Chapter 8: Seepage

In the preceding chapter, we considered some simple cases for which direct application of Darcy's law was required to calculate the flow of water through soil. In many instances, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow. In such cases, the groundwater flow is generally calculated by the use of graphs referred to as flow nets. The concept of the flow net is based on Laplace's equation of continuity, which governs the steady flow condition for a given point in the soil mass. In the following sections of this chapter, the derivation of Laplace's equation of continuity will be presented along with its application to seepage problems.

8.1 Laplace's Equation of Continuity

To derive the Laplace differential equation of continuity, let us consider a single row of sheet piles that have been driven into a permeable soil layer, as shown in Figure (8.1a). The row of sheet piles is assumed to be impervious. The steady state flow of water from the upstream to the downstream side through the permeable layer is a two-dimensional flow. For flow at a point A, we consider an elemental soil block. The block has dimensions *dx*, *dy*, and *dz* (length *dy* is perpendicular to the plane of the paper); it is shown in an enlarged scale in Figure (8.1b). Let v_x and v_z be the components of the discharge velocity in the horizontal and vertical directions, respectively. The rate of flow of water into the elemental block in the horizontal direction is equal to $v_x\, dz\, dy$, and in the vertical direction it is $v_z\, dx\, dy$. The rates of outflow from the block in the horizontal and vertical directions are, respectively,

$$\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dzdy$$

and

$$\left(v_z + \frac{\partial v_z}{\partial z} dz\right) dxdy$$

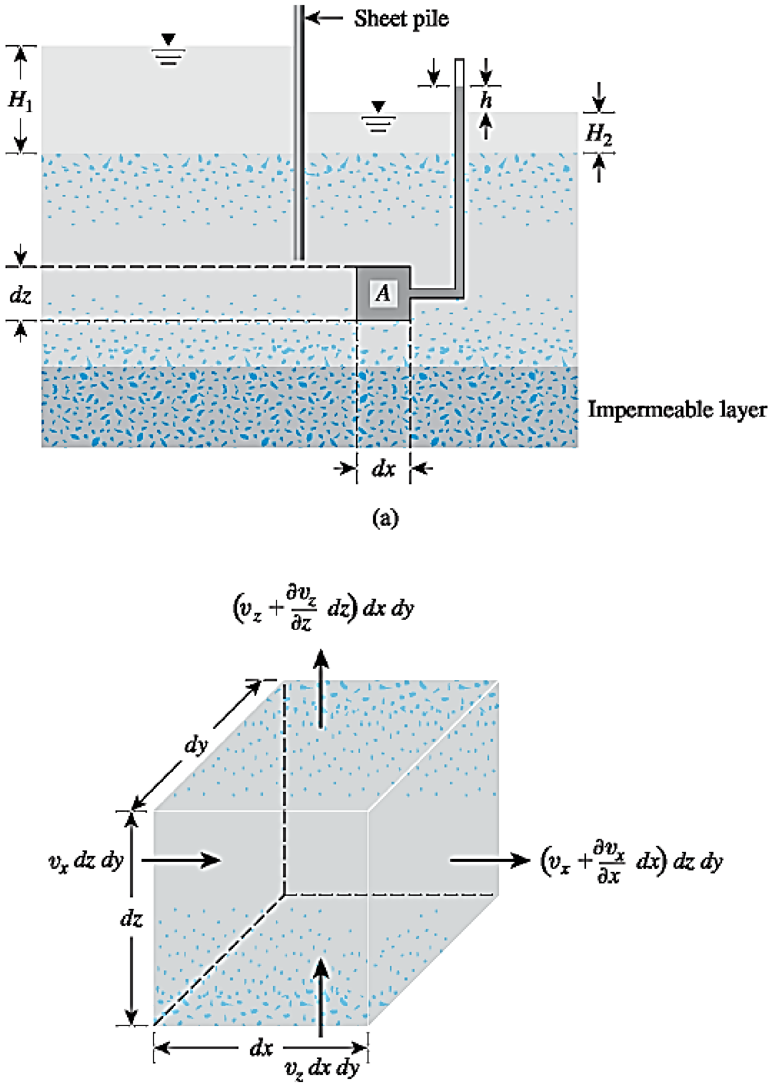


Figure 8.1 (a) Single-row sheet piles driven into permeable layer; (b) flow at A

Assuming that water is incompressible and that no volume change in the soil mass occurs, we know that the total rate of inflow should equal the total rate of outflow. Thus,

$$\left[\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dzdy + \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dxdy\right] - [v_x dzdy + v_z dxdy] = 0$$

or

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \qquad (8.1)$$

With Darcy's law, the discharge velocities can be expressed as

$$v_x = k_x i_x = k_x \frac{\partial h}{\partial x} \quad (8.2)$$

and

$$v_z = k_z i_z = k_z \frac{\partial h}{\partial z} \quad (8.3)$$

where k_x and k_z are the hydraulic conductivities in the horizontal and vertical directions, respectively.

From Eqs. (8.1), (8.2), and (8.3), we can write

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (8.4)$$

If the soil is isotropic with respect to the hydraulic conductivity—that is, $k_x = k_z$ —the preceding continuity equation for two-dimensional flow simplifies to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (8.5)$$

8.2 Flow Nets

The continuity equation [Eq. (8.5)] in an isotropic medium represents two orthogonal families of curves—that is, the flow lines and the equipotential lines. *A flow line* is a line along which a water particle will travel from upstream to the downstream side in the permeable soil medium. An *equipotential line* is a line along which the potential head at all points is equal. Thus, if piezometers are placed at different points along an equipotential line, the water level will rise to the same elevation in all of them. Figure (8.2a) demonstrates the

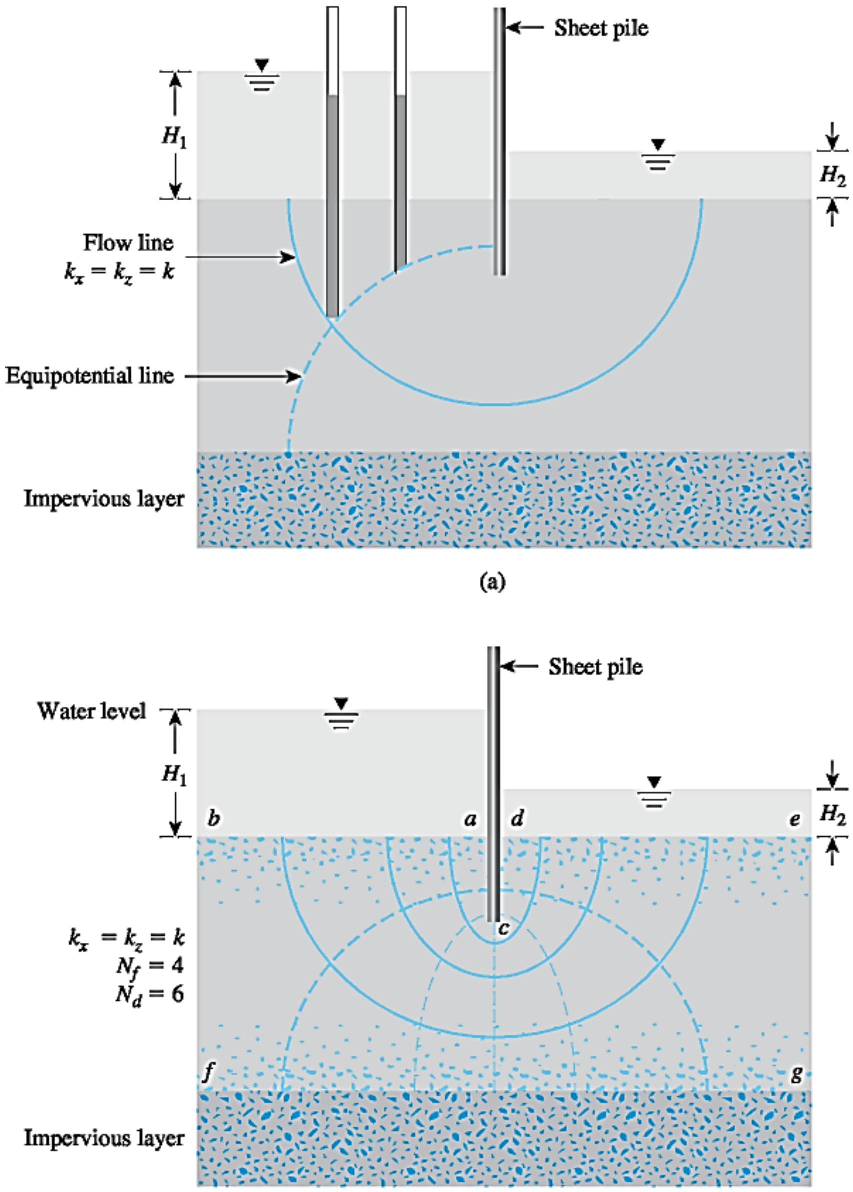


Figure 8.2 (a) Definition of flow lines and equipotential lines; (b) completed flow net

definition of flow and equipotential lines for flow in the permeable soil layer around the row of sheet piles shown in Figure (8.1) (for $k_x = k_z = k$).

A combination of a number of flow lines and equipotential lines is called a *flow net*. As mentioned in the introduction, flow nets are constructed for the calculation of groundwater flow and the evaluation of heads in the media. To complete the graphic construction of a flow net, one must draw the flow and equipotential lines in such a way that

1. The equipotential lines intersect the flow lines at right angles.
2. The flow elements formed are approximate squares.

Figure (8.2b) shows an example of a completed flow net. One more example of flow net in isotropic permeable layer are given in Figure (8.3). In these figures, N_f is the number of flow channels in the flow net, and N_d is the number of potential drops (defined later in this chapter).

Drawing a flow net takes several trials. While constructing the flow net, keep the boundary conditions in mind. For the flow net shown in Figure (8.2b), the following four boundary conditions apply:

Condition 1: The upstream and downstream surfaces of the permeable layer (lines ***ab*** and ***de***) are equipotential lines.

Condition 2: Because ***ab*** and ***de*** are equipotential lines, all the flow lines intersect them at right angles.

Condition 3: The boundary of the impervious layer—that is, line ***fg***—is a flow line, and so is the surface of the impervious sheet pile, line ***acd***.

Condition 4: The equipotential lines intersect ***acd*** and ***fg*** at right angles.

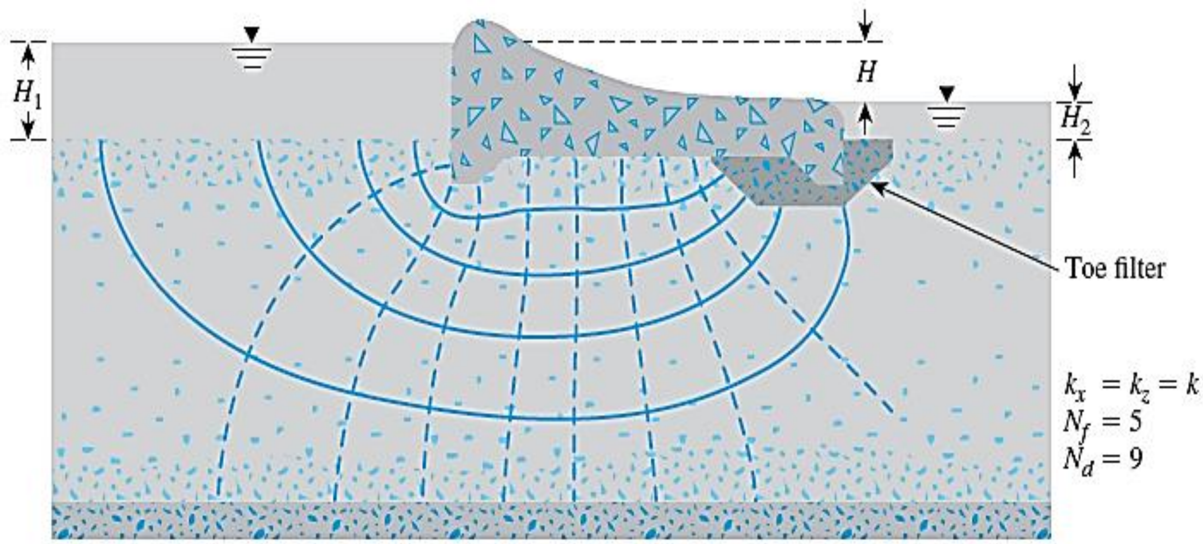


Figure 8.3 Flow net under a dam with toe filter

8.3 Seepage Calculation from a Flow Net

In any flow net, the strip between any two adjacent flow lines is called a *flow channel*. Figure (8.4) shows a flow channel with the equipotential lines forming square elements. Let h_1, h_2, h_3, h_4,.. ., h_n be the piezometric levels corresponding to the equipotential lines. The rate of seepage through the flow channel per unit length (perpendicular to the vertical section through the permeable layer) can be calculated as follows. Because there is no flow across the flow lines,

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \cdots = \Delta q \quad (8.6)$$

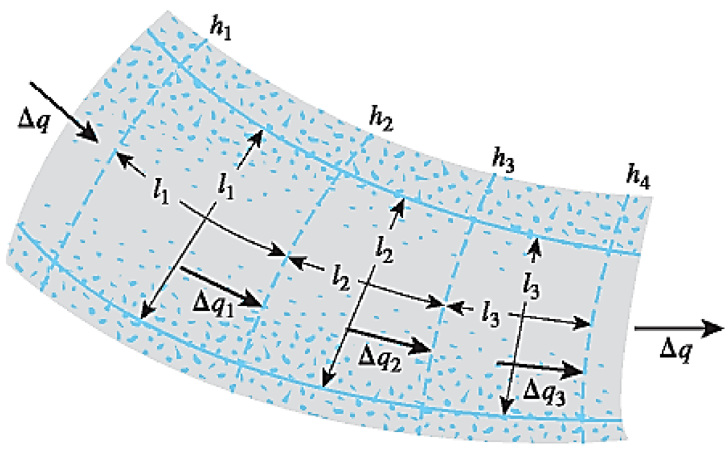


Figure 8.4 Seepage through a flow channel with square elements

From Darcy's law, the flow rate is equal to *kiA*. Thus, Eq. (8.6) can be written as

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right) l_1 = k\left(\frac{h_2 - h_3}{l_2}\right) l_2 = k\left(\frac{h_3 - h_4}{l_3}\right) l_3 = \cdots \quad (8.7)$$

Eq. (8.7) shows that if the flow elements are drawn as approximate squares, the drop in the piezometric level between any two adjacent equipotential lines is the same. This is called the *potential drop*. Thus,

$$\Delta h = h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \cdots = \frac{H}{N_d} \quad (8.8)$$

and

$$\Delta q = k\frac{H}{N_d} \quad (8.9)$$

where H = head difference between the upstream and downstream sides
N_d = number of potential drops

In Figure (8.2b), for any flow channel, $H = H_1 - H_2$ and $N_d = 6$.

If the number of flow channels in a flow net is equal to N_f, the total rate of flow through all the channels per unit length can be given by

$$q = k\frac{HN_f}{N_d} \quad (8.10)$$

8.4 Hydraulic Gradient

The hydraulic gradient over each square in flow net can be calculated by dividing the head loss, Δh, by the length, l, of the cell, that is:

$$i = \frac{\Delta h}{l} \quad (8.11)$$

Where, $$\Delta h = \frac{H}{N_d}$$

Since l is not constant, the hydraulic gradient is not constant. The maximum hydraulic gradient occurs where l is a minimum, that is:

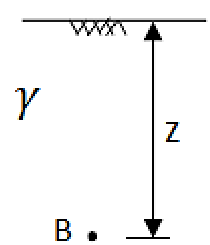
$$i_{max} = \frac{\Delta h}{l_{min}} \quad (8.12)$$

Where l_{min} is the minimum length of the cells within the flow domain. Usually, l_{min} occurs at exit points or around corners.

8.5 Seepage Stress and Seepage Force

The effective stress at a given point in a saturated soil is given by:

$$\acute{\sigma}_z = \sigma_z - u_z = \gamma.z - \gamma_w.z = (\gamma - \gamma_w) = \acute{\gamma}.z = \gamma_{sub}.z$$



If water is seeping, the effective stress at any point in a soil mass will differ from that in the static case. It will increase or decrease, depending on the direction of seepage. The seepage force per unit volume is equal to: $\gamma_w.i$. This force produces a stress with a soil mass at depth z equal to: $\gamma_w.i.z$. Thus, the effective stress will be:

$$\acute{\sigma}_z = \acute{\gamma}.z \pm \gamma_w.i.z \qquad (8.13)$$

+ for downward seepage.
− for upward seepage.

8.6 Static Liquefaction, Heaving, Boiling and Piping

If the effective stress becomes zero, the soil loses its intergranular frictional strength and behaves like a viscous fluid. The soil state at which the effective stress is zero is called static liquefaction (boiling, quicksand, piping or heaving).

$$\acute{\sigma}_z = \acute{\gamma}.z - \gamma_w.i.z = 0 \qquad (8.14)$$

High localized hydraulic gradient statically liquefies the soil, which progresses to the water surface in the form of a pipe, and water then rushes beneath the structure through the pipe, leading to instability and failure.

8.7 Critical Hydraulic Gradient

The hydraulic gradient that brings a soil mass to static liquefaction is called the critical hydraulic gradient:

$$\acute{\sigma}_z = \acute{\gamma}.z - \gamma_w.i.z = 0$$

$$\Rightarrow i = i_{cr} = \frac{\bar{\gamma}}{\gamma_w} = \left(\frac{G_s-1}{1+e}\right)\frac{\gamma_w}{\gamma_w} = \frac{G_s-1}{1+e} \qquad (8.15)$$

Since G_s is constant, the critical hydraulic gradient is a function of the void ratio. In designing structures that are subjected to steady state seepage, it is absolutely essential to ensure that the critical hydraulic gradient cannot develop. The factor of safety against boiling is defined as:

$$(F.S)_{boiling} = \frac{i_{cr}}{i_{exit}} \qquad (8.16)$$

8.8 Uplift Pressure under Hydraulic Structures

The pore water pressure at any point j is calculated as follows:

$$u_j = (h_p)_j \gamma_w \qquad (8.17)$$

where $(h_p)_j$ is the pressure head at point j given by:

$$(h_p)_j = H - (N_d)_j.\Delta h - z_j \qquad (8.18)$$

Example 8.1

A flow net for flow around a single row of sheet piles in a permeable soil layer is shown in Figure (8.5). Given that $k_x = k_z = k = 5 \times 10^{-3}$ cm/sec, determine

a. How high (above the ground surface) the water will rise if piezometers are placed at points *a* and *b*.
b. The total rate of seepage through the permeable layer per unit length
c. The approximate average hydraulic gradient at *c*.

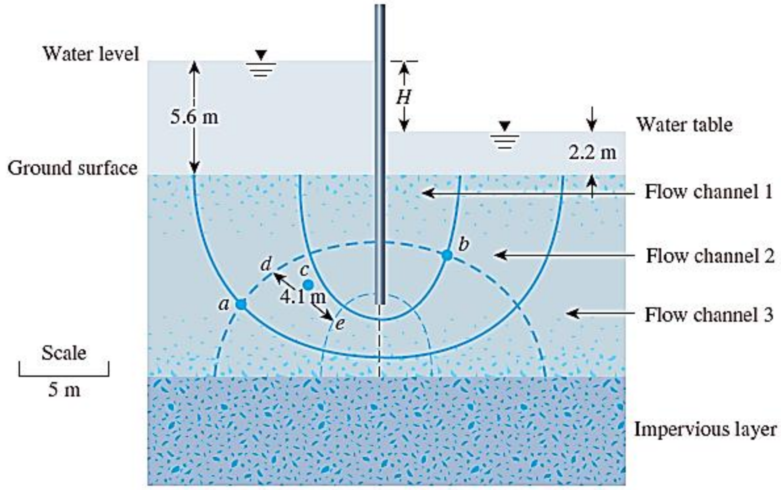


Figure 8.5 Flow net for seepage around a single row of sheet piles

Solution: Part a

From Figure (8.5), we have N_d =6, H_1=5.6m, and H_2=2.2 m. So the head loss of each potential drop is

$H = H_1 - H_2 = 5.6 - 2.2 = 3.4m$

$$\Delta h = \frac{H}{N_d} = \frac{3.4}{6} = 0.567m$$

At point a, we have gone through one potential drop. So the water in the piezometer will rise to an elevation of

(5.6- 0.567) = **5.033 m above the ground surface**

At point b, we have five potential drops. So the water in the piezometer will rise to an elevation of

(5.6- 5 x 0.567) = **2.765 m above the ground surface**

Part b

From Eq. (8.10),

$$q = k\frac{HN_f}{N_d} = (5 \times 10^{-5})(3.4)\frac{2.38}{6} = 6.74 \times 10^{-5}\ m^3/sec/m$$

Part c

The average hydraulic gradient at c can be given as

$$i = \frac{head\ loss}{average\ length\ of\ flow\ between\ d\ and\ e} = \frac{\Delta h}{\Delta l} = \frac{0.567m}{4.1m} = 0.138$$

(Note: The average length of flow has been scaled)

Example 8.2
Draw the uplift pressure distribution under the hydraulic structure shown.

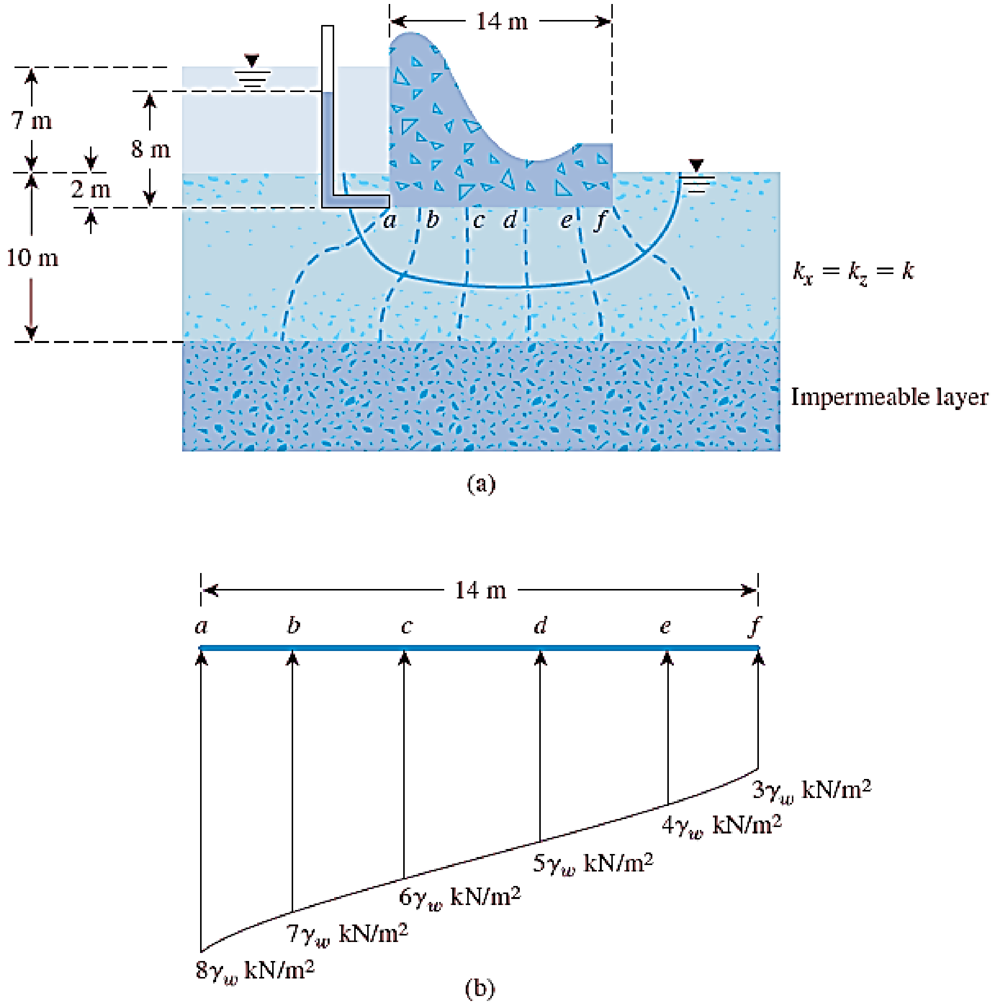


Figure 8.6 (a) A weir; (b) uplift force under a hydraulic structure

H = 7-0 = 7m

N_d = 7 , $\Delta h = \frac{H}{N_d} = \frac{7}{7} = 1.0$ m

point	z_j	$(N_d)_j$	$(h_p)_j = H - (N_d)_j.\Delta h - z_j$	$U_j = (h_p)_j.\gamma_w$
a	-2	1	8	$8\gamma_w$
b	-2	2	7	$7\gamma_w$
c	-2	3	6	$6\gamma_w$
d	-2	4	5	$5\gamma_w$
e	-2	5	4	$4\gamma_w$
f	-2	6	3	$3\gamma_w$

Example 8.3

A dam, shown in Fig.(8.7), retains 10 m of water. A sheet pile wall (cutoff curtain) on the up steam side, which is used to reduce seepage under the dam, penetrates 7 m into a 20.3 m thick silty sand stratum. Below the silty sand is a thick deposit of clay. The average hydraulic conductivity of the silty sand is 2.0×10^{-4} cm/sec. Assume that the silty sand is homogeneous and isotropic.

a. Draw the flownet under the dam
b. Calculate the flow, q.
c. Calculate and draw the porewater pressure distribution at the base of the dam.
d. Determine the uplift force.
e. Determine and draw the porewater pressure distribution on the upstream and downstream faces of the sheet pile wall.

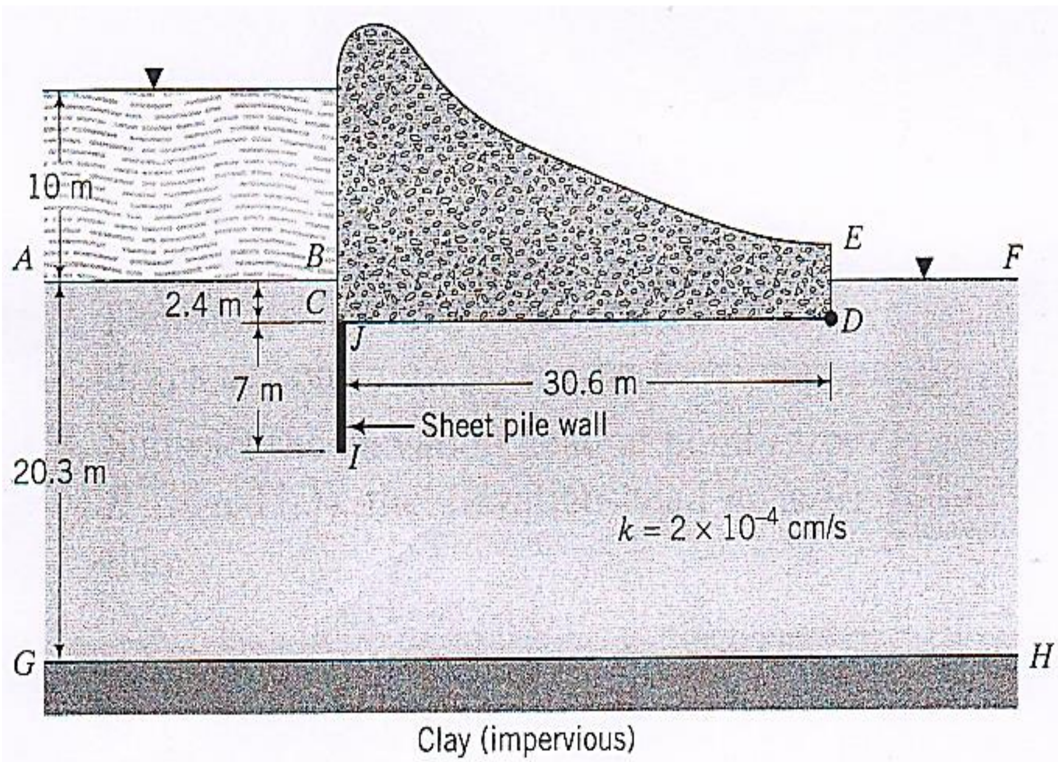


Figure 8.7

f. Determine the resultant lateral force on the sheet pile wall due to the powerwater.
g. Determine the maximum hydraulic gradient.
h. Will piping occur if the void ratio of the silty sand is 0.8.
i. What is the effect of reducing the depth of penetration of the sheet pile wall?

Solution

Step 1: Draw the dam to scale (see Fig. 8.8).

Step 2: Identify the impermeable and permeable boundaries.

With reference to Fig.(8.7), *AB* and *EF* are permeable boundaries and are therefore equipotential lines. *BCIJDE* and *GH* are impermeable boundaries and are therefore flow lines.

Step 3: Sketch the flownet.

Draw about three flow lines and then draw a suitable number of equipotential lines. Remember that flow lines are orthogonal to equipotential lines and the area between two consecutive flow lines and two consecutive equipotential lines is approximately a square. Use a circle template to assist you in estimating the square. Adjust/add/subtract flow lines and equipotential lines until you are satisfied that the flownet consists essentially of curvilinear squares. See sketch of flownet in Fig. (8.8).

Step 4: Calculate the flow.

Select the downstream end, *EF*, as the datum.

H= 10 m

$N_d = 14$, $N_f = 4$

$$q = k\frac{HN_f}{N_d} = 2 \times 10^{-4} \times (10 \times 10^2) \times \frac{4}{14} = 0.057\ cm^3/sec$$

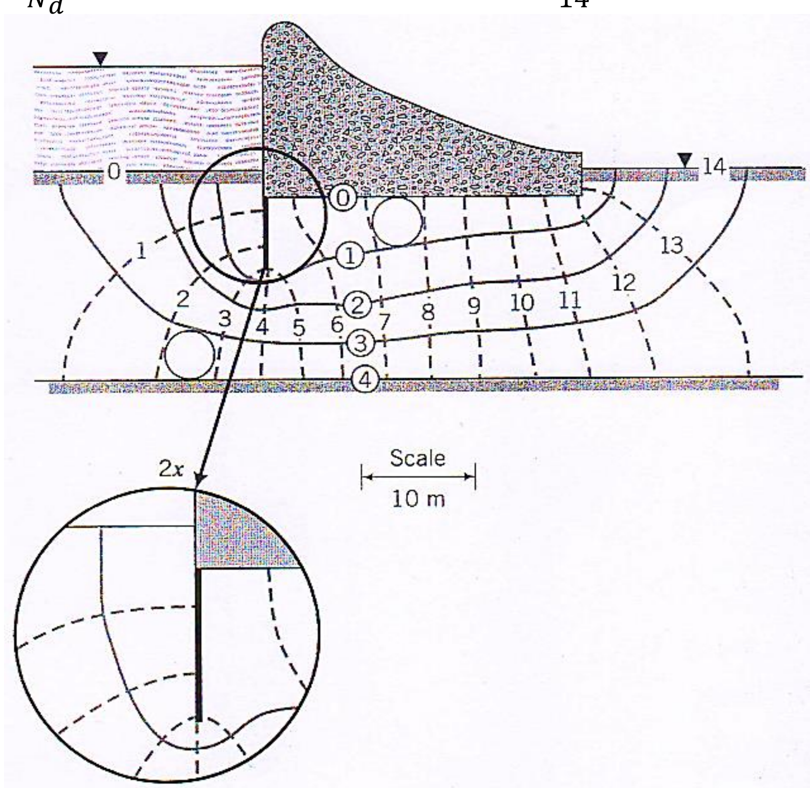


Figure (8.8)

Step 5: determine the porewater pressure under the base of the dam.

Divide the base into a convenient number of equal intervals. Let us use 10 intervals; that is

$$\Delta x = \frac{30.6}{10} = 3.06m$$

Determine the porewater pressure at each nodal point. Use a table for convenience or, better yet, use a spreadsheet.

$$\Delta h = \frac{H}{N_d} = \frac{10}{14} = 0.714m$$

The calculation in the table below was done using a spreadsheet program.

Flow under a dam: *Δh* = 0.714 m

Parameters	Under base of dam										
x (m)	0	3.06	6.12	9.18	12.24	15.3	18.36	21.42	24.48	27.54	30.6
N_d (m)	5.60	5.80	6.20	6.90	7.40	8.00	8.80	9.40	10.30	11.10	12.50
$N_d \Delta h$ (m)	4.00	4.14	4.43	4.93	5.28	5.71	6.28	6.71	7.35	7.93	8.93
z (m)	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40
h_p (m) = $H - N_d \Delta h - z$	8.40	8.26	7.97	7.47	7.12	6.69	6.12	5.69	5.05	4.47	3.48
u (kPa) = $h_p \gamma_w$	82.3	80.9	78.1	73.2	69.7	65.5	59.9	55.7	49.4	43.9	34.1

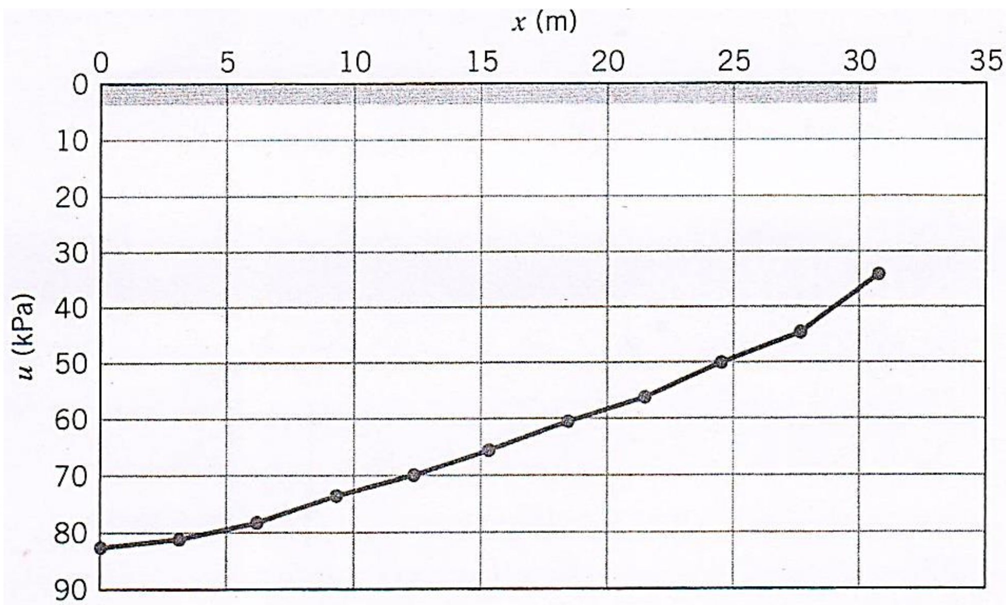


Figure (8.9)

Plot the porewater pressure distribution. See Fig. (8.9)

Step 6: Calculate the uplift force and its location.

Using Simpson's rule, we find

$$A = \frac{\Delta x}{3}\left[y_0 + y_n + 2\sum_{i=2,4,..} y_i + 4\sum_{i=1,3,..} y_i\right]$$

$$\boldsymbol{P_w = \frac{3.06}{3}[82.3 + 34.1 + 2(78.1 + 69.7 + 59.9 + 49.4) + 4(80.9 + 73.2 + 65.5 + 55.7 + 43.9)]}$$

= 1946.4 kN/m

Step 7: Determine the pore water pressure distribution on the sheet pile wall. Divide the front face of the wall into six intervals of 7/6=1.17 m and the back face into one interval. Six intervals were chosen because it is convenient for the scaling using the scale that was used to draw the flownet. The greater the intervals, the greater the accuracy. Only one interval is used for the back face of the wall because there are no equipotential lines that meet there. Use a spreadsheet to compute the porewater pressure distribution and the hydrostatic forces. The distributions of porewater pressure at the front and back of the wall are shown in Figs.(8.10). Use Simpson's rule to calculate the hydrostatic force on the front face of the wall. The porewater pressure distribution at the back face is a trapezoid and the area is readily calculated.

Parameters	Front of wall							Back of wall	
x (m)	0	1.17	2.33	3.50	4.67	5.83	7.00	7.00	0.00
N_d (m)	0.70	1.00	1.30	1.60	1.90	2.40	3.00	5.00	5.60
$N_d \Delta h$ (m)	0.50	0.71	0.93	1.14	1.36	1.71	2.14	3.57	4.00
z (m)	−2.40	−3.57	−4.73	−5.90	−7.07	−8.23	−9.40	−9.40	−2.40
h_p (m) = $H - N_d \Delta h - z$	11.90	12.85	13.81	14.76	15.71	16.52	17.26	15.83	8.40
u (kPa) = $h_p \gamma_w$	116.6	126.0	135.3	144.6	154.0	161.9	169.1	155.1	82.3
	Front	Back	Difference						
P_w (kN/m)	1011.7	830.9	180.8						

Step 8: determine the maximum hydraulic gradient.

The smallest value of l occurs at the exit. By measurement, l_{min} = 2 m

$$i_{max} = \frac{\Delta h}{l_{min}} = \frac{0.714}{2} = 0.36$$

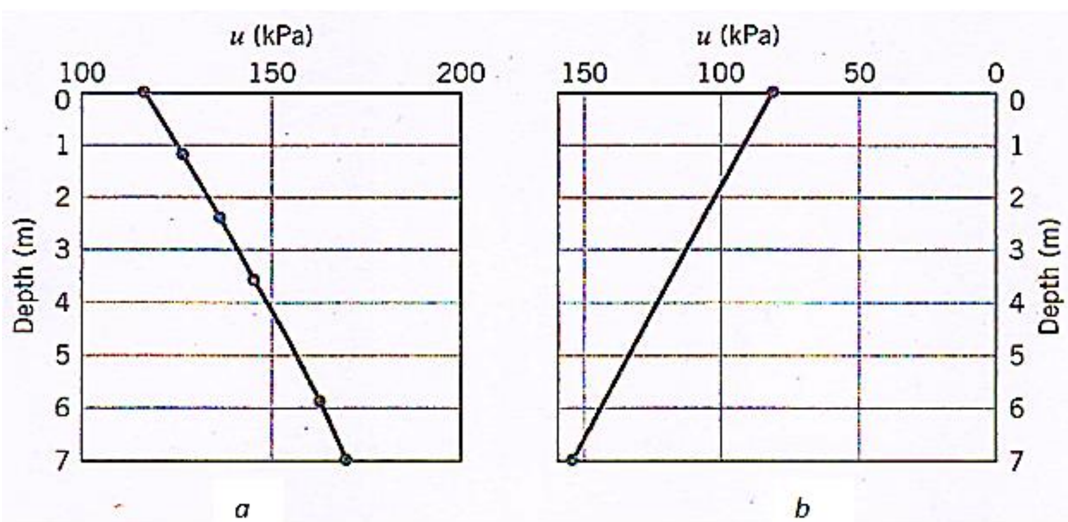


Figure 8.10 porewater pressure distributions (a) at front of wall and (b) at back of wall

Step 9: Determine if piping would occure.

Equation (8.15): $i_{cr} = \frac{G_S - 1}{1+e} = \frac{2.7-1}{1+0.8} = 0.89$

Since $\boldsymbol{i_{max}} < \boldsymbol{i_{cr}}$, piping will not occur.

Factor of safety against piping: $\frac{0.89}{0.36} = 2.5$

Step 10: State the effect of reducing the depth of penetration of the sheet pile wall. If the depth is reduced, the value of $\boldsymbol{\Delta h}$ increases and $\boldsymbol{i_{max}}$ is likely to increase.

Example 8.4

A bridge pier is to be constructed in a river bed by constructing a cofferdam as shown in Fig.(8.11 a). A cofferdam is temporary enclosure consisting of long, slender elements of steel, concrete, or timber members to support the sides of enclosure. After constructing the cofferdam, the water within it will be pumped out. Determine (a) the flow net rate using $k = 1 \times 10^{-4}$ cm/sec and (b) the factor of safety against piping. The void ratio of the sand is 0.59.

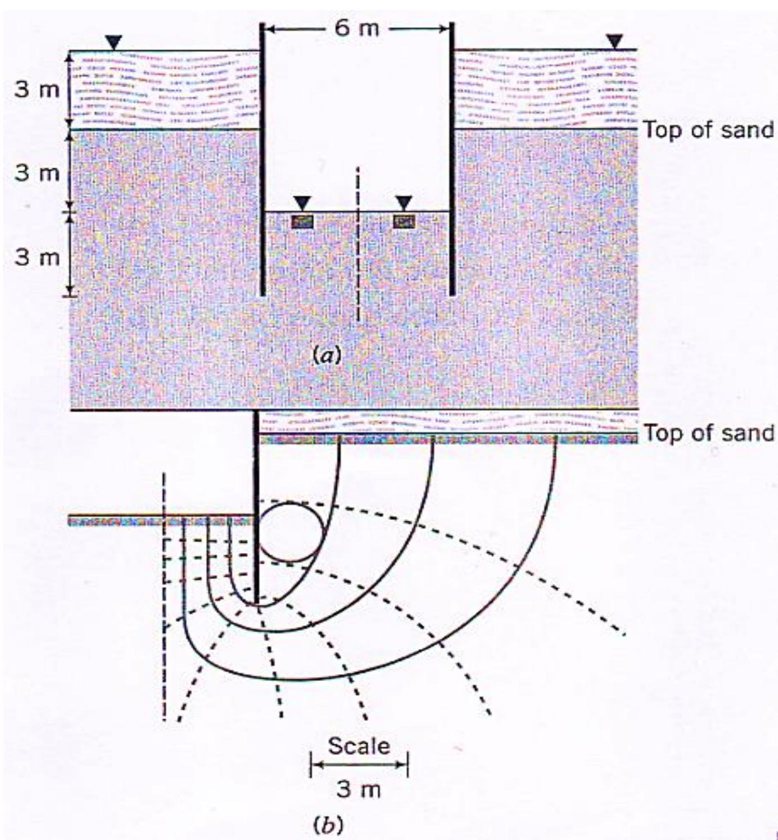


Figure (8.11)

Solution:

Step 1: draw the cofferdam to scale and sketch the flownet. [see Fig.8.11 b]

Step 2: Determine the flow.

$$H = 6\text{m}; \quad N_f = 4, \quad N_d = 10$$

$$q = 2k\frac{HN_f}{N_d} = 2 \times 1 \times 10^{-4} \times 10^{-2} \times 6 \times \frac{4}{10} = 4.8 \times 10^{-6}\ cm^3/sec$$

(Note: the factor 2 is needed because you have to consider both halves of the structure; the factor 10^{-2} is used to convert cm/sec to m/sec).

Step 3: Determine the maximum hydraulic gradient**.**

$l_{min} \approx 0.3$ m (this is an average value of the flow length at the exit of the sheet pile)

$$i_{max} = \frac{\Delta h}{l_{min}} = \frac{H}{N_d l_{min}} = \frac{6}{10 \times 0.3} = 2$$

Step 4: Calculate the critical hydraulic gradient.

$$i_{cr} = \frac{G_S - 1}{1+e} = \frac{2.7-1}{1+0.59} = 1.07$$

Since $\boldsymbol{i_{max}} > \boldsymbol{i_{cr}}$, piping is likely to occur; the factor of safety is 1.07/2 ≈ 0.5

8.9 Flow Nets in Anisotropic Soil

In nature, most soils exhibit some degree of anisotropy. To account for soil anisotropy with respect to hydraulic conductivity, we must modify the flow net construction.

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

or

$$\frac{\partial^2 h}{\partial x^2} + \frac{k_z}{k_x}\frac{\partial^2 h}{\partial z^2} = 0$$

Let; $$c = \sqrt{k_z/k_x} \qquad (8.19)$$

$$\acute{x} = cx \qquad (8.20)$$

$$\Rightarrow \quad \frac{\partial \acute{x}}{\partial x} = c$$

and

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial \acute{x}} . \frac{\partial \acute{x}}{\partial x} = c\,\frac{\partial h}{\partial \acute{x}}$$

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial}{\partial \acute{x}}\left(\frac{\partial h}{\partial x}\right) . \frac{\partial \acute{x}}{\partial x} = c^2\,\frac{\partial^2 h}{\partial \acute{x}^2}$$

Substitute:

$$c^2\,\frac{\partial^2 h}{\partial \acute{x}^2} + c^2\,\frac{\partial^2 h}{\partial z^2} = 0 \qquad \Rightarrow \qquad \frac{\partial^2 h}{\partial \acute{x}^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

To find the equivalent hydraulic conductivity (k_e), we have;

$$v_x = -k_e\,\frac{\partial h}{\partial \acute{x}} = -k_x\,\frac{\partial h}{\partial x}$$

But, $$\frac{\partial h}{\partial x} = c\,\frac{\partial h}{\partial \acute{x}}$$

Thus: $$k_e = k_x . c \qquad \Rightarrow \quad k_e = \sqrt{k_x . k_z} \qquad (8.21)$$

and: $$q = k_e\,\frac{HN_f}{N_d}$$

To construct the flow net for anisotropic soil, use the following procedure:

Step 1: Adopt a vertical scale (that is, z axis) for drawing the cross section.

Step 2: Adopt a horizontal scale (that is, x axis) such that horizontal scale = $\sqrt{k_z/k_x} \times vertical\ scale.$

Step 3: With scales adopted as in Step 1 and 2, plot the vertical section through the permeable layer parallel to the direction of flow.

Step 4: Draw the flow net for the permeable layer on the section obtained from Step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.

Example 8.5

A dam section is shown in Figure (8.12a). The hydraulic conductivity of the permeable layer in the vertical and horizontal directions are 2×10^{-2} mm/s and 4×10^{-2} mm/s, respectively. Draw a flow net and calculate the seepage loss of the dam in m^3/day/m.

Solution

From the given data,

$k_z = 2\times10^{-2}$ mm/sec = 1.73 m/day

$k_x = 4\times10^{-2}$ mm/sec = 3.46 m/day

And H = 20m. For drawing the flow net,

$$c = \sqrt{k_z/k_x} = \sqrt{\frac{2\times10^{-2}}{4\times10^{-2}}} = \frac{1}{\sqrt{2}}$$

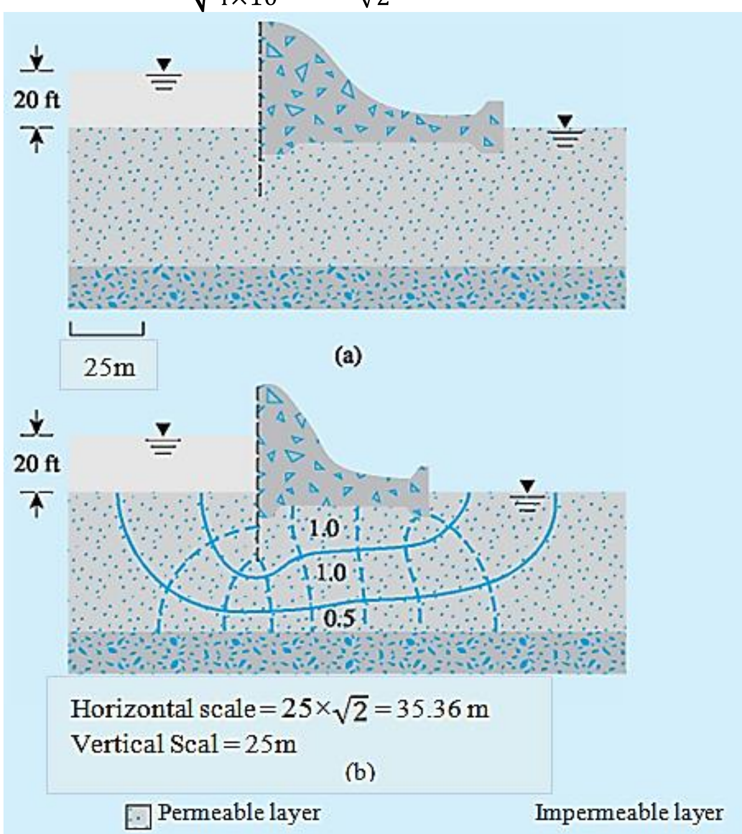


Figure (8.12)

On the basis of this, the dam section is replotted, and the flow net drawn as in Figure (8.12b). The rate of seepage is given by $q = \sqrt{k_x k_z}\frac{HN_f}{N_d}$ .From Figure (8.12b), N_d = 8 and N_f = 2.5 (the lowermost flow channel has a width-to-length ratio of 0.5). So,

$$q = \sqrt{(1.73)(3.46)}\left(\frac{20\times2.5}{8}\right) = 15.3 \quad m^3/day/m$$

8.10 Seepage through an Earth Dam

Flow through earth dams is an important design consideration. We need to ensure that the porewater pressure at the downstream end of the dam will not lead to instability and the exit hydraulic gradient does not lead to piping. The major exercise is to find the top flow line called the "phreatic surface". The pressure head on the phreatic surface is zero, Figure (8.13).

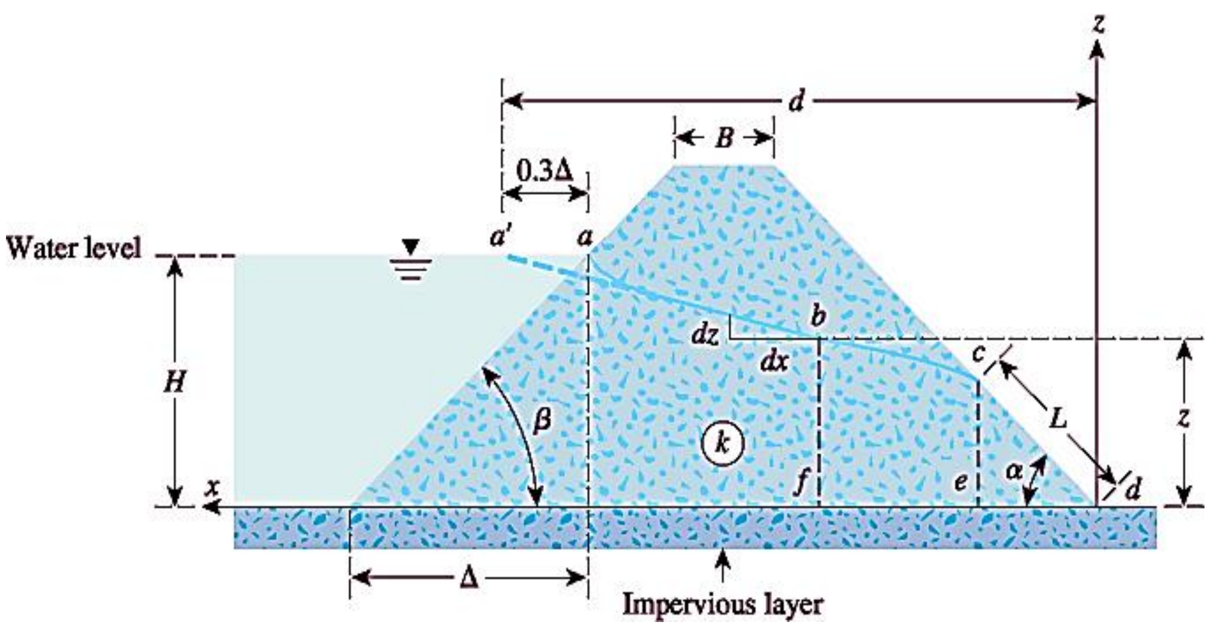


Figure (8.13) Flow through an earth dam constructed over an impervious base

Casagrande (1937) showed that the phreatic surface can be approximated by a parabola. The slop of the free surface can be assumed to be equal to the hydraulic gradient (at *c*):

$$i \approx \frac{dz}{dx} \tag{8.22}$$

Considering the triangle *cde*, we can give the rate of seepage per unit length of the dam (at right angle to the cross section shown in Figure (8.13) as

$$q = kiA$$

$$i = \frac{dz}{dx} = \tan\alpha$$

$$A = (\overline{ce})(1) = L\sin\alpha$$

So

$$q = k(\tan\alpha)(L\sin\alpha) = kL\tan\alpha\sin\alpha \tag{8.23}$$

Again, the rate of seepage (per unit length of the dam) through the section *bf* is

$$q = kiA = k\left(\frac{dz}{dx}\right)(z \times 1) = kz\frac{dz}{dx} \tag{8.24}$$

For continuous flow,

$$q_{Eq.(8.23)} = q_{Eq.(8.24)}$$

or

$$kz\frac{dz}{dx} = kL\tan\alpha\sin\alpha$$

or

$$\int_{z=L\,sin\alpha}^{z=H} kzdz = \int_{x=L\cos\alpha}^{x=d} (kL\tan\alpha\sin\alpha)dx$$

$$\frac{1}{2}(H^2 - L^2 sin^2\alpha) = L\tan\alpha\sin\alpha(d - L\cos\alpha)$$

$$\frac{H^2}{2} - \frac{L^2 sin^2\alpha}{2} = Ld\left(\frac{sin^2\alpha}{\cos\alpha}\right) - L^2 sin^2\alpha$$

$$\frac{H^2\cos\alpha}{2sin^2\alpha} - \frac{L^2\cos\alpha}{2} = Ld - L^2\cos\alpha$$

or

$$L^2\cos\alpha - 2Ld + \frac{H^2\cos\alpha}{sin^2\alpha} = 0$$

So,

$$L = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{cos^2\alpha} - \frac{H^2}{sin^2\alpha}} \tag{8.25}$$

Following is a step-by-step procedure to obtain the seepage rate *q* (per unit length of the dam):

Step 1: Obtain α.
Step 2: Calculate Δ (see Figure 8.13) and then 0.3Δ.
Step 3: Calculate *d*.
Step 4: With known values of α and *d*, calculate *L* from Eq. (8.25).
Step 5: With known value of *L*, calculate *q* from Eq. (8.23).

Example 8.6

Refer to the earth dam shown in Figure 8.13. Given that β = 45°, α = 30°, B = 10 m, H = 20 m, height of dam = 25 m, and $k = 2 \times 10^{-4}$ m/min, calculate the seepage rate, *q*, in m^3/day/m length.

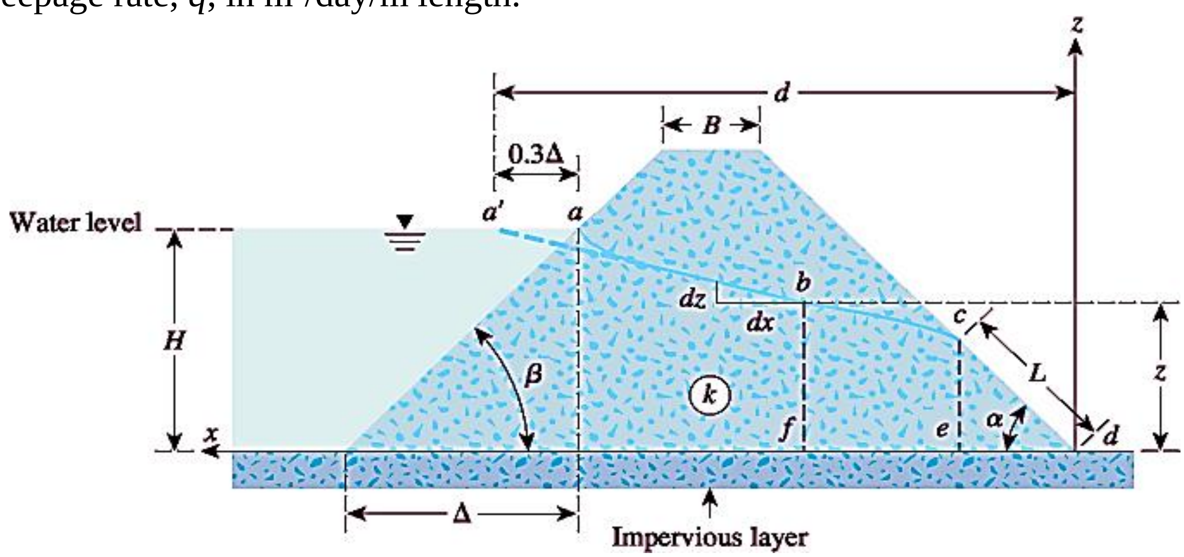


Figure (8.13)

Solution

We know that β = 45° and α= 30°. Thus,

$$\Delta = \frac{H}{\tan\beta} = \frac{20}{tan45^o} = 20m, \quad 0.3\Delta = (0.3)(20) = 6m$$

$$d = 0.3\Delta + \frac{(25-20)}{\tan\beta} + B + \frac{25}{tan\alpha}$$

$$d = 6 + \frac{(25-20)}{tan45^o} + 10 + \frac{25}{\tan 30^o} = 64.3\ m$$

From Eq. (8.25)

$$L = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{cos^2\alpha} - \frac{H^2}{sin^2\alpha}}$$

$$L = \frac{64.3}{\cos 30} - \sqrt{(\frac{64.3}{\cos 30})^2 - (\frac{20}{\sin 30})^2} = 11.7m$$

From Eq. (8.23)

$$q = kL\tan\alpha\sin\alpha = (2\times10^{-4})(11.7)(\tan 30)(\sin 30)$$

$= 6.754\times 10^{-4}\ m^3/min/m = 0.973\ m^3/day/m$

8.11 Earth Dam with Drainage Blanket

Because the exit hydraulic gradient is often large, drainage blankets are used at the downstream end of dams to avoid piping. Seepage is controlled by the gradation of the coarse-grained soils used for the drainage blanket.

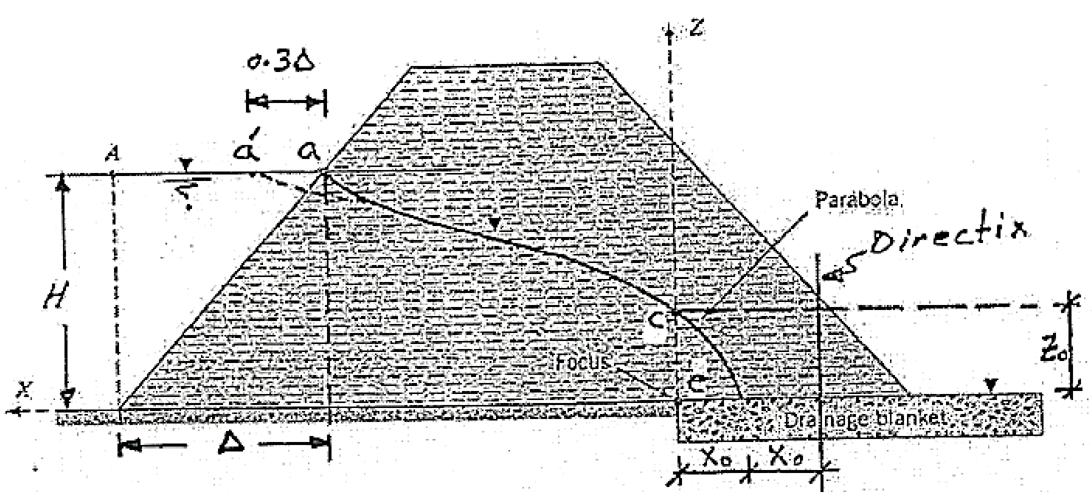


Figure (8.14) A horizontal drainage blanket at the toe of an earth dam

The flow through the dam shown in Figure (8.14) is; (through *ce*):

$$q = kiA = k\frac{dz}{dx}(z_o.1) \tag{8.26}$$

From the geometry of the basic parabola: $z_o = 2x_o$ and the slope of the basic parabola $\{z^2 = 4x_o(x_o + x)\}\ at\ c$ is;

$$\frac{dz}{dx} = \frac{2x_o}{z_o} = \frac{2x_o}{2x_o} = 1 \tag{8.27}$$

Therefore, the flow through a dam with a horizontal drainage blanket is;

$$q = k.1.2x_o = 2x_o k \tag{8.28}$$

Example 8.7

A homogeneous anisotropic embankment dam section is detailed in Figure (8.15), the coefficients of permeability in the x and z directions being 4.5×10^{-8} and 1.6×10^{-8} m/sec, respectively. Construct the flow net and determine the quantity of seepage through the dam. What is the pore water pressure at point P?

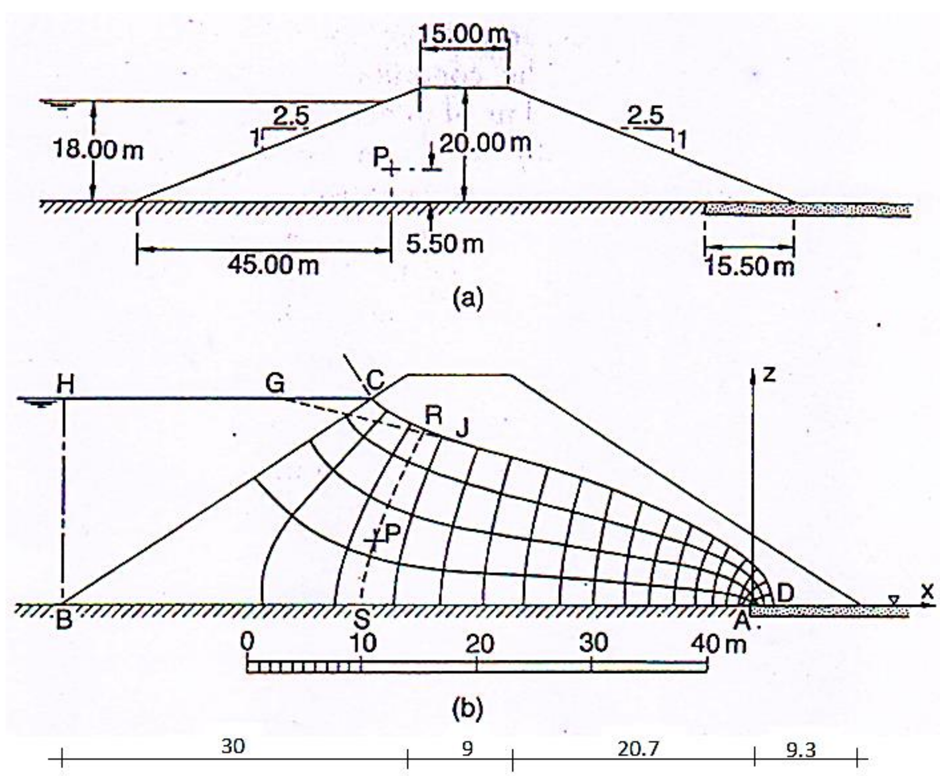


Figure (8.15)

Solution

The scale factor for transformation in the x-direction is,

$$c = \sqrt{\frac{k_z}{k_x}} = \sqrt{\frac{1.6}{4.5}} = 0.6$$

The equivalent isotropic permeability is;

$$k_e = \sqrt{k_x k_z} = \sqrt{(4.5 \times 1.6)} \times 10^{-8} = 2.7 \times 10^{-8}\ m/sec$$

The section is drawn to the transformed scale as in Figure (8.15 b)

$\Delta = 0.6 \times 45 = 27\ m$

$0.3\Delta = 0.3 \times 27 = 8.1\ m$

Thus, the coordinates of $\acute{a}$ are;

$x = 20.7 + 9 + (0.6\times2\times2.5) + 8.1 = 40.8\ m$

$z = 18.0\ m$

Substituting these coordinates in the equation of base parabola:

$$z^2 = 4x_o(x_o + x)$$

$$(18)^2 = 4x_o(x_o + 40.8) \Rightarrow x_o = 1.9m$$

Phreatic surface

Using the equation of basic parabola to draw the phreatic surface:

$$\{z^2 = 4x_o(x_o + x)\} \Rightarrow z^2 = 7.6(1.9 + x)$$

x	-1.9	0	5	10	20	30
z	0	3.8	7.24	9.51	12.9	15.57

The basic parabola is plotted in Figure (8.15 b). The upstream correction is made and flow net completed. In the flow net we have:

$N_f = 3.8$, $N_d = 18$

Hence;

$$q = k_e . H . \frac{N_f}{N_d} = 2.7 \times 10^{-8} \times 18 \times \frac{3.8}{18} = 1.0 \times 10^{-7}\ m^3/sec$$

The quantity of seepage can also be determined from Eq.(8.28)

$$q = 2x_o k = 2 \times 1.9 \times 2.7 \times 10^{-8} = 1.0 \times 10^{-7} m^3/sec$$

At point P,

$z_p = 5.5\ m$

$$\Delta h = \frac{18}{18} = 1.0\ m$$

$(N_d)_p = 2.4$

Thuse;

$$(h_p)_P = H - (N_d)_P.\Delta h - z_P$$

$$= 18 - 2.4 \times 1 - 5.5 = 10.1\ m$$

$$(u)_P = (h_p)_P \times \gamma_w$$

$$= 10.1 \times 9.8 = 99\ kN/m^2$$

Example 8.7 (H.W)

An embankment dam is shown in section in Figure (8.16), the coefficients of permeability in the horizontal and vertical directions being 7.5×10^{-6} and 2.7×10^{-6} m/sec, respectively. Construct the top flow line and determine the quantity of seepage through the dam.

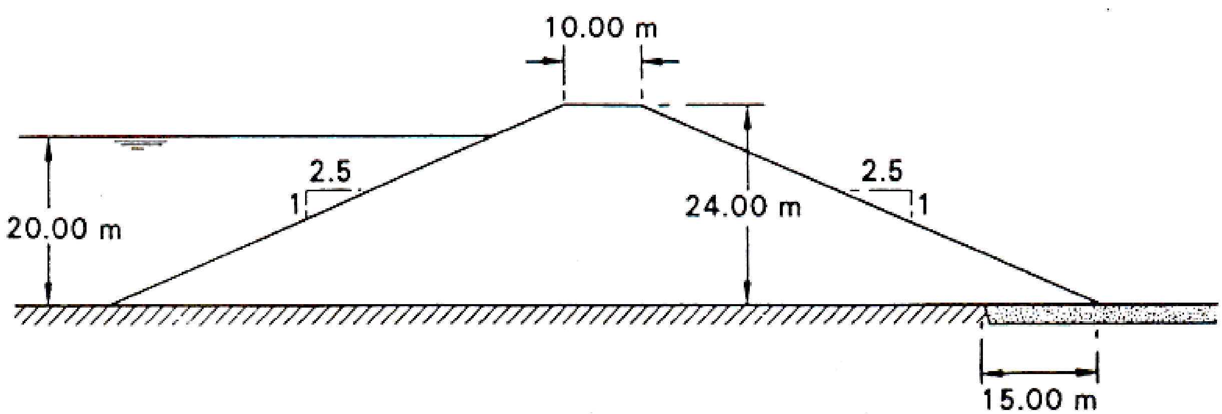


Figure (8.16)

Ans.;

$$\boldsymbol{x = \frac{z^2}{8} - 2.0}$$

$$\boldsymbol{q = 1.8 \times 10^{-5}\ m^3/s/m}$$

8.12 Control of Piping

Where high hydraulic gradients exist there is a possibility that the seeping water may cause internal erosion within the soil. Erosion can work its way back into the soil creating voids in the form of channels or "piping".

In order to increase the factor of safety against piping, two methods can be adopted:

1. The first procedure involves increasing the depth of the sheet pile at the top of the dam. This will increase the length of flow path resulting in drop in the pore pressure at the critical section.
2. The second procedure is to place a surcharge or filter on the top of the downstream side, the weight of which increases the downward force.

8.13 Filter Design

When seepage water flows from a soil with relatively fine grains into a coarser material, there is danger that the fine soil particles may wash away into the coarse material. Over a period of time, this process may clog the void spaces in the coarse material. Hence, the grain-size distribution of the coarse material should be properly manipulated to avoid this situation. For proper selection of the filter material, two conditions should be kept in mind:

1. The size of the voids in the filter material should be small enough to hold the larger particles of the protected material in place.
2. The filter material should have a high hydraulic conductivity to prevent building of large seepage forces and hydrostatic pressure in the filters.

Terzaghi and Peck (1948) provided the following criterion to satisfy condition (1).

$$\frac{D_{15}(F)}{D_{85}(S)} \leq 4\ to\ 5$$

In order to satisfy condition (2), they suggested that;

$$\frac{D_{15}(F)}{D_{15}(S)} \geq 4\ to\ 5$$