Chapter 7: Permeability

Soils are permeable due to the existence of interconnected voids through which water can flow from points of high energy to points of low energy. The study of the flow of water through permeable soil media is important in soil mechanics. It is necessary for estimating the quantity of underground seepage under various hydraulic conditions, for investigating problems involving the pumping of water for underground construction, and for making stability analyses of earth dams and earth-retaining structures that are subject to seepage forces.

7.1 Bernoulli's Equation

From fluid mechanics, we know that, according to Bernoulli’s equation, the total head at a point in water under motion can be given by the sum of the pressure, velocity, and elevation heads, or

$$h = \underbrace{\frac{u}{\gamma_w}}_{\text{Pressure head}} + \underbrace{\frac{v^2}{2g}}_{\text{Velocity head}} + \underbrace{Z}_{\text{Elevation head}} \tag{7.1}$$

where h = total head
u = pressure
v = velocity
g = acceleration due to gravity
γ_w = unit weight of water

Note that the elevation head, Z, is the vertical distance of a given point above or below a datum plane. The pressure head is the water pressure, u, at that point divided by the unit weight of water, γ_w.

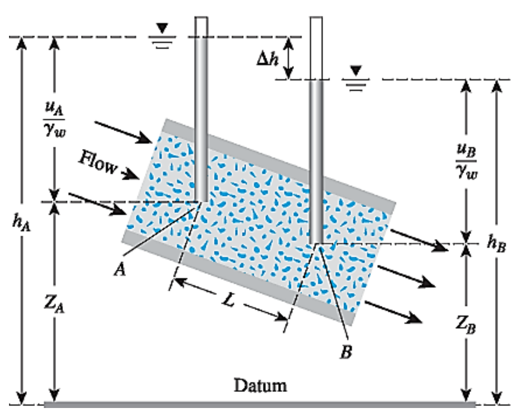

Figure 7.1 Pressure, elevation, and total heads for flow of water through soil

If Bernoulli's equation is applied to the flow of water through a porous soil medium, the term containing the velocity head can be neglected because the seepage velocity is small, and the total head at any point can be adequately represented by

$$h = \frac{u}{\gamma_w} + Z \qquad (7.2)$$

Figure (7.1) shows the relationship among pressure, elevation, and total heads for the flow of water through soil. Open standpipes called *piezometers* are installed at points *A* and *B*. The levels to which water rises in the piezometer tubes situated at points A and B are known as the *piezometric levels* of points *A* and *B*, respectively. The pressure head at a point is the height of the vertical column of water in the piezometer installed at that point.
The loss of head between two points, *A* and *B*, can be given by

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right) \qquad (7.3)$$

The head loss, Δh, can be expressed in a nondimensional form as

$$i = \frac{\Delta h}{L} \qquad (7.4)$$

where *i* = hydraulic gradient
L = distance between points A and B—that is, the length of flow over which the loss of head occurred

7.2 Darcy's Law

Darcy (1856) proposed that average flow velocity through soils is proportional to the gradient of the total head

$$v = ki \qquad (7.5)$$

Where *v* = discharge velocity, which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angle to the direction of flow
k = hydraulic conductivity (otherwise known as the coefficient of permeability)

In Eq. (7.5), *v* is the discharge velocity of water based on the gross cross-sectional area of the soil. However, the actual velocity of water (that is, the seepage velocity) through the void spaces is greater than *v*. A relationship between the discharge velocity and the seepage velocity can be derived by referring to Figure (7.2), which shows a soil of length *L* with a gross cross-sectional area *A*. If the quantity of water flowing through the soil in unit time is q, then

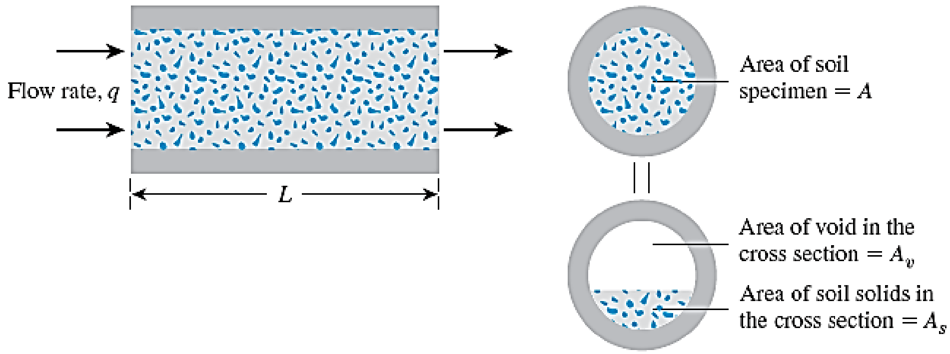

Figure 7.2 Derivation of Eq. (7.7)

$$q = vA = A_v v_s \tag{7.6}$$

Where v_s = seepage velocity
A_v = area of void in the cross section of the specimen

Thus;

$$v_s = \frac{A}{A_v} v = \frac{LA}{LA_v} v = \frac{V}{V_v} v = \frac{v}{n} = v\left(\frac{1+e}{e}\right) \tag{7.7}$$

where e = void ratio
n = porosity

7.3 Hydraulic Conductivity

Hydraulic conductivity is generally expressed in cm/sec. It depends on several factors:

1. Soil type (fine-grained or coarse-grained).
2. Grain size distribution.
3. Void ratio.
4. Pore size distribution.
5. Fluid viscosity.
6. Degree of saturation.
7. Roughness of mineral particles.

7.4 Laboratory Determination of Hydraulic Conductivity

Two standard laboratory tests are used to determine the hydraulic conductivity of soil— the constant-head test and the falling-head test.

Constant-Head Test

A typical arrangement of the constant-head permeability test is shown in Figure 7.3. In this type of laboratory setup, the water supply at the inlet is adjusted in such a way that the difference of head between the inlet and the outlet remains constant during the test period. After a constant flow rate is established, water is collected in a graduated flask for a known duration.

The total volume of water collected may be expressed as

$$Q = Avt = A(ki)t \qquad (7.8)$$

where Q = volume of water collected
A = area of cross section of the soil specimen
t = duration of water collection

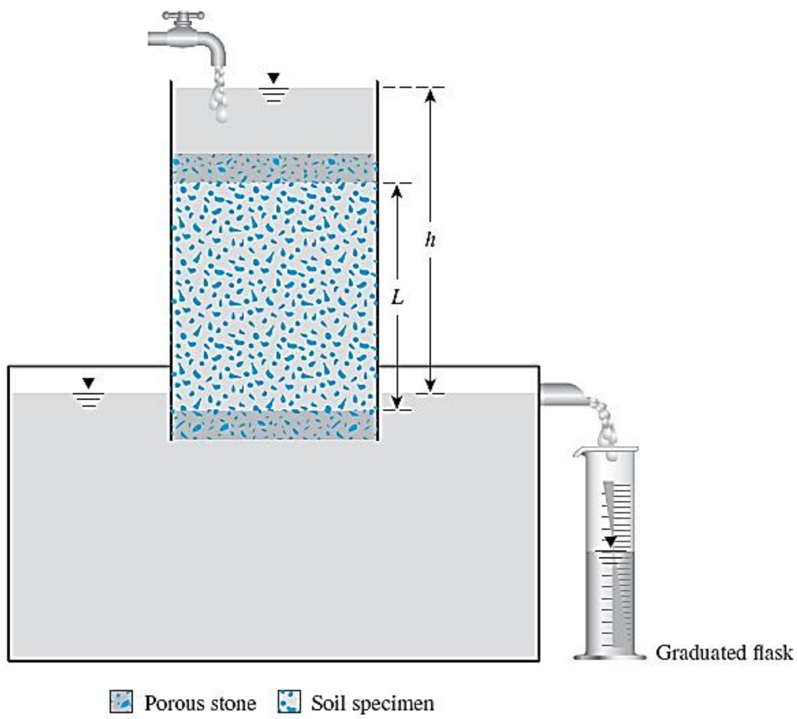

Figure (7.3) Constant-head permeability test

And because

$$i = \frac{h}{L} \qquad (7.9)$$

where L = length of the specimen, Eq. (7.9) can be substituted into Eq. (7.8) to yield

$$Q = A\left(k\frac{h}{L}\right)t \tag{7.10}$$

or

$$k = \frac{QL}{Aht} \tag{7.11}$$

This test is usually used to determine *k* for coarse-grained soils.

Falling-Head Test

A typical arrangement of the falling-head permeability test is shown in Figure 7.4. Water from a standpipe flows through the soil. The initial head difference h_1 at time $t = 0$ is recorded, and water is allowed to flow through the soil specimen such that the final head difference at time $t = t_2$ is h_2.

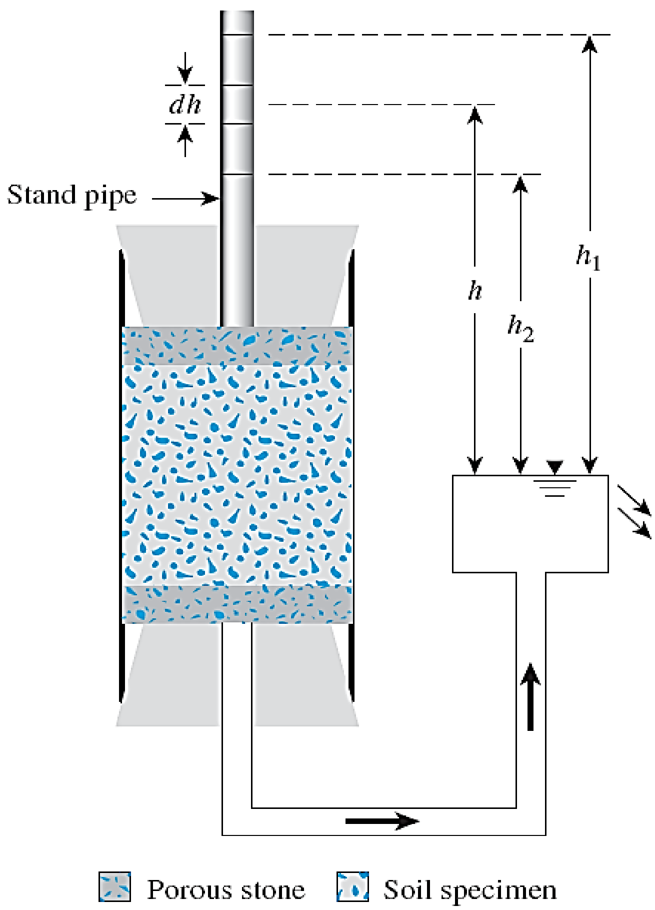

Figure (7.4) Falling-head permeability

The rate of flow of the water through the specimen at any time t can be given by

$$q = k\frac{h}{L}A = -a\frac{dh}{dt} \tag{7.12}$$

where q = flow rate
a = cross-sectional area of the standpipe
A = cross-sectional area of the soil specimen

Rearrangement of Eq. (7.12) gives

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right) \tag{7.13}$$

$$\int_0^t dt = \frac{aL}{Ak}\int_{h_1}^{h_2}\left(-\frac{dh}{h}\right)$$

or

$$k = \frac{aL}{At}ln\left(\frac{h_1}{h_2}\right) = 2.303\frac{aL}{At}log_{10}\frac{h_1}{h_2} \tag{7.14}$$

This test is usually used to determine k for fine-grained soils.

Example 7.1

Refer to the constant-head permeability test arrangement shown in Figure (7.3). A test gives these values:

- L= 30 cm
- A= area of the specimen = 177 cm^2
- Constant-head difference, h= 50 cm
- Water collected in a period of 5 min = 350 cm^3

Calculate the hydraulic conductivity in cm/sec.

Solution:

From eq. (7.11)

$$k = \frac{QL}{Aht}$$

Given Q= 350 cm^3, L=30 cm, A= 177 cm^2, h= 50 cm, and t= 5 min, we have;

$$k\frac{(350)(30)}{(177)(50)(5)(60)} = 3.95 \times 10^3\ cm/sec$$

Example 7.2

For a falling-head permeability test, the following values are given:

- Length of specimen = 20 cm
- Area of soil specimen = 10 cm^2
- Area of standpipe = 0.4 cm^2
- Head difference at time $t_{=0}$ = 50 cm
- Head difference at time $t_{=180\ sec}$ = 30 cm

Determine the hydraulic conductivity of the soil in cm/sec.

Solution:

From Eq. (7.14)**,**

$$k = 2.303 \frac{aL}{At} log_{10} \frac{h_1}{h_2}$$

We are given a= 0.4 cm^2, L= 20 cm, A= 10 cm^2, t= 180 sec, h_1= 50 cm and h_2= 30 cm

$$k = 2.303 \frac{(0.4)(20)}{(10)(180)} log_{10} \left(\frac{50}{30}\right) = 2.27 \times 10^{-3}\ cm/sec$$

Example 7.3

A permeable soil layer is underlain by an impervious layer, as shown in figure (7.5a). With k = 5.3 ×10^{-5} m/sec for the permeable layer, calculate the rate of seepage through it in $m^3/hr/m$ width if H= 3m and α = 8.

Solution: From figure (7.5 b)

$$i = \frac{head\ loss}{length} = \frac{L.tan\alpha}{\left(\frac{L}{cos\alpha}\right)} = sin\alpha$$

$q=kiA = (k)(\sin\alpha)(3\cos\alpha)(1)$

$k = 5.3 \times 10^{-5}$ m/sec

$q = (5.3 \times 10^{-5})(sin8)(3cos8)(3600) = 0.0789\ m^3/hr/m$

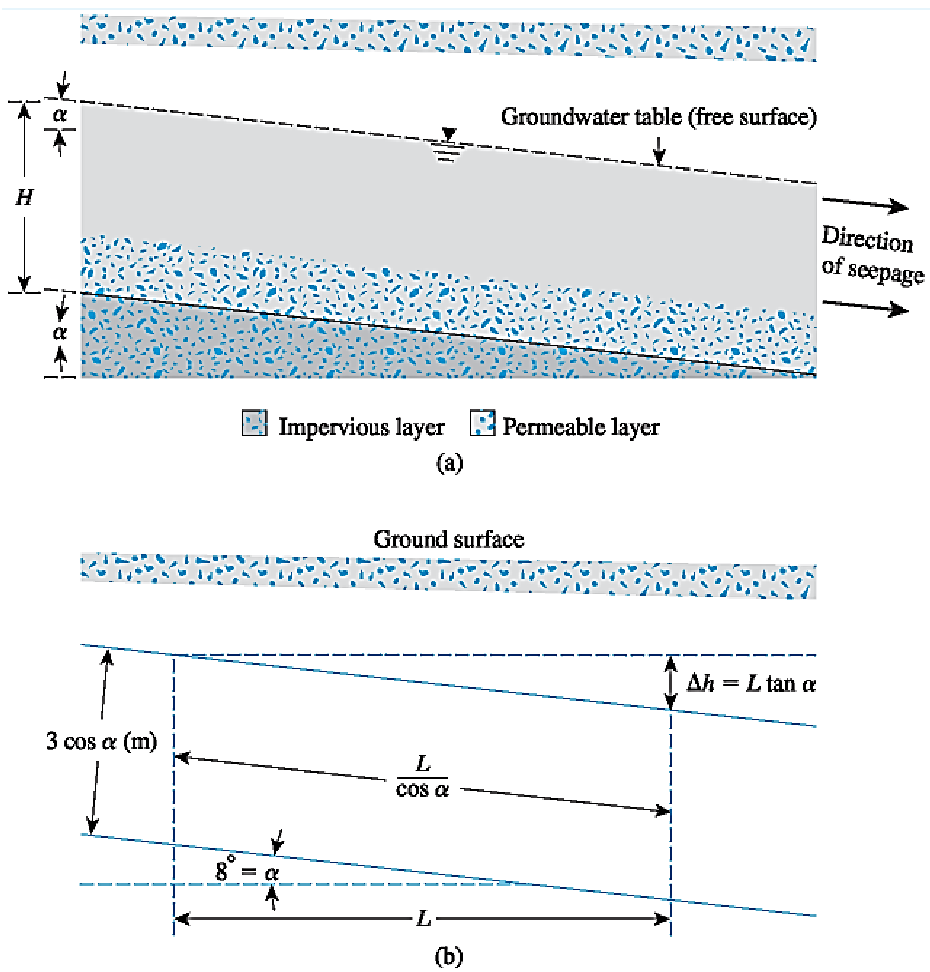

Figure (7.5)

Example 7.4

Find the flow rate in m^3/sec/m length (at right angle to the cross section shown) through the permeable soil layer shown in Figure (7.6) given H = 8 m, H_1= 3m, h = 4m, L = 50m, $\alpha = 8$, and k = 0.08 cm/sec.

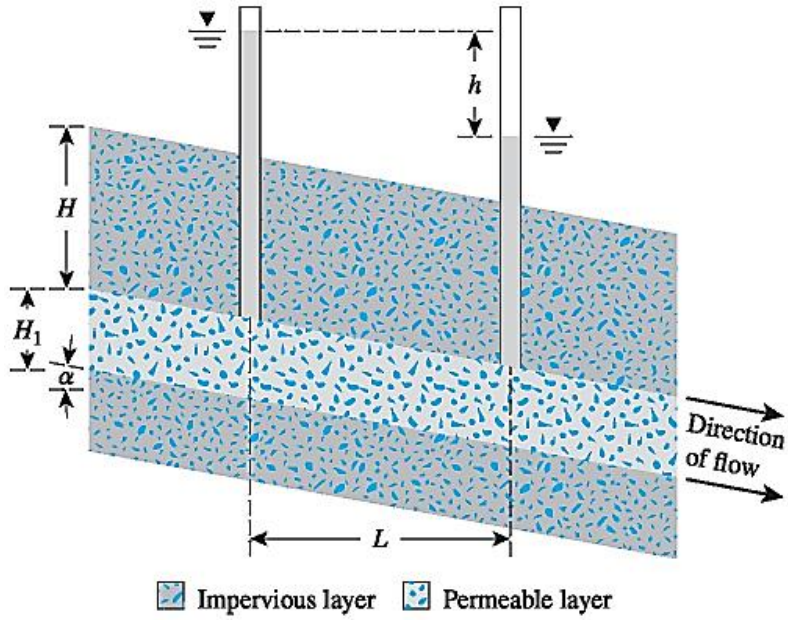

Figure 7.6 Flow through permeable layer

Solution:

Hydraulic gradient $(i) = \frac{h}{\left(\frac{L}{cos\alpha}\right)}$

From Eqs. (7.9) and (7.10)

$q = kiA = (k)\left(\frac{hcos\alpha}{L}\right)(H_1 cos\alpha \times 1)$

$q = (0.08 \times 10^{-2}\ \text{m/sec})\left(\frac{4cos8}{50}\right)(3cos8 \times 1) = 0.19 \times 10^{-3}\ m^3/\text{sec/m}$

Example 7.5

A soil sample 10 cm in diameter is placed in a tube 1m long. A constant supply of water is allowed to flow into one end of the soil at *A* and the outflow at *B* is collected by beaker. The average amount of water collected is 1 cm^3 for every 10 seconds. Determine the:

a. Hydraulic gradient
b. Flow rate
c. Average velocity
d. Seepage velocity, if *e* = 0.6
e. Hydraulic conductivity

Solution :

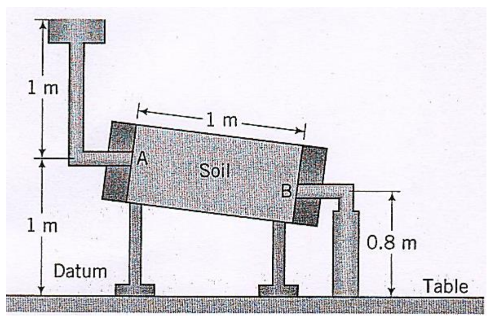

Figure (7.7)

(a) $h_A = Z_A + \frac{U_A}{\gamma_w} = 1 + 1 = 2\text{m}$

$h_B = Z_B + \frac{U_B}{\gamma_w} = 0.8 + 0 = 0.8\text{m}$

$\Delta h = 2 - 0.8 = 1.2\text{m}$

$L = 1.0 \text{ m}$

$i = \frac{\Delta h}{L} = \frac{1.2}{1.0} = 1.2$

(HW) Repeat the solution by choosing the datum passing through point B.

(b) $q = \frac{Q}{t} = \frac{1}{10} = 0.1 \frac{cm^3}{sec}$

(c) $q = vA$

$A = \frac{\pi}{4}(10)^2 = 78.5cm^2$

Thus; $v = \frac{q}{A} = \frac{0.1}{78.5} = 0.0013 \frac{cm}{sec}$

(d) $v_s = \frac{v}{n}$

$n = \frac{e}{1+e} = \frac{0.6}{1+0.6} = 0.38$

Thus; $v_s = \frac{0.0013}{0.38} = 0.0034 \frac{cm}{sec}$

(e) $k = \frac{v}{i} = \frac{0.0013}{1.2} = 10.8 \times 10^{-4} \frac{cm}{sec}$

HW

7.6. A sample of sand, 5 cm in diameter and 15 cm long, was prepared at a porosity of 60% in a constant-head apparatus. The total head was kept constant at 30 cm and the amount of water collected in 5 seconds was 40 cm^3. The test temperature was 20^0C. Calculate the hydraulic conductivity and the seepage velocity.

Ans; $k = 0.2$ cm/sec , $v_s = 0.67$ cm/sec

7.7. The data from a falling-head test on a silty clay are:

Cross-sectional area of soil = 80 cm^2

Length of soil = 10 cm

Initial head = 90 cm, Final head = 84 cm

Duration of test = 15 minutes, Diameter of tube = 6 mm

Determine k.

Ans; $k = 2.7 \times 10^{-6}$ cm/sec

7.5 Equivalent Hydraulic Conductivity in Stratified Soil

In a stratified soil deposit where the hydraulic conductivity for flow in a given direction changes from layer to layer, an equivalent hydraulic conductivity can be computed to simplify calculations.

a. Flow in the Horizontal Direction

Figure (7.8) shows n layers of soil with flow in the *horizontal direction*. Let us consider a cross section of unit length passing through the n layers and perpendicular to the direction of flow. The total flow through the cross section in unit time can be written as

$$\begin{aligned} q &= v.1.H \\ &= v_1.1.H_1 + v_2.1.H_2 + v_3.1.H_3 + \cdots + v_n.1.H_n \end{aligned} \qquad (7.15)$$

Where v = average discharge velocity

$v_1, v_2, v_3, \ldots, v_n$ = discharge velocities of flow in layers denoted by the subscripts

If $\boldsymbol{k_{H1}, k_{H2}, k_{H3}, \ldots, k_{Hn}}$ are the hydraulic conductivities of the individual layers in the horizontal direction and $k_{H(eq)}$ is the equivalent hydraulic conductivity in the horizontal direction, then, from *Darcy's law*,

$$v = k_H(eq) i_{eq}; \quad v_1 = k_{H1} i_1; \; v_2 = k_{H2} i_2; \; v_3 = k_{H3} i_3; \; \ldots \; v_n = k_{Hn} i_n$$

Substituting the preceding relations for velocities into Eq. (7.15) and noting that $i_{eq} = i_1 = i_2 = i_3 = \dots = i_n$ results in

$$k_{H(eq)} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \cdots .. + k_{H_n}H_n) \qquad (7.16)$$

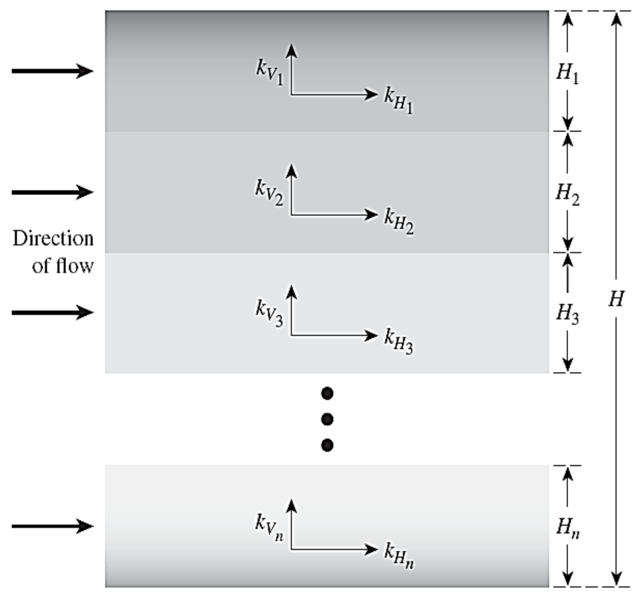

Figure (7.8) Equivalent hydraulic conductivity determination—horizontal flow in stratified soil

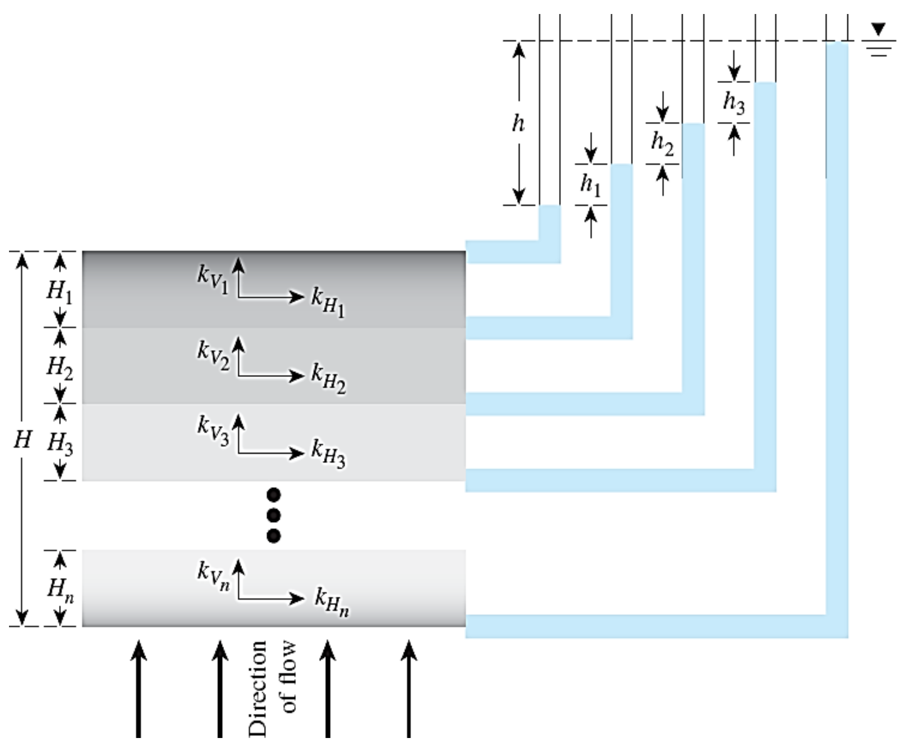

Figure 7.9 Equivalent hydraulic conductivity determination—vertical flow in stratified soil

b. Flow in the Vertical Direction

Figure 7.9 shows n layers of soil with flow in the vertical direction. In this case, the velocity of flow through all the layers is the same. However, the total head loss, *h*, is equal to the sum of the head losses in all layers. Thus,

$$v = v_1 = v_1 = v_1 = \cdots = v_n \qquad (7.17)$$

and

$$h = h_1 + h_2 + h_3 + \cdots + h_n \qquad (7.18)$$

Using Darcy's law, we can rewrite Eq. (7.17) as

$$k_{v(eq)}\left(\frac{h}{H}\right) = k_{v_1} i_1 = k_{v_2} i_1 = k_{v_3} i_3 = \cdots = k_{v_n} i_n \qquad (7.19)$$

where $k_{v_1}, k_{v_2}, k_{v_3}, \ldots, k_{v_n}$ are the hydraulic conductivities of the individual layers in the vertical direction and $k_{v(eq)}$ is the equivalent hydraulic conductivity.

Again, from Eq. (7.18),

$$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdots + H_n i_n \qquad (7.20)$$

Solving Eqs. (7.19) and (7.20) gives

$$k_{v(eq)} = \frac{H}{\left(\frac{H_1}{k_{v_1}}\right) + \left(\frac{H_2}{k_{v_2}}\right) + \left(\frac{H_3}{k_{v_3}}\right) + \cdots + \left(\frac{H_n}{k_{v_n}}\right)} \qquad (7.21)$$

Example 7.8 (H.W)

A layered soil is shown in Figure (7.10). Given:

$H_1 = 2$m $k_1 = 10^{-4}$ cm/sec, $H_2 = 3$m $k_2 = 3.2 \times 10^{-2}$ cm/sec

$H_3 = 4$m $k_3 = 4.1 \times 10^{-5}$ cm/sec

Estimate the ratio of equivalent hydraulic conductivity, $\frac{k_{H(eq)}}{k_{v(eq)}}$

Ans: 139.96

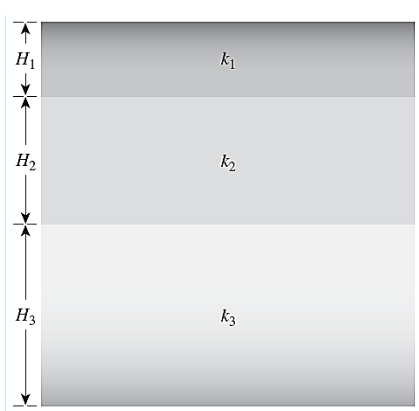

Figure (7.10) A layered soil profile

Example 7.9 (HW)

Figure (7.11) shows three layers of soil in a tube that is 100 mm × 100 mm in cross section. Water is supplied to maintain a constant-head difference of 300 mm across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as follows:

Soil	k(cm/sec)
A	10^{-2}
B	3×10^{-3}
C	4.9×10^{-4}

Find the rate of water supply in cm^3/hr

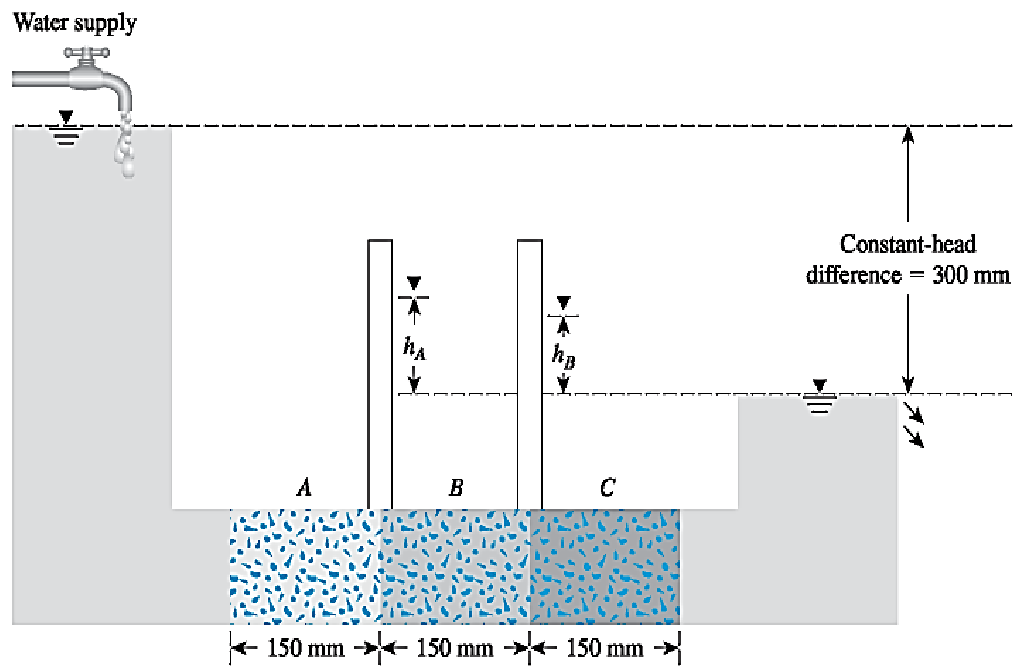

Figure (7.11) Three layers of soil in a tube 100 mm × 100 mm in cross section

Ans: q = 291.24 cm^3/hr

7.6 Permeability Test in the Field by Pumping from Wells

In the field, the average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from wells. Figure (7.12) shows a case where the top permeable layer, whose hydraulic conductivity has to be determined, is unconfined and underlain by an impermeable layer. During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well. Continuous observations of the water level in the test well and in the observation wells are made after the start of pumping, until a steady state is reached. The steady state is established when the water level in the test and observation wells becomes constant. The expression for the rate of flow of groundwater into the well, which is equal to the rate of discharge from pumping, can be written as

$$q = k\left(\frac{dh}{dr}\right)2\pi r h \tag{7.22}$$

or

$$\int_{r_2}^{r_1}\frac{dr}{r} = \left(\frac{2\pi k}{q}\right)\int_{h_2}^{h_1} h\,dh$$

Thus,

$$k = \frac{2.303\,q\,\log_{10}\left(\frac{r_1}{r_2}\right)}{\pi\left(h_1^2 - h_2^2\right)} \tag{7.23}$$

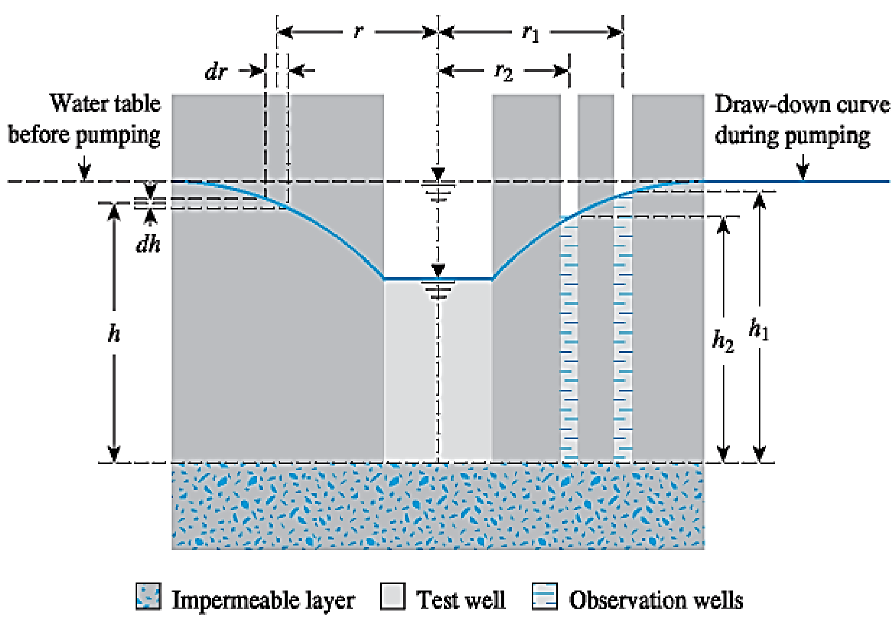

Figure (7.12) Pumping test from a well in an unconfined permeable layer underlain by an impermeable stratum.

7.7 Flow through an Aquifer

The average hydraulic conductivity for a confined aquifer can also be determined by conducting a pumping test from a well with a perforated casing that penetrates the full depth of the aquifer and by observing the piezometric level in a number of observation wells at various radial distances (Figure 7.13). Pumping is continued at a uniform rate q until a steady state is reached.

Because water can enter the test well only from the aquifer of thickness H, the steady state of discharge is

$$q = k\left(\frac{dh}{dr}\right)2\pi rH \tag{7.24}$$

or

$$\int_{r_2}^{r_1}\frac{dr}{r} = \int_{h_2}^{h_1}\left(\frac{2\pi kH}{q}\right)dh$$

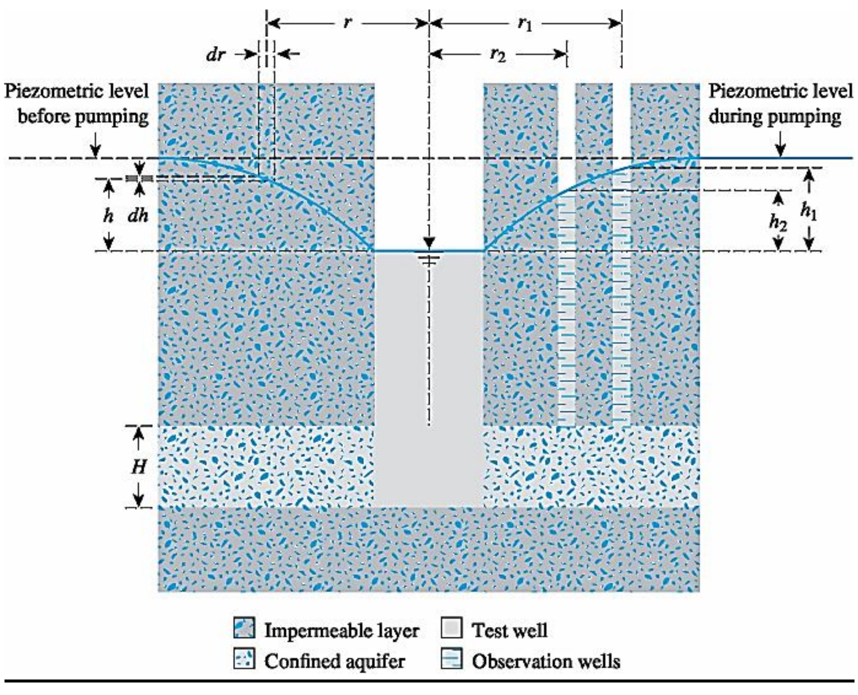

Figure (7.13) Pumping test from a well penetrating the full depth in a confined aquifer

This gives the hydraulic conductivity in the direction of flow as

$$k = \frac{q\log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1-h_2)} \tag{7.25}$$

Problems

7.1 Refer to Figure (7.3). For a constant-head permeability test in sand, the following are given:
* L= 300 mm
* A= 175 cm^2
* h= 500 mm
* Water collected in 3 min = 620 cm^3
* Void ratio of sand = 0.58

Determine

a. Hydraulic conductivity, k (cm/sec)
b. Seepage velocity

Ans: (a) k =0.012 cm/sec (b) v_s = 0.054 cm/sec

7.2 For a falling-head permeability test, the following are given: length of specimen = 380 mm; area of specimen= 6.5 cm^2; k = 0.175 cm/min. What should be the area of the standpipe for the head to drop from 650 cm to 300 cm in 8 min?

Ans: 0.31 cm^2

7.3 A sand layer of the cross-sectional area shown in Figure (7.14) has been determined to exist for a 800-m length of the levee. The hydraulic conductivity of the sand layer is 2.8 m/day. Determine the quantity of water which flows into the ditch in m^3/min.

Ans: 0.206 m^3/min

7.4 A permeable soil layer is underlain by an impervious layer, as shown in Figure (7.15). With $k = 5.2 \times 10^{-4}$ cm/sec for the permeable layer, calculate the rate of seepage through it in m^3/hr/m length if H = 3.8 m and α = 8.

Ans: 0.7739×10^{-6} m^3/hr/m

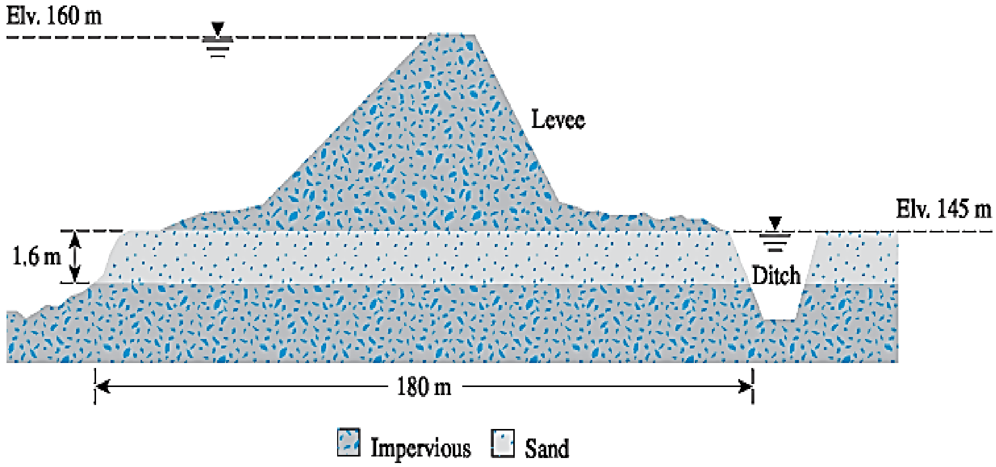

Figure (7.14)

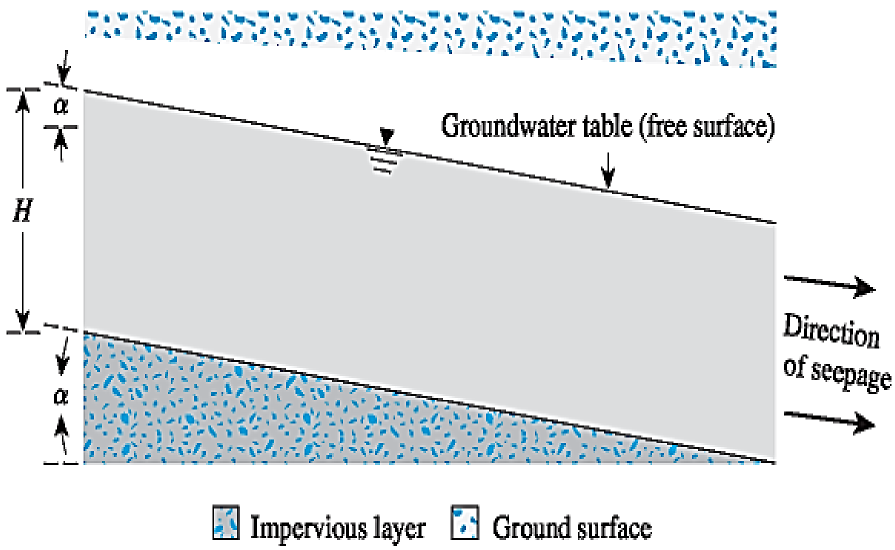

Figure (7.15)

7.5 Refer to Figure (7.16). Find the flow rate in m^3/sec/m length (at right angle to the cross section shown) through the permeable soil layer. Given: H = 5 m, H_1 =2.8 m, h = 3.1 m, L = 60 m, α = 5, k = 0.05 cm/sec.

Ans: 7.18×10^{-5} m^3/sec/m

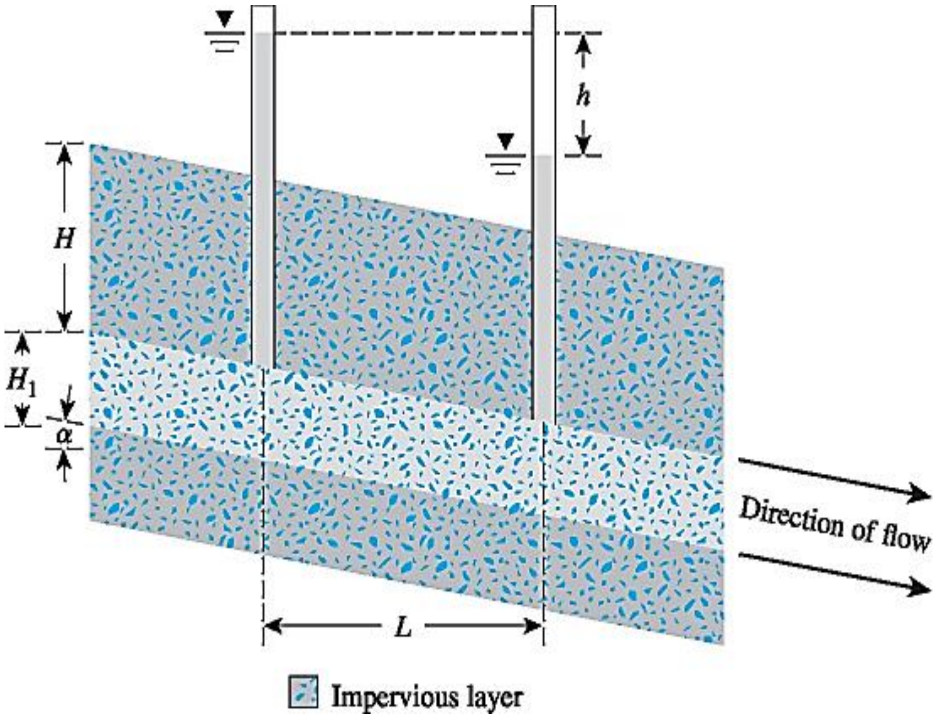

Figure (7.16)

7.6 A layered soil is shown in Figure (7.17). Given that

*****H_1= 1.5m * $k_1 = 10^{-5}$ cm/sec

*****H_2= 2.5m * $k_2 = 3.0\times10^{-3}$ cm/sec

*****H_3= 3.0m * $k_2 = 3.5\times10^{-5}$ cm/sec

Estimate the ratio of equivalent hydraulic conductivity, $\frac{k_{H(eq)}}{k_{v(eq)}}$

Ans: 0.3684

7.7 A layered soil is shown in Figure (7.18). Estimate the ratio of equivalent hydraulic conductivity, $\frac{k_{H(eq)}}{k_{v(eq)}}$

Ans: 3.53

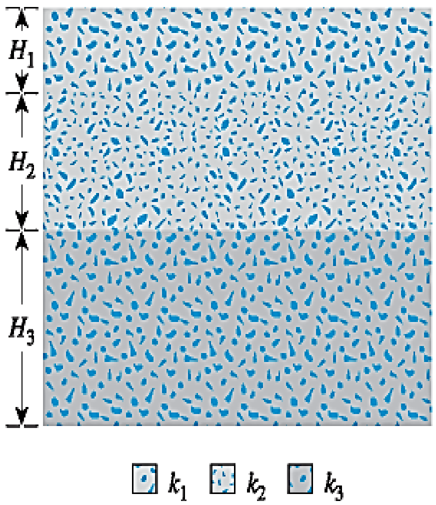

Figure (7.17)

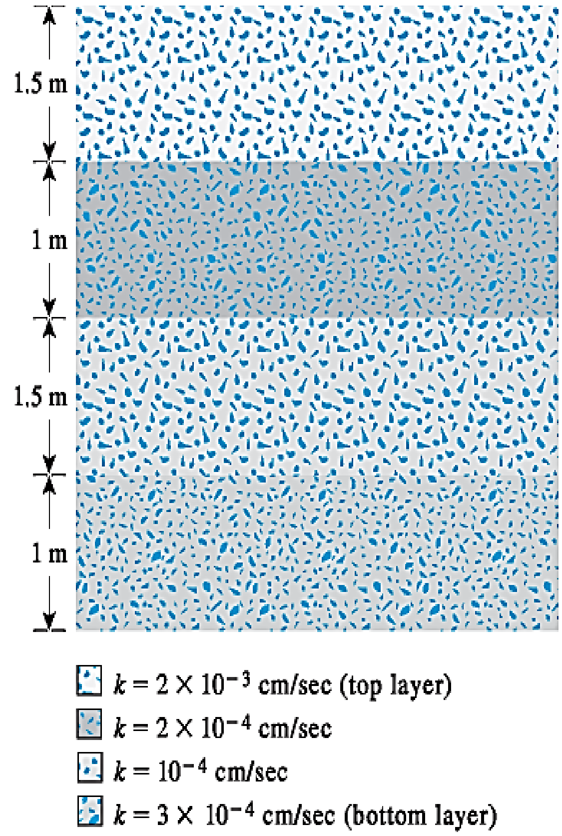

Figure (7.18)