Chapter 11: Compressibility of Soil-Consolidation Settlement

A stress increase caused by the construction of foundations or other loads compresses soil layers. The compression is caused by (a) deformation of soil particles, (b) relocations of soil particles, and (c) expulsion of water or air from the void spaces. In general, the soil settlement caused by loads may be divided into three broad categories:

1. *Elastic settlement* (or *immediate settlement*), which is caused by the elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content. Elastic settlement calculations generally are based on equations derived from the theory of elasticity.
2. *Primary consolidation settlement*, which is the result of a volume change in saturated cohesive soils because of expulsion of the water that occupies the void spaces.
3. *Secondary consolidation settlement*, which is observed in saturated cohesive soils and is the result of the plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

This chapter presents the fundamental principles for estimating the consolidation settlements of soil layers under superimposed loadings.

The total settlement of a foundation can then be given as

$$S_T = S_c + S_s + S_e$$

where S_T = total settlement

S_c = primary consolidation settlement

S_s = secondary consolidation settlement

S_e = elastic settlement

When foundations are constructed on very compressible clays, the consolidation settlement can be several times greater than the elastic settlement.

11.1 Fundamentals of Consolidation

When a saturated soil layer is subjected to a stress increase, the pore water pressure is increased suddenly. In sandy soils that are highly permeable, the drainage caused by the increase in the pore water pressure is completed immediately. Porewater drainage is accompanied by a reduction in the volume of the soil mass, which results in settlement. Because of rapid drainage of the pore water in sandy soils, elastic settlement and consolidation occur simultaneously.

When a saturated compressible clay layer is subjected to a stress increase, elastic settlement occurs immediately. Because the hydraulic conductivity of clay is significantly smaller than that of sand, the excess pore water pressure generated by loading gradually dissipates over a long period. Thus, the associated volume change (that is, the consolidation) in the clay may continue long after the elastic settlement. The settlement caused by consolidation in clay may be several times greater than the elastic settlement.

The time-dependent deformation of saturated clayey soil best can be understood by considering a simple model that consists of a cylinder with a spring at its center. Let the inside area of the cross section of the cylinder be equal to A. The cylinder is filled with water and has a frictionless watertight piston and valve as shown in Figure (11.1a). At this time, if we place a load P on the piston (Figure 11.1b) and keep the valve closed, the entire load will be taken by the water in the cylinder because water is *incompressible*. The spring will not go through any deformation. The excess hydrostatic pressure at this time can be given as

$$\Delta u = \frac{P}{A} \tag{11.1}$$

This value can be observed in the pressure gauge attached to the cylinder.

In general, we can write

$$P = P_s + P_w \tag{11.2}$$

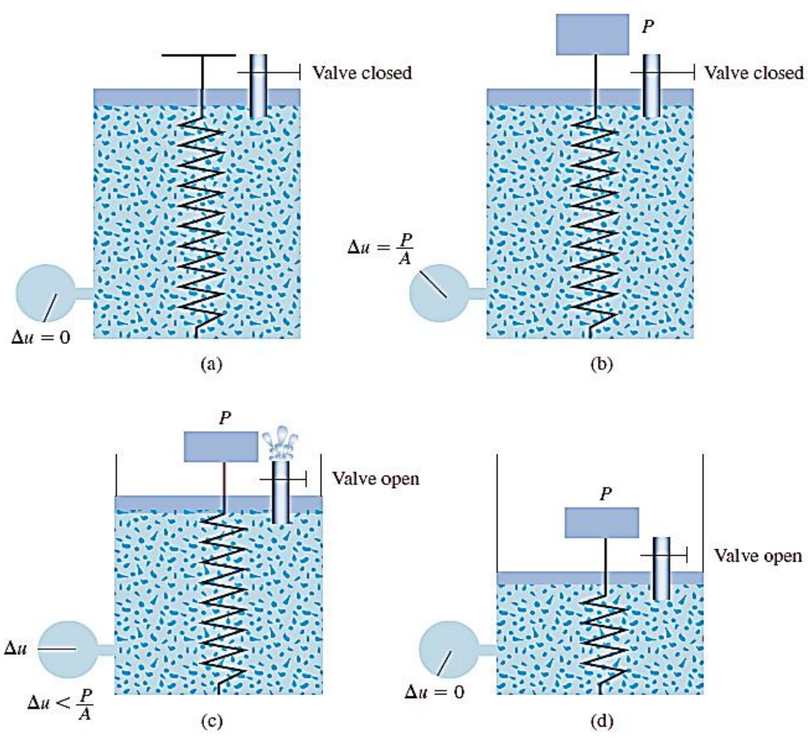


Figure (11.1) Spring-cylinder model

where P_s = load carried by the spring and Pw = load carried by the water.

From the preceding discussion, we can see that when the valve is closed after the placement of the load P,

$$P_s = 0 \quad and \quad P_w = P$$

Now, if the valve is opened, the water will flow outward (Figure 11.1c). This flow will be accompanied by a reduction of the excess hydrostatic pressure and an increase in the compression of the spring. So, at this time, Eq. (11.2) will hold. However,

$$P_s > o \quad and \quad P_w < P \qquad (that\ is, \Delta u < \frac{P}{A})$$

After some time, the excess hydrostatic pressure will become zero and the system will reach a state of equilibrium, as shown in Figure (11.1d). Now we can write

$$P_s = P \quad and \quad P_w = 0$$

And

$$P = P_s + P_w$$

With this in mind, we can analyze the strain of a saturated clay layer subjected to a stress increase (Figure 11.2a). Consider the case where a layer of saturated clay of thickness H that is confined between two layers of sand is being subjected to an instantaneous increase of *total stress* of $\Delta\sigma$. This incremental total stress will be transmitted to the pore water and the soil solids. This means that the total stress, $\Delta\sigma$, will be divided in some proportion between effective stress and porewater pressure. The behavior of the effective stress change will be similar to that of the spring in Figure (11.1), and the behavior of the pore water pressure change will be similar to that of the excess hydrostatic pressure in Figure (11.1). From the principle of effective stress, it follows that

$$\Delta\sigma = \Delta\sigma' + \Delta u \qquad (11.3)$$

where $\Delta\sigma'$ = increase in the effective stress

Δu = increase in the pore water pressure

Because clay has a very low hydraulic conductivity and water is incompressible as compared with the soil skeleton, at time t = 0, the entire incremental stress, $\Delta\sigma$, will be carried by water ($\Delta\sigma = \Delta u$) at all depths (Figure 11.2b). None will be carried by the soil skeleton—that is, incremental effective stress ($\Delta\sigma'$) = 0.

After the application of incremental stress, $\Delta\sigma$, to the clay layer, the water in the void spaces will start to be squeezed out and will drain in both directions into the sand layers. By this process, the excess pore water pressure at any depth in the clay layer

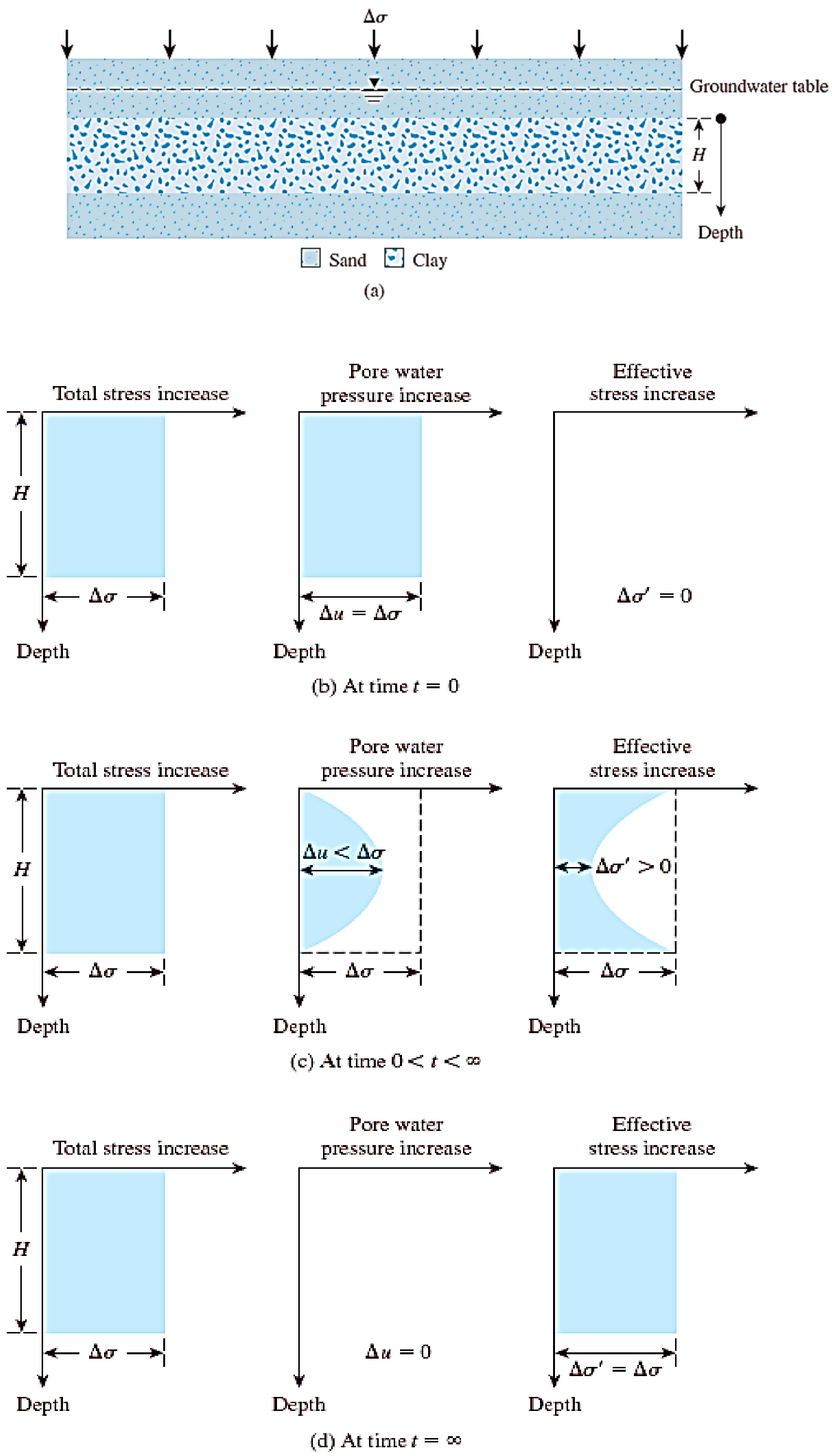


Figure 11.2 Variation of total stress, pore water pressure, and effective stress in a clay layer drained at top and bottom as the result of an added stress, Δσ

gradually will decrease, and the stress carried by the soil solids (effective stress) will increase. Thus, at time $0 < t < \infty$,

$$\Delta\sigma = \Delta\sigma' + \Delta u \qquad (\Delta\sigma' > 0 \quad and \;\; \Delta u < \Delta\sigma)$$

However, the magnitudes of $\Delta\sigma'$ and Δu at various depths will change (Figure 11.2c), depending on the minimum distance of the drainage path to either the top or bottom sand layer.

Theoretically, at time $t = \infty$, the entire excess pore water pressure would be dissipated by drainage from all points of the clay layer; thus, $\Delta u = 0$. Now the total stress increase, $\Delta\sigma$, will be carried by the soil structure (Figure 11.2d). Hence,

$$\Delta\sigma = \Delta\sigma'$$

This gradual process of drainage under an additional load application and the associated transfer of excess pore water pressure to effective stress cause the time-dependent settlement in the clay soil layer.

11.2 One-Dimensional Laboratory Consolidation Test

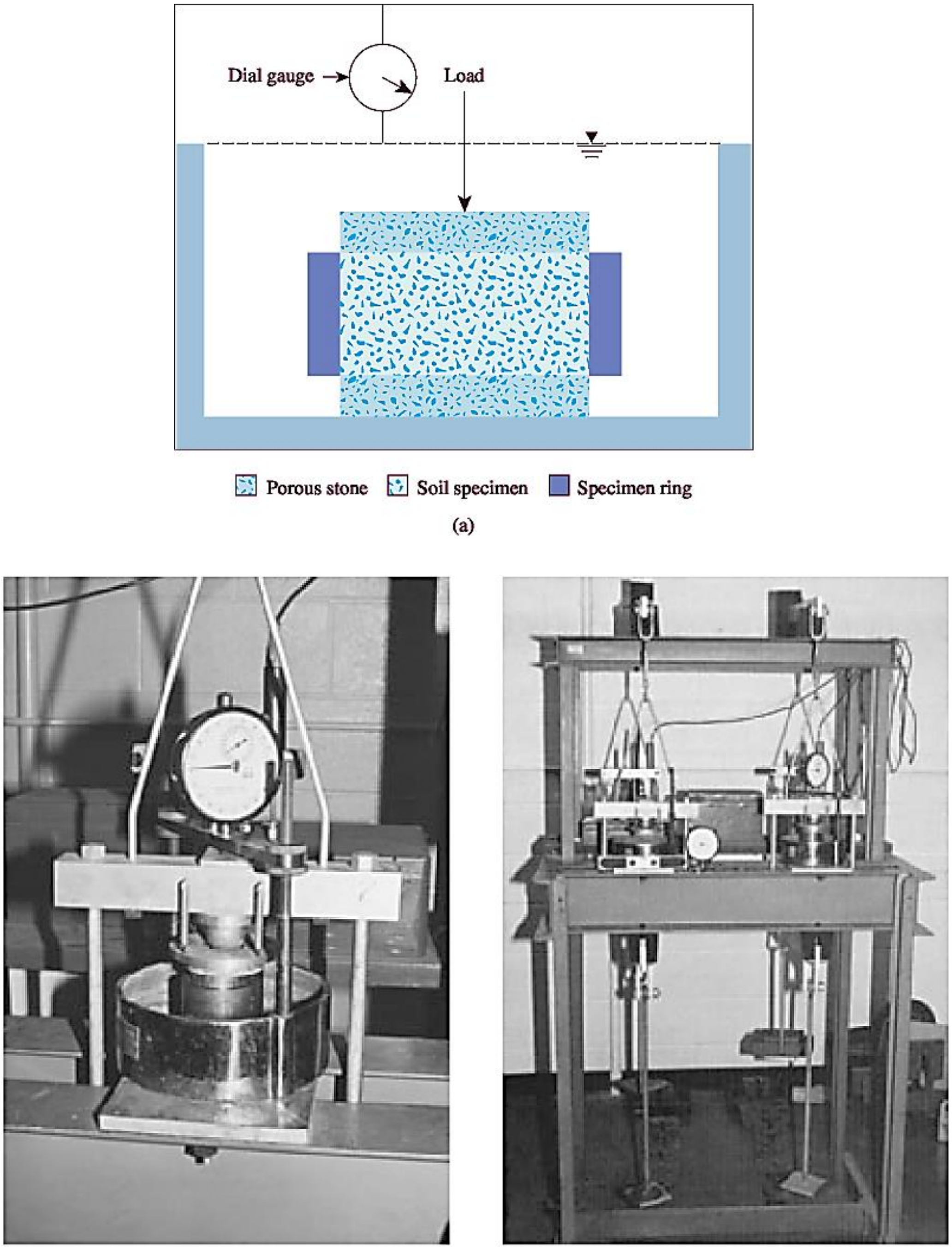
The one-dimensional consolidation testing procedure was first suggested by Terzaghi. This test is performed in a consolidometer (sometimes referred to as an oedometer). The schematic diagram of a consolidometer is shown in Figure (11.3a). Figure (11.3b) shows a photograph of a consolidometer. The soil specimen is placed inside a metal ring with two porous stones, one at the top of the specimen and another at the bottom. The specimens are usually 64 mm (≈ 2.5 in.) in diameter and 25 mm. (≈ 1 in.) thick. The load on the specimen is applied through a lever arm, and compression is measured by a micrometer dial gauge. The specimen is kept under water during the test. Each load usually is kept for 24 hours. After that, the load usually is doubled, which doubles the pressure on the specimen, and the compression measurement is continued. At the end of the test, the dry weight of the test specimen is determined. Figure (11.3c) shows a consolidation test in progress (right-hand side).

The general shape of the plot of deformation of the specimen against time for a given load increment is shown in Figure 11.4. From the plot, we can observe three distinct stages, which may be described as follows:

Stage I: Initial compression, which is caused mostly by preloading.

Stage II: Primary consolidation, during which excess pore water pressure gradually is transferred into effective stress because of the expulsion of pore water.

Stage III: Secondary consolidation, which occurs after complete dissipation of the excess pore water pressure, when some deformation of the specimen takes place because of the plastic readjustment of soil fabric.



(b) (c)

Figure (11.3) (a) Schematic diagram of a consolidometer; (b) photograph of a consolidometer; (c) a consolidation test in progress (right-hand side)

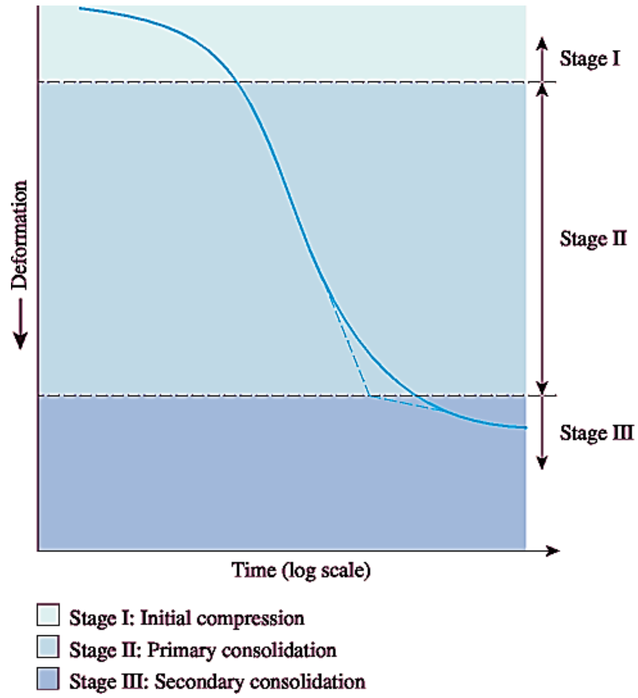


Figure (11.4) Time–deformation plot during consolidation for a given load increment

11.3 Void Ratio–Pressure Plots

After the time–deformation plots for various loadings are obtained in the laboratory, it is necessary to study the change in the void ratio of the specimen with pressure. Following is a step-by-step procedure for doing so:

Step 1: Calculate the height of solids, H_s, in the soil specimen (Figure 11.5) using the equation

$$H_s = \frac{W_s}{AG_s\gamma_w} = \frac{M_s}{AG_s\rho_w} \qquad (11.4)$$

where W_s = dry weight of the specimen
M_s = dry mass of the specimen
A = area of the specimen
G_s = specific gravity of soil solids
γ_w = unit weight of water
ρ_w= density of water

Step 2: Calculate the initial height of voids as

$$H_v = H - H_s \qquad (11.5)$$

Where H = initial height of the specimen

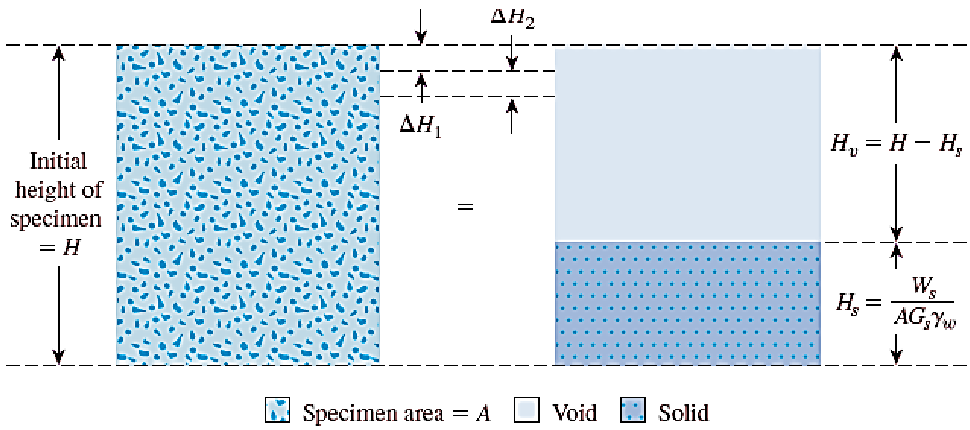


Figure 11.5 Change of height of specimen in one-dimensional consolidation test

Step 3: Calculate the initial void ratio, e_o, of the specimen, using the equation

$$e_o = \frac{V_v}{V_s} = \frac{H_v}{H_s}\frac{A}{A} = \frac{H_v}{H_s} \qquad (11.6)$$

Step 4 : For the first incremental loading, σ_1 (total load/unit area of specimen), which causes a deformation ΔH_1, calculate the change in the void ratio as

$$\Delta e_1 = \frac{\Delta H_1}{H_s} \qquad (11.7)$$

(ΔH_1 is obtained from the initial and the final dial readings for the loading).

It is important to note that, at the end of consolidation, total stress σ_1 is equal to effective stress σ'_1.

Step 5 : Calculate the new void ratio after consolidation caused by the pressure increment as

$$e_1 = e_o - \Delta e_1 \qquad (11.8)$$

For the next loading, σ_2 (note: σ_2 equals the cumulative load per unit area of specimen), which causes additional deformation ΔH_2, the void ratio at the end of consolidation can be calculated as

$$e_2 = e_1 - \frac{\Delta H_2}{H_s} \qquad (11.9)$$

At this time, σ_2 = effective stress, σ'_2. Proceeding in a similar manner, one can obtain the void ratios at the end of the consolidation for all load increments.

The effective stress σ' and the corresponding void ratios (e) at the end of consolidation are plotted on semi logarithmic graph paper. The typical shape of such a plot is shown in Figure (11.6).

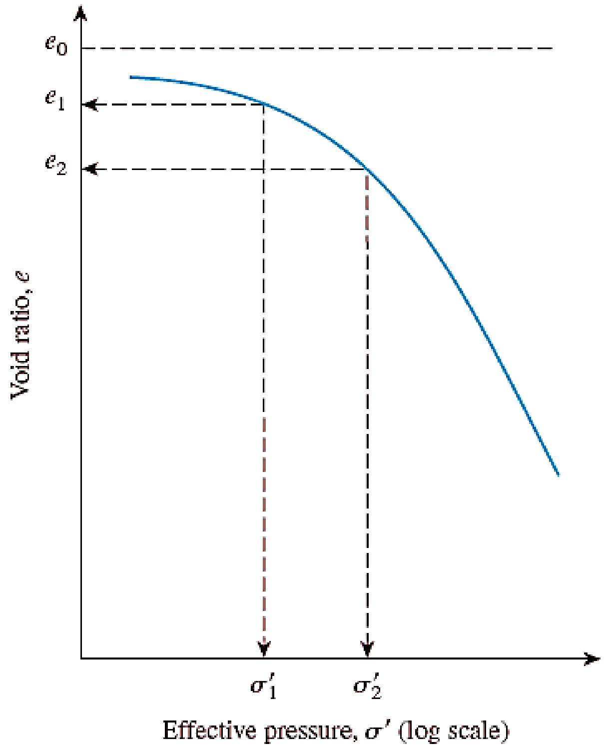


Figure (11.6) Typical plot of e against log σ'

Example 11.1

Following are the results of a laboratory consolidation test on a soil specimen obtained from the field: Dry mass of specimen = 128 *g*, height of specimen at the beginning of the test = 2.54 cm, G_s = 2.75, and area of the specimen = 30.68 cm^2.

Effective pressure, $\bar{\sigma}$ (kN/m^2)	Final height of specimen at the end of consolidation (cm)
0	2.54
0.5	2.488
1	2.465
2	2.431
4	2.389
8	2.324
16	2.225
32	2.115

Make necessary calculations and draw an e versus log σ' curve.

Solution

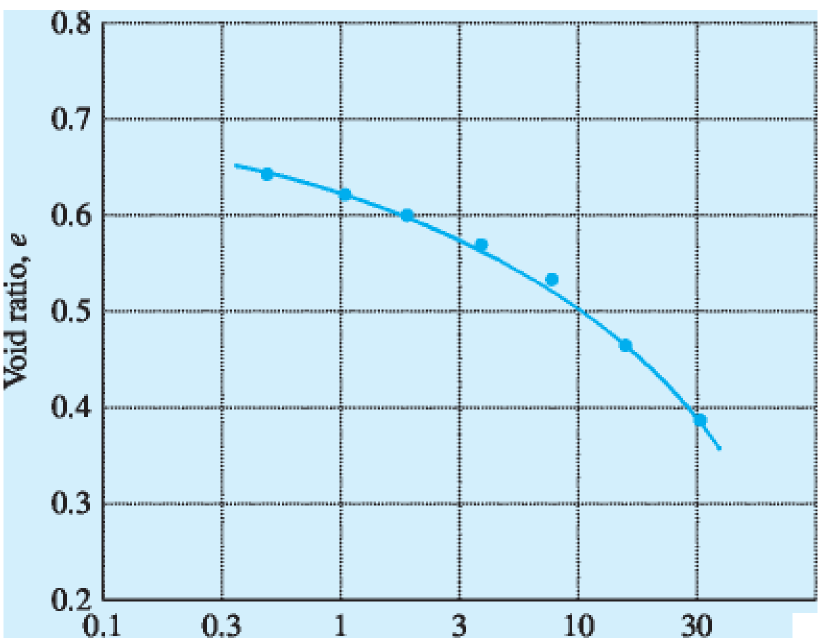
From Eq. (11.4),

$$H_s = \frac{W_S}{AG_S\gamma_w} = \frac{M_S}{AG_S\rho_w} = \frac{128\ g}{(30.68cm^2)(2.75)(\frac{1g}{cm^3})} = 1.52\ cm$$

Now the following table can be prepared.

Effective pressure, σ' (kN /m²)	Height at the end of consolidation, *H* (cm)	$H_v = H - H_s$ (cm)	$e = H_v/H_s$
0	2.540	1.02	0.67 1
0.5	2.488	0.968	0.637
1	2.465	0.945	0.622
2	2.431	0.911	0.599
4	2.389	0.869	0.572
8	2.324	0.804	0.529
16	2.225	0.705	0.464
32	2.115	0.595	0.390

The *e* versus log σ' plot is shown in Figure (11.7)



Effective pressure, σ' (kN/m^2)–log scale

Figure (11.7) Variation of void ratio with effective pressure

11.4 Normally Consolidated and Overconsolidated Clays

Figure (11.6) shows that the upper part of the e–log σ' plot is somewhat curved with a flat slope, followed by a linear relationship for the void ratio with log σ' having a steeper slope. This phenomenon can be explained in the following manner:

A soil in the field at some depth has been subjected to a certain maximum effective past pressure in its geologic history. This maximum effective past pressure may be equal to or less than the existing effective overburden pressure at the time of sampling. The reduction of effective pressure in the field may be caused by natural geologic processes or human processes. During the soil sampling, the existing effective overburden pressure is also released, which results in some expansion. When this specimen is subjected to a consolidation test, a small amount of compression (that is, a small change in void ratio) will occur when the effective pressure applied is less than the maximum effective over-burden pressure in the field to which the soil has been subjected in the past. When the effective pressure on the specimen becomes greater than the maximum effective past pressure, the change in the void ratio is much larger, and the *e*–log σ' relationship is practically linear with a steeper slope.

This relationship can be verified in the laboratory by loading the specimen to exceed the maximum effective overburden pressure, and then unloading and reloading again. The *e* –log σ' plot for such cases is shown in Figure (11.8), in which cd represents unloading and dfg represents the reloading process.

This leads us to the two basic definitions of clay based on stress history:

1. *Normally consolidated*, whose present effective overburden pressure is the maximum pressure that the soil was subjected to in the past.
2. *Overconsolidated*, whose present effective overburden pressure is less than that which the soil experienced in the past. The maximum effective past pressure is called the *preconsolidation pressure*.

Casagrande (1936) suggested a simple graphic construction to determine the pre-consolidation pressure σ'_c from the laboratory *e* –log σ' plot. The procedure is as follows (see Figure 11.9):

1. By visual observation, establish point a, at which the *e* –log σ' plot has a minimum radius of curvature.
2. Draw a horizontal line *ab*.
3. Draw the line ac tangent at *a*.

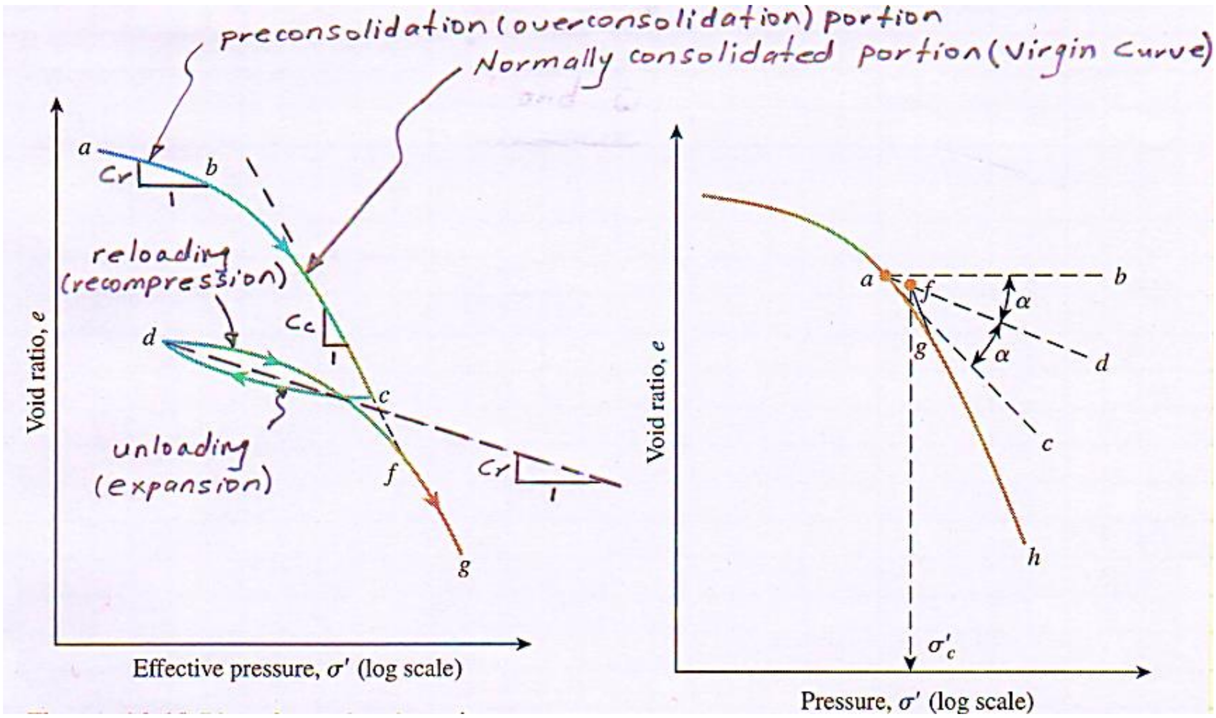


Figure (11.8) Plot of e against log $\bar{\sigma}$ showing loading, unloading, and reloading branches

Figure (11.9) Graphic procedure for determining preconsolidation pressure

4. Draw the line *ad*, which is the bisector of the angle *bac*.
5. Project the straight-line portion *gh* of the *e*–log σ' plot back to intersect line *ad* at *f*.

   The abscissa of point *f* is the preconsolidation pressure, σ'_c.

The overconsolidation ratio (OCR) for a soil can now be defined as

$$OCR = \frac{\sigma'_c}{\sigma'} \tag{11.10}$$

Where σ'_c = preconsolidation pressure of a specimen
σ' = present effective vertical pressure

The compressibility of the clay can be represented by the *compression index* C_c which represents the slope of the linear portion of the *e*–log σ' plot (virgin curve) and is dimensionless. The overconsolidated portion of the *e*–log σ' plot can be approximated to a stright line the slope of which is referred to as the *expansion or swell index* (C_r *or* C_s).

11.5 Calculation of Settlement from One-Dimensional Primary Consolidation

Let us consider a saturated clay layer of thickness H and cross-sectional area A under an existing average effective overburden pressure,σ'_o. Because of an increase of effective pressure, $\Delta\sigma'_o$ let the primary settlement be S_c.

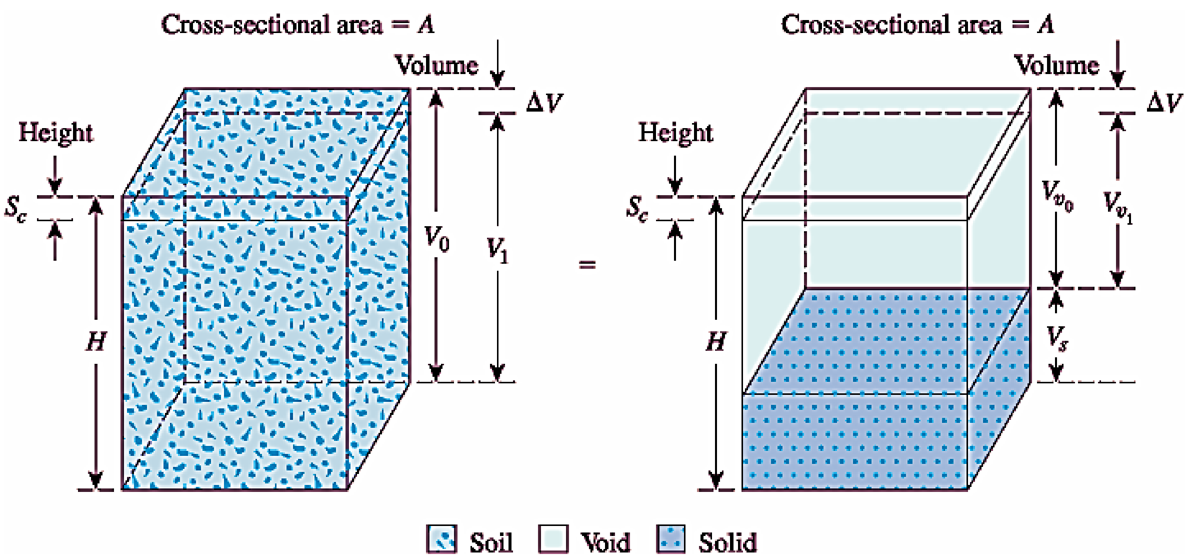


Figure 11.10 Settlement caused by one-dimensional consolidation

$$e_o = \frac{V_{v_o}}{V_s} = \frac{V_o - V_s}{V_s} = \frac{V_o}{V_s} - 1$$

or $\quad V_o = V_s(1 + e_o) \qquad (11.11)$

$$e = \frac{V_v}{V_s} \qquad \Rightarrow \quad \Delta e = \frac{\Delta V_v}{V_s}$$

or $\quad V_s = \frac{\Delta V_v}{\Delta e} \qquad (11.12)$

substitute Eq. (11.11) into Eq. (11.12) gives:

$$V_o = \frac{\Delta V_v}{\Delta e}(1 + e_o) = \frac{\Delta V}{\Delta e}(1 + e_o)$$

or $\quad AH = \frac{S_c A}{\Delta e}(1 + e_o)$

Thus $S_c = H\frac{\Delta e}{1+e_o}$ (11.13)

- For normally consolidated clays (NCC):

$$\Delta e = c_c[\log(\sigma'_o + \Delta\sigma') - log\sigma'_o]$$

or

$$S_c = \frac{C_c H}{1+e_o}\log(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_o}) \quad (11.14)$$

- For overconsolidated clays (OCC):

(i) If $\sigma'_o + \Delta\sigma' \leq \sigma'_c$:

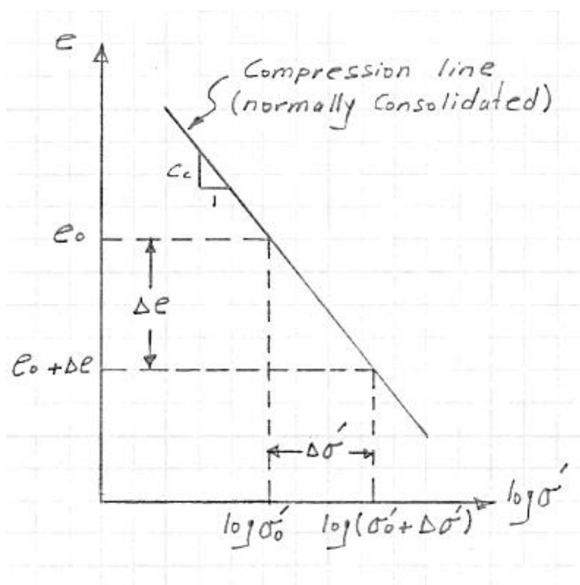
$$\Delta e = c_r[\log(\sigma'_o + \Delta\sigma') - log\sigma'_o]$$

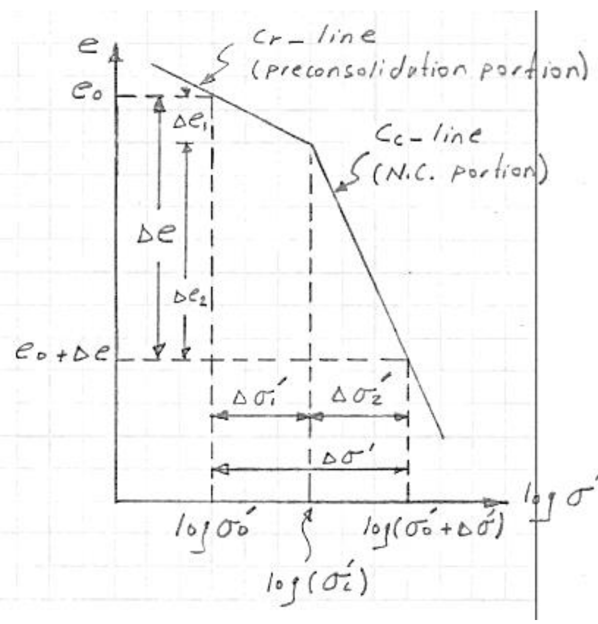
or $S_c = \frac{c_r H}{1+e_o}\log(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_o})$ (11.15)

(ii) If $\sigma'_o + \Delta\sigma' > \sigma'_c$:

$$\Delta e = \Delta e_1 + \Delta e_2 = c_r\left[log\sigma'_c - log\sigma'_o\right] + c_c[\log\left(\sigma'_o + \Delta\sigma'\right) - log\sigma'_c]$$

or $S_c = \frac{c_r H}{1+e_o}log\frac{\sigma'_c}{\sigma'_o} + \frac{c_c H}{1+e_o}\log(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_c})$ (11.16)





Example 11.2

The following are the results of a laboratory consolidation test:

Pressure, σ' (kN /m^2)	Void ratio, e	Remarks	Pressure, σ' (kN /m^2)	Void ratio, e	Remarks
0.25	1.03	Loading	8.0	0.71	Loading
0.5	1.02		16.0	0.62	
1.0	0.98		8	0.635	Unloading
2.0	0.91		4	0.655	
4.0	0.79		2	0.67	

a. Draw an e-logσ'_o graph and determine the preconsolidation pressure, σ'_c
b. Calculate the compression index and the ratio of C_s/C_c
c. On the basis of the average e-log σ' plot, calculate the void ratio at

$$\sigma'_o = 12\ kN/m^2$$

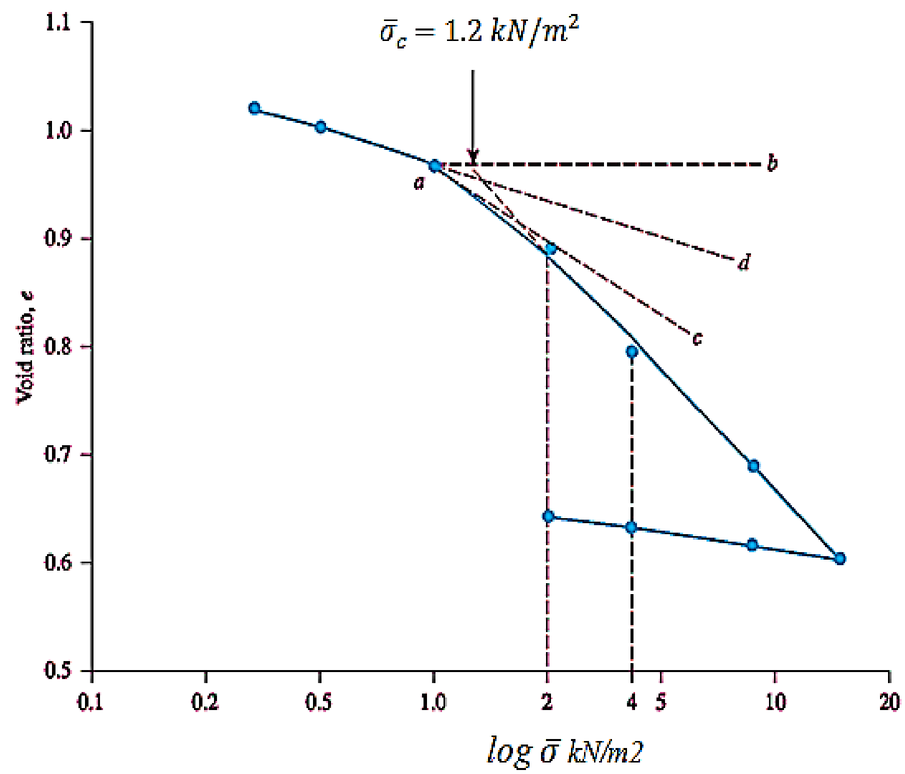


Figure 11.11: Plot of e versus log σ'

Solution

Part a

The e versus log σ' plot is shown in Figure (11.11). Casagrande's graphic procedure is used to determine the preconsolidation pressure:

$$\sigma'_o = 1.2\ kN/m^2$$

Part b

From the average e- log σ' plot, for the loading and unloading branches, the following values can be determined:

Branch	e	σ'_o (kN /m²)
Loading	0.9	2
	0.8	4
Unloading	0.67	2
	0.655	4

From the loading branch,

$$c_c = \frac{e_1 - e_2}{log\frac{\sigma'_2}{\sigma'_1}} = \frac{0.9 - 0.8}{log(\frac{4}{2})} = 0.33$$

From the unloading branch,

$$c_s = \frac{e_1 - e_2}{log\frac{\sigma'_2}{\sigma'_1}} = \frac{0.67 - 0.655}{log(\frac{4}{2})} = 0.05$$

$$\frac{c_s}{c_c} = \frac{0.05}{0.33} = 0.15$$

Part c

$$c_c = \frac{e_1 - e_3}{log\frac{\sigma'_3}{\sigma'_1}}$$

We know that e_1 = 0.9 at $\sigma'_1 = 2kN/m^2$ and that c_c = 0.33 [part (b)]. Let $\sigma'_3 = 12\ kN/m^2$. So,

$$0.33 = \frac{0.9 - e_3}{\log(\frac{12}{2})}$$

$$e_3 = 0.9 - 0.33\log\left(\frac{12}{2}\right) = 0.64$$

Example 11.3

A soil profile is shown in Figure (11.12). If a uniformly distributed load, $\Delta\sigma$, is applied at the ground surface, what is the settlement of the clay layer caused by primary consolidation if

a. The clay is normally consolidated

b. The preconsolidation pressure (σ'_c) = 190 kN/m²

c. $\sigma'_c = 170 kN/m^2$

Use $C_r \approx \frac{1}{6} C_c$

Solution

Part a

The average effective stress at the middle of the clay layer is

$$\sigma'_o = 2\gamma_{dry} + 4[\gamma_{sat(sand)} - \gamma_w] + \frac{4}{2}[\gamma_{sat(clay)} - \gamma_w]$$

$$\sigma'_o = (2)(14) + 4(18 - 9.81) + 2(19 - 9.81) = 79.14\ kN/m^2$$

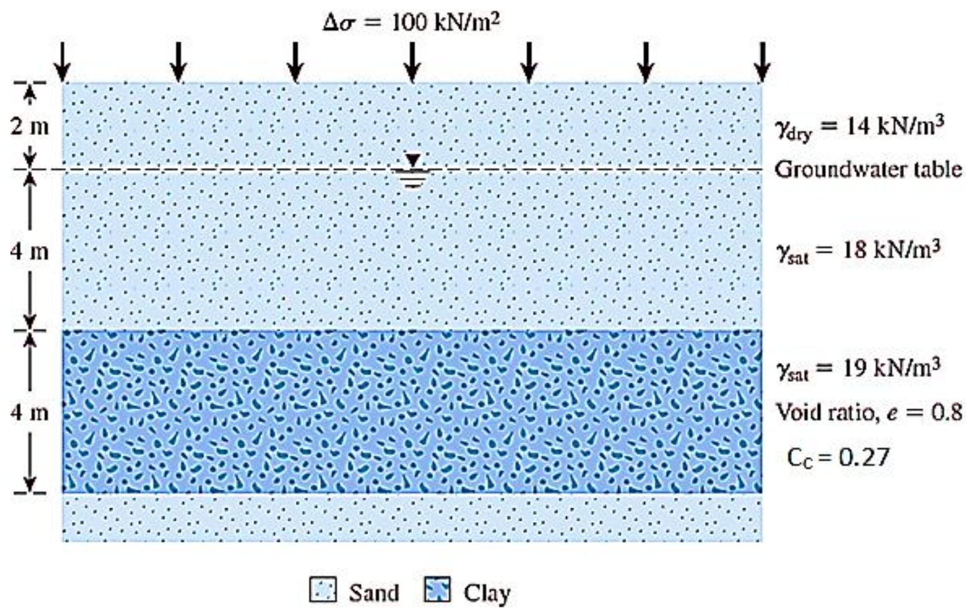


Figure 11.12

From Eq. (11.14),

$$S_c = \frac{C_c H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

$$S_C = \frac{(0.27)(4)}{1+0.8} \log\left(\frac{79.14+100}{79.14}\right) = 0.213m = 213mm$$

Part b

$$\sigma'_o + \Delta\sigma' = 79.14 + 100 = 179.14\ kN/m^2$$

$$\sigma'_c = 190\ kN/m^2$$

Because $\sigma'_o + \Delta\sigma' < \sigma'_c$ use Eq. (11.15):

$$S_c = \frac{c_r H}{1+e_o}\log\left(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_o}\right)$$

$$c_r = \frac{1}{6}c_c = \frac{0.27}{6} = 0.045$$

$$S_c = \frac{(0.045)(4)}{1+0.8}\log\left(\frac{79.14+100}{79.14}\right) = 0.036\ m = 36mm$$

Part c

$$\sigma'_o = 79.14\ kN/m^2$$

$$\sigma'_o + \Delta\bar{\sigma} = 179.14\ kN/m^2$$

$$\sigma'_c = 170\ kN/m^2$$

Because $\sigma'_o < \sigma'_c < \sigma'_o + \Delta\sigma'$, use Eq. (11.16)

$$S_c = \frac{c_r H}{1+e_o} log\,\frac{\sigma'_c}{\sigma'_o} + \frac{c_c H}{1+e_o}\log\left(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_c}\right)$$

$$= \frac{(0.045)(4)}{1.8}\log\left(\frac{170}{79.14}\right) + \frac{(0.27)(4)}{1.8}\log\left(\frac{179.14}{170}\right) \approx 0.0468m = 46.8\ mm$$

Example 11.4

A soil profile is shown in Figure (11.13a). Laboratory consolidation tests were conducted on a specimen collected from the middle of the clay layer. The field consolidation curve interpolated from the laboratory test results is shown in Figure (11.13b). Calculate the settlement in the field caused by primary consolidation for a surcharge of 48 kN/m^2 applied at the ground surface.

Solution

$$\sigma'_o = (5)(\gamma_{sat} - \gamma_w) = 5(18.0 - 9.81) = 40.95\ kN/m^2$$
$$e_o = 1.1$$
$$\Delta\sigma' = 48\ kN/m^2$$
$$\sigma'_o + \Delta\sigma' = 40.95 + 48 = 88.95 kN/m^2$$

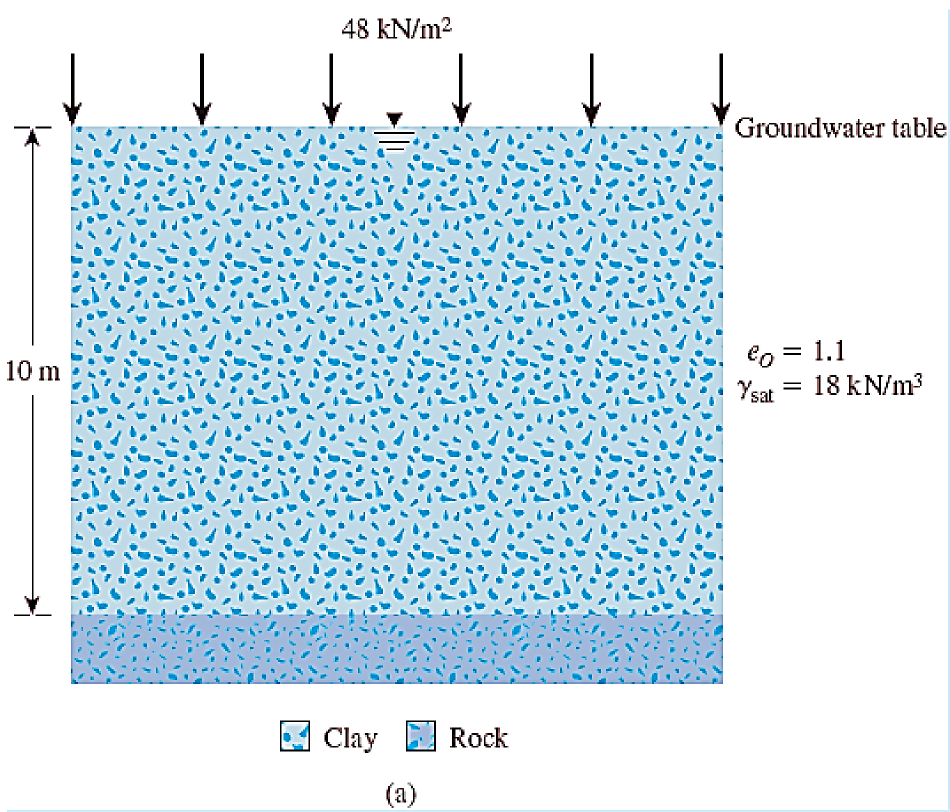


Figure (11.13) (a) soil profile

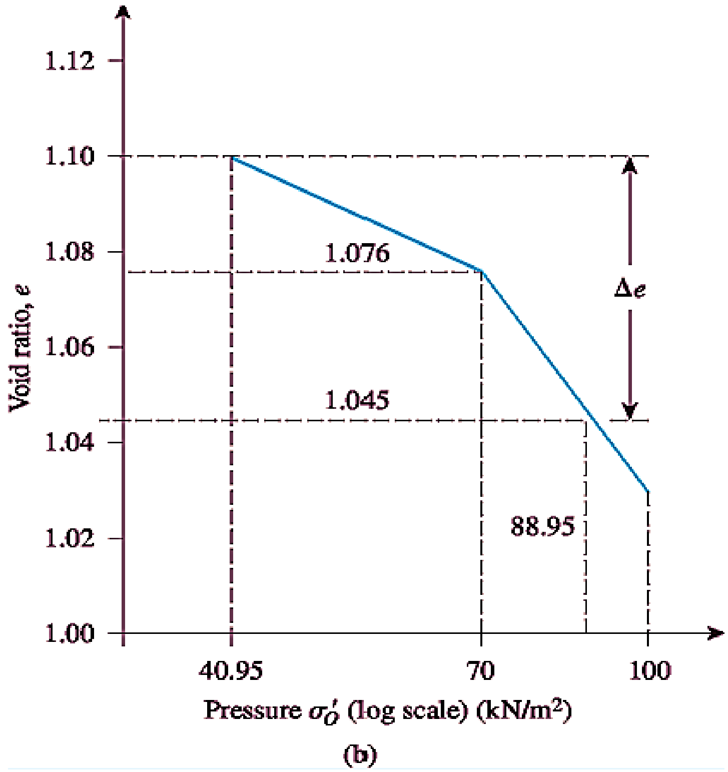


Figure (11.13) (b) field consolidation curve

The void ratio corresponding to 88.95 kN/m^2 (see Figure 11.13b) is 1.045. Hence, Δe = 1.1 - 1.045 = 0.055. We have

Settlement $(S_c) = H \frac{\Delta e}{1+e_o}$ [Eq.(11.13)]

so,

$$S_c = 10 \frac{(0.055)}{1+1.1} = 0.262m = 262mm$$

<u>Another solution</u>

$$c_r = \frac{1.1-1.076}{\log(\frac{70}{40.95})} = 0.103$$

$$c_c = \frac{1.076-1.045}{\log(\frac{88.95}{70})} = 0.298$$

$$S_c = \frac{c_S H}{1+e_o} log \frac{\sigma'_c}{\sigma'_o} + \frac{c_c H}{1+e_o} \log(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_c})$$

= 262mm

11.6 Secondary Consolidation Settlement

Section 11.2 showed that at the end of primary consolidation (that is, after complete dissipation of excess pore water pressure) some settlement is observed because of the plastic adjustment of soil fabrics. This stage of consolidation is called *secondary consolidation*. During secondary consolidation the plot of deformation against the log of time is practically linear (see Figure 11.4). The variation of the void ratio, e, with time t for a given load increment will be similar to that shown in Figure (11.4). This variation is shown in Figure (11.14). From Figure (11.14), the secondary compression index can be defined as

$$C_\alpha = \frac{\Delta e}{log t_2 - log t_1} = \frac{\Delta e}{\log(\frac{t_2}{t_1})} \tag{11.17}$$

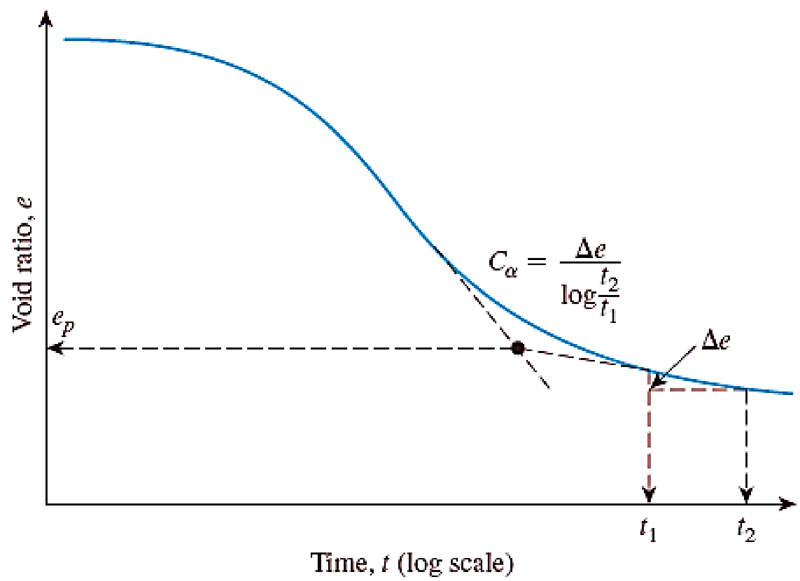


Figure (11.14) Variation of *e* with log *t* under a given load increment and definition of secondary consolidation index

where C_α = secondary compression index

Δe =change of void ratio

t_1,t_2 = time

The magnitude of the secondary consolidation can be calculated as

$$S_s = C'_\alpha H log(\frac{t_2}{t_1}) \tag{11.18}$$

and

$$C'_{\alpha} = \frac{C_{\alpha}}{1+e_p} \tag{11.19}$$

Where e_p = void ratio at the end of primary consolidation (see Figure 10.14)
H= thickness of clay layer

The general magnitudes of C'_{α} as observed in various natural deposits are as follows:
- Overconsolidated clays = 0.001 or less
- Normally consolidated clays = 0.005 to 0.03
- Organic soil =0.04 or more

Example 11.5
For a normally consolidated clay layer in the field, the following values are given:
- Thickness of clay layer = 3.0 m
- Void ratio (e_o) = 0.8
- Compression index (C_c) = 0.28
- Average effective pressure on the clay layer $\sigma'_o = 135\ kN/m^2$
- $\Delta\sigma' = 50\ kN/m^2$
- Secondary compression index (C_{α}) = 0.02

What is the total consolidation settlement of the clay layer five years after the completion of primary consolidation settlement? (*Note*: Time for completion of primary settlement = 1.5 years.)

Solution
From Eq. (11.19)

$$C'_{\alpha} = \frac{C_{\alpha}}{1+e_p}$$

The value of e_P can be calculated as

$$e_p = e_o - \Delta e_{primary}$$

$$\Delta e = C_c \log\left(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_o}\right) = 0.28 \log\left(\frac{135+50}{135}\right) = 0.038$$

$$\text{Primary consolidation}, S_c = \frac{\Delta e H}{1+e_o} = \frac{(0.038)(3\times10^3)}{1+0.8} = 63.3\ mm$$

It is given that e_o= 0.8 , and thus,

e_p = 0.8 – 0.038 = 0.762

Hence,

$$C'_\alpha = \frac{0.02}{1+0.762} = 0.011$$

From Eq. (11.18),

$$S_c = C'_\alpha H log\left(\frac{t_2}{t_1}\right) = (0.011)(3\times10^3)\log\left(\frac{6.5}{1.5}\right) \approx 21.0mm$$

Total consolidation settlement = primary consolidation (S_c) + secondary settlement (S_s). So

Total consolidation settlement = 63.3 + 21.0 = 84.3 mm

Another Solution

$$S_c = \frac{C_cH}{1+e}\log(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_o})$$

$$= \frac{0.28\times3\times10^3}{1+0.8} log\left(\frac{135+50}{135}\right) = 63.3\ mm$$

$$S_s = \frac{C_\alpha H}{1+e_P} Hlog(\frac{t_2}{t_1})$$

$$e_p = e_o - \Delta e$$

$$\Delta e = C_c\log\left(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_o}\right) = 0.28\log\left(\frac{135+50}{135}\right) = 0.038$$

$$e_P = 0.8 - 0.038 = 0.762$$

$$S_S = \frac{0.02\times3\times10^3}{1+0.762} log\left(\frac{6.5}{1.5}\right) = 21.0\ mm$$

$$S_T = S_c + S_s = 63.3 + 21.0 = 84.3\ mm$$

11.7 Time Rate of Consolidation

The total settlement caused by primary consolidation resulting from an increase in the stress on a soil layer can be calculated by the use of one of the three equations—(11.14), (11.15), and (11.16). However, they do not provide any information regarding the rate of primary consolidation. Terzaghi (1925) proposed the first theory to consider the rate of one-dimensional consolidation for saturated clay soils. The mathematical derivations are based on the following six assumptions:

1. The clay–water system is homogeneous.
2. Saturation is complete.
3. Compressibility of water is negligible.
4. Compressibility of soil grains is negligible (but soil grains rearrange).
5. The flow of water is in one direction only (that is, in the direction of compression).
6. Darcy’s law is valid.

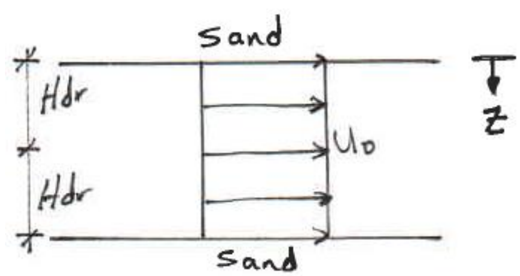
The resulting differential equation of consolidation is given by:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \qquad (11.20)$$

Where:
u = excess p.w.p
c_v = coefficient of consolidation = $\frac{k}{\gamma_w m_v}$ (11.21)
k = coefficient of permeability
m_v = coefficient of volume compressibility

Eq. (11.20) is the basic differential equation of Terzaghi’s consolidation theory and can be solved with the following boundary conditions:

$$z = 0, \quad u = 0$$
$$z = 2H_{dr}, \quad u = 0$$
$$t = 0, \quad u = u_O$$



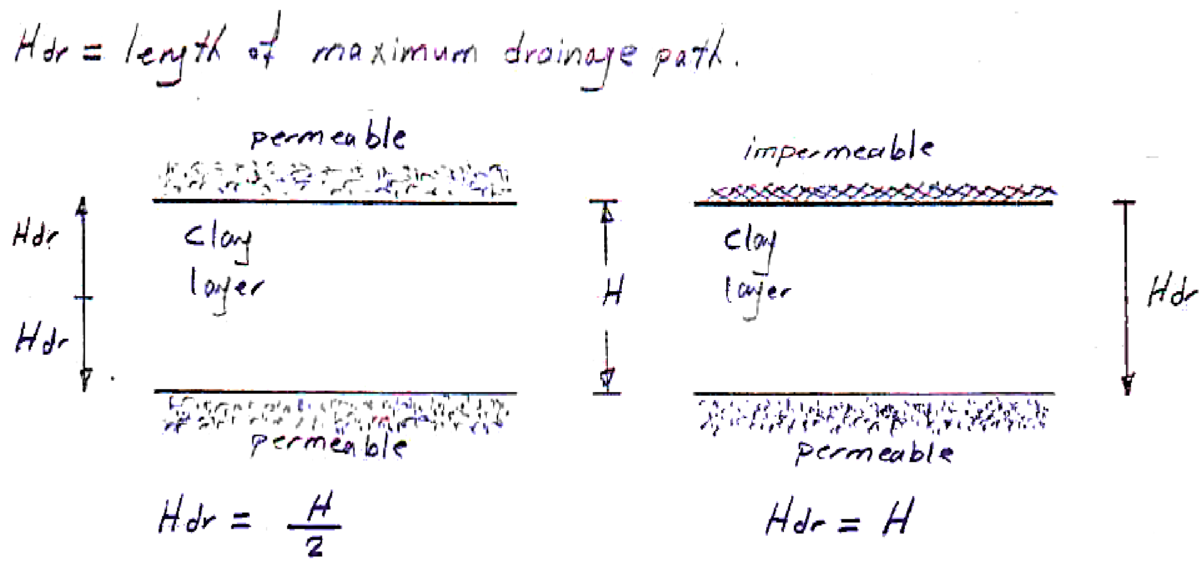
The solution yields

$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_o}{M} \sin\left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v} \qquad (11.22)$$

Where m = an integer
$M = (\frac{\pi}{2})(2m + 1)$
u_o = initial excess pore water pressure

$$T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor} \tag{11.23}$$

The time factor is a nondimensional number.



Because consolidation progresses by the dissipation of excess pore water pressure, the degree of consolidation at a distance *z* at any time *t* is

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o} \tag{11.24}$$

where u_z = excess pore water pressure at time *t*.

Equations (11.22) and (11.23) can be combined to obtain the degree of consolidation at any depth *z*. This is shown in Figure (11.15).

The average degree of consolidation for the entire depth of the clay layer at any time *t* can be written from Eq. (11.24) as

$$U = \frac{S_{c(t)}}{S_c} = 1 - \frac{\left(\frac{1}{2H_{dc}}\right)\int_0^{2H_{dr}} u_z dz}{u_o} \tag{11.25}$$

where U = average degree of consolidation

$S_{c(t)}$ = settlement of the layer at time *t*

S_c = ultimate settlement of the layer from primary consolidation

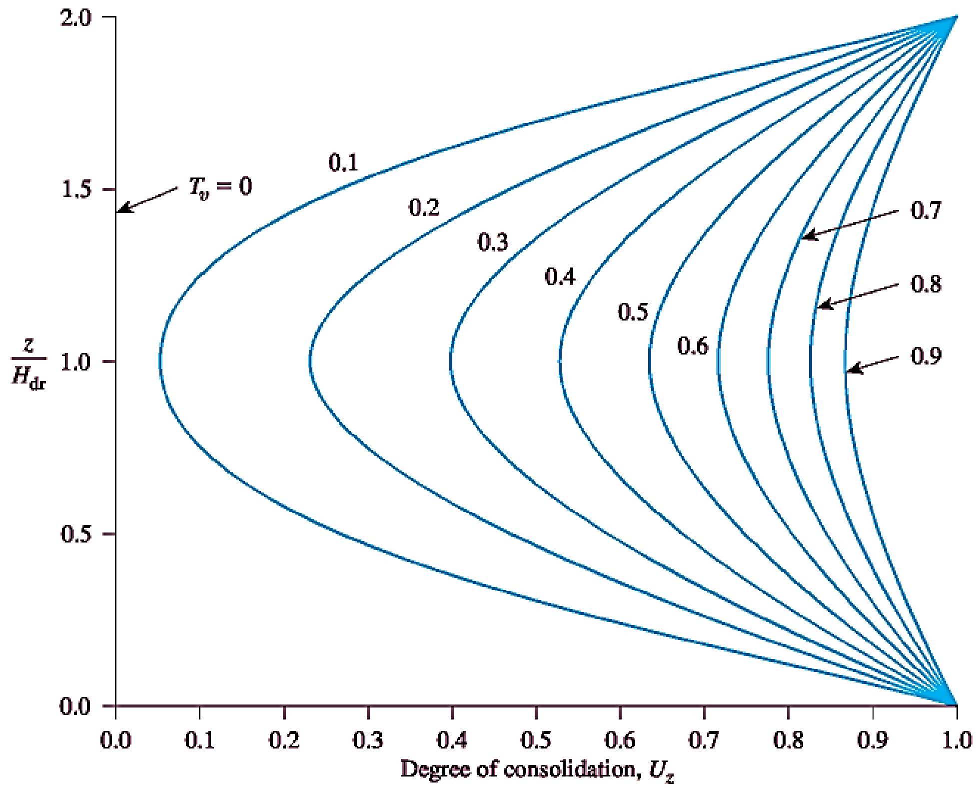


Figure 11.15 Variation of U_z with T_v and z/H_{dr}

The variation in the average degree of consolidation with the nondimensional time factor, T_v, is given in Figure (11.16), which represents the case where u_o is the same for the entire depth of the consolidating layer.

The values of the time factor and their corresponding average degrees of consolidation for the case presented in Figure (11.16) may also be approximated by the following simple relationship:

For $U = 0 \; to \; 60\%, \quad T_v = \frac{\pi}{4}(\frac{U\%}{100})^2$ (11.26)

For $U > 60\%, \; T_v = 1.781 - 0.933 \log(100 - U\%)$ (11.27)

Table (11.1) gives the variation of T_v with U on the basis of Eqs. (11.26) and (11.27).

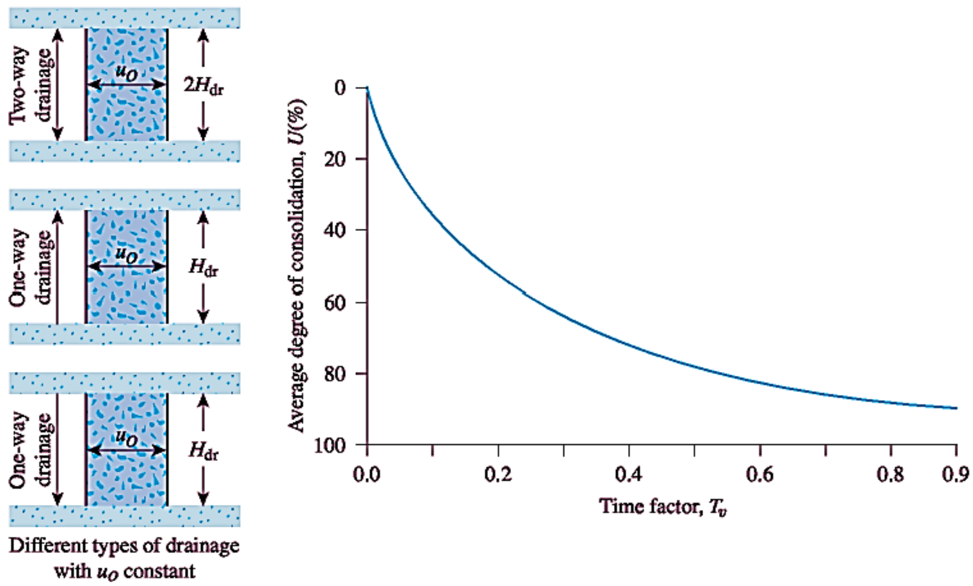


Figure (11.16) Variation of average degree of consolidation with time factor, T_v (u_o constant with depth)

Table (11.1) Variation of T_v with U

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	26	0.0531	52	0.212	78	0.529
1	0.00008	27	0.0572	53	0.221	79	0.547
2	0.0003	28	0.0615	54	0.230	80	0.567
3	0.00071	29	0.0660	55	0.239	81	0.588
4	0.00126	30	0.0707	56	0.248	82	0.610
5	0.00196	31	0.0754	57	0.257	83	0.633
6	0.00283	32	0.0803	58	0.267	84	0.658
7	0.00385	33	0.0855	59	0.276	85	0.684
8	0.00502	34	0.0907	60	0.286	86	0.712
9	0.00636	35	0.0962	61	0.297	87	0.742
10	0.00785	36	0.102	62	0.307	88	0.774
11	0.0095	37	0.107	63	0.318	89	0.809
12	0.0113	38	0.113	64	0.329	90	0.848
13	0.0133	39	0.119	65	0.304	91	0.891
14	0.0154	40	0.126	66	0.352	92	0.938
15	0.0177	41	0.132	67	0.364	93	0.993
16	0.0201	42	0.138	68	0.377	94	1.055
17	0.0227	43	0.145	69	0.390	95	1.129
18	0.0254	44	0.152	70	0.403	96	1.219
19	0.0283	45	0.159	71	0.417	97	1.336
20	0.0314	46	0.166	72	0.431	98	1.500
21	0.0346	47	0.173	73	0.446	99	1.781
22	0.0380	48	0.181	74	0.461	100	∞
23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		

Example 11.6

The time required for 50% consolidation of a 25-mm-thick clay layer (drained at both top and bottom) in the laboratory is 2 min. 20 sec. How long (in days) will it take for a 3-m-thick clay layer of the same clay in the field under the same pressure increment to reach 50% consolidation? In the field, there is a rock layer at the bottom of the clay.

Solution

$$T_{50} = \frac{c_v t_{lab}}{H^2_{dr(lab)}} = \frac{c_v \ t_{field}}{H^2_{dr(field)}}$$

or

$$\frac{t_{lab}}{H^2_{dr(lab)}} = \frac{t_{field}}{H^2_{dr(field)}}$$

$$\frac{140 sec}{(\frac{0.025\ m}{2})^2} = \frac{t_{field}}{(3m)^2}$$

$$t_{field} = 8{,}064{,}000\ sec = 93.33\ days$$

Example 11.7

Refer to Example (11.6). How long (in days) will it take in the field for 30% primary consolidation to occur? Use Eq. (11.26).

Solution

From Eq.(11.26)

$$\frac{c_v \ t_{field}}{H^2_{dr(field)}} = T_v \propto U^2$$

So

$$t \propto U^2$$

$$\frac{t_1}{t_2} = \frac{U_1^2}{U_2^2}$$

or

$$\frac{93.33}{t_2} = \frac{50^2}{30^2} \qquad \Rightarrow \quad t_2 = 33.6\ days$$

11.8 Coefficient of Volume Compressibility (m_v)

σ' = vertical effective stress
ε = vertical strain

$$m_v = \frac{|\Delta\varepsilon|}{\Delta\sigma'} \tag{11.28}$$

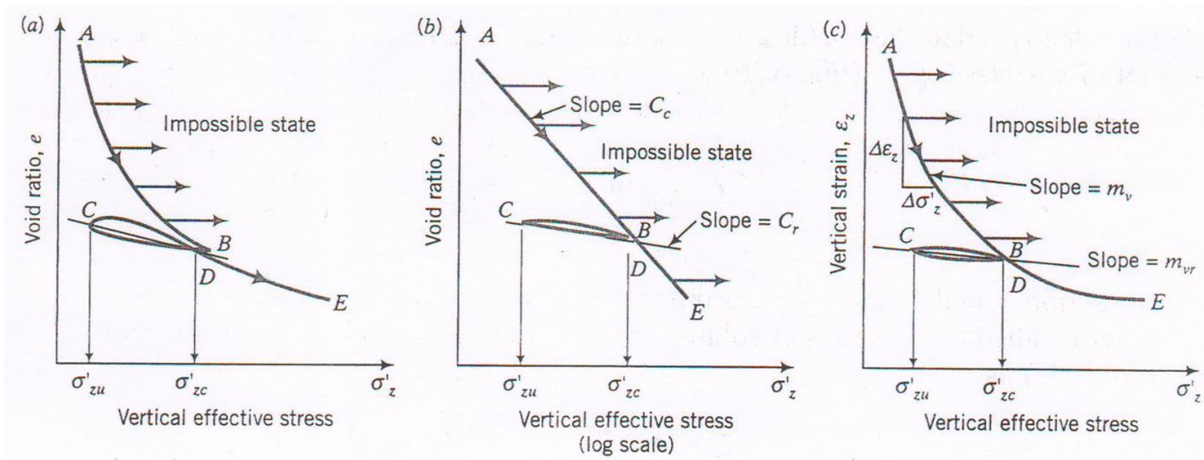


Figure (11.17) Three plots of settlement data from soil consolidation

m_{vr} = coefficient of volume recompressibility

$$\Delta\varepsilon = \frac{\Delta H}{H} = \frac{\Delta V}{V_o} = \frac{\Delta e}{1+e_{av}}$$

$$\Rightarrow\ m_v = \frac{(\frac{\Delta e}{\Delta\sigma'})}{1+e_{av}} \tag{11.29}$$

Settlement:

$$\Delta\varepsilon = m_v\Delta\sigma'$$

$$\frac{\Delta H}{H} = m_v\Delta\sigma'\ \Rightarrow\ \Delta H = S_c = Hm_v\Delta\sigma' \tag{11.30}$$

Example 11.8

A 3-m-thick layer (double drainage) of saturated clay under a surcharge loading underwent 90% primary consolidation in 75 days. Find the coefficient of consolidation of clay for the pressure range.

Solution

$$T_{90} = \frac{c_v\, t_{90}}{H_{dr}^2}$$

Because the clay layer has two-way drainage, H_{dr} = 3 m /2 =1.5 m. Also, T_{90} =0.848 (see Table 11.1). So,

$$0.848 = \frac{c_v(75\times24\times60\times60)}{(1.5\times100)^2}$$

$$c_v = \frac{0.848\times2.25\times10^4}{75\times24\times60\times60} = 0.00294\ cm^2/sec$$

Example 11.9

For a normally consolidated laboratory clay specimen drained on both sides, the following are given:

- $\sigma' = 150\frac{kN}{m^2}$, $e = e_o = 1.1$
- $\sigma'_o + \Delta\sigma' = 300\frac{kN}{m^2}$, $e = 0.9$
- Thickness of clay specimen = 25.4 mm
- Time for 50% consolidation = 2 min

a. Determine the hydraulic conductivity (m/min) of the clay for the loading range.

b. How long (in days) will it take for a 2 m clay layer in the field (drained on one side) to reach 60% consolidation?

Solution

Part a

The coefficient of compressibility is

$$m_v = \frac{a_v}{1+e_{av}} = \frac{\left(\frac{\Delta e}{\Delta\sigma'}\right)}{1+e_{av}}$$

$$\Delta e = 1.1 - 0.9 = 0.2$$

$$\Delta\sigma' = 300 - 150 = 150\ kN/m^2$$

$$e_{av} = \frac{1.1+0.9}{2} = 1.0$$

So

$$m_v = \frac{\frac{0.2}{150}}{1+1.0} = 6.67 \times 10^{-4}\ m^2/kN$$

From Table (11.1), for U = 50%, T_v = 0.197; thus,

$$c_v = \frac{(0.197)(\frac{25.4}{2\times1000})^2}{2} = 1.59 \times 10^{-5} m^2/min$$

$$k = c_v m_v \gamma_w = (1.59 \times 10^{-5})(6.67 \times 10^{-4})(9.81)$$

$$= 1.04 \times 10^{-7}\ m/min$$

Part b

$$T_{60} = \frac{c_v\, t_{60}}{H_{dr}^2}$$

$$t_{60} = \frac{T_{60}\, H_{dr}^2}{c_v}$$

From Table (11.1), for U = 60%, T_v = 0.286,

$$t_{60} = \frac{(0.286)(2)^2}{1.59\times10^{-5}} = 71950 \text{ min} = 50\ days$$

11.9 Coefficient of Consolidation

For a given load increment on a specimen, two graphical methods commonly are used for determining c_v from laboratory one-dimensional consolidation tests. The first is the *logarithm-of-time method* proposed by Casagrande and Fadum (1940), and the other is the *square-root-of-time method* given by Taylor (1942).

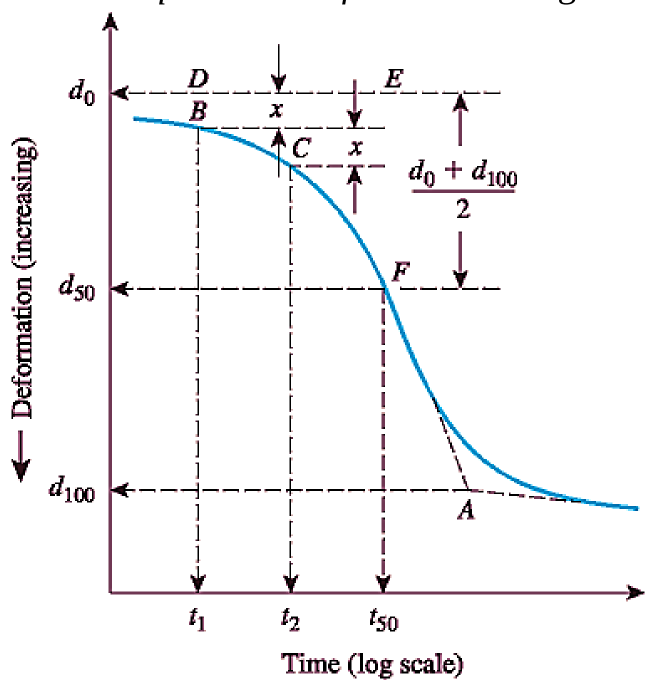


Figure (11.17) Logarithm-of-time method for determining coefficient of consolidation

Logarithm-of-Time Method

For a given incremental loading of the laboratory test, the specimen deformation against log-of-time plot is shown in Figure (11.17). The following constructions are needed to determine c_v.

Step 1: Extend the straight-line portions of primary and secondary consolidations to intersect at *A*. The ordinate of *A* is represented by d_{100}—that is, the deformation at the end of 100% primary consolidation.

Step 2: The initial curved portion of the plot of deformation versus log t is approximated to be a parabola on the natural scale. Select times t_1 and t_2 on the curved portion such that $t_2 = 4t_1$. Let the difference of specimen deformation during time $(t_2 - t_1)$ be equal to x.

Step 3: Draw a horizontal line *DE* such that the vertical distance *BD* is equal to x. The deformation corresponding to the line *DE* is d_0 (that is, deformation at 0% consolidation).

Step 4: The ordinate of point *F* on the consolidation curve represents the deformation at 50% primary consolidation, and its abscissa represents the corresponding time (t_{50}).

Step 5: For 50% average degree of consolidation, T_v = 0.197 (see Table 11.1), so,

$$T_{50} = \frac{c_v\, t_{50}}{H_{dr}^2}$$

or

$$c_v = \frac{0.197\, H_{dr}^2}{t_{50}} \tag{11.31}$$

where H_{dr} = average longest drainage path during consolidation.

For specimens drained at both top and bottom, H_{dr} equals one-half the average height of the specimen during consolidation. For specimens drained on only one side, H_{dr} equals the average height of the specimen during consolidation.

Square-Root-of-Time Method

In the square-root-of-time method, a plot of deformation against the square root of time is made for the incremental loading (Figure 11.18). Other graphic constructions required are as follows:

Step 1: Draw a line *AB* through the early portion of the curve.

Step 2: Draw a line *AC* such that $\overline{OC} = 1.15\, \overline{OB}$. The abscissa of point *D*, which is the intersection of *AC* and the consolidation curve, given the square root of time for 90 % consolidation ($\sqrt{t_{90}}$).

Step 3 : For 90% consolidation, T_{90} =0.848 (see Table 11.1), so

$$T_{90} = 0.848 = \frac{c_v\, t_{90}}{H_{dr}^2}$$

or

$$c_v = \frac{0.848 H_{dr}^2}{t_{90}} \tag{11.32}$$

H_{dr} in Eq. (11.32) is determined in a manner similar to that in the logarithm-of-time method.

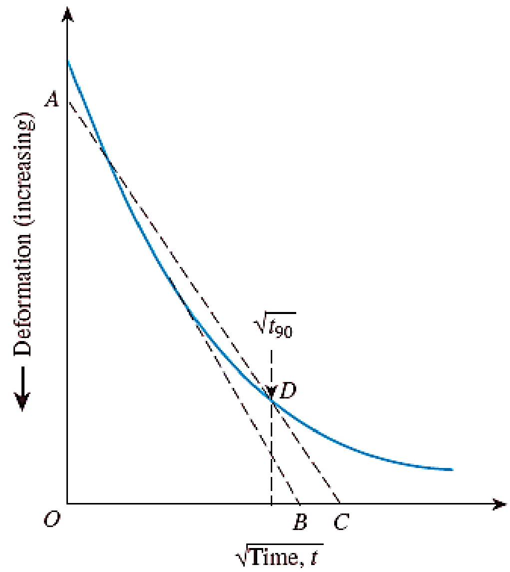


Figure 11.18 Square-root-of-time fitting method

Example 11.10

During a laboratory consolidation test, the time and dial gauge readings obtained from an increase of pressure on the specimen from 50 kN/m^2 to 100 kN/m^2 are given here.

Time (min)	Dial gauge reading (cm × 10^4)	Time (min)	Dial gauge reading (cm × 10^4)
0	3975	16.0	4572
0.1	4082	30.0	4737
0.25	4102	60.0	4923
0.5	4128	120.0	5080
1.0	4166	240.0	5207
2.0	4224	480.0	5283
4.0	4298	960.0	5334
8.0	4420	1440.0	5364

Using the logarithm-of-time method, determine c_v. The average height of the specimen during consolidation was 2.24 cm, and it was drained at the top and bottom.

Solution

The semi-logarithmic plot of dial reading versus time is shown in Figure (11.19). For this, t_1 = 0.1 min, t_2 = 0.4 min to determine d_o. Following the procedure outlined in Figure (11.17), t_{50} = 19 min. From Eq. (11.31)

$$c_v = \frac{0.197H_{dr}^2}{t_{50}} = \frac{0.197(\frac{2.24}{2})^2}{19} = 0.013\frac{cm^2}{min} = 2.17 \times 10^{-4} cm^2/sec$$

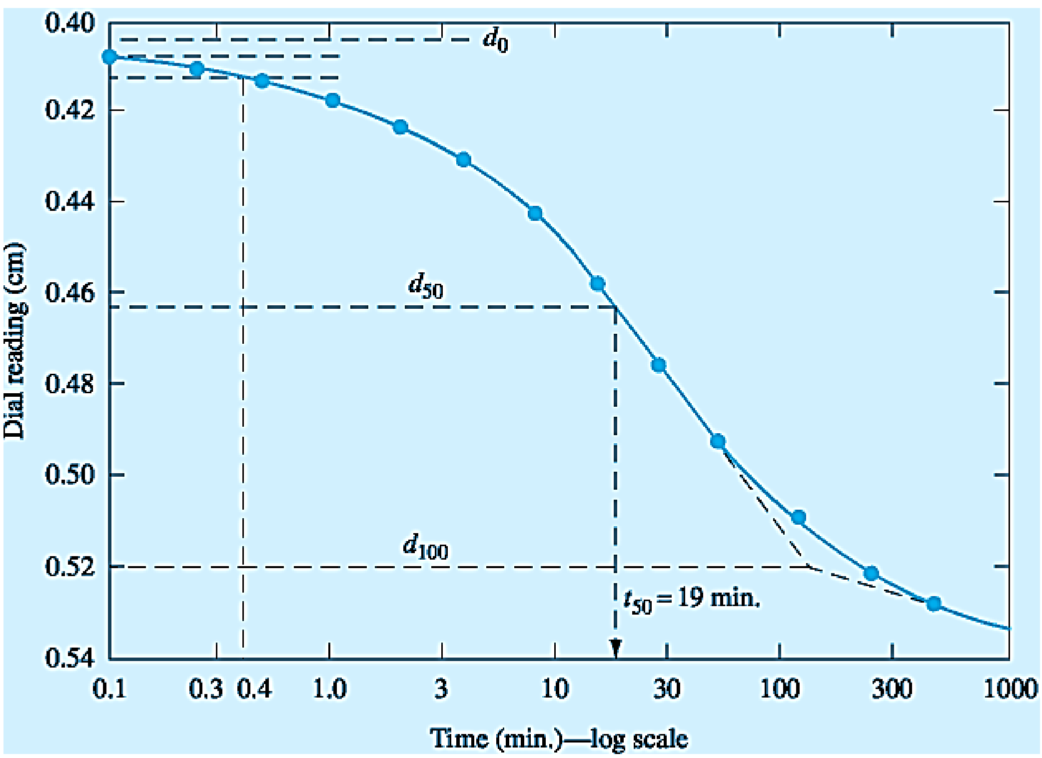


Figure 11.19

11.10 Methods for Accelerating Consolidation Settlement

In many instances, *sand drains* and *prefabricated vertical drains* are used in the field to accelerate consolidation settlement in soft, normally consolidated clay layers and to achieve precompression before the construction of a desired foundation. Sand drains are constructed by drilling holes through the clay layer(s) in the field at regular intervals. The holes then are backfilled with sand.

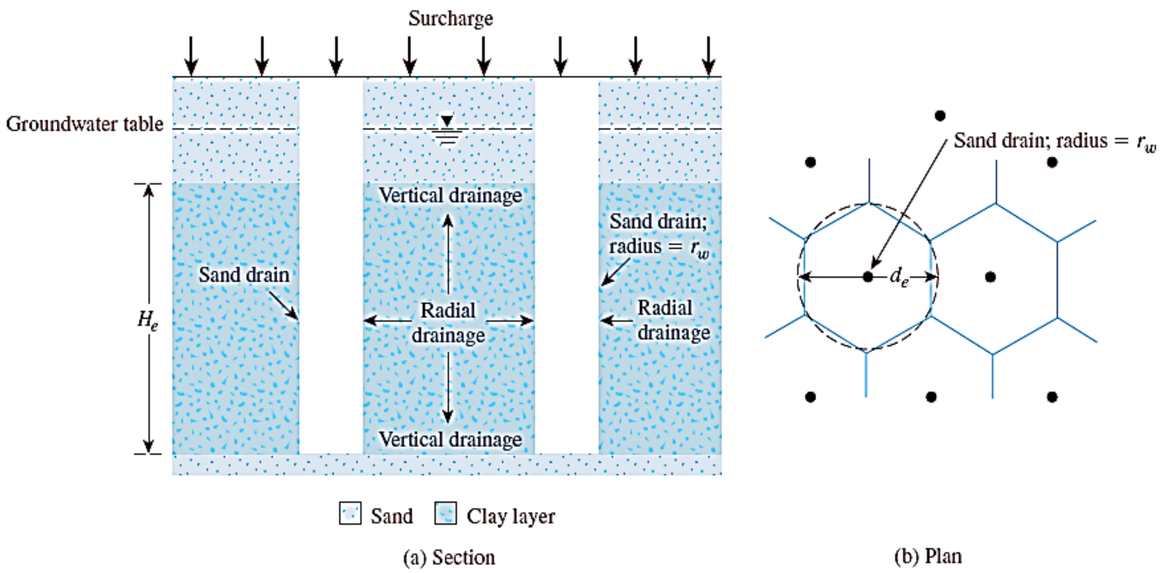


Figure 11.20 Sand drains

Figure (11.20) shows a schematic diagram of sand drains. After backfilling the drill holes with sand, a surcharge is applied at the ground surface. This surcharge will increase the pore water pressure in the clay. The excess pore water pressure in the clay will be dissipated by drainage—both vertically and radially to the sand drains—which accelerates settlement of the clay layer. In Figure (11.20a), note that the radius of the sand drains is r_w. Figure (11.20b) shows the plan of the layout of the sand drains. The effective zone from which the radial drainage will be directed toward a given sand drain is approximately cylindrical, with a diameter of d_e. The surcharge that needs to be applied at the ground surface and the length of time it has to be maintained to achieve the desired degree of consolidation will be a function of r_w, d_e, and other soil parameters.

Prefabricated vertical drains (PVDs), which also are referred to as *wick* or *strip drains*, originally were developed as a substitute for the commonly used sand drain. With the advent of materials science, these drains are manufactured from synthetic polymers such as polypropylene and high-density polyethylene. PVDs normally are manufactured with a corrugated or channeled synthetic core enclosed by a geotextile filter, as shown schematically in Figure (11.21). Installation rates reported in the literature are on the order of 0.1 to 0.3 *m/s*, excluding equipment mobilization and setup time. PVDs have been used extensively in the past for expedient consolidation of low permeability soils under surface surcharge. The main advantage of PVDs over sand drains is that they do not require drilling and, thus, installation is much faster.

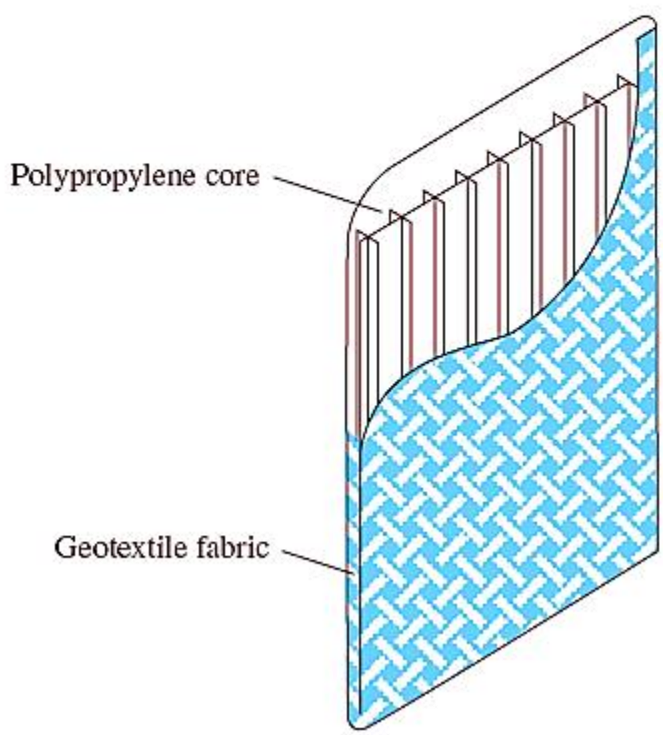


Figure 11.21 Prefabricated vertical drain (PVD)

Problems

11.1 The following are the results of a consolidation test.

e	Pressure, σ' (kN/m^2)
1.1	25
1.085	50
1.055	100
1.01	200
0.94	400
0.79	800
0.63	1600

a. Plot the e-log σ' curve.
b. Using Casagrande's method, determine the preconsolidation pressure.
c. Calculate the compression index, C_c, from the laboratory e-log σ' curve.

Ans: (b) 310 kN/m^2 (c) 0.53

11.2 The results of a laboratory consolidation test on a clay specimen are the following.

Pressure, σ' (kN/m^2)	H (mm)
23.94	17.65
47.88	17.40
95.76	17.03
191.52	16.56
383.04	16.15
766.08	15.88

Given the initial height of specimen = 19.91 mm, G_s = 2.68, mass of dry specimen = 95.2 g, and area of specimen = 3167.7 mm^2
a. Plot the e-log σ' curve
b. Determine the preconsolidation pressure
c. Calculate the compression index, C_c

Ans: (b) 940 kN/m^2 (c) 0.133

11.3 Refer to Figure (11.22). Given: $H_1 = 2.5\ m$, $H_2 = 2.5\ m$, $H_3 = 3\ m$, and $\Delta\sigma = 100\ kN/m^2$. Also,

- Sand: $e = 0.64$, $G_s = 2.65$
- Clay: $e = 0.9$, $G_s = 2.75$, LL= 55 , $C_c = 0.405$

Estimate the primary consolidation settlement of the clay layer assuming that it is normally consolidated.

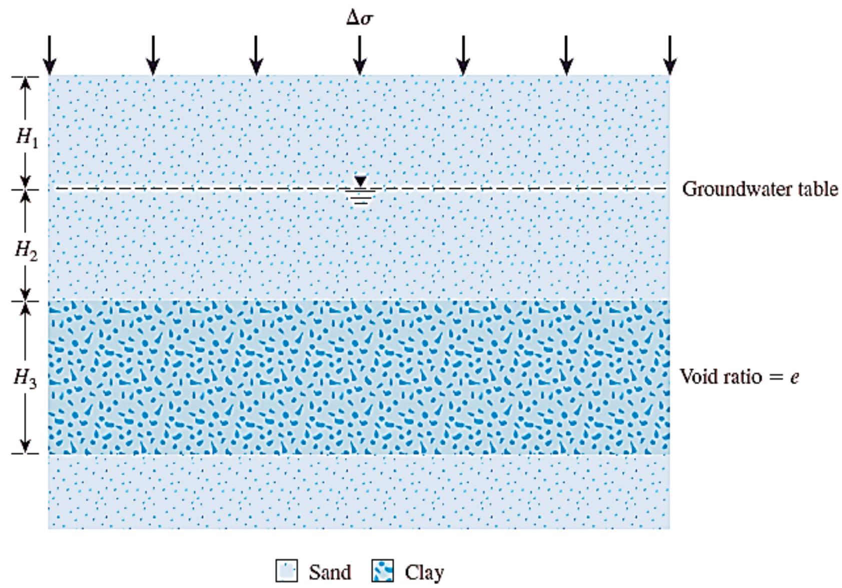


Figure (11.22)

11.4 The coordinates of two points on a virgin compression curve are as follows:

- $e_1 = 0.82$ • $\sigma'_1 = 119.7\ kN/m^2$
- $e_2 = 0.70$ • $\sigma'_2 = 191.5\ kN/m^2$

Determine the void ratio that corresponds to a pressure of 287.3 kN/m^2.

11.5 Refer to Problem 11.3. Given: $c_v = 2.8 \times 10^{-6}\ m^2/min$. How long will it take for 60% consolidation to occur?

Ans: 159.6 days

11.6 The coordinates of two points on a virgin compression curve are as follows:

- $e_1 = 1.7$ • $\sigma'_1 = 150\ kN/m^2$
- $e_2 = 1.48$ • $\sigma'_2 = 400\ kN/m^2$

a. Determine the coefficient of volume compressibility for the pressure range stated.

b. Given that c_v = 0.002 cm^2/sec, determine k in cm/sec corresponding to the average void ratio.

Ans: (a) 0.00034 m^2/kN (b) 6.67×10^{-8} cm/sec

11.7 For a normally consolidated clay, the following are given:

- $\sigma'_o = 191.5\frac{kN}{m^2}$, $e = e_o = 1.21$
- $\sigma'_o + \Delta\sigma' = 383\frac{kN}{m^2}$, $e = 0.96$

The hydraulic conductivity k of the clay for the preceding loading range is 5.5×10^{-5} m/day.

a. How long (*in days*) will it take for a 2.74 m thick clay layer (drained on one side) in the field to reach 60% consolidation?

b. What is the settlement at that time (that is, at 60% consolidation)?

Ans: (a) 240.8 *days* (b) 186 *mm*

11.8 Determine the hydraulic conductivity of the clay for the loading range. The time for 50% consolidation of a 25-*mm* thick clay layer (drained at top and bottom) in the laboratory is 225 *sec*. How long (in days) will it take for a 2-*m* thick layer of the same clay in the field (under the same pressure increment) to reach 50% consolidation? There is a rock layer at the bottom of the clay in the field.

Ans: 66.7 days

11.9 A normally consolidated clay layer is 3 *m* thick (one-way drainage). From the application of a given pressure, the total anticipated primary consolidation settlement will be 80 *mm*.

a. What is the average degree of consolidation for the clay layer when the settle ment is 25 *mm*?

b. If the average value of c_v for the pressure range is 0.002 cm^2/sec, how long will it take for 50% settlement to occur?

c. How long will it take for 50% consolidation to occur if the clay layer is drained at both top and bottom?

Ans: (a) 31.25 % (b) 102.6 days (c) 25.65 days