Chapter 13: Lateral Earth Pressure

Retaining structures such as retaining walls, basement walls, and bulkheads commonly are encountered in foundation engineering as they support slopes of earth masses. Proper design and construction of these structures require a thorough knowledge of the lateral forces that act between the retaining structures and the soil masses being retained. These lateral forces are caused by lateral earth pressure. This chapter is devoted to the study of the earth pressure theory.

13.1 At-Rest, Active, and Passive Pressures

Consider a mass of soil shown in Figure 13.1a. The mass is bounded by a frictionless wall of height *AB*. A soil element located at a depth z is subjected to a vertical effective pressure, σ'_o, and a horizontal effective pressure, σ'_h. There are no shear stresses on the vertical and horizontal planes of the soil element. Let the ratio of σ'_h, to σ'_o, as a nondimensional quantity *K*, or

$$K = \frac{\sigma'_h}{\sigma'_o} \qquad (13.1)$$

Now, three possible cases may arise concerning the retaining wall: and they are described

Case 1 If the wall *AB* is static—that is, if it does not move either to the right or to the left of its initial position—the soil mass will be in a state of static *equilibrium*. In that case, σ'_h is referred to as the *at-rest earth pressure*, or

$$K = K_o = \frac{\sigma'_h}{\sigma'_o} \qquad (13.2)$$

where K_o= at-rest earth pressure coefficient.

Case 2 If the frictionless wall rotates sufficiently about its bottom to a position of $A'B$ (Figure 13.1b), then a triangular soil mass ABC' adjacent to the wall will reach a state of

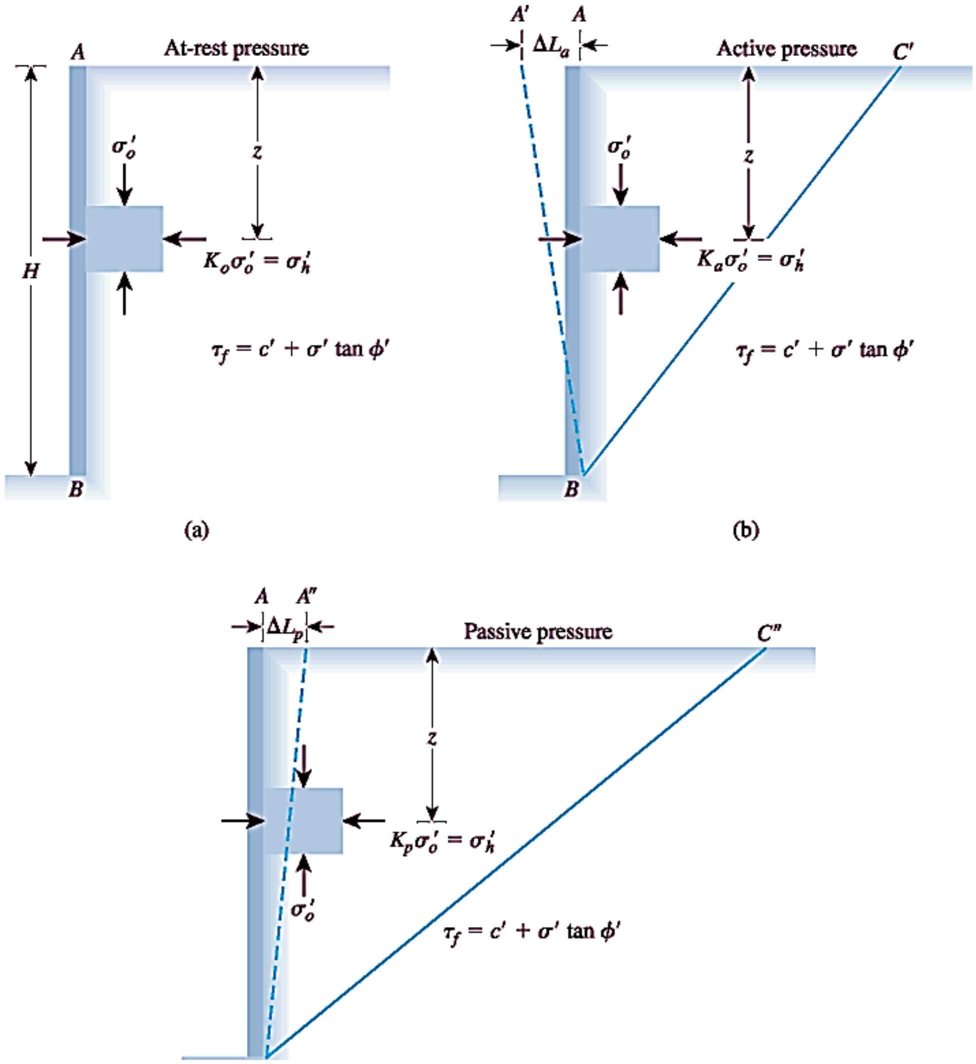


Figure 13.1 Definition of at-rest, active, and passive pressures (Note: Wall AB is frictionless)

plastic equilibrium and will fail sliding down the plane BC'. At this time, the horizontal effective stress, $\sigma'_h = \sigma'_a$, will be referred to as *active pressure*. Now,

$$K = K_a = \frac{\sigma'_h}{\sigma'_o} = \frac{\sigma'_a}{\sigma'_o} \qquad (13.3)$$

where K_a *= active earth pressure coefficient.*

Case 3 If the frictionless wall rotates sufficiently about its bottom to a position $A''B$ (Figure 13.1c), then a triangular soil mass ABC'' will reach a state of *plastic equilibrium* and will fail sliding upward along the plane BC''. The horizontal effective stress at this time will be $\sigma'_h = \sigma'_p$, the so-called passive pressure. In this case,

$$K = K_p = \frac{\sigma'_h}{\sigma'_o} = \frac{\sigma'_p}{\sigma'_o} \tag{13.4}$$

13.2 Earth Pressure at-Rest

The fundamental concept of earth pressure at rest was discussed in the preceding section. In order to define the earth pressure coefficient K_o at rest, we refer to Figure 13.2. Which shows a wall *AB* retaining a dry soil with a unit weight of γ. The wall is static. At a depth z,

Vertical effective stress = $\sigma'_o = \gamma z$

Horizontal effective stress = $\sigma'_h = K_o \gamma z$

So, $$K_o = \frac{\sigma'_h}{\sigma'_o} = at\ rest\ earth\ pressure\ cofficient$$

For coarse-grained soils, the coefficient of earth pressure at rest can be estimated by using the empirical relationship (Jaky, 1944)

$$K_o = 1 - sin\emptyset' \tag{13.5}$$

Where $\emptyset'$ *= drained friction angle.*

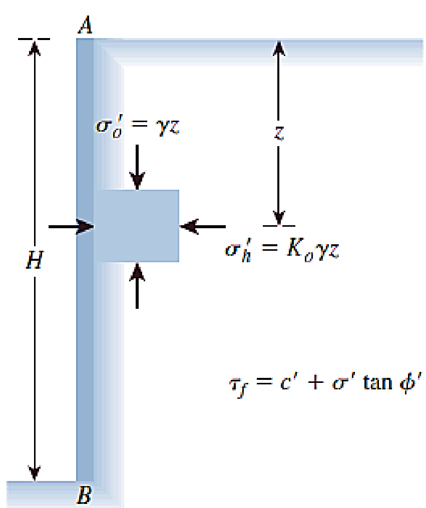


Figure 13.2 Earth pressure at rest

Mayne and Kulhawy (1982), after evaluating 171 soils, recommended a modification to Eq. (13.5). Or

$$k_o = (1 - sin\emptyset')(OCR)^{sin\emptyset'} \qquad (13.6)$$

where

$$OCR = overconsolidation\ ration$$

$$= \frac{preconsolidation\ pressure,\ \sigma_c'}{present\ effective\ overburden\ pressure,\ \sigma_o'}$$

Equation (13.6) is valid for soils ranging from clay to gravel.
For fine-grained, normally consolidated soils, Massarsch (1979) suggested the following equation for K_o:

$$K_o = 0.44 + 0.42\left[\frac{PI(\%)}{100}\right] \qquad (13.7)$$

For overconsolidated clays, the coefficient of earth pressure at rest can be approximated as

$$K_{o(overconsolidation)} = K_{o(normally\ consolidated)}\sqrt{OCR} \qquad (13.8)$$

Figure 13.3 shows the distribution of lateral earth pressure at rest on a wall of height H retaining a dry soil having a unit weight of γ. The total force per unit length of the wall, P_o, is equal to the area of the pressure diagram, so

$$P_o = \frac{1}{2}K_o\gamma H^2 \qquad (13.9)$$

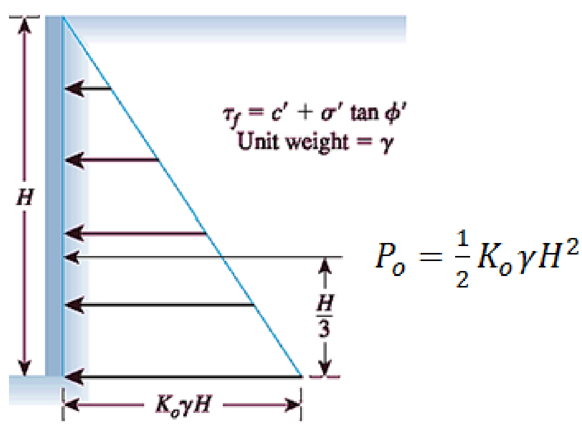
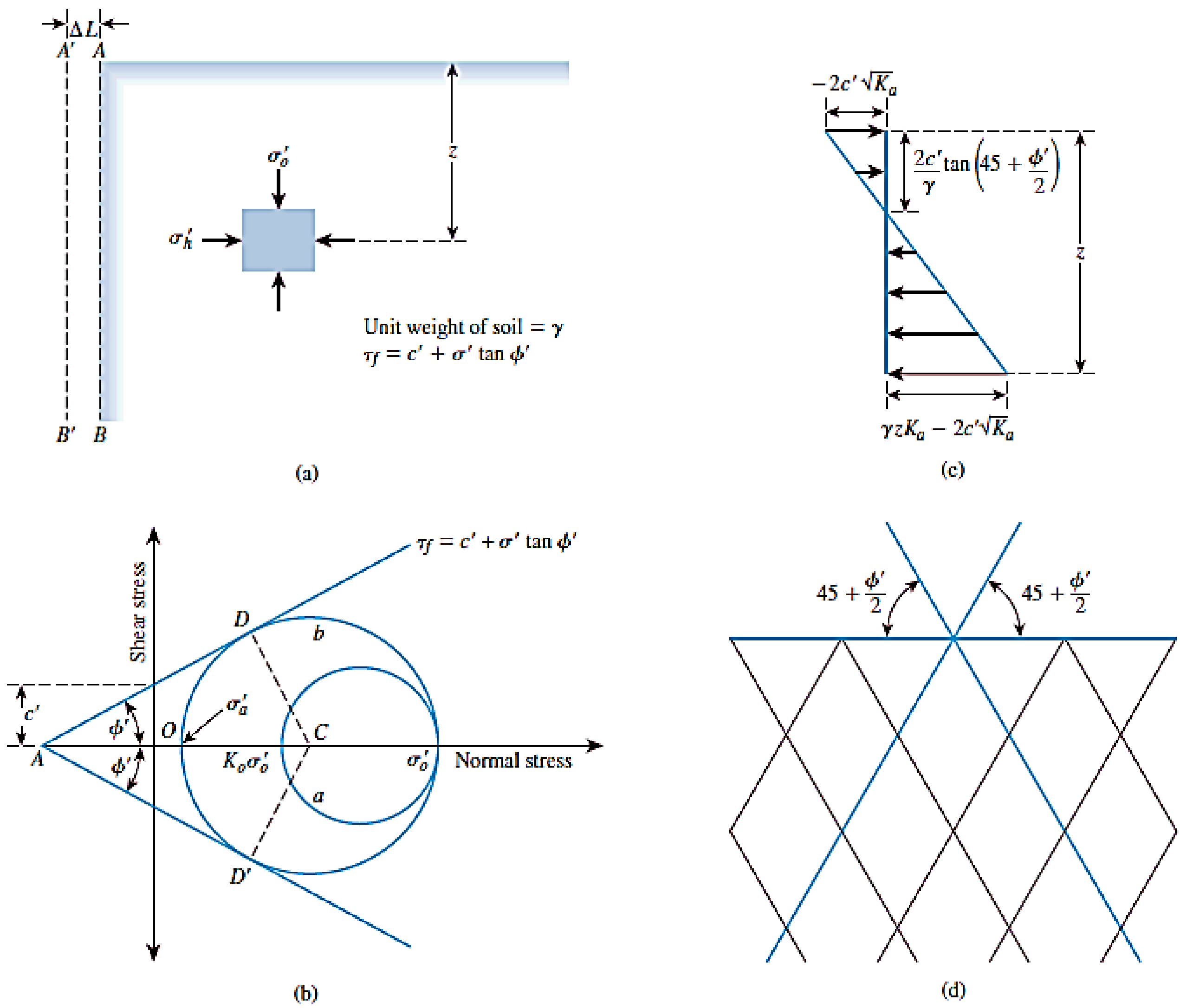


Figure 13.3 Distribution of lateral earth pressure at-rest on a wall

13.3 Rankine's Theory of Active Pressure

The phrase *plastic equilibrium* in soil refers to the condition where every point in a soil mass is on the verge of failure. Rankine (1857) investigated the stress conditions in soil at a state of plastic equilibrium. In this section and in Section 13.4, we deal with Rankine's theory of earth pressure.

Figure 13.4a shows a soil mass that is bounded by a frictionless wall, *AB*, that extends to an infinite depth. The vertical and horizontal effective principal stresses on a soil element at a depth z are σ'_o and σ'_h respectively. As we saw in Section 13.2, if the wall *AB* is not allowed to move, then $\sigma'_h = K_o\sigma'_o$. The stress condition in the soil element can be



13.4 Rankine's active earth Pressure

represented by the Mohr's circle *a* in Figure 13.4b. However, if the wall *AB* is allowed to *move away from the soil mass* gradually, the horizontal principal stress will decrease. Ultimately a state will be reached when the stress condition in the soil element can be represented by the Mohr's circle *b*, the state of plastic equilibrium and failure of the soil will occur. This situation represents *Rankine's active state*, and the effective pressure σ'_a on the vertical plane (which is a principal plane) is Rankine's *active earth pressure*. We next derive σ'_a in terms of $\gamma, z, c', and\ \emptyset'$ from Figure 13.4b

$$sin\emptyset' = \frac{CD}{AC} = \frac{CD}{AO+OC}$$

But

$$CD = radius\ of\ the\ failure\ circle = \frac{\sigma'_o - \sigma'_a}{2}$$

$$AO = c'cot\emptyset'$$

and

$$OC = \frac{\sigma'_o + \sigma'_a}{2}$$

So,

$$sin\emptyset' = \frac{\frac{\sigma'_o - \sigma'_a}{2}}{c'cot\emptyset' + \frac{\sigma'_o + \sigma'_a}{2}}$$

or

$$c'cos\emptyset' + \frac{\sigma'_o + \sigma'_a}{2} sin\emptyset' = \frac{\sigma'_o - \sigma'_a}{2}$$

or

$$\sigma'_a = \sigma'_o \frac{1-sin\emptyset'}{1+sin\emptyset'} - 2c' \frac{cos\emptyset'}{1+sin\emptyset'} \qquad (13.10)$$

But

$$\sigma'_o = vertical\ effective\ overburden\ pressure = \gamma z$$

$$\frac{1-sin\emptyset'}{1+sin\emptyset'} = tan^2\left(45 - \frac{\emptyset'}{2}\right)$$

and

$$\frac{cos\emptyset'}{1+sin\emptyset'} = \tan\left(45 - \frac{\emptyset'}{2}\right)$$

Substituting the preceding values into Eq. (13.10), we get

$$\sigma'_a = \gamma z tan^2\left(45 - \frac{\emptyset'}{2}\right) - 2c' \tan\,(45 - \frac{\emptyset'}{2}) \qquad (13.11)$$

The variation of σ'_a with depth is shown in Figure 13.4c. For cohesionless soils, $c' = 0$ and

$$\sigma'_a = \sigma'_o tan^2\left(45 - \frac{\emptyset'}{2}\right) \qquad (13.12)$$

The ratio of σ'_a to σ'_o is called the *coefficient of Rankine's active earth pressure* and is given by

$$K_a = \frac{\sigma'_a}{\sigma'_o} = tan^2(45 - \frac{\emptyset'}{2}) \qquad (13.13)$$

Again, from Figure 13.4b we can see that the failure planes in the soil make $\pm(45 + \frac{\emptyset'}{2})$ degree angles with the direction of the major principal plane that is, the horizontal. These are called potential *slip planes* and are shown in Figure 13.4d.

It is important to realize that a similar equation for σ_a could be derived based on the total stress shear strength parameters that is, $\tau_f = c + \sigma tan\emptyset$. For this case,

$$\sigma_a = \gamma z tan^2\left(45 - \frac{\emptyset}{2}\right) - 2c\; tan(45 - \frac{\emptyset}{2}) \qquad (13.14)$$

13.4 Theory of Rankine's Passive Pressure

Rankine's passive state can be explained with the aid of Figure 13.5. *AB* is a frictionless wall that extends to an infinite depth (Figure 13.5a). The initial stress condition on a soil element is represented by the Mohr's circle *a* in Figure 13.5b. If the wall gradually is *pushed into the soil mass*, the effective principal stress σ'_h will increase. Ultimately, the wall will reach a situation where the stress condition for the soil element can be expressed by the Mohr's circle *b*. At this time, failure of the soil will occur. This situation is referred to as *Rankine's passive state*. The lateral earth pressure σ'_p, which is the major principal stress, is called *Rankine's passive earth pressure*. From Figure 13.5b, it can be shown that

$$\sigma'_p = \sigma'_o tan^2\left(45 + \frac{\emptyset'}{2}\right) + 2c'\tan\left(45 + \frac{\emptyset'}{2}\right)$$

$$= \gamma z tan^2\left(45 + \frac{\emptyset'}{2}\right) + 2c'\tan\left(45 + \frac{\emptyset'}{2}\right) \quad (13.15)$$

The derivation is similar to that for Rankine's active state.

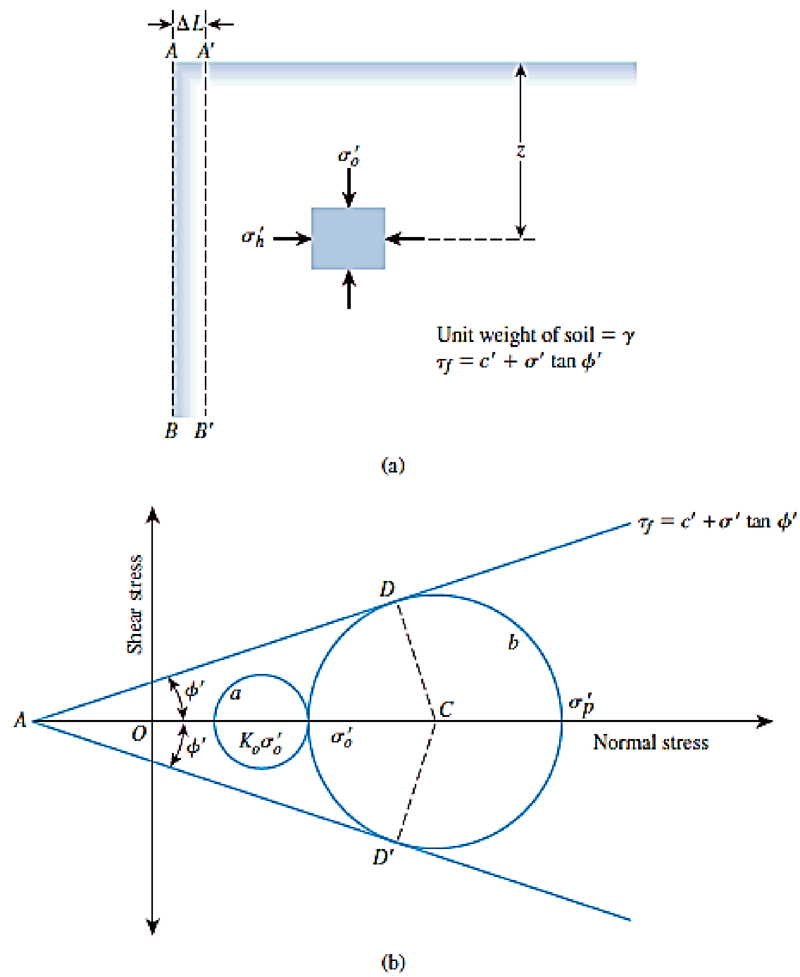
Figure 13.5c shows the variation of passive pressure with depth. For cohesionless soils ($c' = 0$),

$$\sigma'_p = \sigma'_o\, tan^2(45 + \frac{\emptyset'}{2})$$

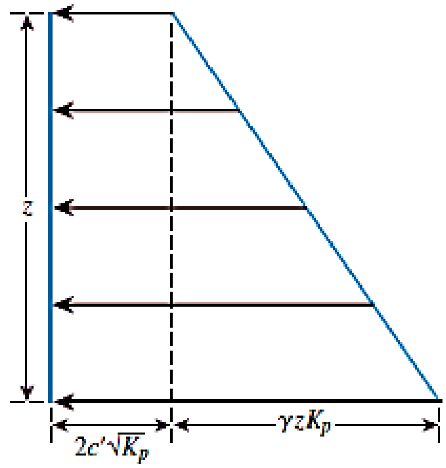
or

$$\frac{\sigma'_p}{\sigma'_o} = K_p = tan^2(45 + \frac{\emptyset'}{2}) \quad (13.16)$$

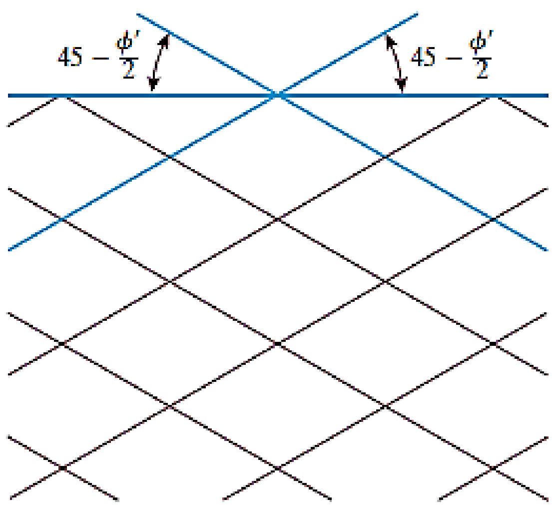
K_p (the ratio of effective stresses) in the preceding equation is referred to as the coefficient of *Rankine's passive earth pressure*.



13.5 Rankine's passive earth Pressure



(c)



(d)

13.5(continued)

Example 13.1

An 6 m high retaining wall is shown in Figure 13.6a. Determine

a. Rankine active force per unit length of the wall and the location of the resultant.
b. Rankine passive force per unit length of the wall and the location of the resultant.

Solution

Part a

Because $c' = 0$, to determine the active force we can use from Eq. (13.13).

$$\sigma'_a = K_a\sigma'_o = K_a\gamma z$$

$$k_a = \frac{1-sin\emptyset'}{1+sin\emptyset'} = \frac{1-sin36}{1+sin36} = 0.26$$

At z=0, $\sigma'_a = 0. at\ z = 6m,$

$$\sigma'_a = (0.26)(16)(6) = 24.96\ kN/m^2$$

The pressure-distribution diagram is shown in Figure 13.6b. The active force per unit length of the wall is as follows:

$$p_a = \frac{1}{2}(6)(24.96) = 74.88\ km/m$$

Also,

$$\bar{z} = 2m$$

Part b

To determine the passive force, we are given that $c' = 0$. So, from Eq. (13.16),

$$\sigma'_p = K_p\sigma'_o = K_p\gamma z$$

$$k_p = \frac{1+sin\emptyset'}{1-sin\emptyset'} = \frac{1+sin36}{1-sin36} = 3.85$$

At z=0, $\sigma'_p = 0, at\ z = 6m,$

$$\sigma'_p = (3.85)(16)(6) = 369.6\ kN/m^2$$

The pressure-distribution diagram is shown in Figure 13.6c. The passive force unit length of the wall is as follows.

$$p_p = \frac{1}{2}(6)(369.6) = 1108.8\ km/m$$

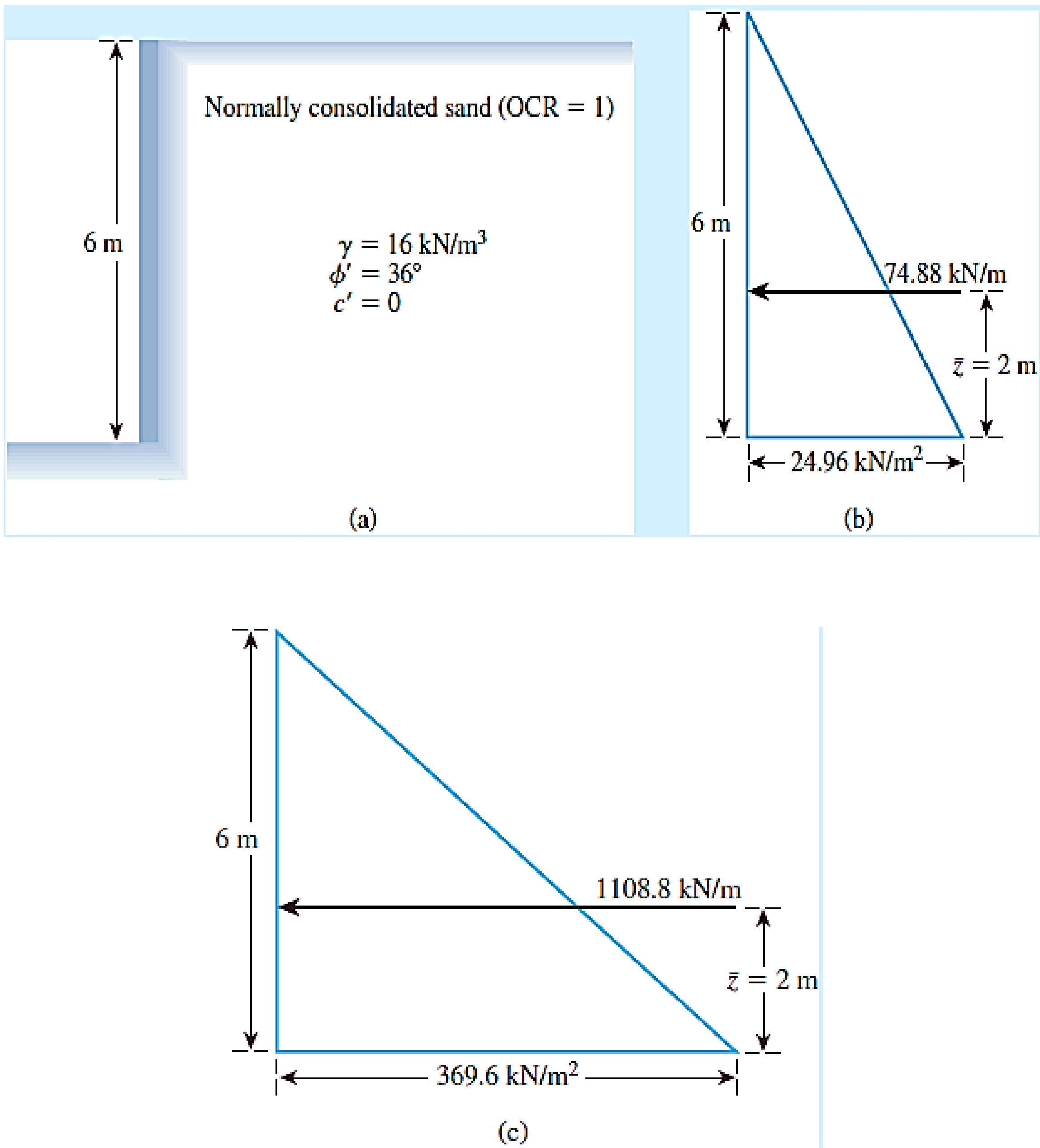


Figure 13.6 Diagrams of determining active and passive forces

Example 13.2

For the retaining wall shown in Figure 13.7a, determine the force per unit length of the wall for Rankine's active state. Also find the location of the resultant.

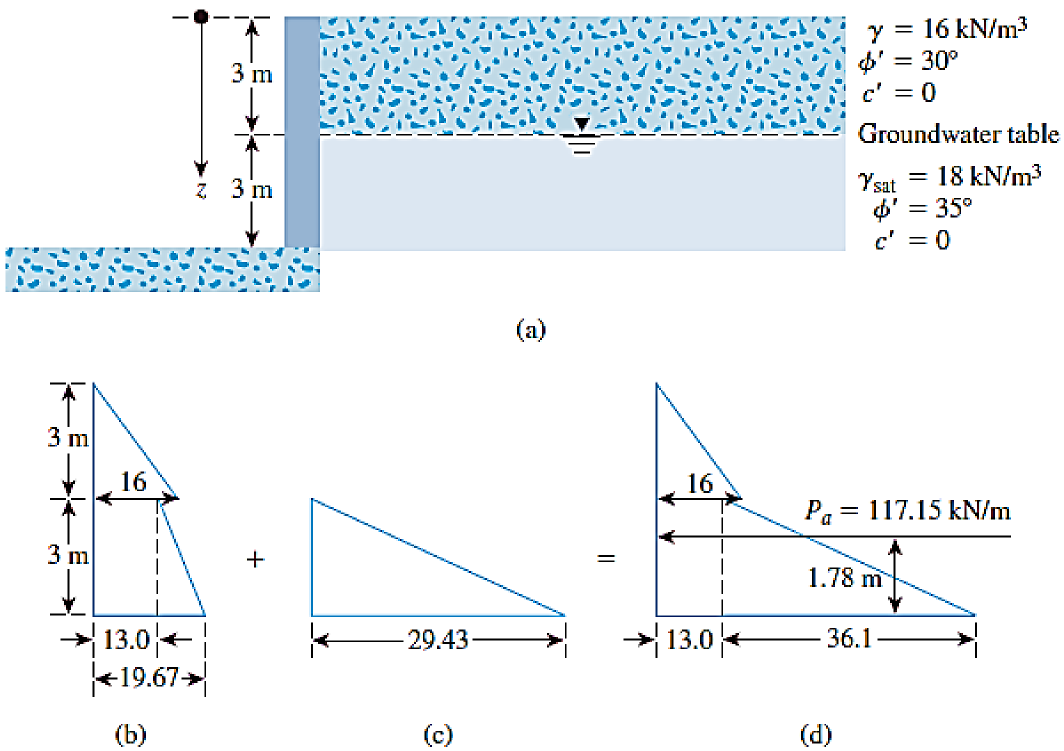


Figure 13.7 Retaining wall and pressure diagrams for determining Rankine's active earth pressure. (Note: The units of pressure in (b), (c), and (d) are kN/m^2)

Solution

Given that $c' = 0$, we known that $\sigma'_a = \mathrm{k_a}\sigma'_o$. For the upper layer of the soil, Rankine's active earth pressure coefficient is

$$k_a = K_{a(1)} = \frac{1-sin30^o}{1+sin30^o} = \frac{1}{3}$$

For the lower layer,

$$k_a = K_{a(2)} = \frac{1-sin35^o}{1+sin35^o} = 0.271$$

At z=0, $\sigma'_o = 0$. At $z = 3\ m\ (just\ inside\ the\ bottom\ of\ the\ upper\ layer)$,

$\sigma'_a = 3 \times 16 = 48\ kN/m^2$. So

$\sigma'_a = K_{a(1)}\sigma'_o = \frac{1}{3} \times 48 = 16\ kN/m^2$

Again, at z=3 m (in the lower layer), $\sigma'_o = 3 \times 16 = 48\frac{kN}{m^2}$, and

$\sigma'_a = K_{a(2)}\sigma'_o = 0.271 \times 48 = 13.0\ kN/m^2$

At $z = 6\ m$,

$\sigma'_o = 3 \times 16 + 3(18 - 9.81) = 72.57\ kN/m^2$

and

$\sigma'_a = K_{a(2)}\sigma'_o = 0.271 \times 72.57 = 19.67\ kN/m^2$

The variation of σ'_a with depth is shown in Figure 13.7b.

The lateral pressures due to the pore water are as follows.

$At\ z = 0$: $u = 0$
$At\ z = 3m$: $u = 0$
$At\ z = 6m$: $u = 3 \times \gamma_w = 3 \times 9.81 = 29.43\ kN/m^2$

The variation of u with depth is shown in Figure 13.7c, and that for σ_a (total active pressure) is shown in Figure 13.7d. Thus,

$$P_a = \left(\frac{1}{2}\right)(3)(16) + 3(13.0) + \left(\frac{1}{2}\right)(3)(36.1) = 24 + 39.0 + 54.15 = 117.15kN/m$$

The location of the resultant can be found by taking the moment about the bottom of the wall:

$$\bar{z} = \frac{24\left(3+\frac{3}{3}\right)+39.0\left(\frac{3}{2}\right)+54.15\left(\frac{3}{3}\right)}{117.15} = 1.78m$$

Example 13.3

A retaining wall that has a soft, saturated clay backfill is shown in Figure 13.8a. For the undrained condition ($\emptyset = 0$) of the backfill, determine

a. Maximum depth of the tensile crack
b. P_a before the tensile crack occurs
c. P_a after the tensile crack occurs

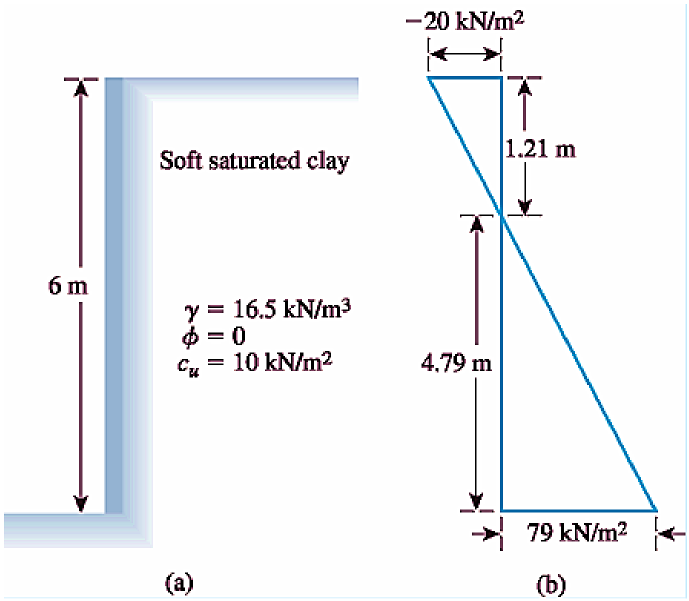


Figure 13.8 Rankine's active earth pressure due to a soft, saturated clay backfill

Solution

For $\emptyset = 0, K_a = tan^2 45 = 1$ and $c = c_u$. From eq. (13.14),

$$\sigma_a = \gamma z - 2c_u$$

At z=0

$$\sigma_a = -2c_u = -(2)(10) = -20\ kN/m^2$$

At z=6 m

$$\sigma_a = (16.5)(6) - (2)(10) = 79\ kN/m^2$$

The variation of σ_a with depth is shown in Figure 13.8b.

Part a

From the equation; $\sigma_a = \gamma z k_a - 2c\sqrt{k_a}$

For cohesive soils ($k_a = 1$), the depth of tensile crack equals:

$$z_o = \frac{2c_u}{\gamma} = \frac{(2)(10)}{16.5} = 1.21m$$

Part b

$$P_a = P_1 + P_2$$

$P_1 = \frac{1}{2}(4.79)(79) = 189.2\ kN/m$

$P_2 = \frac{1}{2}(1.21)(-20) = -12.1\ kN/m$

$\therefore P_a = 177.1\ kN/m$

Part c

$P_a = P_1 = 189.2\ kN/m$

Problems

13.1 Figure 13.9 shows a retaining wall that is restrained from yielding. Determine the magnitude of the lateral earth force per unit length of the wall. Also, state the location of the resultant, $\bar{z}$, measured from the bottom of the wall.

$If\ H = 7m, \emptyset' = 38^o, \gamma = 17\ kN/m^3, OCR = 2.5$

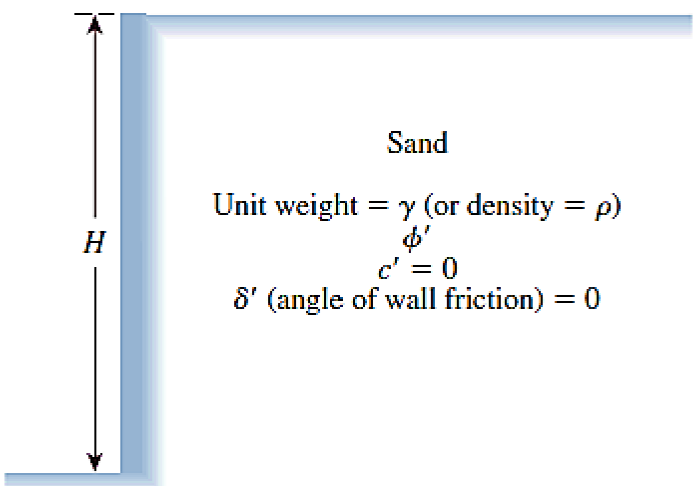


Figure 13.9

Ans. $P_o = 281.55 \frac{kN}{m}, \quad \bar{z} = 2.33m$

13.2 Assume that the retaining wall shown in Figure 13.9 is frictionless. Determine the Rankine active force per unit length of the wall, the variation of active earth pressure with depth, and the location of the resultant.

$If\ H = 4m, \overline{\emptyset} = 36^o\ and\ \gamma = 18\ kN/m^3$

Ans. $P_o = 37.44 \frac{kN}{m}, \quad \bar{z} = 1.33m$

13.3 Assume that the retaining wall shown in Figure 13.9 is frictionless. Determine the Rankine passive force per unit length of the wall, the variation of lateral earth pressure with depth, and the location of the resultant.

$If\ H = 5m,\ \overline{\emptyset} = 35^o\ and\ \gamma = 14\ kN/m^2$

Ans. $P_p = 645.8\ \frac{kN}{m}, \quad \bar{z} = 1.67m$

13.4 A retaining wall is shown in Figure 13.10. Determine the Rankine active force, P_a, per unit length of the wall and the location of the resultant.

$If\ H = 6m, H_1 = 3m, \gamma_1 = 15.5 \frac{kN}{m^3},\ \gamma_2 = 19.0 \frac{kN}{m^3}, \overline{\emptyset}_1 = 30^o, \overline{\emptyset}_2 = 36^o and\ q = 15\ kN/m^2$

Ans. $P_a = 141.1\ \frac{kN}{m}, \quad \bar{z} = 2.04m$

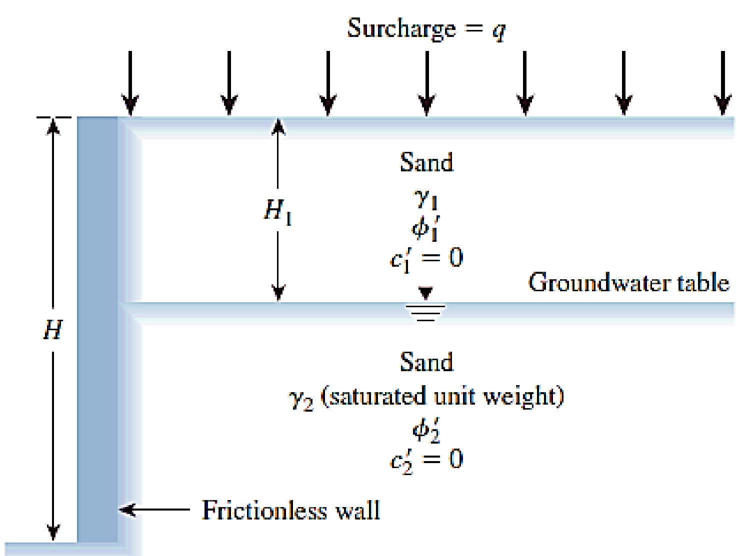


Figure (13.10)

13.5 A 5-m-high retaining wall with a vertical back face has a $c' - \emptyset'$ soil for backfill. For the backfill, $\gamma = 19\ kN/m^3$, c' = 26 kN/m^2, and $\emptyset' = 16^o$. Considering the existence of the tensile crack, determine the active force, P_a, on the wall for Rankine's active state.

Ans. $P_a = 10.02\ \frac{kN}{m}$