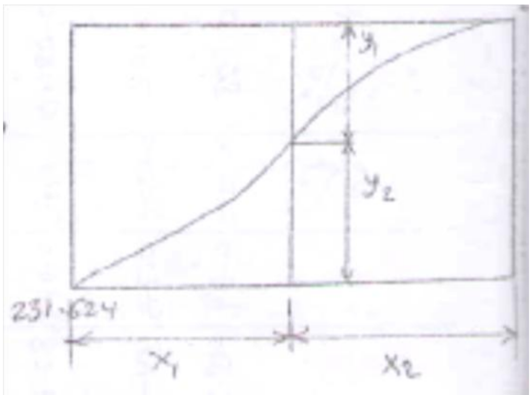
=231.7572+0.061=231.8182m

W.S=231.8182-0.1328=231.6854m

B.L=231.6854-water depth =231.6854-1.7=229.9854m

Loss of head in contraction between sec (1-1) &sec (2-2) =$k1\left(\frac{v2^2 - v1^2}{2g}\right)$

=0.2*($\frac{1.61^2 - 0.69^2}{2*9.81}$) =0.00216m



At sec (1-1)

T.E.L=231.8182+0.00216=231.815m

W.S=231.815-0.0284=231.815m

B.L=231.815-1.7=230.115m

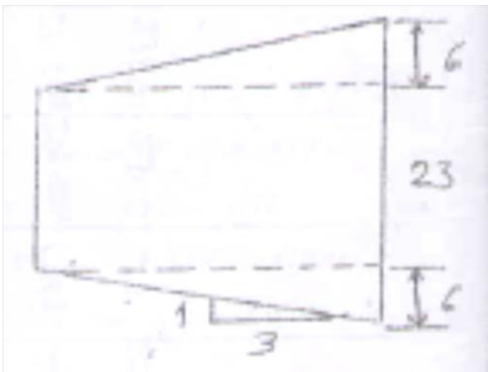
Water surface

Transition of expansion

(23-11)/2=6

L= (23-11)/2*3=18m length of expansion bed width

X1=l/2=18/2=9 m



$\Delta y =$

Y1=0.0753/2=0.0377

X1=9

Y= $(y1/x1^2) * x^2$.

C=0.0377/9*9=0.000465

Y=$0.000465x^2$

Dist. From (3-3)	Change in Water level Δ y	Change in Velocity head	9Hv M	V=√2gh m/sec	A=Q/v	W.S.L m	B.L	Water depth y	Mean width A/y=B	Side slope	Bed Width
1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0.1328	1.6	18.7	231.644	229.924	1.7	11	0:1	11
3	0.042	0.006	0.1268	1.57	19.1	231.6286	229.93	1.69	11.25	0.1:1	11.1
6	0.0167	0.0238	0.109	1.459	20.5	231.6411	229.94	1.69	12.1	0.25:1	11.7
9	0.0378	0.0549	0.0788	1.24	24.14	231.6622	229.96	1.7	14.18	0.45:1	13.4
12	0.0589	0.084	0.0488	0.974	30.7	231.6833	229.97	1.7	17.95	0.8:1	16.6
15	0.0714	0.102	0.0308	0.775	38.7	231.6958	229.98	1.76	22.61	1.1:1	20.6
18	0.0756	0.108	0.0248	0.69	43.2	231.7	230	1.7	25.45	1.5:1	23
	Δ y $=cx^2$	Δhv =Δy/0.7	$V^2/2g$								

Procedure

for explaining the table of expansion transition

Line (1)

Col (1) distance of expansion transition each 3.0 m.

Col (2) find $\Delta y = 0$

Col (3) $\Delta hv = 0$

Col (4) hv=0.1328 at sec (3-3)

Col (5) v= $\sqrt[2]{2ghv}$ $= \sqrt[2]{2 * 9.81 * 0.1328}$

Col (6) A=Q/V=30/1.6=18.7 m2

Col (7) water surface elevation=231.6244 m

Col (8) bed elevation =231.6244-1.7=229.9224 m

Col (9) water depth =1.7 m

Col (10) mean width=A/Y=18.7/1.7=11 m

Col (11) side slope =0:1

Col (12) bed width =B=Bm-zy=11-0*y=11 m

B.L = 229.9244+0.0126=229.937 m

Col. 9 = 231.6286-229.937=1.6916 m

Line (2)

Col (1) x1= 3.0 m.

Col (2) $\Delta y = 0.000465 * 3 * 3 = 0.0042$

Col (3) $\Delta hv = 0.042/0.7$

Col (4) hv-Δhv =$0 = 0.1328 - 0.0.006$

Col (5) v= $\sqrt[2]{2ghv} = \sqrt[2]{2 * 9.81 * 0.128}$

Col (6) A=Q/V=30/1.57=19.1m2

Col (7) water surface elevation=231.6286 m

Col (8) bed elevation =

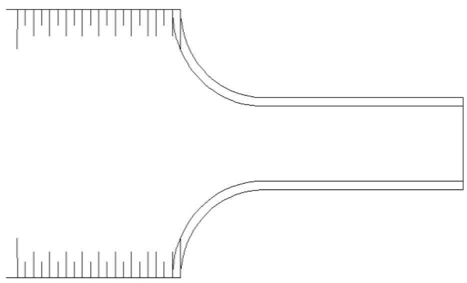
B.L at distance x1=(230-229.9244)x1/18=0.0126

B.L=229.9244+0.0126=229.937 m

Col (9) Water depth=W.S-B.L=231.6286-229.937=1.6916 m

Types of transition most commonly use in subcritical flow:

1-cylinder quadrant:



It is common one simple construction .

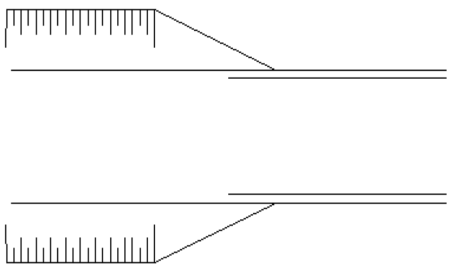
it is use for low velocity &for small structure.

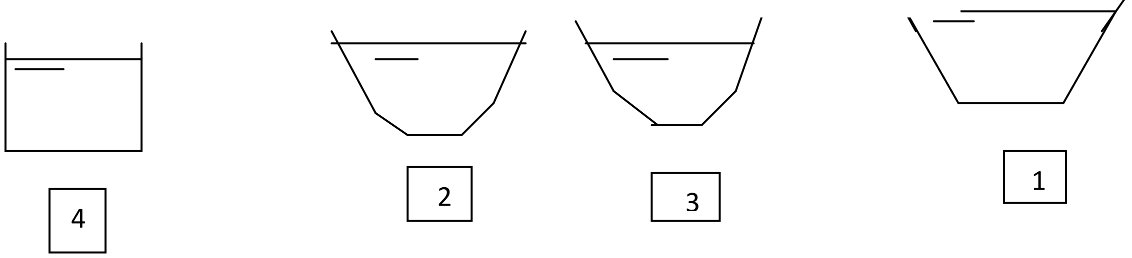
K1=0.15 , k2=0.25

2-wedge type transition:

It is use for small velocity .it is efficient transition usually use for medium regulator.

K1=0.14 k2=0.24

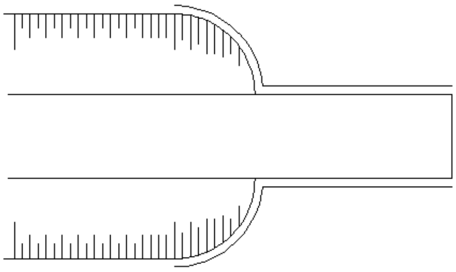




3-warped transition:

it is a most efficient transition but it is difficult to construct .it is use for medium and big structure

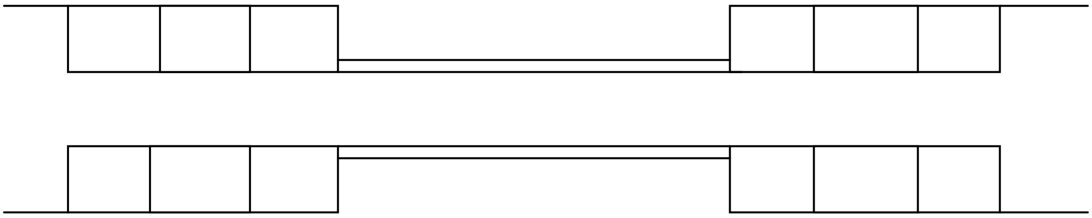
k1=0.2 k2=0.3



4-square corner entrance:

It is recommended.

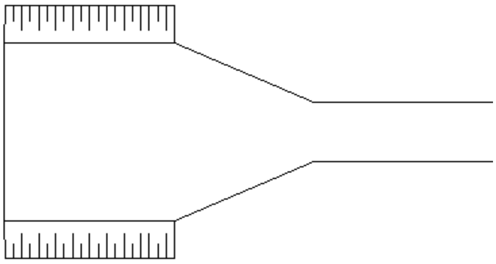
K1=0.3 k2=0.75



5-vertical straight line:

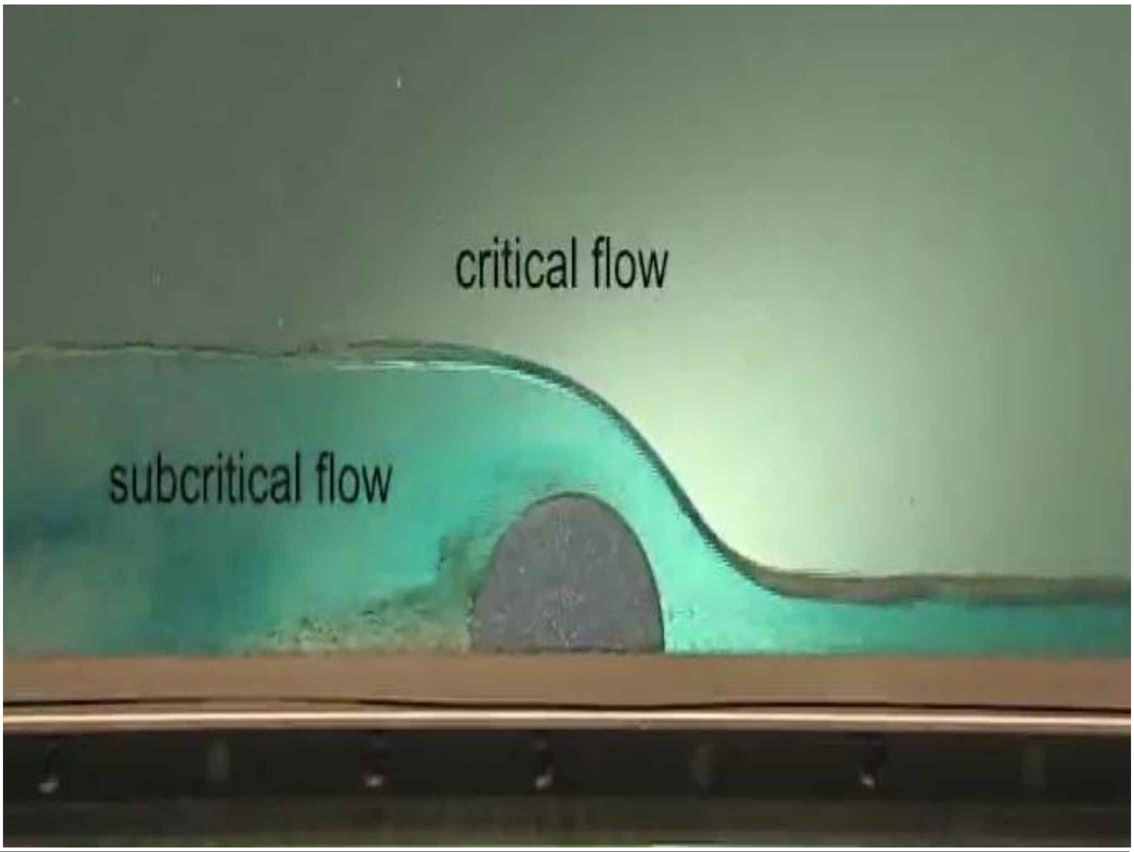
It is recommended.

K1=0.2 k2=0.5



Lecture Two :

WEIRS



WEIRS

Introduction

Any flow taking place over a hydraulic structure (over shoot) under free surface conditions is analyzed with the weir formula.

Weirs are good flow measuring devices .

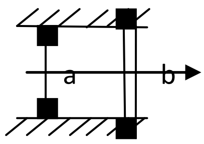
Weirs are commonly constructed as :

1) Shape – crested weirs : with opening of the shapes :
   a) Rectangular b) triangular c) trapezoidal (cipolletti weir)

2) Broad – crested weirs :
   Where the flow is significantly influenced by viscous drag which is enumerated in the form of A discharge – coefficient.

Weirs also classified as :

a) Contracted weirs : where the width of the channel is greater than the width of the weir opening
b) Suppressed weirs : where the width of the channel and the width of the weir opening are equal.



Hydraulics and theories of weirs :

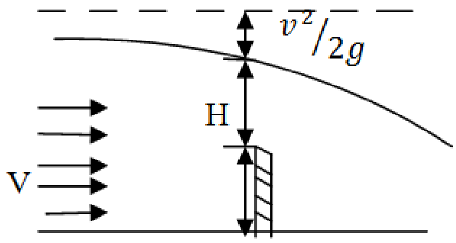
1) Rectangular sharp – crested weirs :
   Hydraulic equation of this type can be simplified to :
   (theoretical)

$$q = \frac{2}{3}\sqrt{2g}\, H^{3/2} \qquad \dots \text{(ideal)}$$

For real weir flow

$$q = cd \,.\, \frac{2}{3}\sqrt{2g}\, H^{3/2} \qquad \dots \text{(real)}$$

Cd : is an experimentally determined coefficient including the effects of the many simplifications disregard in the derivation of eq. , i.e. , C is the factor which transforms the simplified weir flow in to the rear weir flow (on is primarily coeff. of contraction)



The experimental work of rehbock
Led to an empirical formula for
C of well- ventilated sharp- crested rectangular weir :

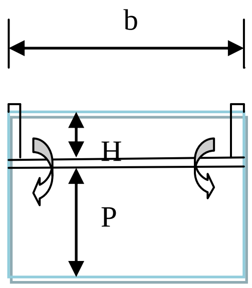
$C = 0.611 + 0.08\,\frac{H}{P}$

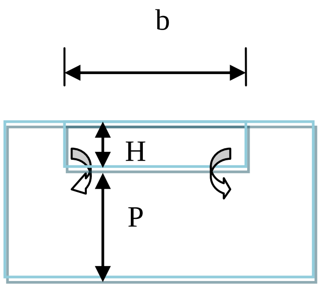
for water ; true only for values of $\frac{H}{P}$ up to approximately (5)

The effective width (b):

this is considered as :

1) (b) for suppressed rectangular weir (R.W)
2) (b – n* 0.1 H) for contracted (R.W)
   n : number of contractions (usually one to each side)

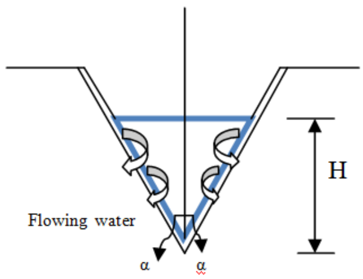




2- triangular weir or V- notch :

this type is widely used as measuring device for small flow rates.
A simplified analysis yield the fundamental formula :

$$Q = C * \frac{8}{15} \tan\alpha \sqrt{2g}\;\; H^{5/2}$$



C ≈ 0.59 for weir of $2\alpha = 90^\circ$
Coeff. (c) for lenz is :

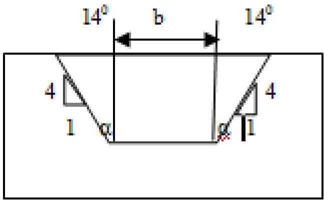
$$C = 0.56 + \frac{0.7}{IR^{0.165} W^{0.17}}$$

IR : Reynolds No. , W : Surface tension
The conditions of (c) for Lenz :

1- H > 0.06 2- IR > 300 3- W > 300

3-Trapezoidal weir (cipolletti weir) : (S.S 1: 4) :

This can be considered as one rectangular notch of width (b) and two half V-notch (angle = α =14) The discharge eq. may be written as :



$$Q = Cd_1 \frac{2}{3}\sqrt{2g}\,(b - 0.2H)\,H^{3/2} + \frac{8}{15}\,Cd_2\sqrt{2g}\,\tan\propto\,H^{5/2}$$

due to presence of 2 ends contractions.

-when we use this type of weir , we can obtained W.L more stability than The type of rectangular Weir because that (b) increase with increase of the depth & give a greater discharge & keep the W.L At stable , therefore ; its use in the escape weir .

Broad – crested weirs :

assume that a flow runs over top of a high frictionless broad – crested weir : The flow rate will remain increases with the gate is lifted clear of the flow (position C). With constant Specific energy exist , the flow occurring without gate is maximum , & that the depth on the crest is ,therefore ,the critical depth :

$y_c = \frac{2}{3}H$

We have also

$$q = \sqrt{g y_c^3} = \sqrt{g\left(\frac{2}{3}H\right)^3} = 0.577 * \frac{2}{3}\sqrt{2g}\,H^{3/2}$$

$$\left(\frac{2}{3}\right)^{3/2} = \left(\frac{2}{3}\right)^{2/2} * \left(\frac{2}{3}\right)^{1/2} = \left(\frac{2}{3}\right)^{2/2} * \left(\frac{1}{3}\right)^{1/2} * \left(\frac{2}{1}\right)^{1/2}$$

$$q = 0.577 * \frac{2}{3}\sqrt{2g}\,H^{3/2}$$

This eq. is compared with the standard weir eq. i.e

$q = Cd * \frac{2}{3}\sqrt{2g}\,H^{3/2}$ for real flow……..(1)

The weir coefficient c=0.577 is higher than those obtained in experimental (which lies between 0.5 & 0.57) This is because of neglect of friction in the analysis .

*Above eq. in case H=E

However , broad – crested weir as an inline canal structure and flow measuring device :

$q = \sqrt{g\left(\frac{2}{3}E\right)^3} = \left(\frac{2}{3}\right)^{3/2}\sqrt{g}\,E^{3/2}$ for ideal flow……..(2)

Now : $q_{real} = Ci\ \ q_{ideal}$

Or Eq.(1) = Ci Eq.(2) i.e. $Cd * \frac{2}{3}\sqrt{2g}\, H^{3/2} = Ci \left(\frac{2}{3}\right)^{3/2} \sqrt{g}\, E^{3/2}$

$\frac{Cd}{Ci} * \frac{2}{3}\sqrt{2g}\, H^{3/2} = \left(\frac{2}{3}\right)^{3/2} \sqrt{g}\, E^{3/2}$ ⇒ $\bar{C} * \frac{2}{3}\sqrt{2g}\, H^{3/2} = \left(\frac{2}{3}\right)^{3/2} \sqrt{g}\, E^{3/2}$

$$\bar{C} = \frac{1}{\sqrt{3}} \left(\frac{E}{H}\right)^{3/2}$$

Now :

1) For very high weir :

$P/H = \infty$, $E = H$, $\frac{E}{H} = 1$

& $\bar{C} \rightarrow \frac{1}{\sqrt{3}} = 0.577$ represented reservoir case

2) For lower weir :

$P/H = \infty$, $E > H$, $\frac{E}{H} > 1$

$\bar{C} > 0.577$

Notes :

1) Weirs may be classified as free flow or submerged a low the first allows air to circulate between the weir & the under site of the nappe . however , in a suppressed weir the sides of the structure prevent the air from circulating under the nappe so the under side has to be vented .

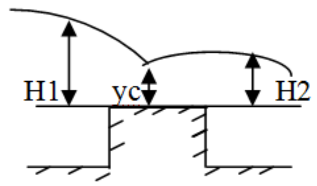
2) Weirs should be designed to discharge freely rather than submerged because of grater measurement accuracy although a slight submergence dose not appreciably effect the discharge as much as the lock of ventilation under the nappe.

3) In rectangular broad – crested weirs when the D/S water level (H_2) exceeds the crest height , it may influence the discharge over the weir , preventing water from passing by free fall.

There are two such conditions :

i)As H2 increases gently , H1 is used in the discharge eq.

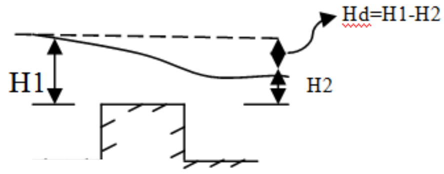
$$Q = C\frac{2}{3}\, b\,\sqrt{2g}\, H1^{3/2}$$



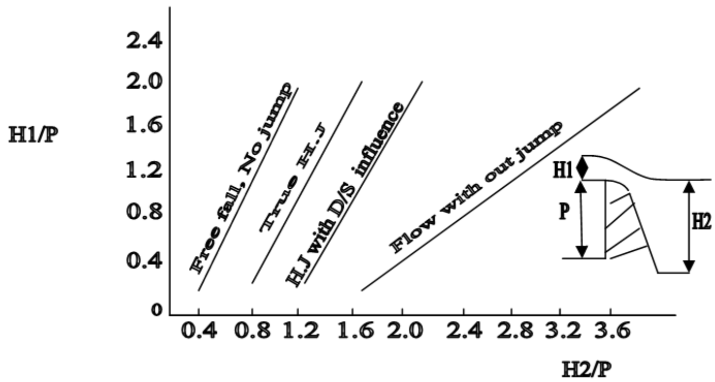
ii)with further increase of H2:

Hd = H1 - H2 is used in the discharge eq

$$Q = C\frac{2}{3}\, b\,\sqrt{2g}\, Hd^{3/2}$$



The following fig. explain the D/S effect on the discharge :

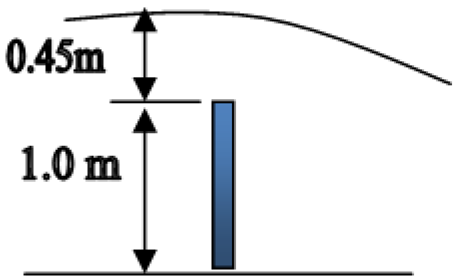


For one typical weir shape , the range of free over fall , free or submerged hydraulic jump, or subcritical over flow can be shown as in the fig. above

EXAMPLE :

Determine the discharge over a sharp crested weir 4.5m long with no lateral constrictions(suppressed) the measured head over the crest being 0.45m & the sill height of the weir is 1m ?

Sol :



$$Q = C\frac{2}{3}\, b\,\sqrt{2g}\, H^{3/2}$$

$$Cd = 0.611 + 0.08\,\frac{H}{P}$$

$$= 0.611 + 0.08\,\frac{0.45}{1} = 0.647$$

$$Q = 0.647\,\frac{2}{3}\sqrt{2 * 9.81} * 4.5\,(0.45)^{3/2} \approx 2.61$$

EXAMPLE :

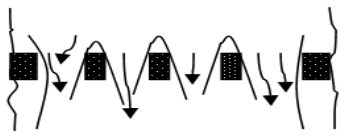
A 6m long weir was measured to carry a 1.4 m3/ sec discharge when the creast is over topped by 0.2m of water. Determine the discharge coefficient of the weir ?Sol: $Cd = \dfrac{Q}{\frac{2}{3}\, b\,\sqrt{2g}\, H^{3/2}} = \dfrac{1.4}{\frac{2}{3} * 6 * \sqrt{2 * 9.81} * (0.2)^{3/2}} = 0.883$

EXAMPLE :

A 30m long weir is divided into 10 equal bays by vertical posts each 0.6m wide . calculate the discharge over the weir , under an effective head of 1m ? Cd = 0.623

Solution:-

Sometimes the total length of a weir is divided into a number of bars or span by vertical posts in such case , the number of bays , or span , into which the weir is divided.



No.of bays = 10 m (30 m length of weir)

Width of each post = 0.6m

Effective length L = (30 -9* 0.6) = 24.6m

No. of end contractions, n = 2*10 = (one bay has two end contraction)

$Q = \frac{2}{3}\, Cd\, (L - 0.1nH)\sqrt{2g}\; H^{3/2}$

$= 2/3 * 0.623 * \sqrt{2g}\; (24.6 - 0.1*20*1) * 1^{3/2}$

$= 41.6\; m^3/sec$

EXAMPLE :

A submerged sharp crested weir 0.8m heigh stands clear across a channel having vertical sides and a width of 3m . the depth of water in the channel of approach is 1.25m , and 10m downstream from the weir , the depth of water is 1m. determine the discharge in litters per minute ? Cd =0.6

Solution

Depth of water on the upstream side of weir H1 = 1.25-0.8 = 0.45m

Depth of water on the downstream of weir H2 = 1-0.8 = 0.2m

Q1 = discharge through the free portion, and

Q2 = discharge over the submerged portion

$Q1 = \frac{2}{3}\, Cd.L.\sqrt{2g}\; (H1 - H2)^{3/2}$

$= \frac{2}{3} * 0.6 * 3 * \sqrt{2 * 9.81} * (0.45 - 0.2)^{3/2}$ = 0.664 m^3/sec = 664 L/sec

$Q2 = Cd.L.\sqrt{2g(H1 - H2)^{3/2}}$

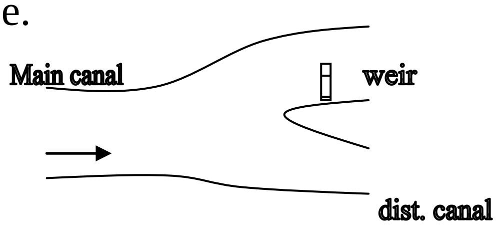
$= 0.6 * 3 * 0.2\sqrt{2 * 9.81(0.45 - 0.2)^{3/2}}$ = 0.797 m^3/sec = 797 L/sec

$Q_{total} = Q_1 + Q_2$ = 1461 L/sec = 87660 littres/min

Practical purpose of weirs :-

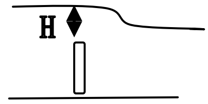
weirs are used for the following purposes:

1) To maintain high water level in order to divert water into a diversion channel for irrigation or Power purpose.

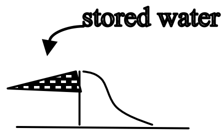


2) To gauge the discharge of branch channel at their intakes, the discharge of drains at their escape & the discharge , of canals funding power houses

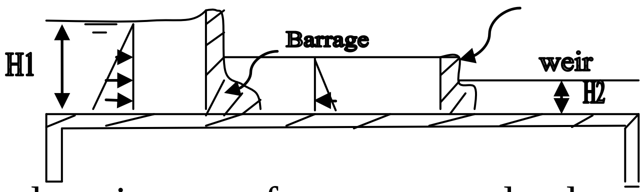
$Q = f(H)^{3/2}$



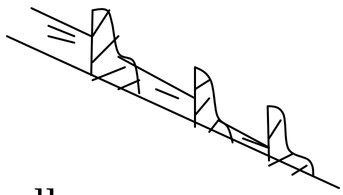
3) Water can be stored for a short period



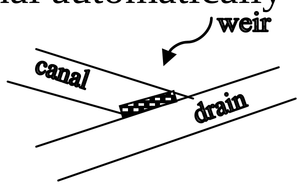
4) To reduce the head acting on a barrage



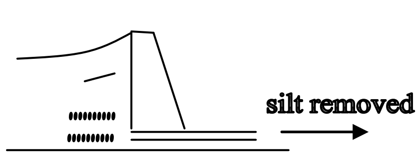
5) To reduce the water slope in case of a very steep land



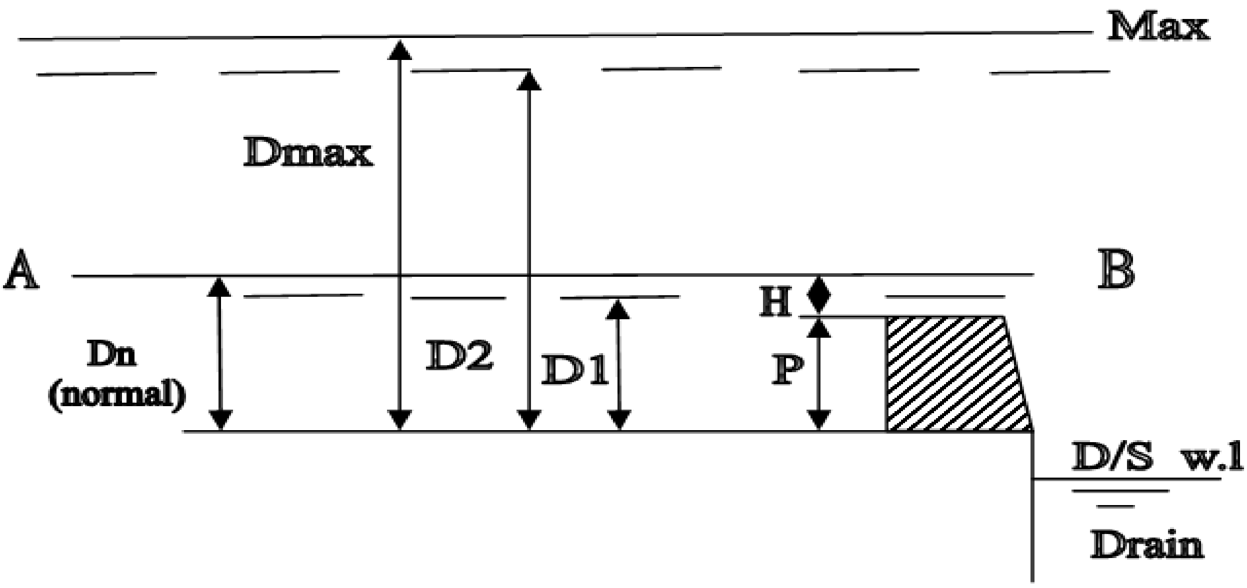
6) To escape the water in canal automatically



7) To control silt movement into the canal system , we can use the weir to many purpose at the same time.



Design of Escape weir :-



Let AB be the normal water line in the canal which should be ended at a natural drain of point B . to avoid a sudden drop in water a weir is constructed (escape weir). For the determination of the dimensions of this weir , knowing (alternatively) either b or p the remaining dimension (p or b) can be directly obtained from the weir formula :

$$Q = Cd * \frac{2}{3}\sqrt{2g}\, H^{3/2}$$

Regarding that

P = D – H or H = D – P

As Q & H are given

on the other hand , knowing both the canal properties & the range of max. & min. water depth (D_{max} ,D_{min})

then 2 cases at four D ranges can be chosen :

$D_1 = D_{min} + \frac{1}{4}\ (D_{max} - D_{min})$

$D_2 = D_{min} + \frac{3}{4}\ (D_{max} - D_{min})$ or $D_2 = D_{max} - \frac{1}{4}\ (D_{max} - D_{min})$

Now : if the corresponding discharges for D_1 & D_2 are Q_1 & Q_2 and corresponding depth over sill are

H_1 & H_2

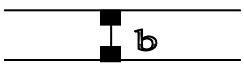
$$Q_1 = Cb\, H_1^{3/2} = Cb\,(D_1 - P)^{3/2} \quad \text{..........(1)}$$

$$Q_2 = Cb\, H_2^{3/2} = Cb\,(D_2 - P)^{3/2} \quad \text{..........(2)}$$

Where $C = Cd \cdot \frac{2}{3}\sqrt{2g}$

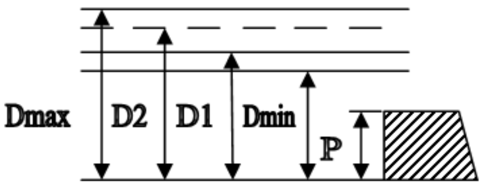
From which b & P are found

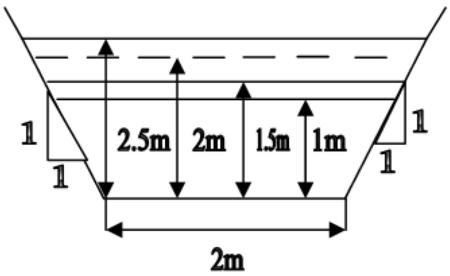
EX : Given an open channel of the following properties :
Min water depth = 1m (0.7 Q)
Max water depth = 2.5m (1.2Q)
Bed width = 2m
Bed slope = 15 cm/km
Side slope = 1.1
Manning's roughness = n = 0.03



It is required to design an escape weir at the end of this channel assuming that : $C = \frac{2}{3}\, Cd\, \sqrt{2g} = 1.6$

Solution :





$D_1 = D_{min} + \frac{1}{4}\,(D_{max} - D_{min}) = 1 + \frac{1}{4}\,(2.5 - 1) = 1.375$ say 1.5m

$D_2 = D_{min} + \frac{3}{4}\,(D_{max} - D_{min}) = 1 + \frac{3}{4}\,(2.5 - 1) = 2.125$ say 2.0m

Using the manning's eq. with the canal properties :

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$; $A = b.\,d + z\,d^2$ or $(b + zd)d$

$$Q_1 = \frac{1}{0.03}(2 + 1 * 1.5)1.5 \left[\frac{(2 + 1.5 * 1) * 1.5}{2 + 2\sqrt{2} * 1.5}\right]^{2/3} * (0.00015)^{1/2}$$

$Q_1 = 1.91\ m^3/sec$ (min)

$$Q_1 = \frac{1}{0.03}(2 + 2 * 1)\,2 \left[\frac{(2 + 2 * 1) * 1.5}{2 + 2\sqrt{2} * 2}\right]^{2/3} * (0.00015)^{1/2}$$

$Q_2 = 3.364\ m^3/sec$

Using the weir formula :

$Q_1 = Cb\, H_1^{3/2} = Cb\,(D_1 - P)^{3/2}$

$Q_2 = Cb\, H_2^{3/2} = Cb\,(D_2 - P)^{3/2}$

$1.91 = 1.6 * b * (1.5 - P)^{1.5}$ ……………(1)

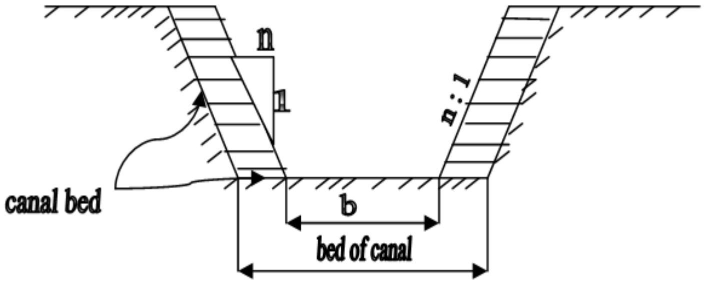
$3.364 = 1.6 * b * (2 - P)^{1.5}$ ……………(2)

$(1.76)^{2/3} = \frac{2-P}{1.5-P}$

From which P = 0.41 m & then b = 1.865 m

Trapezoidal Notch – Fall Escape :

In the previous type of escape weir the canal cannot be drained totally unless a pipe is provided at bed level. This pipe adds a certain variable to the prob .

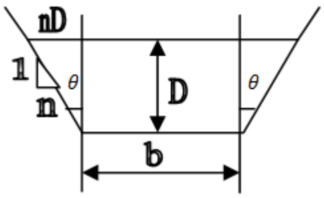


A solution which is more simple being to adopt a v – notch Fall on which the inclination of the sides takes the place of the depth (p) as a variable :

$$\tan\theta = \frac{nD}{D} = n \qquad or \qquad n = \tan\theta$$

Two values of Q & D must be known to
Determine the two unknowns (b) & (h) from an equation which may be derived as follows:
The eq. of the trapezoidal weir was previously given as :

$$Q = C_1 \frac{2}{3}\sqrt{2g}(b - 0.2H)H^{3/2} + \frac{8}{15}C_2\sqrt{2g}(tan\theta)H^{5/2}$$



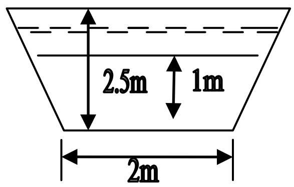
For a very low rectangular weir :
C_1 =1.06 (1+P/H) P=0
=1.06 (1+0) = 1.06 ≈ 1.0
Assume $C_2 \approx 0.6$
Their the last eq. may approximate in SI units as:

$$Q = 1 * \frac{2}{3}\sqrt{2 * 9.81} * b * H^{3/2} + \frac{8}{15} * 0.6 * \sqrt{2 * 9.81} * n * H^{5/2}$$

In which $n = \tan\theta$
Or :
$Q = 2.95\ b\ H^{3/2} + 1.417\ H^{5/2}$
Rearranging (out of brackets)
$Q = 2.95\ H^{3/2}\ (b + 0.48\ n\ H)$
But we have :
P=0 no. sill
H = D
Finally :
$Q = 2.95\ D^{3/2}\ (b + 0.48\ n\ D)$

EX : for the previous example it is required to design a trapezoidal notch escape ?

Solution: from manning's equation it has been found that :



For :

$D_1 = 1.5$ m $\quad Q_1 = 1.91\ m^3/sec$

$D_2 = 2.0$ m $\quad Q_2 = 3.364\ m^3/sec$

Using equation :

$Q = 2.95\ D^{3/2}\ (b + 0.48\ n\ D)$

$1.91 = 2.95 * 1.5^{3/2}\ (b + 0.48\ n * 1.5)$ ………….(1)

$3.364 = 2.95 * 2^{3/2}\ (b + 0.48\ n * 2)$ ……………..(2)

From eq. (1) & (2)

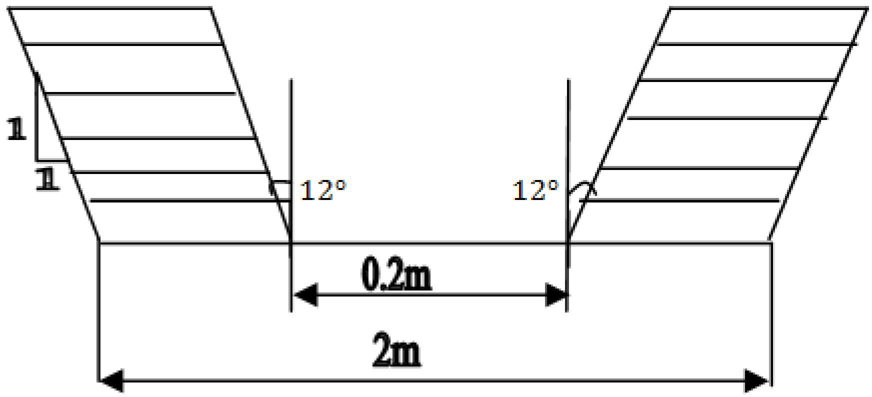
$b = 0.352 - 0.72\ n$ …………(1)

$b = 0.403 - 0.96\ n$ …………..(2)

from which $n = 0.2125 = \tan\theta$

$\theta = 12^0$

& $b = 0.352 - 0.72 * 0.2125 = 0.2$ m



Lecture Three :

Dams

Dams

Introduction

The dam is a barrier constructed across the river to store water on its upstream side due to construction of a dam the water level large areas lying upstream of the dam get submerged. Dams are constructed to store the river water in form of on artificial lake or reservoir. The stored water can be utilized for generation of hydro-electric power, water supply, irrigation or for any other purpose.

Classification Of Dams:

Dams may be classified in several way as follows:

1-Classification based on materials of construction:

a- Earth fill dams.
b- Rock fill dams.
c- concrete dams.
d- Masonry dams.
e- Steel dams.
f- Timber Dams.

2- Classification based on flow over its top:

a- Over flow dams.
b-Non over flow dams.

3- Classification based on the use of the dams:

a- Storage dams.
b- Diversion dams.
c- Detention dams.
d-Multi-purposes dams.

4- Classification based on the mode or resistance offered by the dams against external forces:

a- Gravity dams.
b- Buttress dams.
c-Arch dams

5- Classification based on rigidity of the dams:

a- Rigid dams.
b- Non- rigid dam.

Advantages and Disadvantages of Gravity dams:

Advantage:

1-Maintenace cost is negligible.
2- They are specially suitable for deep steep valley conditions where no other dam is possible.
3- If suitable foundation is available, such dams can be constructed for very large heights.
4- Because they can be constructed in very large heights, they can store more amount of water.
5- If suitable separate place is not available for installation of spillways, they can be installed in the dam section itself.
6- This dam gives prior indication of instability. If remedial measures are taken in time-unsafe dams may even be rendered safe. Even if they cannot be made safe they give sufficient time for the people to move out the area likely to be submerged due to failure of the dam.
7- silting rate of the reservoir can be reduced considerably by installing under sluices in the dam near the bed of the reservoir. Sluices can be operated from time to time and silt may be scoured out of the reservoir.
8- They are not affected by very heavy rainfall. Earth dams cannot sustain very heavy rainfall because of heavy erosion.

Disadvantages:

1-They are very costly in initial construction.
2-They take lot of time to construct.
3-They require skilled laborer for construction.
4-Such dams can be constructed only on good foundation.
5-If height of the dam is to be raised, it cannot be done unless provision for it had been made in the construction of the lower part of the dam.

Earth fill and Rock fill Dams:

Advantage:

1-They can be constructed on any type of foundation.
2- They can be constructed in comparatively less time.
3-they do not require skilled lab our.
4-Initial cost of construction is low as locally available soils, and rock boulders are normally used.
5-Their height can be increased without any difficulty.
6-They are specially suitable for condition where slopes of river banks are very flat. Gravity dams under such conditions are not found suitable.

Disadvantage:

1-The fail all of a sudden without giving any per-warning.
2-Flood water affect the dam safety.
3-spillways have to be located independent of the dam.
4-They cannot be constructed as over flow dams.
5-They require continuous maintenance.
6-They cannot be constructed in narrow steep valleys.
7-They cannot with stand heavy rains unless properly protected.
8-They cannot be constructed in large height. The usual height is 30m for which most of the earthen dams are

Factors governing selection types of Dams:

1-Topography: such as V-shape nor row valley select arch dam.(Top width of the valley less than 1\4 height).
* Narrow U-shape valley indicates choice of over flow concrete dam.
*A low, rolling plan suggest earth dam.
2-Geology&Foundation:
*Solid rock-foundation: select any type.
*Gravel &coarse sand foundation: select earth dam or Rock fill dam.
*Silt & fine sand foundation: select earth dam or low concrete dam up to 8m.
*Clay foundation: select earth dam with special treatment.
3-Availability of materials of construction: *If sand ,gravel and stone is available , concrete gravity dam may be suitable.
*If coarse and fine grained soils are available, on earth dam may be suitable.

4-Length and height of dam

If the height of the dam is very long and its height is low, an earth dam would be a better choice. If the length is small and height is more, gravity dam is preferred.

5-Spillways :

Separate spillway -----earth dam

Large spillway with dam concrete gravity dam and

6-Road way over the dam:

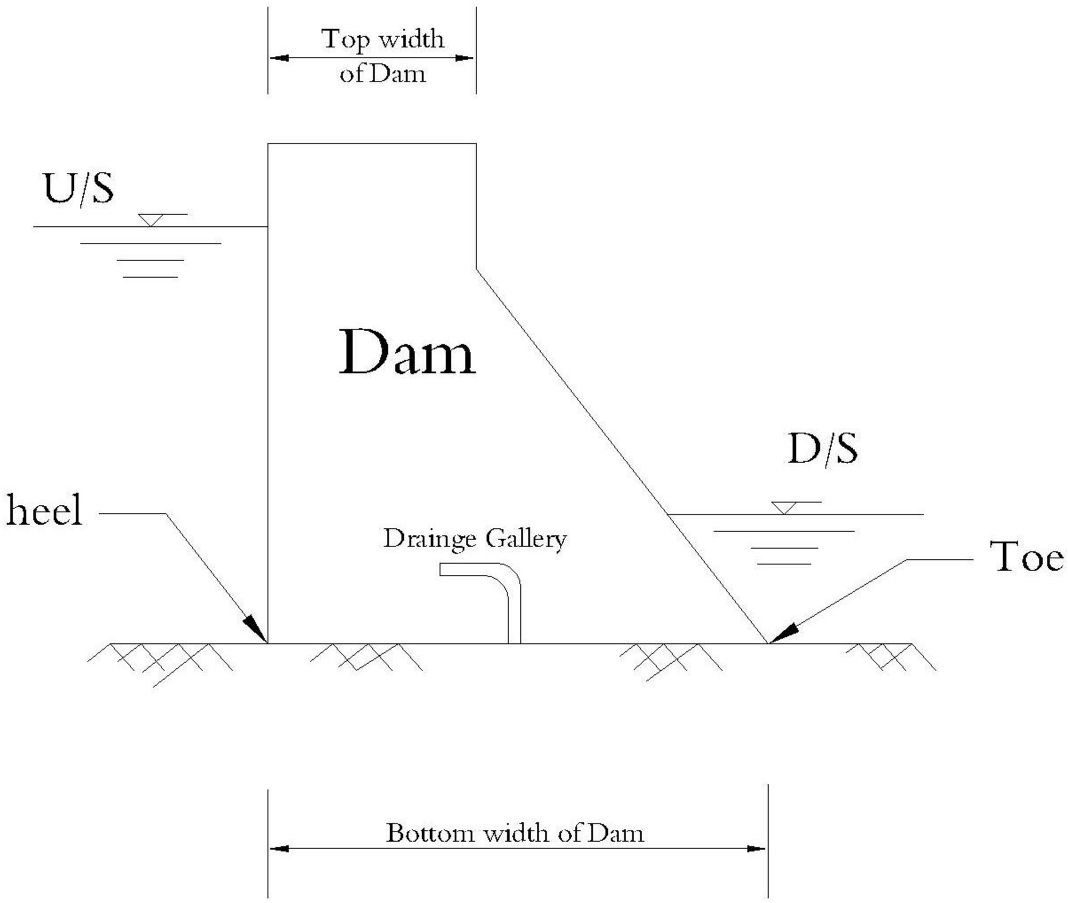
We can constructed earth or gravity dam

7-Generation of hydro-electric power:

Concrete or masonry gravity dams because can be constructed at-height level and develop sufficient head for running the turbines

Concrete Dams:

Is a structure which is designed in such a way that its weight resist the force exerted up on it. It may constructed of concrete or masonary

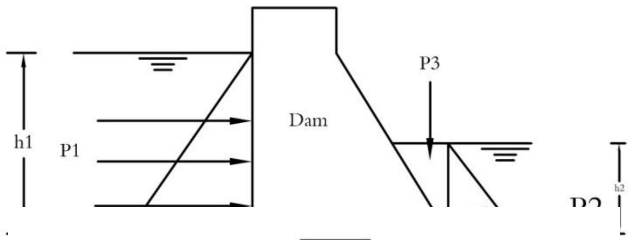


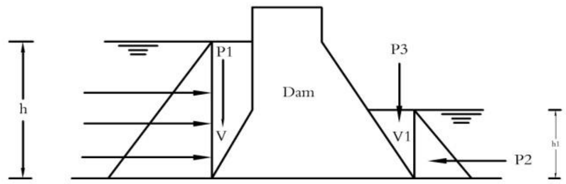
Forces acting on gravity dams:

1-Water pressure.
2-Uplift pressure.
3-Silt pressure.
4-Wave pressure
5-Pressur due to earth quake force
6-Ice pressure
7-Weight of the dam

1-Water pressure

P=1/2 ɣh2*
If the upstream face Is partly vertical and partly inclined
The resultant water pressure can be Solved in tow components





→ $P=1/2\ \gamma h^2$ & $P_1=\gamma v$ ↓

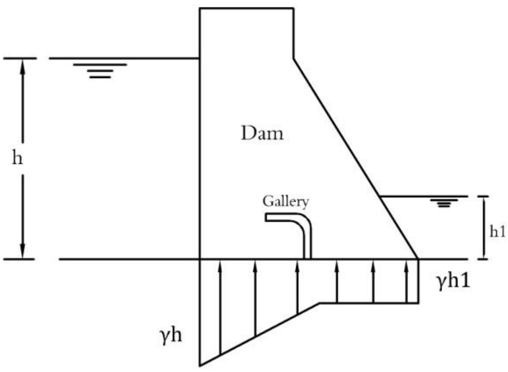
← $P_2=1/2\gamma h_1^2$ & $P_3=\gamma v_1$ ↓

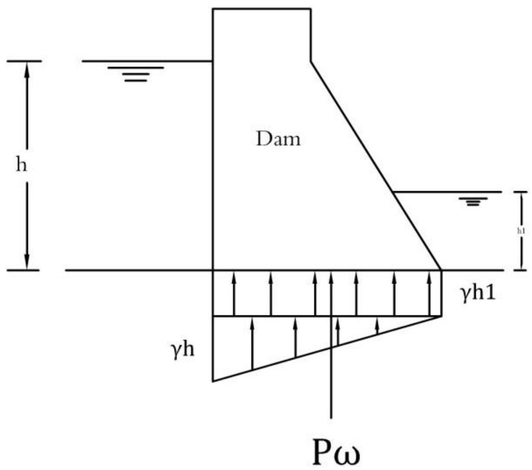
2-Uplift pressure

Without gallery

With gallery

$$\gamma h1+\frac{1}{3}(\gamma h-\gamma h1)=\gamma(\frac{(zh1+h)}{3})$$





3- Earth Quack Forces:

a- Effect of vertical acceleration:

when the acceleration is vertically upward the inertia force ; F = W K (where W = weight of the dam and K = coff. Of earth quake) acts vertically downwards , these increasing the downwards weights .

when the acceleration is vertically downward the inertia force acts upwards and decrease the downward weight .

Net Weight = W(1 ± kv) ; kv coff. Of earth quake of ver. dir.

\+ for acceleration is ver. upward .

\- for acceleration is ver. downward .

b- Effect Of Horizontal acceleration

I- Hydrodynamic pressure

The horizontal acceleration of the dam and foundation towards the reservoir causes a momentary increase in the water pressure .

The increase in water pressure (Pe) is given by :

$Pe = 0.55\ Kh\ \gamma\ h^2$ acts at $4h / 3\Pi$

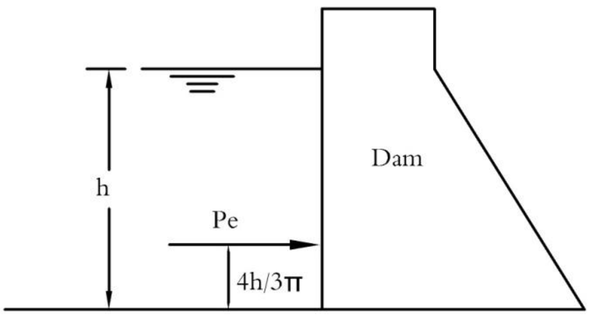
Kh=coff. Of earth quake horizontal dir.

II- Horizontal Inertia force

The inertia force acts in a direction opposite to the acceleration imparted by the earth quake forces

$FH = W \cdot Kh$ W = weight of the dam

This force can be considered at the center of gravity of the mass .



4- Wave Pressure:

Wave pressure depends on the height of the wave (hw) developed .

$hw = 0.032\sqrt{V*F} + 0.763 - 0.27\sqrt[4]{F}$ ….. for $F \le 32$ km

$hw = 0.032\sqrt{V*F}$ …..for $F > 32$ km

where : hw = height of the wave in (meter)

V = wind velocity in (km / hr)

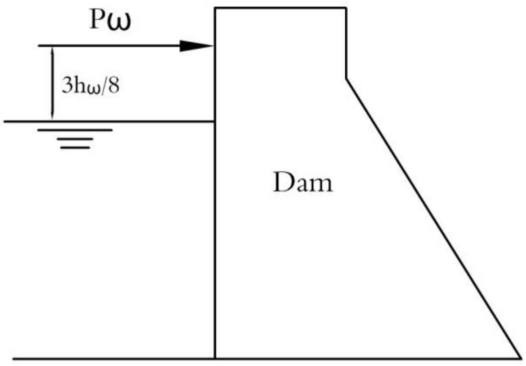
F = straight length of water expanse in (km)

∴ wave pressure is

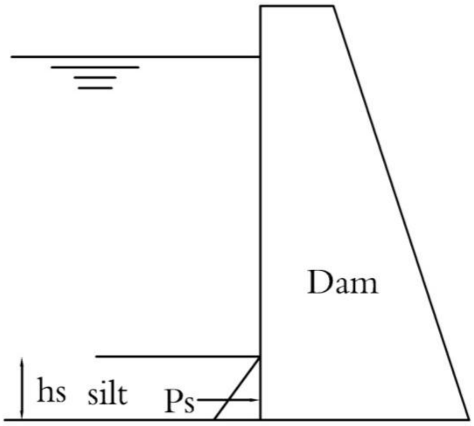
$PW = 2000\ \gamma\ hw2$ kg / m

$= 2\ \gamma\ hw2$ Ton / m

This force acts at distance (3 hw / 8) above the reservoir surface .



5- Silt Pressure:



$P_{silt} = \frac{1}{2}\ \gamma s\ hs2\ ka$

Where ka = (1-SINϕ) / (1+SINϕ)

γs = submerged unit weight of silt material

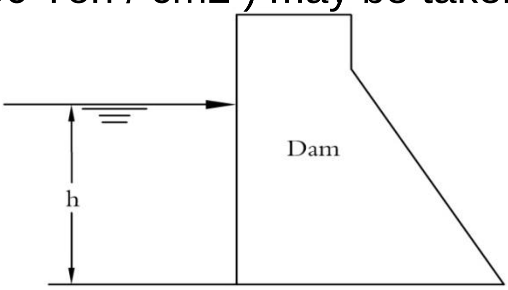
hs = height of silt

If the upstream face of the dam is inclined , the vertical weight of silt supported on the slope also acts as vertical force .

6- Ice Force:

The coefficient of the thermal expansion of ice being five times more than that of concrete . the ice force acts linearly along the length of the dam at the reservoir level.

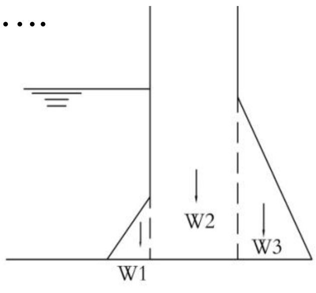
The average value of (5 Kg / cm2) or (50 Ton / cm2) may be taken as an ice force 5kg/cm2



7- Weight of the dam:

$W_1 = V_1\ \gamma_{con}$; $W_2 = V_2\ \gamma_{con}$ …..

$W_{dam} = \sum W = W_1 + W_2 + W_3 + \ldots$



STRUCTURAL STABILITY OF GRAVITY DAMS

Failure and inertia for structural stability of concrete gravity dam due to the following reasons :

1- overturning of the dam
2- compression or crushing of the dam
3- sliding of the dam
4- development of tension in the dam

1- Failure by overturning :

If the resultant of all forces acting on a dam at any section of its sections passes outside the toe , the dam shall rotates and overturning about the toe .

The factor of safety against overturning is :

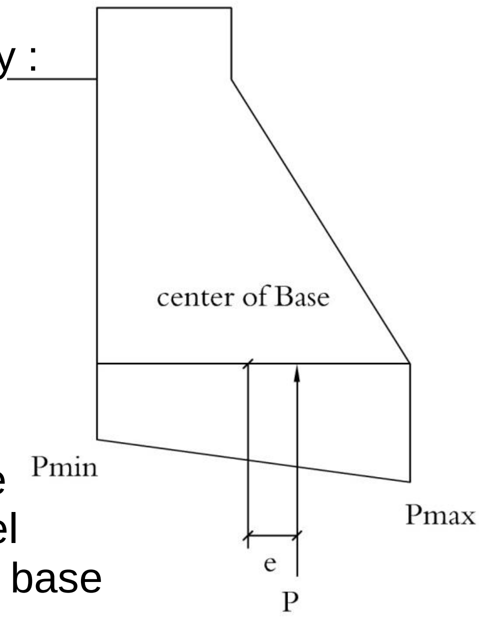
(F.S)overturning = (∑ Righting moments) / (∑ Overturning moments)

(F.S)overturning = (∑ MR) / (∑ MO)

The value of F.S against overturning should not be less than (1.5)

2- Compression or crushing :

A dam may fail by the failure of its materials , the compressive stress produced may exceed the allowable stress and dam material may get crushed .



The vertical stress distribution at the base is given by :

$\pm \frac{M\,C}{I}$ Pmax,min = $\frac{\sum V}{B}$

) $= \frac{\sum V}{B}$ ($1 \pm \frac{6\,e}{B}$

Where : $\frac{\sum V}{B}$ = direct stress

$=$ $\pm \frac{M\,C}{I}$

Bending stress = $\frac{V\,e}{\frac{1}{6} B^2}$ = $\frac{6\,V\,e}{B^2}$

+ will be used for calculating normal stress at the toe
- will be used for calculating normal stress at the heel

e : eccentricity of the resultant from the center of the base

∑ V : total vertical force

B : base width of the dam

3 – Sliding (Shear failure):

Sliding occur when the net horizontal force at the base of the dam exceeds the frictional resistance developed at the level

F.S sliding = $(\mu \sum(V-U)) / (\sum H) \quad > 1.0$

Where : $\sum(V-U)$ = net vertical force = $\sum V$

$\sum H$ = sum of horizontal forces causes the sliding

μ : coefficient of friction = (0.65 – 0.75)

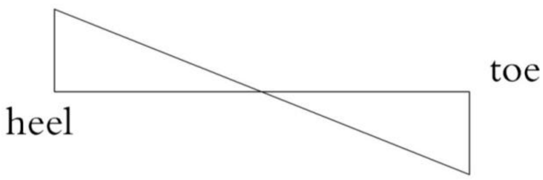
4- Tension

From the present equation :

P heel = $(\sum V / B)\,(1 - ((6e)/B))$

If $e > (B/6)$,the normal stress at the heel will be (-ve) or tension

No tension should be permitted at any point of the dam



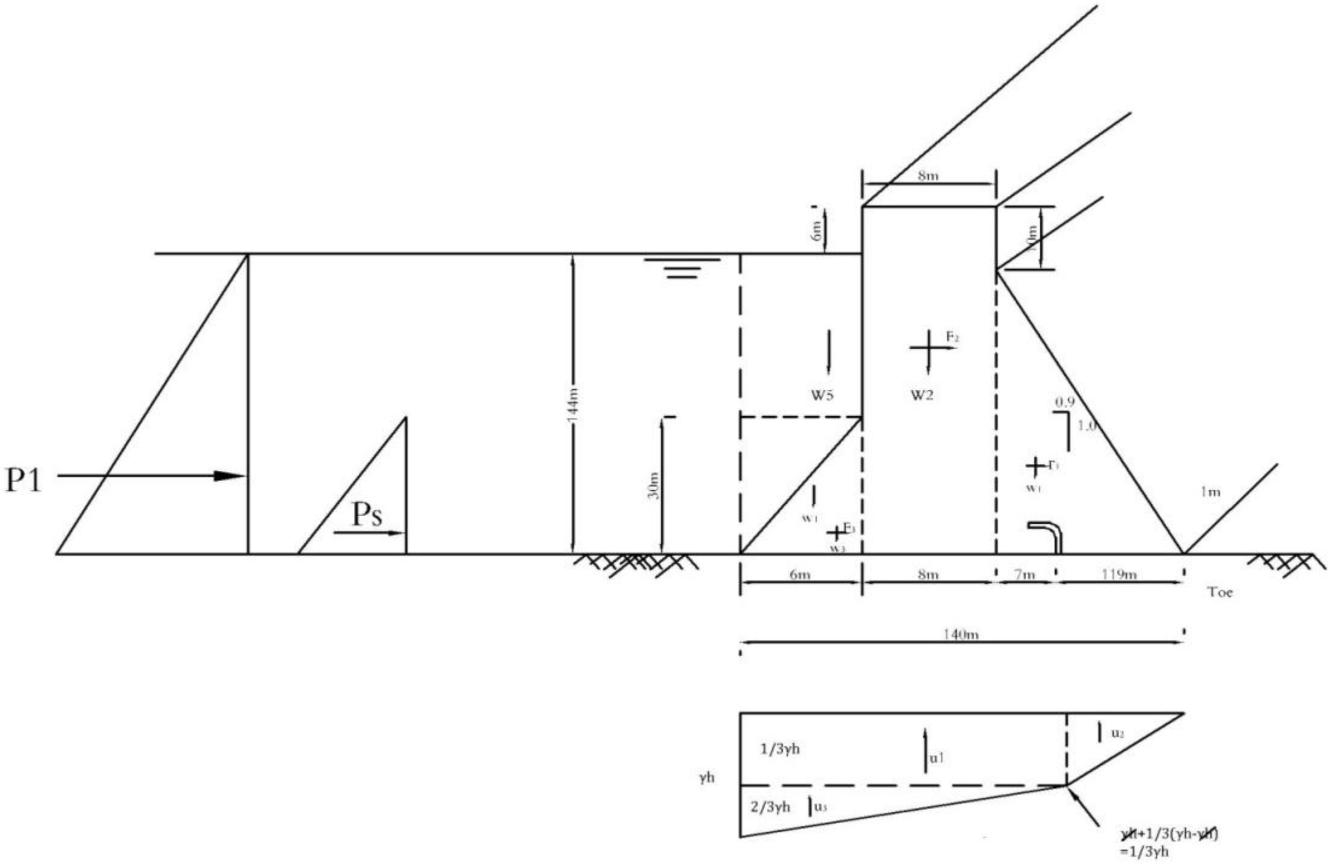
The eccentricity (e) should be less than B/6($e < B/6$) → the resultant should always lie within the middle third.

Example:

Determine the heel and toe stresses and the sliding factor for the dam section shown in the figure for the following loading conditions :

- horizontal earth quake (Kh) = 0.1
- normal uplift pressure (drain working)
- silt deposit up to 30 m height
- no wave pressure and no ice pressure
- unit weight of concrete = 2.4 Ton/m3
- unit weight of silty water = 1.4 Ton/m3
- submerged weight of silt = 0.9 Ton/m3
- coeff. Of friction = 0.65
- angle of reporse = 25°

(+↓) (-↑) (-⟶) (+⟵) (+)



Name of forces	Magnitude	L_a (m)	Moment toe
1- vertical forces W_1 W_2 W_3 W_4 W_5	 (126+140) * 0.5 * 1*2.4 = 21168 +↓ 150 * 8 *1*2.4 = 2889+↓ (30+6)*0.5*1*2.4 = 216 +↓ (30+6)*0.5*1*1.4 = 126 +↓ 114 *6*1*1 = 684 +↓	 84.3 130 136 138 137	 +1778112 374400 29376 17388 93708
∑ W	+ 25074		∑ M = 229284
2- uplift pressure U_1 U_2 U_3	 ⅓ (1)(144)(21)(1) = - 1008 ↑ ½ (⅓ *1*144)*119*1 = - 2856 ↑ ½ (⅔*1*144)*21*1 = -1008 ↑	 129.5 79.33 133	 - 130536 - 226576 - 134065
∑ U	- 4872		∑ M = -491176
3- horizontal forces $P_1 = ½\gamma h^2$ $P_s = ½\gamma_s h_s^2 k_a$	 0.5 * 144^2 * 1*1 = - 10368 0.5 *0.9 * 30^2 *0.4058 = - 164.4	 144/3 30/3	 - 497664 - 164.4
∑ H	- 10532.4		- 499014
4- earth quake $F_1 = W_1 * K_h$ $F_2 = W_2 * K_h$ $F_3 = W_3 * K_h$ Hydrodynamic (P_e) ($0.55\ k_h\ \gamma\ h^2$)	 21168 *0.1= - 2116.8 2880*0.1 = - 288 216 *0.1 = - 21.6 0.55 * 0.1 *1 * 144^2 = - 1150.848	 140/3 150/2 30/3 144/3π	 - 98784 - 21600 - 216 - 703346
∑ E	- 3577.284		∑ M = - 190934.6

$\sum M = +2292984 - 499014 - 190934.6 - 491176 = 1111859.4$ Ton . m

$\sum V = \sum W - \sum U = 25074 - 4872 = 20202$ Ton

$e = (B/2) - X' = (B/2) - (\sum M / \sum V)$

$e = (140/2) - (1111859.4 / 20202) = 15$ m

$P_{max}, _{min} = (\sum V / B)(1 \pm ((6e)/B))$

$= (20202/140)(1 \pm ((6*15)/140))$

$P_{max} = 237.06$ Ton / m2

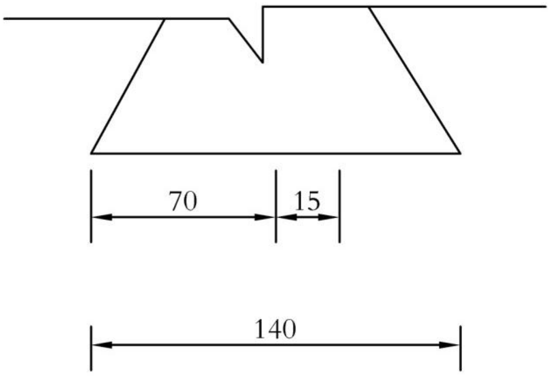
$P_{min} = 51.54$ Ton / m2

$F.S_{sliding} = (\mu \sum(V-U)) / (\sum H) > 1.0$

= 0.933 < 1 not o.k

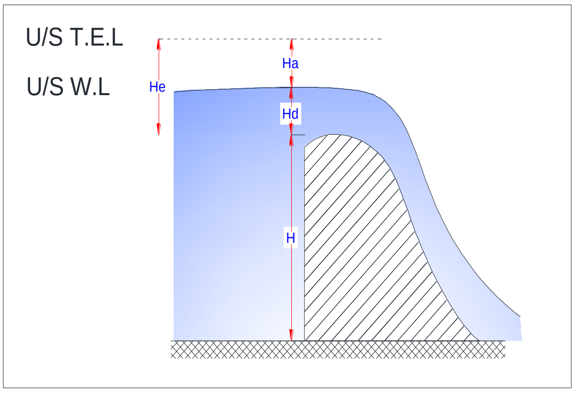
$(F.S)_{overturning} = (\sum MR) / (\sum MO)$

$= 2292984 / (Mu + MH + ME)$



Lecture Four :

Spillway



Spillway

Introduction :

A spillway is a structure constructed at a dam site , for effectively disposing of the dam surplus water from upstream and downstream just after the reservoir gets filled up. up to the normal pool level , water starts flowing over the top of the spillway crest (which is generally kept at normal pool level) . A spillway is essentially a safety valve from a dam .

A spillway can be located either within the body of the dam or at one end of it or entirely away from it .

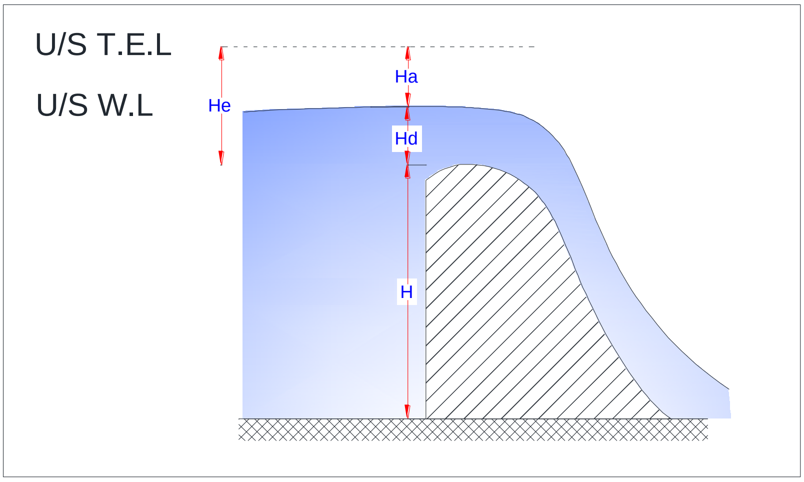
The spillways can be classified of the following major type , depending upon the type of the structure constructed for disposing of the surplus water :

1- Straight drop spillway
2- Ogee spillway (overflow spillway)
3- Chute spillway (open channel spillway)
4- Side channel spillway
5- Shaft spillway
6- Siphon spillway

Ogee spillway (overflow spillway) :

Ogee spillway is an improvement upon the free over fall spillway , and is widely used with concrete , arch and buttress dams.

Discharge formula for the ogee spillway :



The discharge passing over the ogee spillway is given by the equation:

$$Q = C\ Le\ He^{3/2}$$

Where :

Q : discharge

Le :effective length of the spillway crest

C : coefficient of discharge

He : total head over the crest (Ha + Hd)

$Ha = Va^2/2g$

Va = Q / (H + Hd)B where B : width of canal

Le = L – 2 [Kp . N + Ka] He

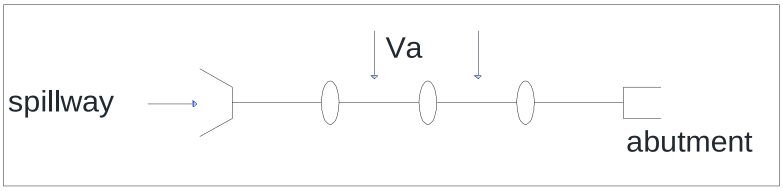
Where :

L : the net clear length of the spillway crest

Kp : pier contraction coeff.

N : number of piers

Ka : abutment contraction coeff.



Pier shape	*Kp*
Square nosed piers without any rounding	**0.10**
Square nosed piers with rounding on radius = 0.1 of pier thickness	**0.02**
Rounded nose piers and 90^0 cut water nosed piers	**0.01**
Pointed nose piers	**0.00**

abutment shape	*Ka*
Square abutment with head wall at 90^0 to the direction of flow	**0.2**
Rounded abutment with head wall at 90^0 to the direction of flow	**0.1**

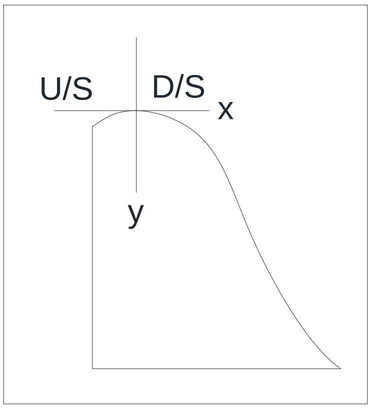
For high spillway the velocity is very small and it can be neglected (ha=0) → (He = Hd)

If $\frac{H}{Hd} > 1.33$ (High ogee spillway)

Crest of Spillway :

For spillway having a vertical face the D/S crest is given by :

$$X^{1.85} = 2\ Hd^{0.85}\ y$$



The U/S profile may be designed as the equation :

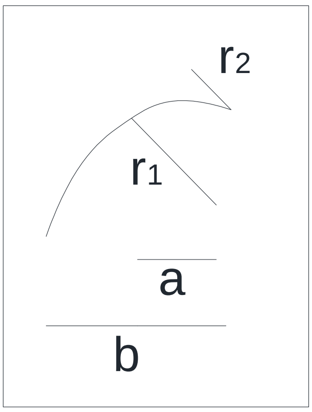
$$y = \frac{0.724\ (x + 0.27\ Hd)^{1.85}}{Hd^{0.85}} + 0.126\ Hd - 0.4315\ Hd^{0.375}\ (x + 0.27\ Hd)^{0.625}$$

a= 0.175 H

b= 0.282 H

r1= 0.5 H

r2= 0.2 H



vertical U/S

Example :

Design ogee spillway for concrete gravity dam having downstream face sloping at (0.781) , the discharge for the spillway is (8000 m3/sec) the height of spillway crest is kept at level (204 m) the average river level at the site is (100 m) , the spillway length consist (6) span having clear width (10 m) each , kp=0.01 , ka=0.1 , C=2.2

Solution :

L = 6 * 10 = 60 m

Le = L – 2 [N Kp + Ka] He = 60 – 2 [5 (0.01) + 0.1] He = 60 – 0.3 He

$Q = C\ Le\ He^{3/2}$

$8000 = 2.2\ (60 – 0.3\ He)\ (He^{3/2})$

By trial and error

He = 16.3 m

Let He = Hd

$\frac{H}{Hd} = \frac{204 – 100}{16.3} = 6.4 > 1.33$ » High spillway

H/Hd أسوء حالة تعطي أقل رقم للمقدار

D/S profile

$X^{1.85} = 2\ Hd^{0.85}\ y$

$y = X^{1.85} / 2\ Hd^{0.85}$

$= X^{1.85} / 2\ (16.3)^{0.85}$

$= X^{1.85} / 21.45$

$dy/dx = (1.85\ X^{0.85}) / 21.45$

$1/0.781 = 0.0862\ X^{0.85}$

X = 22.4 m

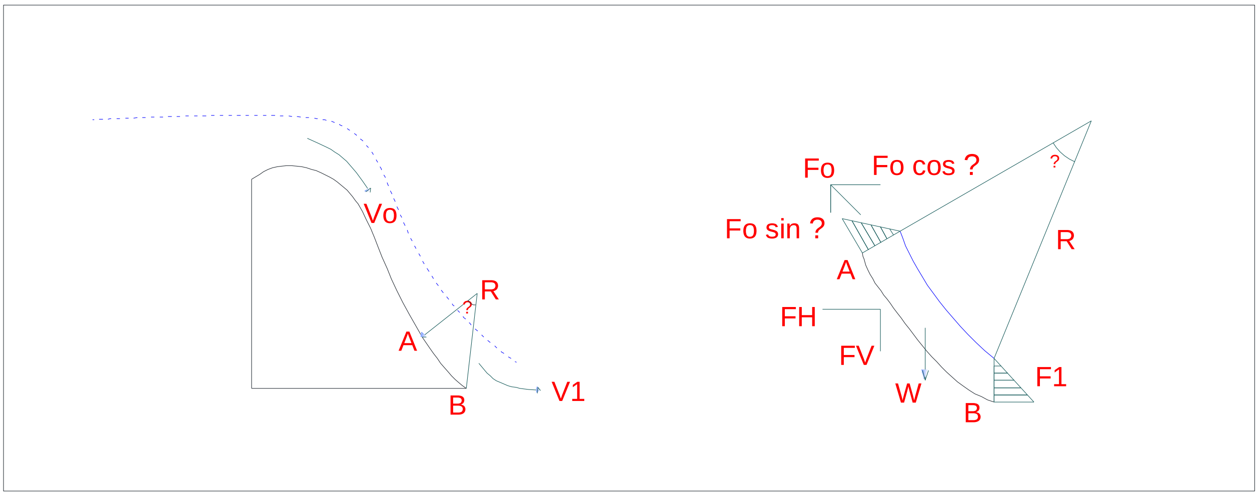
وهكذا يرسم المقدم عمودي والمؤخر لكل مسافة X نسقط y ونوصلها لينتج مقطع المسيل المائي .

X	$y = X^{1.85} / 21.45$
1	0.046
2	0.166
3	0.354
22.4	14.6

Dynamic force on spillway :

When water flows over the curved surface of ogee spillway , there is continuous change of velocity and hence there is change in momentum from section to section .

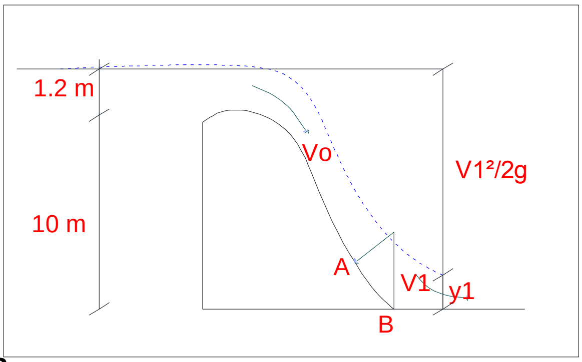
According to Newton's second low of motion , this change in momentum , causes a force on the spillway structure this force is known as the (dynamic force) .



Example :

The ogee spillway shown in the figure which discharge water with head of 1.2m over crest , taking C=2.2 .

Compute the dynamic force on the curved section AB which has a constant radius of 3m and ϴ = 60o



Solution :

$Q = C\, L_e\, H^{3/2}$

$q = Q/L = 2.2\, L\, (1.2)^{3/2} / L = 2.9\ m^3/sec/m$

$(V_{u/s})^2 / 2g + P_{u/s}/\gamma + Z_{u/s} = (V_o)^2 / 2g + P_o/\gamma + Z_o = (V_1)^2 / 2g + P_1/\gamma + Z_1$

$11.2 = (V_o)^2 / 2g + y_o \cos 60 + Z_o = (V_1)^2 / 2g + P_1/\gamma\ (y_1) + Z_1$

$11.2 = (V_o)^2 / 2g + y_o \cos 60 + 1.5 = (V_1)^2 / 2g + y_1$

$V_o y_o = V_1 y_1 = 2.9$

$V_o = 2.9 / y_o$ and $V_1 = 2.9 / y_1$

$11.2 = (2.9)^2 /19.62 \; y_o + 0.5\, y_o + 1.5$

$y_o = 0.212m$ and $V_o = 13.7$ m/s

$11.2 = (2.9)^2 /19.62 \; y_1 + y_1$

$Y_1 = 0.197m$ and $V_o = 14.7$ m/s

$F_o = 1/2\, \gamma\, y_o^2$

$F_{ox} = 1/2\, \gamma\, y_o^2 \cos\Theta = 0.5 * 9.81 * 0.212^2 * 0.5 = 0.11$ kN

$F_{oy} = 1/2\, \gamma\, y_o^2 \sin\Theta = 0.5 * 9.81 * 0.212^2 * \sin 60 = 0.19$ kN

$F_{1x} = 1/2\, \gamma\, y_o^2 = 0.5 * 9.81 * 0.197^2 = 0.19$ kN $\leftarrow$

$W = V \gamma = 2 * 3\pi * (0.212 + 0.197)/2 * 60 / 360 * 9.81 = 6.3 \downarrow$

$\sum F_x = \rho\, Q\, (- V_o \cos 60 + V_1)$

$0.11 – 0.19 + F_H = 1 * 2.9 * (- 13.7 \cos 60 + 14.7)$

$F_H = 22.845$ KN $\rightarrow$

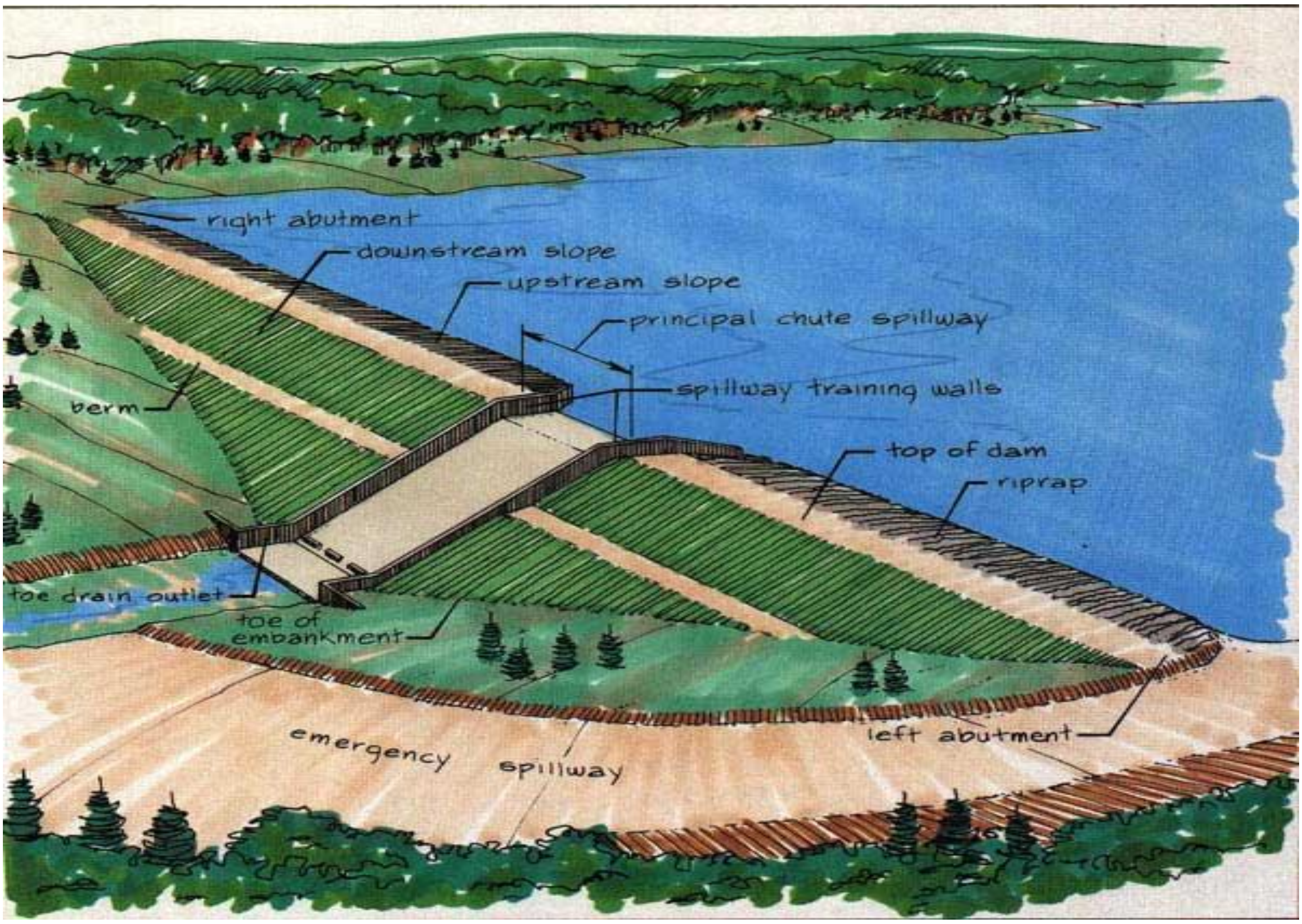
$\sum F_y = \rho\, Q\, (- V_o \sin 60 + 0)$

$- 6.3 – 0.19 + F_v = 1 * 3.9 * (- 13.7 \sin 60)$

$F_v = - 27.917$ KN

$F = \sqrt{F_H^2 + F_y^2} = 36.07$ KN

$\alpha = \tan^{-1} F_y/F_x = 50.7^o$



Hydraulics of Culverts:-

Lecture Five :

Hydraulics of Culvert

Hydraulics of Culvert

Introduction

A culvert is a conduit passing under a road or highway . In Section , culverts may be circular , rectangular , or oval .Culverts may operate with either a submerged entrance (fig. 1) or a free entrance (fig. 2).

Submerged Entrance:-

In the case of submerged entrance there are three possible regimes of flow as indicated in fig. 1 . Under conditions (a) and (b) of the figure the culvert is said to be flowing under outlet control , while condition (c) represents entrance control . In (a) the outlet is submerge possibly because of inadequate channel capacity downstream or due to back water from a connecting stream . In (b) the normal depth y0 of the is greater than the culvert height D , causing the culvert to flow full . The same equation is applicable to both cases (a) and (b) , namely

$$\Delta h = h_e + h_f + h_v$$

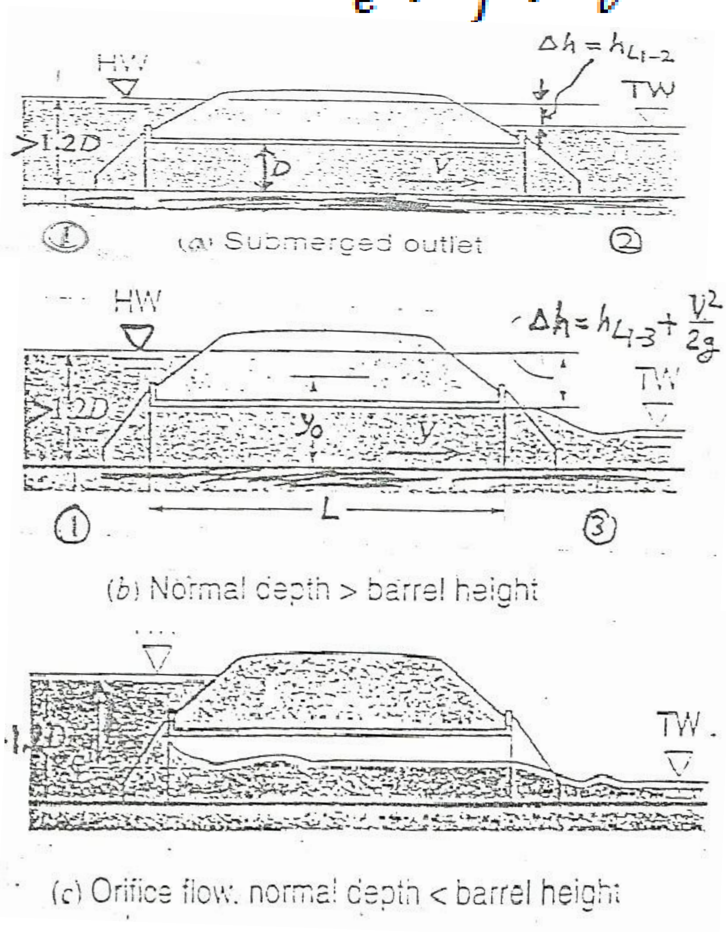


Figure (1) Flow condition in culverts with submerged entrance.

Where Δh is define in fig.1a , and fig.1b , he is the entrance head loss , h_f is the friction head loss in the culvert barrel , and h_v is the velocity head loss at submerged discharge in case (a) or the residual velocity head at discharge in case (b).

Entrance loss is a function of the velocity head in the culvert , while friction loss may be computed using Manning's equation . Thus , in BG units ,

Outlet control

Case (a) or (b) :

$$\Delta h = K_e \frac{V^2}{2g} + \frac{n^2 V^2 L}{2.21 R_h^{4/3}} + \frac{V^2}{2g}$$

This expression in BG units can be reduced to outlet control ,

$$\Delta h = \left(K_e + \frac{29.2 n^2 L}{R_h^{4/3}} + 1\right)\frac{V^2}{2g}$$

The entrance coefficient K_e is about 0.5 for a square-edged entrance and about 0.05 if the entrance is well rounded .

If the outlet is submerged , the head loss may be reduced somewhat by flaring the culvert outlet so that the outlet velocity is reduced and some of the velocity head recovered. Tests show that the flare angle should not exceed about 60 for maximum effectiveness .

To determine which of cases (b) or (c) occurs when the outlet is free (not submerged) , we need to find if normal flow in the barrel will fill it . Usually the discharge is known or assumed. For rectangular culverts the normal depth can be solved in the usual way from Manning's eq. by trial and error . For circular cross sections it is ** easier to use Manning's equation to find the diameter which would just flow full $\left(R_{h=}\frac{D}{4}\right)$, and to compare that with the actual or proposed diameter . If alternative slops are being considered with a giber barrel diameter . If alternative slops are being considered with a given barrel diameter , the algebra can be rearranged to solve for the slope that just causes the barrel to flow full.

If normal depth in the culvert is less than the barrel condition (c) illustrated in fig. 1c will normally result .

This culvert is said to be flowing under entrance control. i.e , the entrance will not admit water fast enough to fill the barrel and the discharge is determined by the entrance conditions . the inlet functions like an orifice for which Entrance control,

Case (c) :

$$Q = C_d A\sqrt{2gh}$$

Where h is the head on the center of the orifice and Cd , is the orifice coefficient of discharge . the head required for a given flow Q is therefore Entrance control,

Case (c) :

$$h = \frac{1}{C_d^2}\frac{Q^2}{2gA^2}$$

It is impractical to cite appropriate values of Cd , because of the wide variety of entrance conditions which may be encountered : for a specific design this must be determine from model tests or tests of similar entrances . For a sharp-edged entrance without suppression of the contraction Cd = 0.62 , while for a well-rounded entrance Cd approaches unity . If the culvert is set with its invert at stream-bed level , the contraction is suppressed at the bottom. Flared wing walls may also cause partial suppression of the side contractions .

Free Entrance :

Some box culverts may be designed so that the top of the box forms the road way . In this case the head water show not submerge inlet and one of the flow conditions of Fig.2 (free entrance) will exist. In cases (a) and (b) critical depth in the barrel controls the head water elevation , while in case (c) the tail water elevation is the control . In all cases the head water elevation may be computed using the principles of open-channel flow discussed in this chapter with an allowance for entrance and exit losses .

When the culvert is on a steep slope [case (b)] , critical depth will occur at about 1.4yc downstream from the entrance. The water surface will impinge on the head water when the head water depth is about 1.2D if yc is 0.8D or more. Since it would be inefficient to design a culvert with yc much less than 0.8D , a head water depth of 1.2D is approximately the boundary between free-entrance conditions (Fig.2) and submerged-entrance conditions (Fig.1).

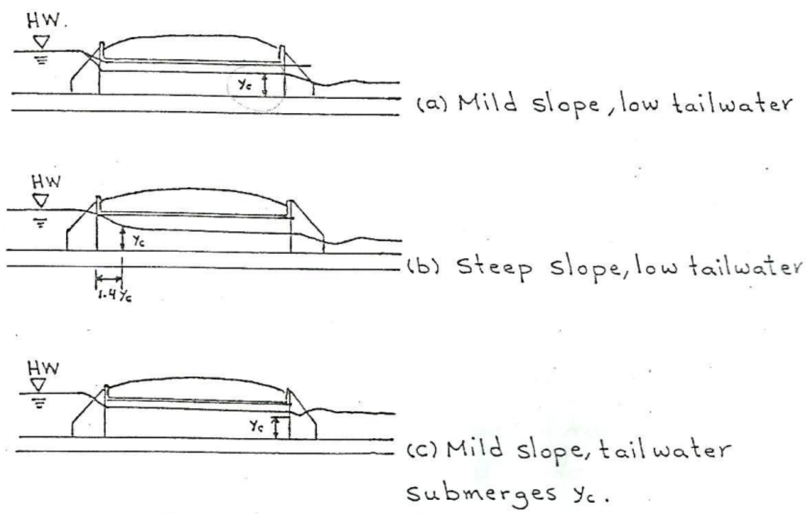


Figure (2) flow conditions in culverts with free entrance.

Entrance and outlet structures:-

The geometry of the entrance structure is an important aspect of culvert design. Entrance structures (Fig.3) serve to protect the embankment from erosion and , if properly designed , may improve the hydraulic characteristic of the culvert. The straight end wall (a) is used for small culverts on flat slopes when the axis of the stream coincides with the culvert axis. If an abrupt change in flow direction is necessary , the L end wall (b) is used. The U-shaped end wall (c) is the least efficient form from the hydraulic view point and has the sole advantage of economy of construction . where flows are large , the flared wing wall (d) is preferable.

The flare should , however , be made from the axis of the approaching stream (e) rather than from the culvert axis. Some hydraulic advantage is gained by warping wing walls into a smooth transition , but the gain is not usually sufficient to offset the cost of the complex forming required for such warped surfaces.

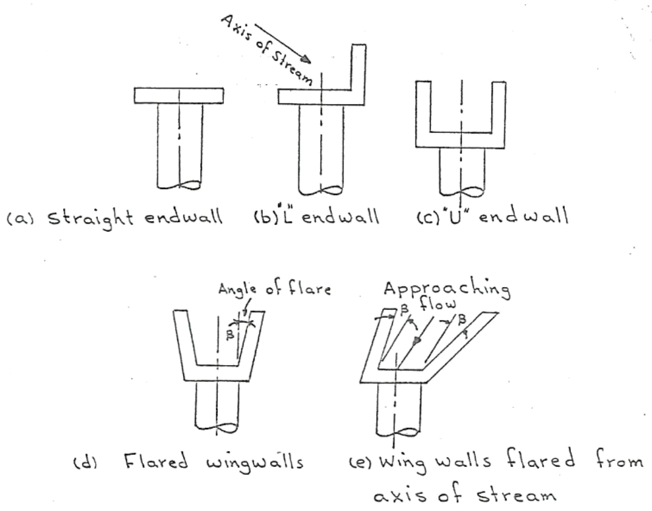


Figure (3) : culvert end walls and wing walls.

The purpose of the culvert outlet is to protect the downstream slope of the fill from erosion and prevent undercutting of the culvert barrel . where the discharge velocity is low or the channel below the outlet is not subject to erosion , a straight end wall or a U-shaped end wall may be quite sufficient at higher velocities , lateral scour of the embankment or channel banks may result from eddies at the end of the walls , especially when the culvert is much narrower than the outlet channel. With moderate velocities , flaring of outlet wing walls is helpful, but the flare angel β must be small enough so that the stream from the culvert will adhere to the walls of the transition.

Example :

A culvert under a road must be 30m long , have slope of 0.003, and carry 4.3m3/s. If the max . permissible head water level is 3.6m above the culvert invert , what size of corrugated-pipe culvert (n=0.025) would you select? Neglect velocity of approach . Assume a square-edged inlet with Ke=0.5 and Cd=0.65 . The outlet will discharge freely.

Solution:

Assume D<3m , i.e , that head water depth /D>3.6/3=1.2 , i.e. , assume that the entrance is submerged.

Given the discharge is free , so Fig.1.a cannot apply.

Conditions are those of either Fig.1.b or 1.c .

Assume case (b) , Fig.1 , i.e. , that the barrel flows full

$$V = \frac{Q}{A} = \frac{4.3}{\frac{\pi D^2}{4}} = \frac{5.47}{D^2} \; ; \quad R = \frac{D}{4}$$

Fig.1.b: $\Delta h = h_{L1-3} + \frac{V^2}{2g} = (y_1 - y_3) + (z_1 - z_3) + \frac{V^2}{2g}$

$$= y_1 - y_3 + S_0 L + \frac{\left(\frac{5.47}{D^2}\right)^2}{2g}$$

$$= 3.6 - D + 0.003(30) + \left(\frac{5.47}{D^2}\right)^2 * \frac{1}{2(9.81)}$$

$$\Delta h = 3.69 - D + \frac{1.528}{D^4} \dots\dots\dots\dots (1)$$

$$\Delta h = \left[0.5 + \frac{19.62(0.025)^2 * 30}{\left(\frac{D}{4}\right)^{4/3}} + 1\right] * \frac{5.47^2}{2(9.81)D^4} \dots\dots\dots\dots (2)$$

$$= \left(1.5 + \frac{2.34}{D^{4/3}}\right) * \frac{1.528}{D^4}$$

Equation the Δh expressions and simplifying:-

$$3.69 = D + \left(0.5 + \frac{2.34}{D^{4/3}}\right) * \frac{1.528}{D^4}$$

By trial and error or by equation solver, D=1.107 m.

Thus the first assumption (D<3m) is ok.

Now to determine is we have case (b) or case (c) ; find the diameter d0 that just flows full with normal (uniform) flow:-

$$4.3 = \frac{1}{0.025} \frac{\pi d_0^2}{4} \left(\frac{d_0}{4}\right)^{2/3} (0.003)^{1/2}$$

$$\Rightarrow d_0 = 1.994\, m.$$

As $d_0 > D$, the culvert runs full , we do have case (b) , as assumed . The above assumption and analysis are valid.

D=1.107 m. use standard D=1.2m

Example :-

A culvert under a road must carry 4.5 m^3/sec. (a) If the culvert length is 32m , the slope is 0.004 , and the max. permissible headwater level above the culvert invert is 3.8m . what size of corrugated-pipe culvert (n=0.025) would you select ? the outlet will discharge freely . Neglect velocity of approach . Assume square-edged entrance with K_e= 0.5 , C_d=0.65. (b) Repeat for a culvert length of 110m.

Types of flow in the barrel of a culvert

Type	H/D	Exit depth (y_2)	Flow Type	Length (L)	Slop S_0	Control	Remarks
Submerged entrance conditions							
1	>1.0	>D	Full	Any	Any	outlet	Pipe flow
2	>1.2	<D	Full	Long	Any	Outlet	Pipe flow
3	>1.2	<D	Part Full	Short	Any	outlet	Orifice
Free entrance conditions							
4	<1.2	<D <critical	Part Full	Any	mild	outlet	Subcritical
5	<1.2	<D <critical	Part Full	Any	mild	outlet	Subcritical
6	<1.2	<D <critical	Part Full	Any	mild	Inlet	Subcritical
		>critical	Formation of hyd. Jump in barrel				

Lecture Six :

Bridges

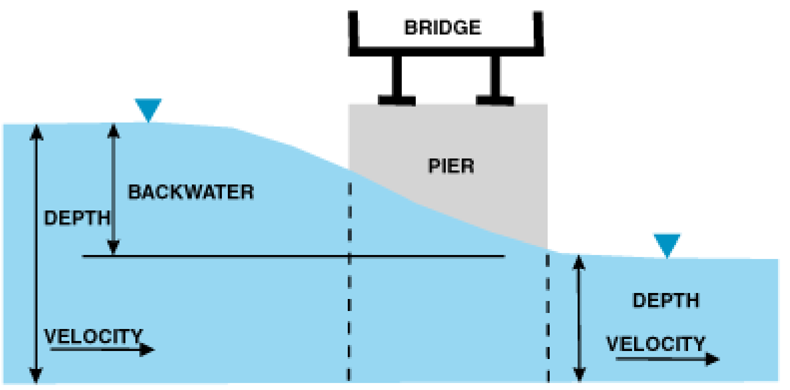
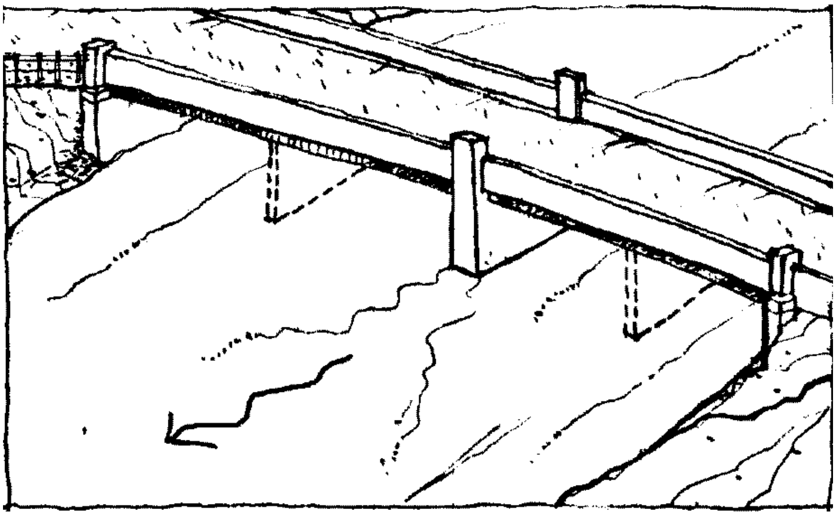


Figure 5. Side view of stream showing backwater and hydraulic variables at and upstream from bridge.



Bridges

Introduction

The presence of a bridge across a stream Creates Constricted flow Through its opening because of :

1-The reduction in the width of the stream due to piers and their associated end Contractions .

2-The fluming of The stream itself (in The case of wide streams with flood plains) to reduce The Costs of The structure .

Apart from (local) Scour around The piers and Possible bed erosion ,There is a considerable back water effect of The bridge . The corresponding afflux (rise in upstream water level) depends on The type of flow (Subcritical or supercritical), As most bridges are designed for subcritical flow conditions in order to minimize scour and chocking problems , further discussions here are mainly confined to subcritical flow.

The establishment of afflux levels is extremely important for design of upstream dykes and other protective works and also for the location of safe bridge deck levels (to avoid the flooding of the deck and any consequent structural damage) .

It is equally important to determine The minimum Clear length of span (economic Considerations) which will not Cause undesirable afflux levels .In order to establish permissible upstream stage levels , detailed investigations of The properties along the stream have be investigated .

Downstream of the bridge the water level are only influenced by The nearest Control section Blew the bridge . These levels Can therefore be established by backwater Computation .

Backwater Level Bridges at long Contractions

In The Case where the bridge has number of larger piers and/or long approach embankments contracting The water width ,The backwater effect is Considerable .

Yarnell's experimental data on The flow through bridge Piers resulted in The following empirical equation

$$\Delta y / y3 = K\, Fr_3^2 \;\; (K + 5Fr_3^2 - 0.6)(\alpha + 15\alpha^4)$$

Where $\alpha = 1-\sigma = 1-b/B$ (show Fig. (1))

and K is a function of the pier shape (Table (1))

Δy: Afflux , Y3 : downstream depth

Table (1) values of K as a function of pier shape

Pier shape	***K***
Semicircular	0.9
Lens – shaped	0.9
Cylinder	0.95
90º triangular	1.05
Square nose	1.25

The above equation is valid only if σ is large the Contraction Cannot setup critical flow conditions between piers and choke the flow .If the flow becomes chocked by excessive Contraction the afflux increases Substantially (Fig.(1))The limiting value of σ (assuming uniform velocity at section 2) for critical flow at section (2) Can be written as

$$\sigma = (2 + 1/\sigma)^3\, Fr_3^4 / (1 + 2Fr_3^2)^3$$

In the case of chocked flow the energy loss between Section (1) and (2) Fig (1) was given.

$$E1 - E2 = CL\ V1^2/2g$$

Where CL = 0.35 for square pier
= 0.18 for rounded pier
*For a pier length ; width ratio to (4) .

From the last eq. upstream y1 Can be Calculated From which The afflux Δ y is obtained as (y1-y3)

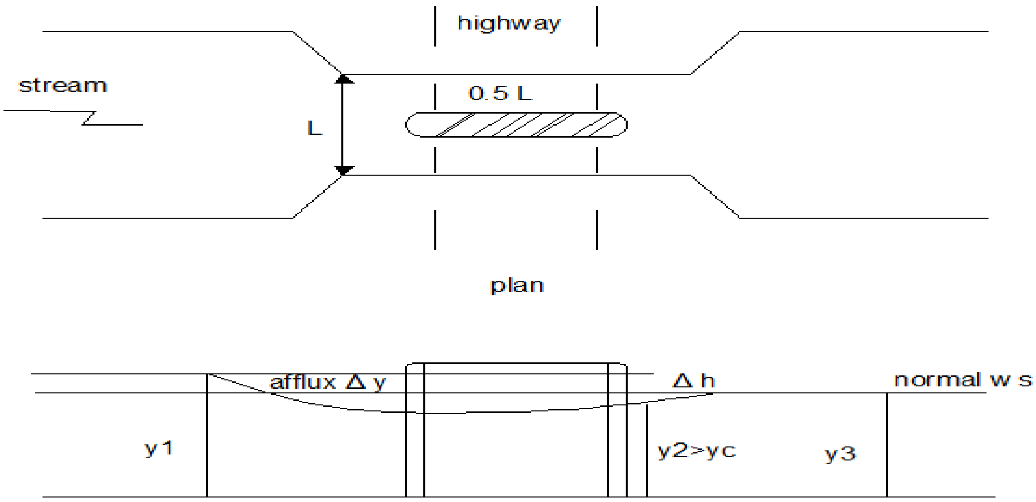


Fig (1) Flow profile through bridge with contracted channel (subcritical flow)

Example

A road bridge of seven equal span lengths crosses a 106 m wide river. The piers are 2.5m thick , each with Semicircular noses and tails, and their length :breadth ratio is 4. The stream flow data are given as Follows: discharge=500m3/sec, depth of flow downstream of the bridge=2.50 m. Determine the afflux upstream of the bridge.

Solution

The velocity at The downstream section V3=500/106*2.5

$V3=1.887 m/sec \rightarrow Fr_3= V_3/\sqrt{gy_2} = 0.318$

Flow condition within The pier as follow;

The limiting value of $\sigma =(2+1/\sigma)^3 Fr_3^{4}/(1+2Fr_3^{2})^3$

$\sigma= 0.55$

While The value of σ provided=b / B=13/ 15.5=0.839 Since the value of σ provided is more than the limiting (σ) value , subcritical flow conditions exist between the piers

$\Delta y/y3 =K\ Fr32\ (K+5Fr32-0.6)(\alpha+15\alpha4)$

With K= 0.9 (from table 1) and $\alpha=1-\sigma = 0.161$

The afflux , $\Delta y=5.41*10-2$

Scour depth under the bridge

If the Contracted width (i.e. The bridge length ,L)is less Than The regime width (w) (The minimum stable width of an alluvial Channel (as eq. B=4.75Q1/2) where (B) is The want way width (measured along The water surface and right angles to The banks in m, and Q The max .flood discharge in m3 ⁄ sec) , the normal Scour depth ,Dn, under the bridge is ;

$Dn=Rs(W/L)^{0.61}$

Where Rs=regime scour depth=$0.475(Q/f)^{1/3}$

If B>regime width

$Rs=1.35(q^2/f)^{1/3}$ B< regime width

F=lacys silt Factor=1.76 √ d

The max Scour depth in single span bridge (no piers) with a straight approach is about (25%) more than Dn, where as in Case of multispan str. With Curve Approach it is (100%) Than Dn.

If The Construction is predominant;

$Dmax=Rs(w/L)^{1.56}$

W=regim width

L= length of bridge

Local scour around bridge piers

Several formula based on experimental results have been proposed to predict The max .scour depth (ys, below bed level) around bridge piers, In general ,these assume The relationship

$ys/b = F(y_0 / b, Fr ,d / b)$

Where b is the pier width ,y_0 =the u ⁄s flow depth , d=sediment size

$ys / b=4.2 (y_0 / b)^{0.78} Fr^{0.52}$ Laursens formula

$ys/y_0=(B/b)^{5/7} -1$ live bed (bed load transport)

Although The presence of Scour tends to reduce The backwater levels upstream of the bridge ,the damage to the foundations of the structure may far outweigh the possible benefit . Hense protective measures , both to minimize the scour and prevent under mining of the foundations , have to be taken. piers with base diaphragms (horizontal rings) and multiple cylinder type piers have been found to Minimize the Scour Considerably. The normal practice for protection of the foundation is to provide thick protective layers of stone or Concrete aprons around the piers .

On the abutment can applied khassaf's formula

$Ds/y = 16.826\ Fr^{2/3}\ \alpha^{-1}\ \theta^{-0.325}$

$Ds/y = 3.96\ Fr^{2/3}\ \alpha^{-1}$ Khassafs formula

θ= Argle of inclination of abutment with the direction of flow(any of attack)

α=opening ratio B-b/B

$S/d_1=1.1(a/d_1)^{0.4}\ Fr_1^{\ 0.33}$

a= length of abutment

d_1=u/s water depth

S=scour depth

Rang of fornd No. 0.09 → 0.22 sub

D50→ 0.25 mm

Opening ratio o.6→ 0.85

Width= 6 b

Length= 7 b d= 0.06-0.033

Thick=⅓ b

Lecture Seven :

Dimensional Analysis

and Hydraulic Similitude

Dimensional Analysis and Hydraulic Similitude

Introduction

Dimensional Analysis is a mathematical technique which deals with the dimensions of the physical quantities involved in the phenomenon. The physical quantities are measurable entities such as length, time, electric current, temperature, density, specific weight. Dimensional analysis reduces the number of variables in a fluid phenomenon by combining some variable to form non dimensional parameters. Instead of observing the effect of individual variables.

Dimensional Analysis is widely used in research work for developing design criteria and also for conducting model tests.

Dimensions

The fundamental dimensions are mass (M) length (L) and time (T) in M.L.T system, and force (F), length (L) and time (T) in F.L.T system. The two systems are interrelated by Newton's second law of motion:

$$(F) = MX\left(\frac{L}{T^2}\right)$$

$$M = \left[\frac{FT^2}{L}\right]$$

Temperature(∅) is taken as the fundamental dimension.

Dimensional Homogeneity

A physical equation is said to be dimensionally homogeneous if the quantities on both sides of the equation have identical dimensions.

Let us consider the following two equations:

$$P = \delta h - - - - - (1)$$

$$V = \frac{1}{N} R^{2/3} S^{1/2} - - - - - (2)$$

eq. (1) is dimensionally homogeneous:

$$\left(\frac{F}{L^2}\right) = \left(\frac{F}{L^3}\right)(L) = \left(\frac{F}{L^2}\right)$$

eq. (2) is dimensionally non-homogeneous:

Left- hand side= $\left(\frac{L}{T}\right)$

Right – hand side = $\left[L^{\frac{2}{3}}\right][1]$

Note:

1- The quantities which are dimensionless are represented by (1) such as represented by (1) such as slope.
2- Dimensionally homogeneous equation is applicable to all system of units. On the hand a dimensions ally non- homogenous equation applicable only to the system of units for which it had been derived.
3- Two dimensionally homogeneous equation con be multiplied or divided without affecting. But the equations can't be added or subtracted (as the resulting equation may not be dimensionally homogeneous).
4- The principle of dimensional analysis can be system to other system.

Example:

Convert the pressure from kg/cm2 to Ib/in2

Sol.

1 Ib= 453.6 gm
1 kg= 2.204 lb
1 inch= 2.54 cm

Conversion factor=$\frac{\mathrm{kg/cm^2}}{\mathrm{Ib/inch^2}} = \frac{\mathrm{kg}}{\mathrm{Ib}} \times \frac{(\mathrm{inch})^2}{(\mathrm{cm})^2}$

$= 2.204 \times 2.54 \times 2.54 = 14.22$

Therefore, the pressure in kg/cm2 can be converted into Ib/in2 by multiplying it with (14.22). In other words.
Pressure in (Ib/in2)= 14.22*(pressure in kg/cm2).

Example:

Find the dimensions of the following quantities in both M-L-T and F-L-T systems:
(a) Discharge (b) momentum (c) Energy
Sol.

(a) Discharge = $\frac{\mathrm{volume}}{\mathrm{time}} = (\frac{\mathrm{L}^3}{\mathrm{T}})$ in both system

(b) Momentum= mass * velocity

$= (\mathrm{M}) \times \left(\frac{\mathrm{L}}{\mathrm{T}}\right) = \left[\frac{\mathrm{ML}}{\mathrm{T}}\right]$

Sub. The dimensions of Min terms of F

Momentum= $\left[\frac{\mathrm{FT}^2}{\mathrm{L}} \times \frac{\mathrm{L}}{\mathrm{T}}\right] = [\mathrm{FT}]$

(c) Energy = WORK = Force* Distance

$= \mathrm{F} \times \mathrm{L} = [\mathrm{FL}]$

$\mathrm{Energy} = \left[\frac{\mathrm{ML}}{\mathrm{T}^2} \times \mathrm{L}\right] = \left[\frac{\mathrm{ML}^2}{\mathrm{T}^2}\right]$

H.W

(a) Torque
(b) Power
(c) Surface tension
(d) Specific Weight

H.W

Convert (viscosity = 2 kg (f). sec/m2 to S.I units also 50 h.p to s.I system.

Example:

Check the dimensional homogeneity of the following common equatins in the field of hydraulics.

(1) Q= Cd. a$\sqrt{2gH}$ (2) V= C$\sqrt{mi}$

Sol.

(1) Q= Cd. a$\sqrt{2gH}$

$\mathrm{L^3T^{-1}} = [1][\mathrm{L}^2](\mathrm{LT^{-2}} \times \mathrm{L})^{1/2}$

$\mathrm{L^3T^{-1}} \therefore$ Dimensionally homogenous

(2) V= C$\sqrt{mi}$

$\mathrm{LT^{-1}} = [1][\mathrm{L} \times 1]^{1/2}$

$= \mathrm{L}^{1/2}$ ∴ the ∴ equation is not dimensioal

Methods of dimensional analysis:

The following two methods of dimensional analysis:
(1) Rayleigh's method (1899).
(2) Buckingham's π thereon.

The first method of dimensional analysis becomes cumbersome when a large number of variables are involved; therefore; it will study the second method because it's a general method.

Buckingham's π- theorem

The Buckingham's π- theorem states that if there are (n) variable in a dimensionally homogenous equation and if these variable contain (m) fundamental dimensions (such as M, L, T), they may be grouped into (n-m) non-dimensional parameters, Buckingham called these non-dimensional parameters as π- terms.

The Buckingham method can be summarized as follows:

1. List the entire (n) variable which affects the phenomenon.
2. Choose (m) repeating variables. When selecting the repeating variable, the points mentioned above must be kept in view.
3. Write the general equation giving the functional relationship in the form of eq.$f\,(\pi_1, \pi_2, \pi_3, \ldots = \text{constant}.$
4. Write separate expressions for each π-term in the form of

eq.$\pi_1 = x_1^{a1} x_2^{b1} x_3^{c1} \ldots$

$\pi_2 = x_1^{a2} x_2^{b2} x_3^{c2} \ldots$

⋮

$\pi_{n-m} = x_1^{an-m} x_2^{bn-m} x_3^{cn-m} \ldots$

Each π-term contains the repeating variables and one of the remaining variables. The repeating variables are written in exponential form.

5. Determine the exponents of the repeating variables on the basis that each π-term is non- dimensional.
6. After the π - term have been determined, the functional relationship $[f(\pi_1, \pi_2, \pi_3, \ldots, \pi_{m-n}) = \text{const.}]$ can be written and the required expression can be obtained.

The following artifice, if necessary, can be used to get the required expression.

(A) Any π- term may be replaced by any power of it because the power of a non-dimensional term is also non-dimensional. For example π, may be replaced by$\pi_1^2, \pi_1^3, \text{or } \frac{1}{\pi_1}$.

(B) Any π- term may be replaced by the product of that term with an absolute numerical constant. For example (π_1) can be replaced by ($2\pi_1$), ($3\pi_1$)or$\frac{\pi_1}{2}$.

(C) Any π- term can be replaced by another term which is the product or quotient of two π- term. For example (π_1) can be replaced by$(\pi_1 \times \pi_2)\text{or } (\frac{\pi_1}{\pi_2})$.

(d) Any π- term can be replaced by another π- term which is the sun or difference of the π- term with an absolute numerical constant. For example, π_1 can be replaced $(\pi_1 - 1)\text{or } (\pi_1 + 1)$.

Note: the repeating variables selected are ρ, V and L i, e, the first representing the fluid property, the second representing the flow characteristics and third representing the geometrical characteristics of the body, this in general.

Example: Assume that the friction factor (f) depends upon the diameter of pipe (D), density (ρ), viscosity (μ), the height of roughness (K) and the velocity (V). Driveπ- term of the non-dimensional variables of the friction factor of the flow through the pipe?

Sol.

$f = \emptyset(D, \rho, \mu, V, K)$

$F(f, D, \rho, \mu, V, K)$

Where F stands for "a function f "

The number of primary dimensions involved is (3), i.e., m=3 (M, L, T)

The number of variable is (6), i. e., n=6

Therefore, the number of π- terms 6-3=3 thus $F(\pi_1, \pi_2, \pi_3) = \text{const}$

Now taking ρ, V and D as repeating variables

$\therefore\ \pi_1 = \rho^{a1}V^{b1}D^{c1}\mu$

$\pi_2 = \rho^{a2}V^{b2}D^{c2}K$

$\pi_3 = \rho^{a3}V^{b3}D^{c3}F$

The exponents can be determined as under:

$\pi_1 = \rho^{a1}V^{b1}D^{c1}\mu$

Writing the dimensions

$$[M^{\circ}L^{\circ}T^{\circ}] = \left[\frac{M}{L^3}\right]^{a1}\left[\frac{L}{T}\right]^{b1}[L]^{c1}\left[\frac{M}{LT}\right]$$

EQUATING EXPONENTS OF M, L and T

For $M: 0 = a_1 + 1 \quad \text{or } a_1 = -1$

$T: 0 = -b_1 - 1 \quad \text{or } b_1 = -1$

$L: 0 = -3a_1 + b_1 + c_1 - 1 \quad \text{or } c_1 = -1$

Therefore $\pi_1 = \frac{\mu}{\rho VD}$

As the reciprocal of a non-dimensional parameter is also non-dimensional, the expression for $\pi_1$can be written as

$$\pi_1 = \frac{\rho VD}{\mu}$$

Likewise, writing the dimensions in expression for π_2

$\pi_2 = \rho^{a2}V^{b2}D^{c2}K$

$$[M^\circ L^\circ T^\circ] = \left[\frac{M}{L^3}\right]^{a2} \left[\frac{L}{T}\right]^{b2} [L]^{c2}[L]$$

Equating exponents of M, L and T

ForM: $0 = a_2$ or $a_2 = 0$

T: $0 = -b_2$ or $b_2 = 0$

L: $0 = -3a_2 + b_2 + c_2 + 1$ or $c_2 = -1$

Therefore $\pi_2 = \frac{k}{D}$

Likewise, $\pi_3 = \rho^{a3} V^{b3} D^{c3} f$

$$[M^\circ L^\circ T^\circ] = \left[\frac{M}{L^3}\right]^{a3} \left[\frac{L}{T}\right]^{b3} [L]^{c3}[M^\circ L^\circ T^\circ]$$

Equation exponents

For

M: $0 = a_3$ or $a_3 = 0$

T: $0 = -b_3 + 0$ or $b_3 = 0$

L: $0 = -3a_3 + b_3 + c_3 + 0$ or $c_3 = 0$

It may be noted that the non-dimensional variable, such as f, itself becomes the π- term

Thus the functional relationship becomes

$$F\left[\frac{\mu}{\rho VD}, \frac{K}{D}, f\right] = \text{constant}$$

$$f = \emptyset\left[\frac{\rho VD}{\mu}, \frac{K}{D}\right] \rightarrow f = \theta\left[IR, \frac{K}{D}\right]$$

∴ Friction factor depends upon IR and $\frac{K}{D}$ (roughness ratio)

H.W:

1) The discharge Q OF a centrifugal pump depends upon the mass density of fluid (ρ), the speed of the pump (N), the diameter of the impeller (D), the monometric head (Hm) and the viscosity of fluid (u) show that:

$$Q = ND^3 \emptyset\left(\frac{gH}{N^2D^2}, \frac{\mu}{\rho ND^2}\right)$$

2) Show that the velocity through a circular orifice is given by:

$V = \sqrt{2gH}\, f\left(\frac{D}{H}, \frac{\mu}{\rho VH}\right)$ H= head causing flow

D= dia of orifice

Example:

If the drag force on the ship (Fd) depended on length of ship (L), Viscosity of the liquid (M), Velocity of ship (V) and (ρ), (g), Explain the terms of dimensionless parameter.

Sol. By Rayleigh's method

$$Fd = F_1(L, \rho, \mu, V, g)$$

$$Fd = KL^a \rho^b \mu^c V^d g^e$$

$$\frac{ML}{T^2} = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{LT}\right)^c \left(\frac{L}{T}\right)^d \left(\frac{L}{T^2}\right)^e$$

M: 1=b+c

L: a-3b-c+d+e

T: -2=-c-d-ze

Solve the equation in terms C and e:

a=z+e-c, b=1-c, c=c, d=z-c-ze, e=e

$$\therefore Fd = KL^{2+e-c} \rho^{1-c} \mu^c V^{2-c-ze} g^e$$

$$Fd = K\left(\frac{V^2}{gL}\right)^{-e} \left(\frac{VLP}{\mu}\right)^{-C} \rho L^2 V^2 \qquad \because Fr = \frac{V}{\sqrt{gL}}$$

$$\therefore Fd = f_2(Fr, Re)\rho L^2 V^2 \qquad Re = \frac{VLP}{\mu}$$

$$\therefore \frac{Fd}{\rho L^2 V^2} = f2(Fr, Re)$$

Note: solve the problem by π- term

Buckingham method

$$F_1(fd, L, \rho, \mu, V, g) = 0$$

$$6 - 3 = 3 \quad \text{repeating var.} = \rho, V, L$$

$$F_2(\pi_1, \pi_2, \pi_3) = 0$$

$$\pi_1 = \rho^a V^b L^c fd$$

In term

$$MLT \rightarrow \pi_1 = \left(\frac{fd}{\rho L^2 V^2}\right)^2$$

$$\pi_2 = \mu \rho L^2 V^2 \rightarrow \pi_2 = \frac{VLP}{\mu}$$

$$\pi_g = g \rho^a L^b V^c \rightarrow \pi_3 = \frac{V}{\sqrt{gL}}$$

$$\therefore F_2 = \left(\frac{Fd}{\rho L^2 V^2}, Re, Fr\right) = 0$$

$$\frac{Fd}{\rho L^2 V^2} = f(Re, Fr)$$

H.W:

A capillary rise (h) of a liquid in tube varies with tube diameter (d) gravity (g), fluid densityρ, and surface tension (σ) and the contact angle (θ). Find a dimensionless statement of this relation.

Comparison of Rayleigh and Buckingham method:

In Rayleigh method, a services of terms is terms is used. The variables are expressed in exponential forms. The values of the exponents are determined from the principle of dimensional homogeneity.
The method became tedious when a large number of variables affected the phenomenon.
In Buckingham method, there is no series of terms, the expression for π- term are simple and straight forward, irrespective of the number of variables involved. In flow phenomena in which the number of variables involved, in flow phenomena in which the number of variables is large, this method is more convenient than the Rayleigh method. It may, however, be noted that both the methods are intrinsically the same. Both methods are based on the principle of dimensional homogeneity. The algebraic steps in the two methods are essentially the same.

Dimensional analysis of a General Flow problem

Let us now derive a general functional relationship of a flow phenomenon which depends upon the following variables:
Massdesity (ρ), Viscosity (μ), Acceleration due to gravity (g), bulk modulus (k), pressure (p), velocity (v), surface teasion(σ) and two characteristic length (L) and (D).
The functional relationship can be written as:
f ($\rho, \mu, g, K, P, V, \sigma, L, D$)= const.
n=q and m=3 therefore
F($\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6$) = const.
Takingρ, L and V as repeating variables.

$$\pi_1 = \rho^{a1} L^{1} V^{c1} M$$

$$[M^\circ L^\circ T^\circ] = \left[\frac{M}{L^3}\right]^{a1} [L]^{b2} \left[\frac{L}{T}\right]^{c1} \left[\frac{M}{LT}\right]$$

a1=-1, b1=-1 and c1=-1

$$\therefore \pi_1 = \frac{\mu}{\rho L V}$$

$$\therefore \pi_1 = \frac{\rho L V}{\mu} = \text{IR (Reynolds number)} *$$

Likewise

$$\pi_2 = \rho^{a2} L^{b2} V^{c2} g \rightarrow a2 = 0, b2 = 1, c2 = -1$$

$$\pi_2 = \frac{Lg}{V^2} \rightarrow \pi_2 = \frac{V}{\sqrt{gL}} = \text{Fr (froude number)}$$

Likewise

$$\pi_3 = \rho^{a3} L^{b3} V^{c3} K$$

$$[M^\circ L^\circ T^\circ] = \left[\frac{M}{L^3}\right]^{a3} [L]^{b3} \left[\frac{L}{T}\right]^{c3} \left[\frac{M}{LT^2}\right]$$

a2 =-1 , b2=0 and c2-=-2

$$\pi_3 = \frac{K}{\rho V^2} \rightarrow \pi_3 = \frac{V}{\sqrt{K/\rho}} = \text{IM (Mach Number)} *$$

= Mn

Likewise

$$\pi_4 = \rho^{au} L^{bu} V^{cu} P \rightarrow au = -1, bu = 0, cu = -2$$

$$\pi_4 = \frac{P}{\rho V^2} \rightarrow \pi_4 = \frac{V}{\sqrt{P/\rho}} = \text{Eules number } *$$

= En

Further

$$\pi_5 = \rho^{a5} L^{b5} V^{c5} \sigma \qquad \left[\sigma = \frac{M}{L_2}\right]$$

$$\left.\begin{array}{l} a_2 = -1 \\ \\ c_2 = -2 \end{array}\right\} \qquad b_2 = -1 \rightarrow \pi_5 = \frac{\sigma}{\rho V^2 L}$$

$$\pi_5 = \frac{V}{\sqrt{\sigma/\rho L}}$$

Likewise $\pi_6 = \rho^{a6} L^{b6} V^{c6} D$

a6=0 , b6=-1 and c6=0→ $\pi_6 = \frac{D}{L}$

Thus; therefore; the general functional relationship can be written as:

$$F\left(\frac{\rho LV}{\mu}, \frac{V}{\sqrt{gL}}, \frac{V}{\sqrt{\frac{K}{\rho}}}, \frac{V}{\sqrt{\frac{P}{\rho}}}, \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}, \frac{D}{L}\right) = c$$

$$F\left(Re, Fr, Mn, En, Wn, \frac{D}{L}\right) = c$$

∴ The number of variable always of π- term will be less than 6 if there is one more characteristic length such as (B), which effect the phenomenon, the ratio $\left(\frac{B}{L}\right)$ will be effect as (π- term), and it will be (Z) π-term.

Hydraulic Similitude

Model analysis is frequently used to study the flow phenomenon which are complex and are not amenable to mathematical analysis.

In model analysis, investigations are made on a model which is similar to the full-size structure known as prototype. Model study of proposed hydraulic structures and machines are generally under taken to predict the behavior of the prototypes.

The civil Engineering model such as dams, spillways, canals to know the working of full size structures.

It is not essential that the model should always be smaller than its prototype. Sometimes a full size a full size model or even a model larger than the prototype is used.

In fact, a model is a mechanical analog of the prototype. The advantages of model testing is for most economical and for safe design in case of its failure.

Types of hydraulic similarity:

To complete working and behavior of the prototype, from its model, there should be a complete similarity is known as hydraulic similitude or hydraulic similarity.

Following three types of hydraulic similarities:

(a) Geometric Similarity

The model and its prototype are geometrically similar when they are identical in shape but differ only in size.
Thus the scale ratio (Lr) is:

$$L_r = \frac{L_m}{L_p} = \frac{B_m}{B_p} = \frac{t_m}{t_p}$$

Similarly, area ratio and volume ratio:

$$A_r = \frac{L_m . B_m}{L_p . B_p} = L_r^3$$

(b) Kinematic Similarity

Kinematic Similarity if the prototype and model have identical motions or velocities, on the other words, the Kinematic Similarity is said to exist between the model and the prototype, if the ratio of the corresponding velocities at corresponding points are equal.
If V1 , V2 the velocity of liquid at points 1 and 2.
If the Kinematic Similarity exists, then the velocity ratio of the prototype to the model (Vr):

$$V_r = \frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = \text{velocity ratio}$$

$M = \text{model}$

$P = \text{prototype}$

$$a_r = \frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} = \text{acceleration ratio}$$

(c) Dynamic Similarity

Dynamic Similarity means the similarity of forces. The corresponding points of prototype and model are equal.

$$F_r = \frac{(F_1)_m}{(F_1)_p} = \frac{(F_2)_m}{(F_2)_p}$$

The forces mean: gravity force (Fg), pressure force (Fp), viscosity force (Fv), elasticity force (Fe) and the surface tension force (Fs):

$$\frac{(F_g)_m}{(F_g)_p} = \frac{(F_p)_m}{(F_p)_p} = \frac{(F_r)_m}{(F_r)_p} = \frac{(F_e)_m}{(F_e)_p} = \frac{(F_s)_m}{(F_s)_p}$$

The techniques of hydraulic model involve the selection of suitable scale, operation of hydraulic model, and correct prediction.

Example:

A spillway design is to be studied by means of a geometrically similar model constructed to a scale of 1:20. Neglecting viscous and surface tension effects, calculate:
(a) The discharge in the model corresponding to a discharge of 1500 cumecs in the prototype.
(b) The velocity in the prototype corresponding to a velocity of Zm/sec in the model.
(c) Theheight of the hydraulic jump corresponding to a jump of 5 cm in the model.
(d) The energy dissipated in the prototype corresponding to 0.25 H.P.in the model.

Solution:

a) $\frac{Q_m}{Q_p} = \frac{(VL^2)_m}{(VL^2)_p}$ $\because$ $(Fr)_m = (Fr)_p \rightarrow \frac{V_m}{\sqrt{L_m g_m}} = \frac{V_p}{\sqrt{L_p g_p}} \rightarrow V_r = \sqrt{L_r g_r}$

$$\therefore \frac{V_m}{V_p} = L_r^{\frac{1}{2}}$$

$$\therefore \frac{Q_m}{Q_p} = L_r^{\frac{5}{2}} = \frac{1}{(20)^{5/2}} = \frac{1}{1790} \rightarrow \therefore Q_m = \frac{1500}{1790} = 0.838 m^3/sec$$

b) $\frac{V_m}{V_p} = L_r^{\frac{1}{2}} = \frac{1}{(20)^{1/2}} = \frac{1}{4.47} \rightarrow \therefore V_p = 4.47 \times V_m = 4.47 \times 2 = 8.94\ {}^m/sec$

c) Height of hydraulic jump, from geometric similarity,

$$\frac{J_m}{J_p} = L_r = \frac{1}{20}$$

Jp= 20Jm= 20*0.05=1m

d) energy/sec = δQH

Therefore, $\frac{E_m}{E_p} = \frac{\delta_m Q_m H_m}{\delta_p Q_p H_p}$

$$\frac{(H.P)_m}{(H.P)_p} = \frac{Q_m}{Q_p} \cdot \frac{H_m}{H_p}$$

$$= \frac{1}{1790} \times \frac{1}{20} = \frac{1}{35800}$$

$\therefore\ (H.P)_P = 35800 \times 0.25 = 8950\ h.p$

H.W:

The mean velocities in the river and its model are respectively 3m/sec and 1m/sec. if the slopes in the river and model are 1:2500 and 1:200 respectively. Calculate the length ratio.

Classification of models:

All the hydraulic models may be broadly classified into the following two types:

1- Undistorted model, and

2- Distorted model.

Undistorted models: which is geometrically similar to the prototype (i.e. havinggeometric similarly in length, breadth, and height and head of water etc.).The protection of undistorted model is comparatively easy and some of the results, obtained from the models, can be easy transferred to the prototype as the basic condition (of geometric similarity) is satisfied.

Example:

The discharge over a model, which is the corresponding discharge in the prototype?

Solution:

Model scale

$$\frac{1}{S} = \frac{1}{100} \text{ or } s = 100$$

$\frac{1}{S}$ = scale ratio of the model to prototype = L_r

q in model = 1.5 l/sec

Q= over prototype

$$Q = q \times S^{5/2}$$

$$= 1.5 \times 100^{5/2}$$

$$= 150 m^3/sec$$

$$q = a \times v \rightarrow Q = A \times V$$

$$\frac{q}{Q} = \frac{a \times V_m}{A \times V_p} = \frac{a}{A} \times \frac{V_m}{V_p} = \frac{1}{S^2} \times \frac{1}{\sqrt{S}} = \frac{1}{S^{5/2}}$$

Distorted models:

A model is said to be perfect or undistorted when it is perfectly similar to its prototype. In undistorted models because the conditions of similitude are completely satisfied, the results of model investigations can be directly used for the prototype.

A model is said to be distorted when one or more characteristics of the model are not identical with their corresponding characteristics in the prototype. In distorted model, the results obtained are primarily qualitative. However, in certain cases, by applying the laws of distortion, the results obtained from model investigation can be transferred to prototype quantitatively as well.

Distortion of the model can be due to any one of the following models:

(1) Geometric distortion: This type of distortion occurs when the scale ratio is not constant. Usually, the vertical model scale is exaggerated.

(2) Distortion of configuration: In this type of distortion, the configuration of the model is different from the configuration of prototype, such as the slope of model is different from the slope of prototype, although the geometric similarity exists.

(3) Material distortion: in this case, the material used in model used in the model is not of the required grade.

(4) Distortion of hydraulic quantities: in such a distortion, there is a distortion of some hydraulic quantity such as discharge, velocity, and time. It means that the required scale ratio of that quantity is not maintained .Examples of distorted model: river model in movable bed models, surface roughness.

The disadvantages of distorted model is the pressure distribution, velocity distribution and wave patterns are not correctly reproduced and it is difficult to prepare distorted models of some structure such as river bends, earth cuts and dikes.

The following table represents a scale ratio for distorted models for some quantity:

Quantity	*Symbol*	*Scale ratio*	
		Reynold's law	*Froude's law*
Length, Breadth, height	L_r , B_r , H_r	*Lr, Dr*	*Lr, Dr*
Horizontal Area	*Ahr*	L_r^2	L_r^2
Vertical Area	*Avr*	*LrDr*	*LrDr*
Slope	*Sr*	$\frac{Dr}{Lr}$	$\frac{Dr}{Lr}$
Velocity	*Vr*	$\frac{\mu r}{\rho r Dr}$	$D^{1/2} g_r^{1/2}$
Discharge	*Qr*	$Lr\frac{\mu r}{\rho r}$	$LrD^{3/2} g_r^{1/2}$

Example:

A distorted model of a rigid bed river has a horizontal scale ratio of 1:1000 and a vertical scale ratio of 1 in 100.What is the flow in the model corresponding to a discharge of 5000 cumecs in the river? Also calculate the value of manning roughness (N) for the model if that for the river is 0.03.If the flood peak requires 1 hour traveling through 100m in the model, how much time would the flood peak take to travel the corresponding distance in the river?

Solution:

According the Froude: $V_r = \sqrt{Dr} = \sqrt{\frac{1}{100}} = \frac{1}{10}$

$$Q_r = A_{vr} V_r = \frac{1}{100 \times 1000} \times \frac{1}{10} = \frac{1}{10^6}$$

$$Q_m = \frac{Q_p}{10^6} = \frac{5000}{10^6} = \frac{1}{200} \text{cumecs} = 5L/SEC$$

$N_r = \frac{D_r^{2/3}}{L_r^{1/2}}$From manning's equation to prototype and model for

$R_r \approx D_r$ for river

$$N_r = \frac{\left(\frac{1}{100}\right)^{2/3}}{\left(\frac{1}{1000}\right)^{1/2}} = 1.47$$

$$N_m = N_p \times 1.47 = 0.03 \times 1.47 = 0.0441$$

Time ratio: $\frac{L_r}{D_r^{\frac{1}{2}}} = \frac{\frac{1}{1000}}{\sqrt{\frac{1}{100}}} = \frac{1}{100}$

Time in prototype=$Tm \times 100 = 1 \times 100 = 100hrs$