Chapter 9: In Situ Stresses

As described in Chapter 3, soils are multiphase systems. In a given volume of soil, the solid particles are distributed randomly with void spaces between. The void spaces are continuous and are occupied by water and/or air. To analyze problems (such as compressibility of soils, bearing capacity of foundations, stability of embankments, and lateral pressure on earth-retaining structures), we need to know the nature of the distribution of stress along a given cross section of the soil profile.

9.1 Stresses in Saturated Soil without Seepage

Figure (9.1) shows a column of saturated soil mass with no seepage of water in any direction. The total stress at the elevation of point A can be obtained from the saturated unit weight of the soil and the unit weight of water above it. Thus,

$$\sigma = H\gamma_w + (H_A - H)\gamma_{sat} \tag{9.1}$$

where σ = total stress at the elevation of point A

γ_w = unit weight of water

γ_{sat} = saturated unit weight of the soil

H = height of water table from the top of the soil column

H_A = distance between point A and the water table

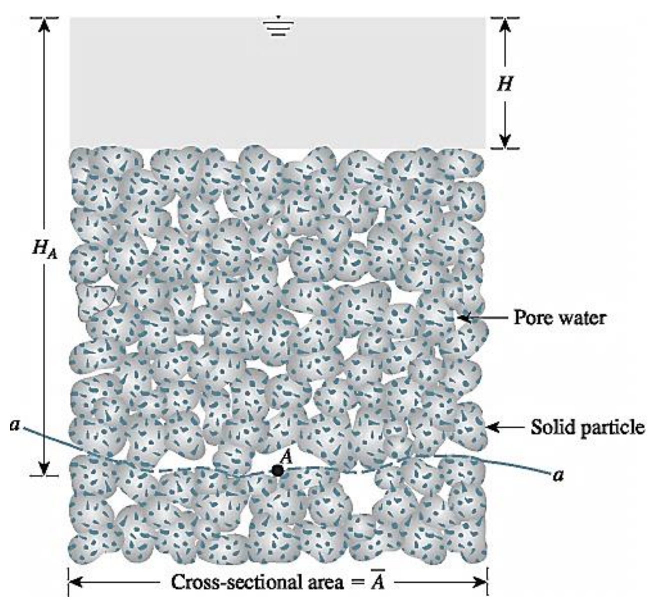

Figure 9.1 Effective stress considerations for a saturated soil column without seepage.

The total stress, σ, given by Eq. (9.1) can be divided into two parts:

1. A portion is carried by water in the continuous void spaces. This portion acts with equal intensity in all directions.
2. The rest of the total stress is carried by the soil solids at their points of contact. The sum of the vertical components of the forces developed at the points of contact of the solid particles per unit cross-sectional area of the soil mass is called the *effective stress*.

$$\sigma = \sigma' + u \qquad (9.2)$$

or

$$\sigma' = \sigma - u \qquad (9.3)$$

Where u is the hydrostatic pressure at A (also referred to as pore water pressure or neutral stress):

$$u = H_A \gamma_w \qquad (9.4)$$

Substitution of Eq. (9.1) for σ in Eq. (9.4) gives

$$\begin{aligned} \sigma' &= \left[H\gamma_w + (H_A - H)\gamma_{sat}\right] - H_A\gamma_w \\ &= (H_A - H)(\gamma_{sat} - \gamma_w) \\ &= (Height\ of\ the\ soil\ column) \times \gamma' \end{aligned} \qquad (9.5)$$

where $\gamma' = \gamma_{sat} - \gamma_w$ equals the submerged unit weight of soil. Thus, we can see that the effective stress at any point A is independent of the depth of water, H, above the submerged soil.

In summary, effective stress is approximately the force per unit area carried by the soil skeleton. The effective stress in a soil mass controls its volume change and strength. Increasing the effective stress induces soil to move into a denser state of packing.

Example 9.1
A soil profile is shown in Figure (9.2). Calculate the total stress, pore water pressure, and effective stress at points *A*, *B*, and *C*.

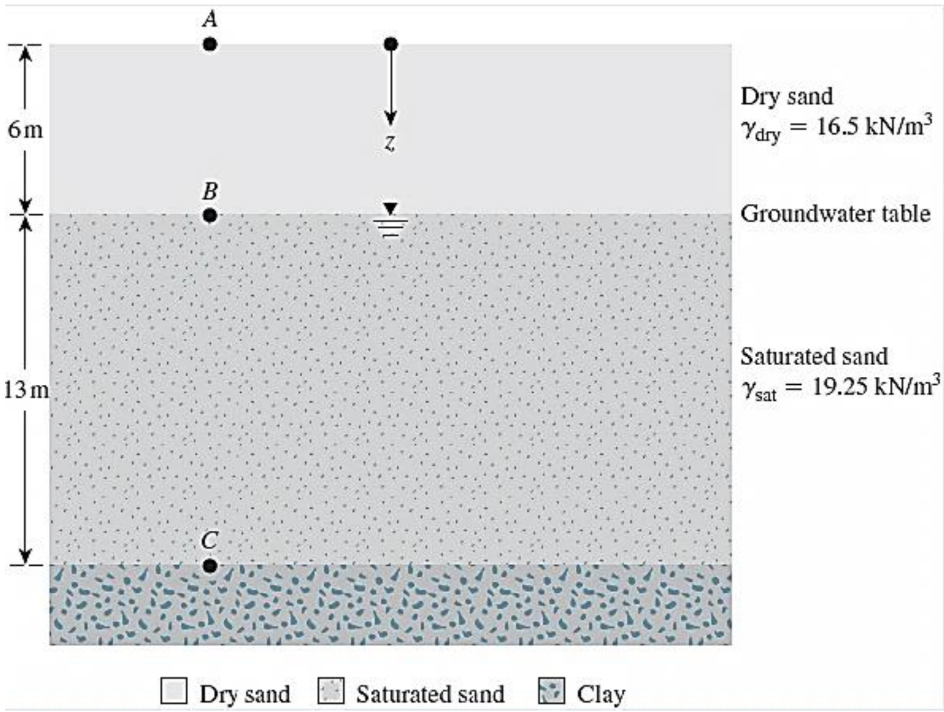

Figure 9.2 Soil profile

Solution
At Point *A*,

Total stress: $\sigma_A = 0$
Pore water pressure : $u_A = 0$
Effective stress: $\sigma'_A = 0$

At point *B*,

$$\sigma_B = 6\gamma_{dry(sand)} = 6 \times 16.5 = 99 kN/m^2$$
$$u_B = 0\ kN/m^2$$
$$\sigma'_B = 99 - 0 = 99 kN/m^2$$

At point *C*,

$$\sigma_c = 6\gamma_{dry(sand)} + 13\gamma_{sat(sand)}$$
$$= 6 \times 16.5 + 13 \times 19.25$$
$$= 99 + 250.25 = 349.25\ kN/m^2$$
$$u_C = 13\gamma_w = 13 \times 9.81 = 127.53 kN/m^2$$
$$\sigma'_C = 349.25 - 127.53 = 221.72 kN/m^2$$

9.2 Stresses in Saturated Soil with Upward Seepage

If water is seeping, the effective stress at any point in a soil mass will differ from that in the static case. It will increase or decrease, depending on the direction of seepage.

Figure (9.3) shows a layer of granular soil in a tank where upward seepage is caused by adding water through the valve at the bottom of the tank. The rate of water supply is kept constant. The loss of head caused by upward seepage between the levels of *A* and *B* is *h*.

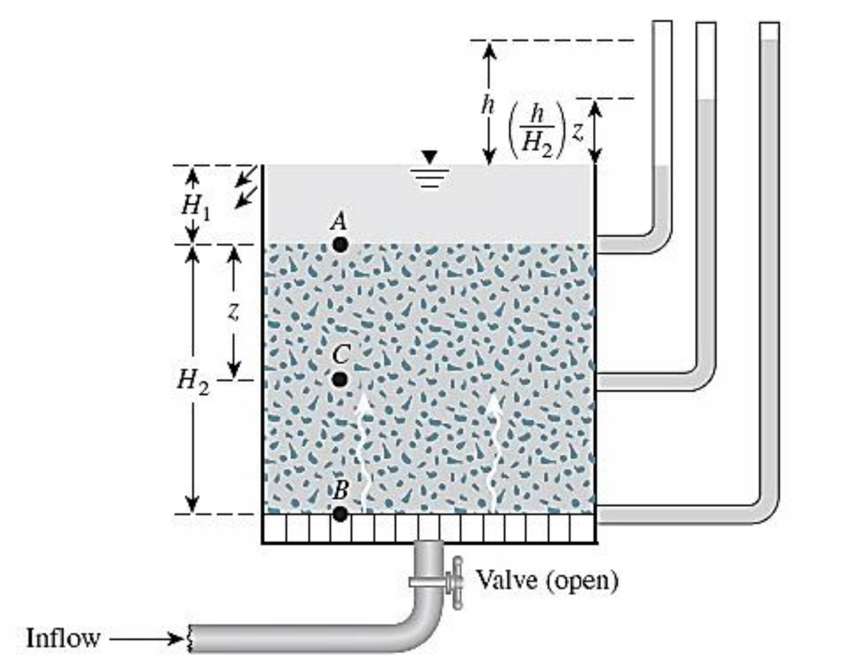

Figure 9.3 Layer of soil in a tank with upward seepage

The effective stress at a point *C* located at a depth *z* below the top of the soil surface can be calculated as follows:

- The total stress: $\sigma_C = H_1\gamma_w + z\gamma_{sat}$
- Pore water pressure: $u_c = (H_1 + z + \frac{h}{H_2}z)\gamma_w$
- Effective stress: $\sigma'_C = \sigma_C - u_C$

$$= z(\gamma_{sat} - \gamma_w) - \frac{h}{H_2}z\gamma_w$$

$$= z\gamma' - \frac{h}{H_2}z\gamma_w$$

Note that ***h/H₂*** is the hydraulic gradient ***i*** caused by the flow, and therefore,

$$\sigma'_C = z\gamma' - iz\gamma_w \qquad (9.6)$$

Example 9.2

A 6m thick layer of stiff saturated clay is underlain by a layer of sand (Figure 9.4). The sand is under artesian pressure. Calculate the maximum depth of cut H that can be made in the clay.

Solution

Due to excavation, there will be unloading of the overburden pressure. Let the depth of the cut be H, at which point the bottom will heave. Let us consider the stability of point A at that time:

$$\sigma_A = (6 - H)\gamma_{sat}$$

$$u_A = 3.75\gamma_w$$

For heave to occur σ'_A should be 0. So

$$\sigma_A - u_A = (6 - H)\gamma_{sat(clay)} - 3.75\gamma_w = 0$$

or

$$(6 - H)18.9 - 3.75 \times 9.81 = 0$$

$$H = \frac{6\times18.9-3.75\times9.81}{18.9} = 4.05m$$

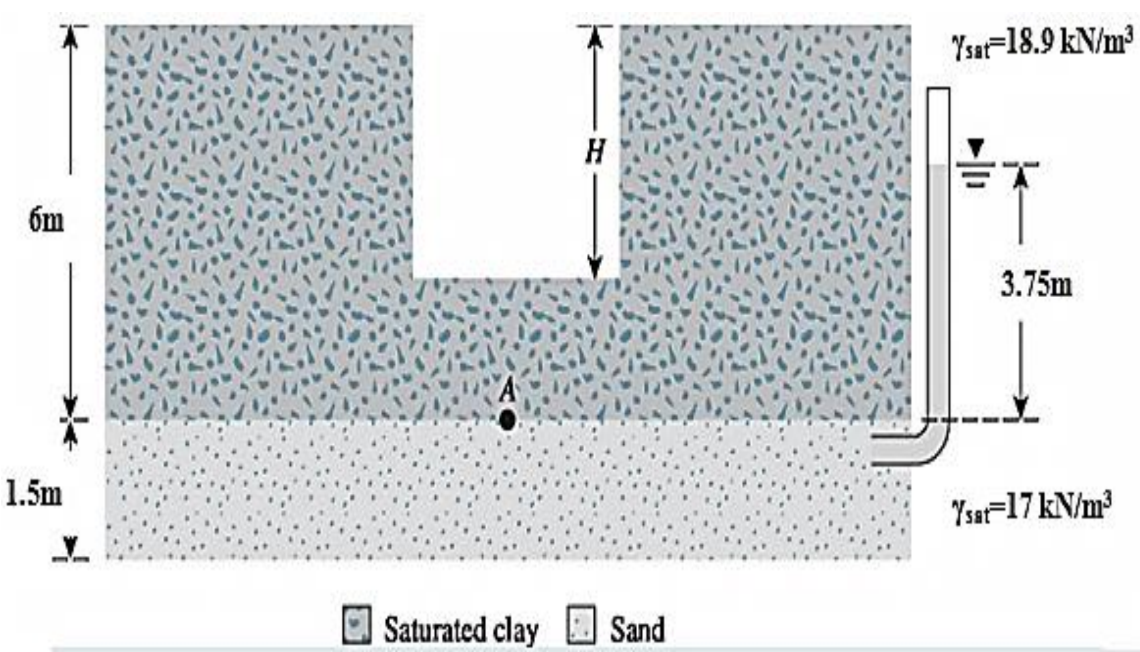

Figure (9.4)

9.3 Stresses in Saturated Soil with Downward Seepage

The condition of downward seepage is shown in Figure (9.5). The water level in the soil tank is held constant by adjusting the supply from the top and the outflow at the bottom.

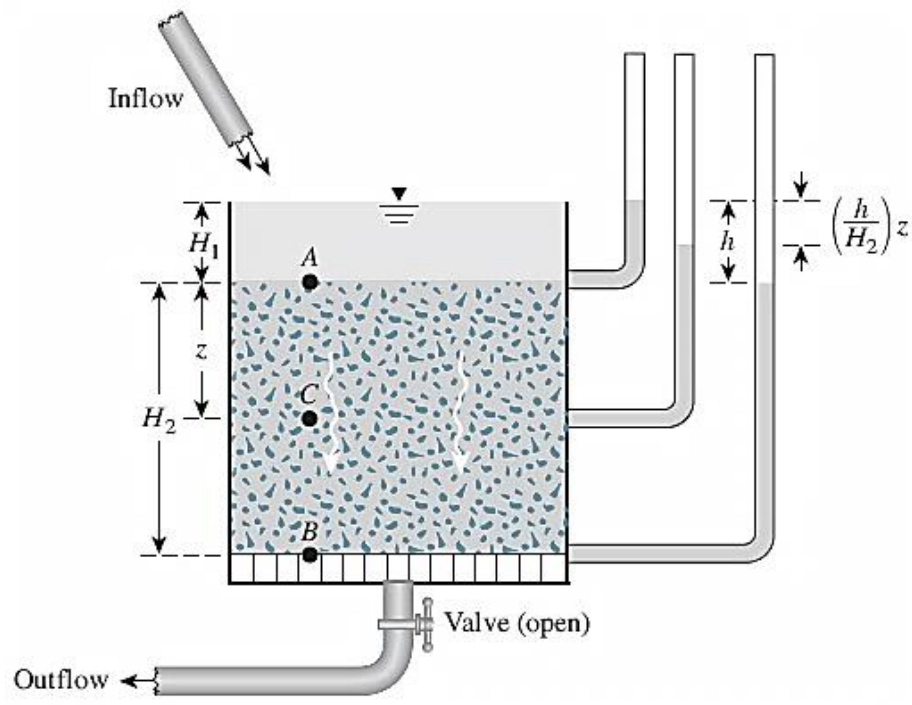

Figure 9.5 Layer of soil in a tank with downward seepage

The hydraulic gradient caused by the downward seepage equals $i = h/H_2$. The total stress, pore water pressure, and effective stress at any point C are, respectively,

$$\sigma_C = H_1\gamma_w + z\gamma_{sat}$$

$$u_C = (H_1 + z - iz)\gamma_w$$

$$\sigma'_C = (H_1\gamma_w + z\gamma_{sat}) - (H_1 + z - iz)\gamma_w$$

$$= z\gamma' + iz\gamma_w \quad (9.7)$$

Example 9.3

Consider the upward flow of water through a layer of sand in a tank as shown in Figure (9.6). For the sand, the following are given: void ratio (e) = 0.52 and specific gravity of solids = 2.67.

a. Calculate the total stress, pore water pressure, and effective stress at points *A* and *B*.

b. What is the upward seepage force per unit volume of soil?

Solution

Part a

The saturated unit weight of sand is calculated as follows:

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.67 + 0.52)9.81}{1 + 0.52} = 20.59 kN/m^3$$

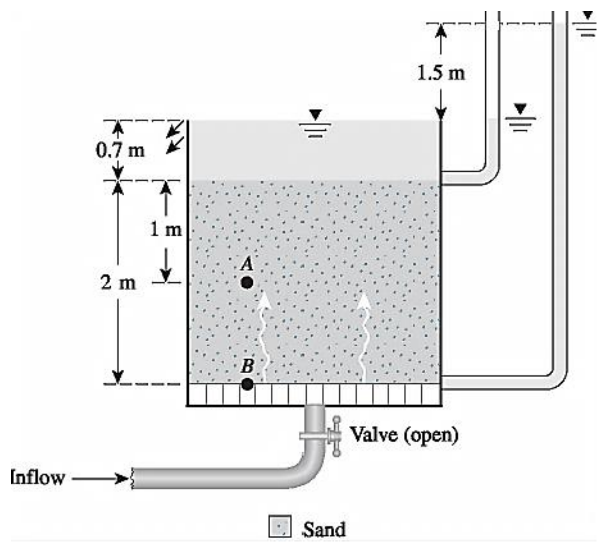

Figure 9.6 Upward flow of water through a layer of sand in a tank

Now, the following table can be prepared:

point	Total stress,σ (kN/m^2)	Pore water pressure, u (kN/m^2)	Effective stresss, $\sigma' = \sigma - u$ (kN/m^2)
A	$0.7\gamma_w + 1\gamma_{sat} = (0.7)(9.81) + (1)(20.59) = 27.46$	$[(1 + 0.7) + (1.5/2)(1)]\gamma_w = (2.45)(9.81) = 24.03$	3.43
B	$0.7\gamma_w + 2\gamma_{sat} = (0.7)(9.81) + (2)(20.59) = 48.05$	$(2 + 0.7 + 1.5)\gamma_w = (4.2)(9.81) = 41.2$	6.85

Part b

Hydraulic gradient (i) = 1.5/2 = 0.75. Thus, the seepage force per unit volume can be calculated as

$$i\gamma_w = (0.75)(9.81) = 7.36 \text{ kN/m}^3$$

Problems

9.1 and **9.2** Refer to Figure (9.7). Calculate σ, u, and σ' at *A*, *B*, *C*, and *D* for the following cases and plot the variations with depth. (Note: e = void ratio, w = moisture content, G_s = specific gravity of soil solids, γ_d = dry unit weight, and γ_{sat} = saturated unit weight.)

Details of soil layer			
Problem	I	II	III
9.1	$H_1 = 3\ m$ $\gamma_d = 15\ kN/m^3$	$H_2 = 4\ m$ $\gamma_{sat} = 16\ kN/m^3$	$H_3 = 5\ m$ $\gamma_{sat} = 18\ kN/m^3$
9.2	$H_1 = 4\ m$ $e = 0.6$ $G_s = 2.65$	$H_2 = 3\ m$ $e = 0.52$ $G_s = 2.68$	$H_3 = 1.5\ m$ $w = 40\ \%$ $e = 1.1$

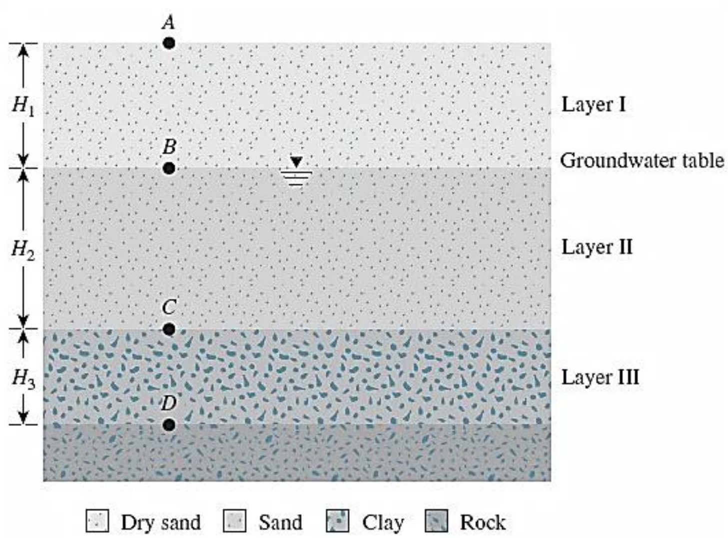

Figure 9.7

Ans

9.1

	kN/m²		
Point	σ	u	σ'
A	0	0	0
B	45	0	45
C	109	39.24	69.76
D	199	88.29	110.71

9.2

	kN/m²		
Point	σ	u	σ'
A	0	0	0
B	65	0	65
C	126.95	29.43	97.52
D	153.94	44.158	109.79

9.3 Refer to the soil profile shown in Figure (9.8).

a. Calculate the variation of σ, u, and σ' with depth.

b. If the water table rises to the top of the ground surface, what is the change in the effective stress at the bottom of the clay layer ?

c. How many meters must the groundwater table rise to decrease the effective stress by 15 kN/m^2 at the bottom of the clay layer ?

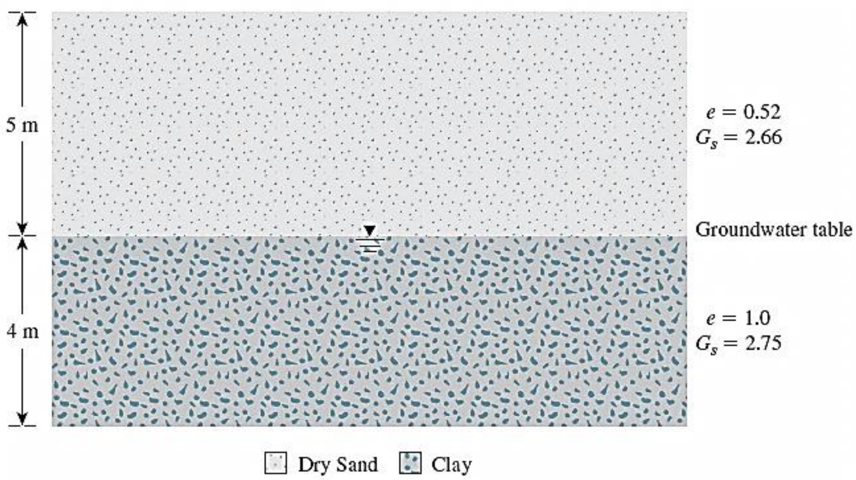

Figure (9.8)

Ans:

a.

Depth(m)	σ (kN/m²)	u (kN/m²)	$\bar{\sigma}$ (kN/m²)
0	0	0	0
5	85.85	0	85.85
9	159.41	39.24	198.65

b. σ' = 110.78 (kN/m²)

c. h = 42.85 m

9.4 An exploratory drill hole was made in a stiff saturated clay (see Figure 9.9). The sand layer underlying the clay was observed to be under artesian pressure. Water in the drill hole rose to a height of 4 *m* above the top of the sand layer. If an open excavation is to be made in the clay, how deep can the excavation proceed before the bottom heaves?

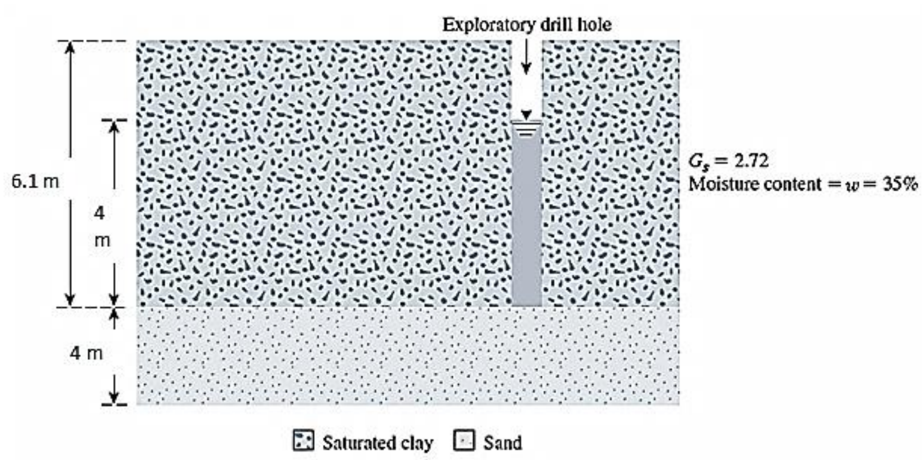

Figure (9.9)

Ans: 3.97 m

9.5 A cut is made in stiff saturated clay that is underlain by a layer of sand. (See Figure 9.10). What should be the height of the water, *h*, in the cut so that the stability of the saturated clay is not lost?

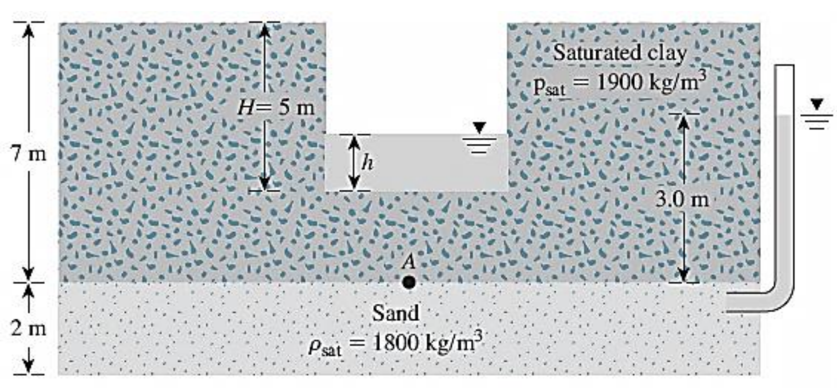

Figure (9.10)

Ans: *h = 3.6 m*

9.6 Refer to Figure (9.3). If $H_1 = 1.5$ m, $H_2 = 2.5$ m, $h = 1.5$ m, $\gamma_{sat} = 18.6$ kN/m^3, hydraulic conductivity of sand $(k) = 0.12$ cm/sec, and area of tank = 0.45 m^2, what is the rate of upward seepage of water (m^3/min)?

Ans: $\boldsymbol{q = 0.019\ m^3/min}$

9.7 Refer to Figure (9.3). Given: $H_1 = 1\ m$, $H_2 = 2\ m$, $h = 1.2\ m$, void ratio of sand (e) = 0.55, specific gravity of soil solids $(G_s) = 2.68$, area of the tank = $0.5\ m^2$, and hydraulic conductivity of sand = $0.1\ cm/sec$.
a. What is the rate of upward seepage?
b. If $h = 1.2\ m$, will boiling occur? Why?
c. What should be the value of h to cause boiling?

Ans: a. $\boldsymbol{q = 0.018\ m^3/min}$ b. No boiling occur c. h= 1.2m