

Chapter 3: Weight-Volume Relationships (Phase Relationships)

Partially saturated soil (three-phase soil) is composed of solids (soil particles), liquids (usually water), and gases (usually air). The spaces between the solids are called voids. The soil water is commonly called pore water and it plays a very important role in the behavior of soils under load. If all voids are filled with water, the soil is saturated (two-phase). Otherwise, the soil is unsaturated. If all the voids are filled with air, the soil is said to be dry (two-phase).

3.1 Weight-Volume Relationships

Figure (3.1a) shows an element of soil of volume V and weight W as it would exist in a natural state. To develop the weight–volume relationships, we must separate the three phases (that is, solid, water, and air) as shown in Figure (3.1b). Thus, the total volume of a given soil sample can be expressed as

$$V = V_s + V_v = V_s + V_w + V_a \quad (3.1)$$

where V_s = volume of soil solids

V_v = volume of voids

V_w = volume of water in the voids

V_a = volume of air in the voids

Assuming that the weight of the air is negligible, we can give the total weight of the sample as

$$W = W_s + W_w \quad (3.2)$$

where W_s = weight of soil solids

W_w = weight of water

The *volume relationships* commonly used for the three phases in a soil element are *void ratio*, *porosity*, and *degree of saturation*. *Void ratio* (e) is defined as the ratio of the volume of voids to the volume of solids. Thus,

$$e = \frac{V_v}{V_s} \quad (3.3)$$

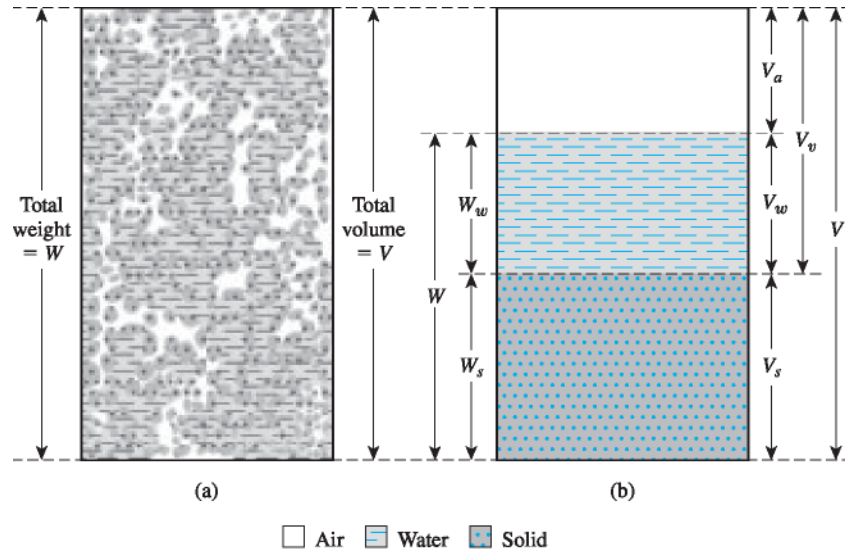


Figure (3.1a) Soil element in natural state; (b) three phases of the soil element

Porosity (n) is defined as the ratio of the volume of voids to the total volume, or

$$n = \frac{V_v}{V} \quad (3.4)$$

The degree of saturation (S) is defined as the ratio of the volume of water to the volume of voids, or

$$S = \frac{V_w}{V_v} \quad (3.5)$$

It is commonly expressed as a percentage.

The relationship between void ratio and porosity can be derived from Eqs. (3.1), (3.3), and (3.4) as follows:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{\left(\frac{V_v}{V}\right)}{1 - \left(\frac{V_v}{V}\right)} = \frac{n}{1 - n} \quad (3.6)$$

Also, from Eq. (3.6),

$$n = \frac{e}{1 + e} \quad (3.7)$$

The common terms used for *weight relationships* are *moisture content* and *unit weight*. *Moisture content* (w) is also referred to as *water content* and is defined

as the ratio of the weight of water to the weight of solids in a given volume of soil:

$$w = \frac{W_w}{W_s} \quad (3.8)$$

Unit weight (γ) is the weight of soil per unit volume. Thus,

$$\gamma = \frac{W}{V} \quad (3.9)$$

The unit weight can also be expressed in terms of the weight of soil solids, the moisture content, and the total volume. From Eqs. (3.2), (3.8), and (3.9),

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s \left[1 + \left(\frac{W_w}{W_s} \right) \right]}{V} = \frac{W_s (1 + w)}{V} \quad (3.10)$$

Soils engineers sometimes refer to the unit weight defined by Eq. (3.9) as the *moist unit weight*.

Often, to solve earthwork problems, one must know the weight per unit volume of soil, excluding water. This weight is referred to as *the dry unit weight*, γ_d . Thus,

$$\gamma_d = \frac{W_s}{V} \quad (3.11)$$

From Eqs. (3.10) and (3.11), the relationship of unit weight, dry unit weight, and moisture content can be given as

$$\gamma_d = \frac{\gamma}{1 + w} \quad (3.12)$$

Sometimes it is convenient to express soil densities in terms of mass densities (ρ). The SI unit of mass density is kilograms cubic meter (kg/m^3). We can write the density equations [similar to Eqs. (3.9) and (3.11)] as

$$\rho = \frac{M}{V} \quad (3.13)$$

and

$$\rho_d = \frac{M_s}{V} \quad (3.14)$$

where ρ = density of soil (kg/m^3)

ρ_d = dry density of soil (kg/m^3)

M = total mass of the soil sample (kg)

M_s = mass of soil solids in the sample (kg)

The unit of total volume, V , is m^3 .

The unit weight in kN/m^3 can be obtained from densities in kg/m^3 as

$$\gamma \text{ (kN}/\text{m}^3) = \frac{g\rho \left(\frac{\text{kg}}{\text{m}^3}\right)}{1000}$$

and

$$\gamma_d \text{ (kN}/\text{m}^3) = \frac{g\rho_d \left(\frac{\text{kg}}{\text{m}^3}\right)}{1000}$$

where g = acceleration due to gravity = $9.81 \text{ m}/\text{sec}^2$.

Note that unit weight of water (γ_w) is equal to $9.81 \text{ kN}/\text{m}^3$.

3.2 Relationships among Unit Weight, Void Ratio, Moisture Content, and Specific Gravity

To obtain a relationship among unit weight (or density), void ratio, and moisture content, let us consider a volume of soil in which the volume of the soil solids is one, as shown in Figure 3.2. If the volume of the soil solids is one, then the volume of voids is numerically equal to the void ratio, e [from Eq. (3.3)]. The weights of soil solids and water can be given as

$$W_s = G_s \gamma_w$$

$$W_w = wW_s = wG_s \gamma_w$$

where G_s = specific gravity of soil solids

w = moisture content

γ_w = unit weight of water

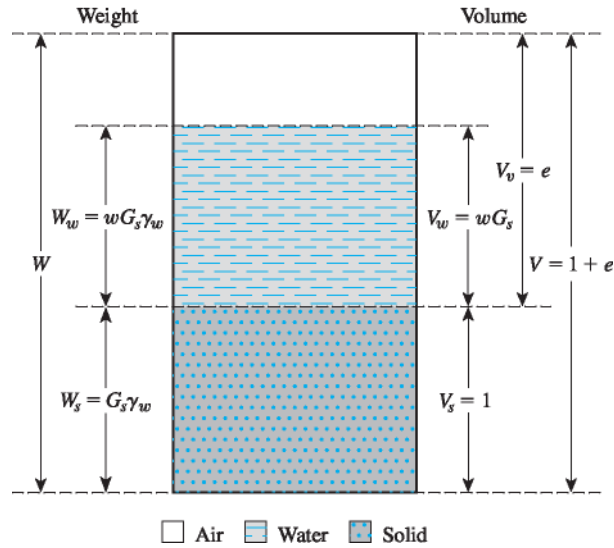


Figure 3.2 Three separate phases of a soil element with volume of soil solids equal to one

Now, using the definitions of unit weight and dry unit weight [Eqs. (3.9) and (3.11)], we can write

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + w G_s \gamma_w}{1 + e} = \frac{(1 + w) G_s \gamma_w}{1 + e} \quad (3.15)$$

and

$$\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1 + e} \quad (3.16)$$

or

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 \quad (3.17)$$

Because the weight of water for the soil element under consideration is $w G_s \gamma_w$, the volume occupied by water is

$$V_w = \frac{W_w}{\gamma_w} = \frac{w G_s \gamma_w}{\gamma_w} = w G_s$$

Hence, from the definition of degree of saturation [Eq. (3.5)],

$$S = \frac{V_w}{V_v} = \frac{w G_s}{e}$$

or

$$Se = wG_s \quad (3.18)$$

This equation is useful for solving problems involving three-phase relationships.

If the soil sample is saturated—that is, the void spaces are completely filled with water (Figure 3.3)—the relationship for saturated unit weight (γ_{sat}) can be derived in a similar manner:

$$\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + e \gamma_w}{1 + e} = \frac{(G_s + e) \gamma_w}{1 + e} \quad (3.19)$$

Also, from Eq. (3.18) with $S=1$,

$$e = wG_s \quad (3.20)$$

As mentioned before, due to the convenience of working with densities in the SI system, the following equations, similar to unit-weight relationships given in Eqs. (3.15), (3.16), and (3.19), will be useful:

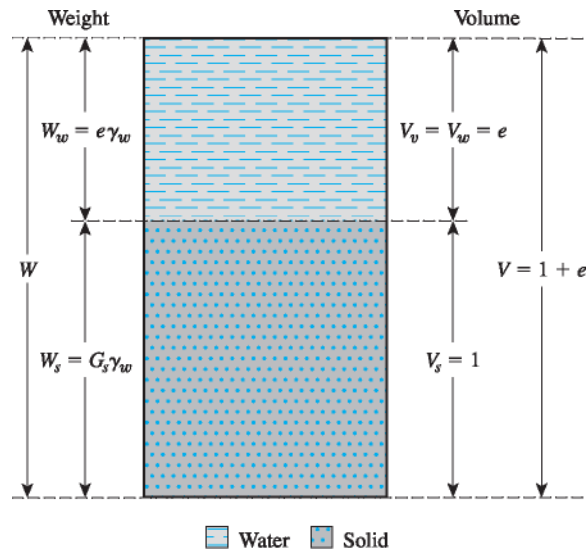


Figure (3.3) Saturated soil element with volume of soil solids equal to one

$$\text{Density} = \rho = \frac{(1+w)G_s\rho_w}{1+e} \quad (3.21)$$

$$\text{Dry density} = \rho_d = \frac{G_s\rho_w}{1+e} \quad (3.22)$$

$$\text{Saturated density} = \rho_{sat} = \frac{(G_s+e)\rho_w}{1+e} \quad (3.23)$$

where ρ_w = density of water = 1000 kg/m³.

Equation (3.21) may be derived by referring to the soil element shown in Figure 3.4, in which the volume of soil solids is equal to 1 and the volume of voids is equal to e .

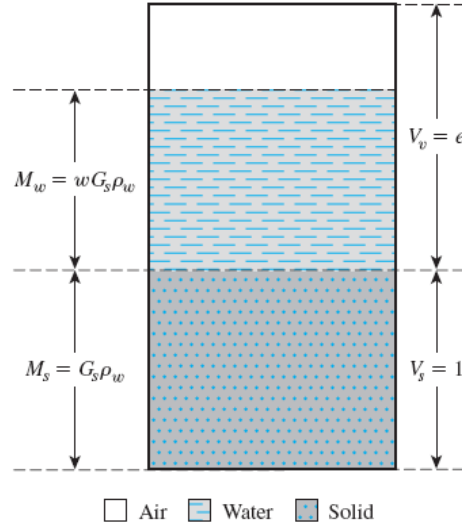


Figure (3.4) Three separate phases of a soil element showing mass–volume relationship

Hence, the mass of soil solids, M_s , is equal to $G_s\rho_w$. The moisture content has been defined in Eq. (3.8) as

$$\begin{aligned} w &= \frac{W_w}{W_s} = \frac{(\text{mass of water}).g}{(\text{mass of soil}).g} \\ &= \frac{M_w}{M_s} \end{aligned}$$

where M_w = mass of water.

Since the mass of soil in the element is equal to $G_s\rho_w$, the mass of water

$$M_w = wM_s = wG_s\rho_w$$

From Eq. (3.13), density

$$\begin{aligned} \rho &= \frac{M}{V} = \frac{M_s + M_w}{V_s + V_v} = \frac{G_s\rho_w + wG_s\rho_w}{1 + e} \\ &= \frac{(1+w)G_s\rho_w}{1+e} \end{aligned}$$

Equations (3.22) and (3.23) can be derived similarly.

3.3 Relationships among Unit Weight, Porosity, and Moisture Content

The relationship among *unit weight, porosity, and moisture content* can be developed in a manner similar to that presented in the preceding section. Consider a soil that has a total volume equal to one, as shown in Figure 3.5. From Eq. (3.4),

$$n = \frac{V_v}{V}$$

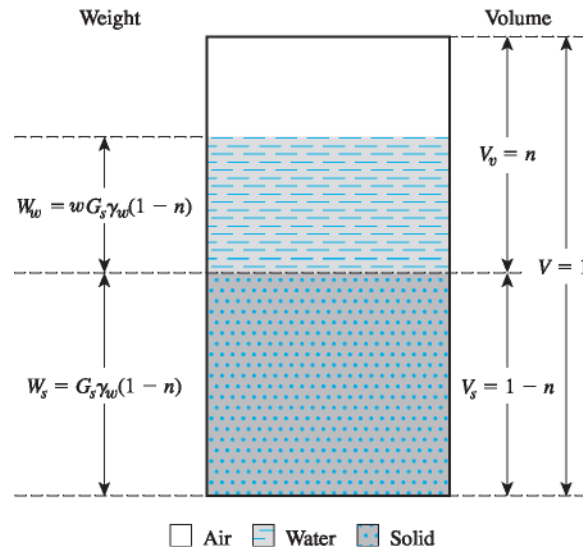


Figure (3.5) Soil element with total volume equal to one

If V is equal to 1, then V_v is equal to n , so $V_s = 1 - n$. The weight of soil solids (W_s) and the weight of water (W_w) can then be expressed as follows:

$$W_s = G_s \gamma_w (1 - n) \quad (3.24)$$

$$W_w = w W_s = w G_s \gamma_w (1 - n) \quad (3.25)$$

So, the dry unit weight equals

$$\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w (1 - n)}{1} = G_s \gamma_w (1 - n) \quad (3.26)$$

The moist unit weight equals

$$\gamma = \frac{W_s + W_w}{V} = G_s \gamma_w (1 - n) (1 + w) \quad (3.27)$$

Figure (3.6) shows a soil sample that is saturated and has $V = 1$. According to this figure,

$$\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{(1-n)G_s\gamma_w + n\gamma_w}{1} = [(1-n)G_s + n]\gamma_w \quad (3.28)$$

The moisture content of a saturated soil sample can be expressed as

$$W = \frac{W_w}{W_s} = \frac{n\gamma_w}{(1-n)\gamma_w G_s} = \frac{n}{(1-n)G_s} \quad (3.29)$$

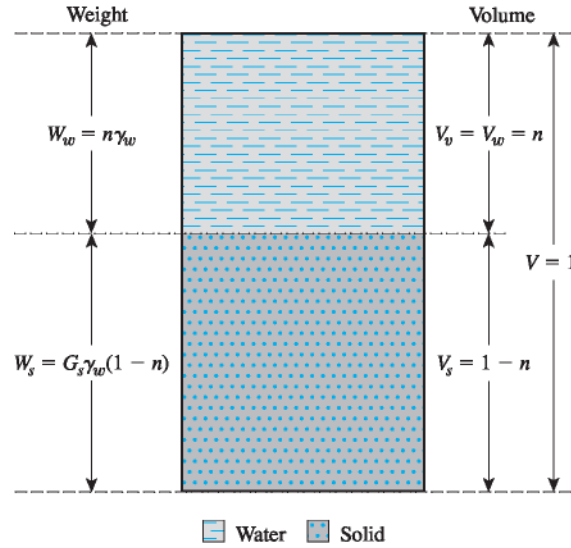


Figure (3.6) Saturated soil element with total volume equal to one

3.4 Various Unit-Weight Relationships

In Sections 3.2 and 3.3, we derived the fundamental relationships for the moist unit weight, dry unit weight, and saturated unit weight of soil. Several other forms of relationships that can be obtained for γ , γ_d , and γ_{sat} are given in Table (3.1). Some typical values of void ratio, moisture content in a saturated condition, and dry unit weight for soils in a natural state are given in Table (3.2).

Table 3.1 Various Forms of Relationships for γ , γ_d , and γ_{sat}

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	Relationship	Given	Relationship	Given	Relationship
w, G_s, e	$\frac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\frac{\gamma}{1+w}$	G_s, e	$\frac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\frac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\frac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat}-\frac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d+\left(\frac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat}-n\gamma_w$	γ_d, n	$\gamma_d+n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	γ_d, S	$\left(1-\frac{1}{G_s}\right)\gamma_d+\gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

Table 3.2 Void Ratio, Moisture Content, and Dry Unit Weight for Some Typical Soils in a Natural State

Type of soil	Void ratio, e	Natural moisture content in a saturated state (%)	Dry unit weight, γ_d	
			lb/ft ³	kN/m ³
Loose uniform sand	0.8	30	92	14.5
Dense uniform sand	0.45	16	115	18
Loose angular-grained silty sand	0.65	25	102	16
Dense angular-grained silty sand	0.4	15	121	19
Stiff clay	0.6	21	108	17
Soft clay	0.9–1.4	30–50	73–93	11.5–14.5
Loess	0.9	25	86	13.5
Soft organic clay	2.5–3.2	90–120	38–51	6–8
Glacial till	0.3	10	134	21

Example 3.1

For a saturated soil, show that

$$\gamma_{\text{sat}} = \left(\frac{e}{w} \right) \left(\frac{1+w}{1+e} \right) \gamma_w$$

Solution

From Eqs. (3.19) and (3.20),

$$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1 + e} \quad (\text{a})$$

and

$$e = wG_s$$

or

$$G_s = \frac{e}{w} \quad (\text{b})$$

Combining Eqs. (a) and (b) gives

$$\gamma_{\text{sat}} = \frac{\left(\frac{e}{w} + e \right) \gamma_w}{1 + e} = \left(\frac{e}{w} \right) \left(\frac{1 + w}{1 + e} \right) \gamma_w \quad \blacksquare$$

Example 3.2

For a moist soil sample, the following are given.

- Total volume: $V = 1.2 \text{ m}^3$
- Total mass: $M = 2350 \text{ kg}$
- Moisture content: $w = 8.6\%$
- Specific gravity of soil solids: $G_s = 2.71$

Determine the following.

- a. Moist density
- b. Dry density
- c. Void ratio
- d. Porosity
- e. Degree of saturation
- f. Volume of water in the soil sample

Solution

Part a

From Eq. (3.13),

$$\rho = \frac{M}{V} = \frac{2350}{1.2} = 1958.3 \text{ kg/m}^3$$

Part b

From Eq. (3.14),

$$\rho_d = \frac{M_s}{V} = \frac{M}{(1+w)V} = \frac{2350}{\left(1 + \frac{8.6}{100}\right)(1.2)} = 1803.3 \text{ kg/m}^3$$

Part c

From Eq. (3.22),

$$\rho_d = \frac{G_s \rho_w}{1+e}$$

$$e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{(2.71)(1000)}{1803.3} - 1 = 0.503$$

Part d

From Eq. (3.7),

$$n = \frac{e}{1+e} = \frac{0.503}{1+0.503} = 0.335$$

Part e

From Eq. (3.18),

$$S = \frac{wG_s}{e} = \frac{\left(\frac{8.6}{100}\right)(2.71)}{0.503} = 0.463 = 46.3\%$$

Part f

Volume of water:

$$\frac{M_w}{\rho_w} = \frac{M - M_s}{\rho_w} = \frac{M}{\rho_w} - \frac{M}{(1+w)\rho_w} = \frac{2350}{1000} - \left(\frac{2350}{1 + \frac{8.6}{100}}\right) = 0.186 \text{ m}^3$$

Alternate Solution

Refer to Figure 3.7.

Part a

$$\rho = \frac{M}{V} = \frac{2350}{1.2} = 1958.3 \text{ kg/m}^3$$

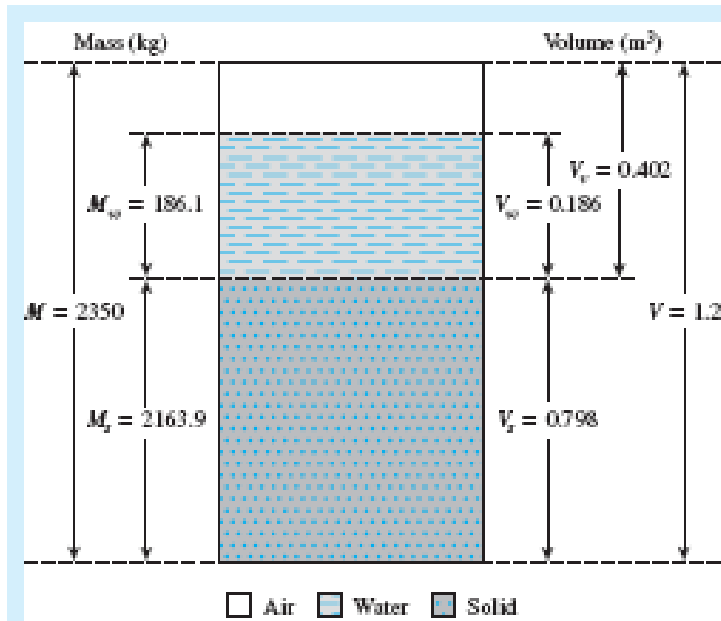


Figure 3.7

Part b

$$M_s = \frac{M}{1 + w} = \frac{2350}{1 + \frac{8.6}{100}} = 2163.9 \text{ kg}$$

$$\rho_d = \frac{M_s}{V} = \frac{M}{(1 + w)V} = \frac{2350}{\left(1 + \frac{8.6}{100}\right)(1.2)} = 1803.3 \text{ kg/m}^3$$

Part c

$$\text{The volume of solids: } \frac{M_s}{G_s \rho_w} = \frac{2163.9}{(2.71)(1000)} = 0.798 \text{ m}^3$$

$$\text{The volume of voids: } V_v = V - V_s = 1.2 - 0.798 = 0.402 \text{ m}^3$$

$$\text{Void ratio: } e = \frac{V_v}{V_s} = \frac{0.402}{0.798} = 0.503$$

Part d

$$\text{Porosity: } n = \frac{V_v}{V} = \frac{0.402}{1.2} = 0.335$$

Part e

$$S = \frac{V_w}{V_v}$$

$$\text{Volume of water: } V_w = \frac{M_w}{\rho_w} = \frac{186.1}{1000} = 0.186 \text{ m}^3$$

Hence,

$$S = \frac{0.186}{0.402} = 0.463 = 46.3\%$$

Part f

From Part e,

$$V_w = 0.186 \text{ m}^3$$

Example 3.3

The following data are given for a soil:

- Porosity: $n = 0.4$
- Specific gravity of the soil solids: $G_s = 2.68$
- Moisture content: $w = 12\%$

Determine the mass of water to be added to 10 m^3 of soil for full saturation.

Solution

Equation (3.27) can be rewritten in terms of density as

$$\rho = G_s \rho_w (1 - n)(1 + w)$$

Similarly, from Eq. (3.28)

$$\rho_{sat} = [(1 - n)G_s + n]\rho_w$$

Thus,

$$\rho = (2.68)(1000)(1 - 0.4)(1 + 0.12) = 1800.96 \text{ kg/m}^3$$

$$\rho_{sat} = [(1 - 0.4)(2.68) + 0.4](1000) = 2008 \text{ kg/m}^3$$

Mass of water needed per cubic meter equals

$$\rho_{sat} - \rho = 2008 - 1800.96 = 207.04 \text{ Kg/m}^3$$

So, total mass of water to be added equals

$$207.04 \times 10 = 2070.4 \text{ kg}$$

Example 3.4

A saturated soil has a dry unit weight of 16.2 kN/m^3 . Its moisture content is 23%

Determine:

- Saturated unit weight, γ_{sat}
- Specific gravity, G_s
- Void ratio, e

Solution**Part a: saturated Unit Weight**

From Eq. (3.12),

$$\gamma_{sat} = \gamma_d(1 + w) = (16.2) \left(1 + \frac{23}{100}\right) = 19.93 \frac{kN}{m^3}$$

Part b: Specific Gravity, G_s

From Eq. (3.16),

$$\gamma_d = \frac{G_s \gamma_w}{1 + e}$$

Also from Eq(3.20) for saturated soils, $e = wG_s$. Thus,

$$\gamma_d = \frac{G_s \gamma_w}{1 + wG_s}$$

So,

$$16.2 = \frac{G_s(9.81)}{1 + (0.23)(G_s)}$$

or

$$16.2 + 3.726G_s = 9.81G_s$$

$$G_s = 2.66$$

Part c: Void Ratio, e

For saturated soils,

$$e = wG_s = (0.23)(2.66) = 0.61$$

3.5 Relative Density

The term relative density is commonly used to indicate the *in situ* denseness or looseness of granular soil. It is defined as

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \quad (3.30)$$

where D_r = relative density, usually given as a percentage

e = in situ void ratio of the soil

e_{max} = void ratio of the soil in the loosest state

e_{min} = void ratio of the soil in the densest state

The values of D_r may vary from a minimum of 0% for very loose soil to a maximum of 100% for very dense soils. Soils engineers qualitatively describe the granular soil deposits according to their relative densities, as shown in Table (3.3). In-place soils seldom have relative densities less than 20 to 30%. Compacting a granular soil to a relative density greater than about 85% is difficult.

Table 3.3 Qualitative Description of Granular Soil Deposits

Relative density (%)	Description of soil deposit
0–15	Very loose
15–50	Loose
50–70	Medium
70–85	Dense
85–100	Very dense

The relationships for relative density can also be defined in terms of porosity, or

$$e_{max} = \frac{n_{max}}{1-n_{max}} \quad (3.31)$$

$$e_{min} = \frac{n_{min}}{1-n_{min}} \quad (3.32)$$

$$e = \frac{n}{1-n} \quad (3.33)$$

where n_{max} and n_{min} = porosity of the soil in the loosest and densest conditions, respectively. Substituting Eqs. (3.31), (3.32), and (3.33) into Eq. (3.30), we obtain

$$D_r = \frac{(1-n_{min})(n_{max}-n)}{(n_{max}-n_{min})(1-n)} \quad (3.34)$$

By using the definition of dry unit weight given in Eq. (3.16), we can express relative density in terms of maximum and minimum possible dry unit weights. Thus,

$$D_r = \frac{\left[\frac{1}{\gamma_{d(min)}} \right] - \left[\frac{1}{\gamma_d} \right]}{\left[\frac{1}{\gamma_{d(min)}} \right] - \left[\frac{1}{\gamma_{d(max)}} \right]} = \left[\frac{\gamma_d - \gamma_{d(min)}}{\gamma_{d(max)} - \gamma_{d(min)}} \right] \left[\frac{\gamma_{d(max)}}{\gamma_d} \right] \quad (3.35)$$

where $\gamma_{d(min)}$ = dry unit weight in the loosest condition (at a void ratio of e_{max})

γ_d = in situ dry unit weight (at a void ratio of e)

$\gamma_{d(max)}$ = dry unit weight in the densest condition (at a void ratio of e_{min})

In terms of density, Eq. (3.35) can be expressed as

$$D_r = \left[\frac{\rho_d - \rho_{d(min)}}{\rho_{d(max)} - \rho_{d(min)}} \right] \frac{\rho_{d(max)}}{\rho_d} \quad (3.36)$$

ASTM Test Designations D-4253 and D-4254 (2007) provide a procedure for determining the maximum and minimum dry unit weights of granular soils so that they can be used in Eq. (3.35) to measure the relative density of compaction in the field. For sands, this procedure involves using a mold with a volume of 2830 cm³ (0.1 ft³). For a determination of the *minimum dry unit weight*, sand is poured loosely into the mold from a funnel with a 12.7 mm ($\frac{1}{2}$ in.) diameter spout. The average height of the fall of sand into the mold is maintained at about 25.4 mm (1 in.). The value of $\gamma_{d(min)}$ then can be calculated by using the following equation

$$\gamma_{d(min)} = \frac{W_s}{V_m} \quad (3.37)$$

Where W_s = weight of sand required to fill the mold

V_m = volume of the mold

The *maximum dry unit weight* is determined by vibrating sand in the mold for 8 min. A surcharge of 14 kN/m² (2 lb/in²) is added to the top of the sand in the mold. The mold is placed on a table that vibrates at a frequency of 3600 cycles/min and that has an amplitude of vibration of 0.635 mm (0.025 in.). The value of $\gamma_{d(max)}$ can be determined at the end of the vibrating period with knowledge of the weight and volume of the sand.

Example 3.5

For a given sandy soil, $e_{max}=0.75$ and $e_{min}=0.4$. Let $G_s=2.68$. In the field, the soil is compacted to a moist density of 17.63 kN/m³ at a moisture content of 12%. Determine the relative density of compaction.

Solution

From Eq. (3.21)

$$\rho = \frac{(1+w)G_s\rho_w}{1+e}$$

or

$$e = \frac{G_s\gamma_w(1+w)}{\gamma} - 1 = \frac{(2.68)(9.81)(1+0.12)}{17.63} - 1 = 0.67$$

From Eq. (3.30),

$$D_r = \frac{e_{max}-e}{e_{max}-e_{min}} = \frac{0.75-0.67}{0.75-0.4} = 0.229 = 22.9\%$$

Problems

3.1- For a given soil, show that

$$\gamma_{sat} = \gamma_d + n\gamma_w$$

3.2- For a given soil, show that

$$\gamma_{sat} = \gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$$

3.3 - For a given soil, show that

$$\gamma_d = \frac{eS\gamma_w}{(1+e)w}$$

3.4 - A 0.4 m³ moist soil sample has the following:

* Moist mass = 711.2 Kg

* Dry mass = 623.9 Kg

* Specific gravity of soil solids = 2.68

Estimate:

a-Moisture content

b-Moist density

c-Dry density

d-Void ratio

e-Porosity

Ans: (a) 13.99% (b) 1778 kg/cm³ (c) 1559.75 kg/cm³ (d) 0.72 (e) 0.42

3.5- The moist weight of 5600 cm³ of a soil is 102.3N. The moisture content and the specific gravity of the soil solids are determined in the laboratory to be 11% and 2.7, respectively. Calculate the following:

a- Moist unit weight

b- Dry unit weight

c- Void ratio

d- Porosity

e- Degree of saturation (%)

f- Volume occupied by water

Ans: (a) 18.27 kN/m³ (b) 16.46 kN/m³ (c) 0.61 (d) 0.38 (e) 48.7% (f) 0.297

3.6 - The saturated unit weight of a soil is 19.8 kN/m³. The moisture content of the soil is 17.1%. Determine the following:

a- Dry unit weight

b- Specific gravity of soil solids

c-Void ratio

Ans: (a) 16.91 kN/m³ (b) 2.44 (c) 0.417

3.7 - The unit weight of a soil is 14.84 kN/m³. The moisture content of this soil is 19.2% when the degree of saturation is 60%. Determine:

a-Void ratio

b-Specific gravity of soil solids

c-Saturated unit weight

Ans: (a) 0.68 (b) 2.13 (c) 16.41 kN/m³

3.8 - For a given soil, the following are given: $G_s = 2.67$; moist unit weight, $\gamma = 17.5 \text{ kN/m}^3$; and moisture content, $w = 10.8\%$. Determine:

- a-Dry unit weight
- b-Void ratio
- c-Porosity
- d- Degree of saturation

Ans: (a) 15.8 kN/m^3 (b) 0.658 (c) 0.397 (d) 44%

3.9 - The moist density of a soil is 1680 kg/m^3 . Given $w = 18\%$ and $G_s = 2.73$, determine:

- a- Dry density
- b- Porosity
- c- Degree of saturation
- d- Mass of water, in kg/m^3 , to be added to reach full saturation

Ans: (a) 1423.7 kg/cm^3 (b) 0.479 (c) 53.5% (d) 222.06 kg

3.10 - The dry density of a soil is 1780 kg/m^3 . Given $G_s = 2.68$, what would be the moisture content of the soil when saturated?

Ans: 19%

3.11 - The porosity of a soil is 0.35. Given $G_s = 2.69$, calculate:

- a- Saturated unit weight (kN/m^3)
- b- Moisture content when moist unit weight = 17.5 kN/m^3

Ans: (a) 20.59 kN/m^3 (b) 2%

3.12 - A saturated soil has $w = 23\%$ and $G_s = 2.62$. Determine its saturated and dry densities in kg/m^3 .

Ans: $\rho_{\text{sat}} = 2012.5 \text{ kg/m}^3$, $\rho_d = 1637.5 \text{ kg/m}^3$

3.13 - A soil has $w = 18.2\%$, $G_s = 2.67$, and $S = 80\%$. Determine the moist and dry unit weights of the soil.

Ans: 19.23 kN/m^3 , 16.27 kN/m^3

3.14 - The moist unit weight of a soil is 17.55 kN/m^3 at a moisture content of 10%. Given $G_s = 2.7$, determine:

- a- e
- b- Saturated unit weight

Ans: (a) 0.66 (b) 19.86 kN/m^3

3.15 - For a given sand, the maximum and minimum void ratios are 0.78 and 0.43, respectively. Given $G_s = 2.67$, determine the dry unit weight of the soil in kN/m^3 when the relative density is 65%.

Ans: 16.87 kN/m^3

3.16 - For a given sandy soil, $e_{\text{max}} = 0.75$, $e_{\text{min}} = 0.46$, and $G_s = 2.68$. What will be the moist unit weight of compaction (kN/m^3) in the field if $D_r = 78\%$ and $w = 9\%$?

Ans: 18.8 kN/m^3

3.17 - The moisture content of a soil sample is 18.4%, and its dry unit weight is 15.63 kN/m^3 . Assuming that the specific gravity of solids is 2.65,

a- Calculate the degree of saturation.

b- What is the maximum dry unit weight to which this soil can be compacted without change in its moisture content?

Ans: (a) 74% (b) 17.5 kN/m^3

3.18 - A loose, uncompacted sand fill 1.83m in depth has a relative density of 40%. Laboratory tests indicated that the minimum and maximum void ratios of the sand are 0.46 and 0.90, respectively. The specific gravity of solids of the sand is 2.65.

a- What is the dry unit weight of the sand?

b- If the sand is compacted to a relative density of 75%, what is the decrease in thickness of the 1.83m fill?

Ans: (a) 15.1 kN/m^3 (b) 16.1cm