<u>Lectures 1</u>

Introduction

In this chapter we discuss the concept of dimension and the choices of units commonly used in physics. We discuss dimensional analysis and order of magnitude calculations, and define some mathematical notation

Standards of Length, Mass and Time

There are five **basic quantities**:

- length(L)
- mass (M)
- time (t)
- electric current (I)
- temperature (T)

Systems of Units

- SI units (used mostly in physics):
    - length: meter (m)
    - mass: kilogram (kg)
    - time: second (s)

  This system is also referred to as the **mks** sytem for **m**eter-**k**ilogram-**s**econd.
- Gaussian units (used mostly in chemistry):
    - length: centimeter (cm)
    - mass: gram (g)
    - time: second (s)

  This system is also referred to as the **cgs** system for **c**entimeter-**g**ram-**s**econd.
- British engineering system:
    - length: foot (ft)

- mass: slug
- time: second (s)

Scientific Notation

It is sometimes convenient to express large or small numbers in **scientific notation**.

For Example: 5,000 = 5×10^3 and .0004 = 4×10^{-4}.

Commonly used prefixes for powers of 10 used with metric units are given below in Table 1.1.

Table 2.1: Prefixes used with metric units.

Power	Prefix	Abbreviation
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M

For Example:

a) 60,000 m = 6×10^4 m = 60 km

b) 0.003 s = 3×10^{-3} s = 3 ms

Dimensional Analysis

Definition: The **Dimension** is the qualitative nature of a physical quantity (length, mass, time).

Square brackets denote the dimension or units of a physical quantity:

Table 2.2: Dimensions

quantity	dimension	SI units
area	$[A] = L^2$	m^2

volume	$[V]=L^3$	m^3
velocity	$[v] = L/t$	m/s
acceleration	$[a] = L/t^2$	m/s^2
mass	$[m] = M$	kg

Idea: Dimensional analysis can be used to derive or check formulas by treating dimensions as algebraic quantities. Quantities can be added or subtracted only if they have the same dimensions, and quantities on two sides of an equation must have the same dimensions.
Note: Dimensional analysis can't give numerical factors. **For Example:** The distance (x) travelled by a car in a given time (t) , starting from rest and moving with constant acceleration (a) is given by, $x = \frac{1}{2}at^2$. We can check this equation with dimensional analysis:

$$l.h.s. \Rightarrow [x] = L$$

$$r.h.s. \Rightarrow \left[\frac{1}{2}at^2\right] = \frac{1}{2}[a][t^2] = \frac{L}{t^2}t^2 = L.$$

Since the dimension of the left hand side (l.h.s.) of the equation is the same as that on the right hand side (r.h.s.), the equation is said to be *dimensionally consistent*.

Conversion of Units

Idea: Units can be treated as algebraic quantities. For example, we can use the conversion factor 1 in = 2.54 cm to rewrite 15 inches in centimeters.

$$15\ in = 15\ in \left(\frac{2.54\ \text{cm}}{1\ \text{in}}\right) = 38.1\ cm \qquad (1)$$

Order of Magnitude Calculations

Idea: An order of magnitude calculation is an *estimate* to determine if a more precise calculation is necessary. We round off or guess at various inputs to obtain a result that is usually reliable to within a factor of 10. Specifically, to get the order of magnitude of a given quantity, we round off to the closest power of 10 (example: 75 kg $\rightarrow 10^2$ kg).

Mathematical Notation

1- $\propto$ - proportional to
2- < or > - less than or greater than

3- $\ll$ or $\gg$ - much less than or much greater than
4- $\simeq$ - approximately equal to
5- $\equiv$ - defined as
6- Δx - change in the quantity x
7- Σ - summation sign
8- $|x|$ - absolute value of x

Coordinate Systems and Frames of Reference

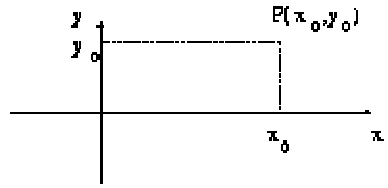
The location of a point on a line can be described by one coordinate; a point on a plane can be described by two coordinates; a point in a three dimensional volume can be described by three coordinates. In general, *the number of coordinates equals the number of dimensions.* A coordinate system consists of:

1- a fixed reference point (*origin*)
2- a set of *axes* with specified directions and scales
3- instructions that specify how to *label* a point in space *relative to the origin and axes.*

For Example:

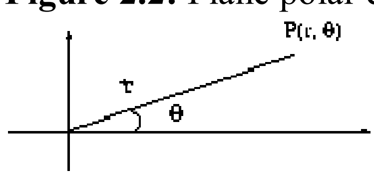
- **Cartesian coordinate system** (rectangular coordinate system): (x,y)

Figure 2.1: Cartesian coordinate system



- **Plane polar coordinates**: (r, θ)

Figure 2.2: Plane polar coordinates



Definition: The **position vector** ($\vec{r}$) in any given coordinate system specifys the position of a given point within that coordinate system relative to the origin.

Problems

PROBLEM 2.1

The diameter of the earth, measured at the equator, is 7930 mi. Express the diameter a) in meters and b) in kilometers. Use scientific notation when expressing your answers.

Solution:

a)

$$d = 7930\ mi \cdot \left[\frac{1609\ \text{m}}{1.0\ \text{mi}}\right] = 1.28 \text{ x } 10^7\ m$$

b)

$$d = 1.28 \text{ x } 10^7\ m \cdot \left[\frac{1\ \text{km}}{1 \times 10^3\ \text{m}}\right] = 1.28 \text{ x } 10^4\ km$$

PROBLEM 2.2

The period of a simple pendulum, defined as the time for one complete oscillation, is measured in time units and is given by:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where l is the length of the pendulum and g is the acceleration due to gravity, in units of length divided by time squared. Show that this equation is dimensionally consistent; that is, show that the right hand side of this equation gives units of time.

Solution:

$$\left[2\pi\sqrt{\frac{l}{g}}\right] = \sqrt{\frac{\text{L}}{\text{L}/\text{t}^2}} = \sqrt{\text{t}^2} = t$$

PROBLEM 2.3

A point is located in a polar co-ordinate system by the co-ordinates $r = 2.5$ m and $\theta = 35^{o}$. Find the x - and y - co-ordinates of this point, assuming that the two co-ordinate systems have the same origin.

Solution:

$$\frac{x}{2.5} = \cos 35^{o}$$

$$x = 2.5\cos 35^{o} = 2.05 \ m$$

$$\frac{y}{2.5} = \sin 35^{o}$$

[2*mm*]

$$y = 2.5\sin 35^{o} = 1.43 \ m$$

PROBLEM 2.4

A truck driver moves up a straight mountain highway, as shown in the figure. Elevation markers at the beginning and ending points of the trip show that he has risen vertically 0.530 km, and the mileage indicator on the truck shows that he has travelled a total distance of 3.00 km during the ascent. Find the angle of incline of the hill, θ.

Figure 2.3: Problem 1.4

Solution:

$$\sin \theta = \frac{0.530}{3.00} = 0.177$$

$$\theta = \sin^{-1}(0.177) = 10.2^{o}$$

Lectures 2

Motion in One Dimension

In this chapter we discuss motion in one dimension. We introduce definitions for displacement, velocity and acceleration, and derive equations of motion for bodies moving in one dimension with constant acceleration. We apply these equations to the situation of a body moving under the influence of gravity alone.

Displacement

Definition: **Displacement** is change in position, $\Delta \vec{x} = \vec{x}_f - \vec{x}_i$ where $\vec{x}_f$ is the final position and $\vec{x}_i$ is the initial position. The arrow indicates that displacement is a vector quantity: it has direction and magnitude. In 1 dimension, there are only two possible directions which can be specified with either a plus or a minus sign. Other examples of vectors are velocity, acceleration and force. In contrast, scalar quantities have only magnitude. Some examples of scalars are speed, mass, temperature and energy.

Average Velocity

Definition: **Average Velocity** is displacement over total time. Mathematically:

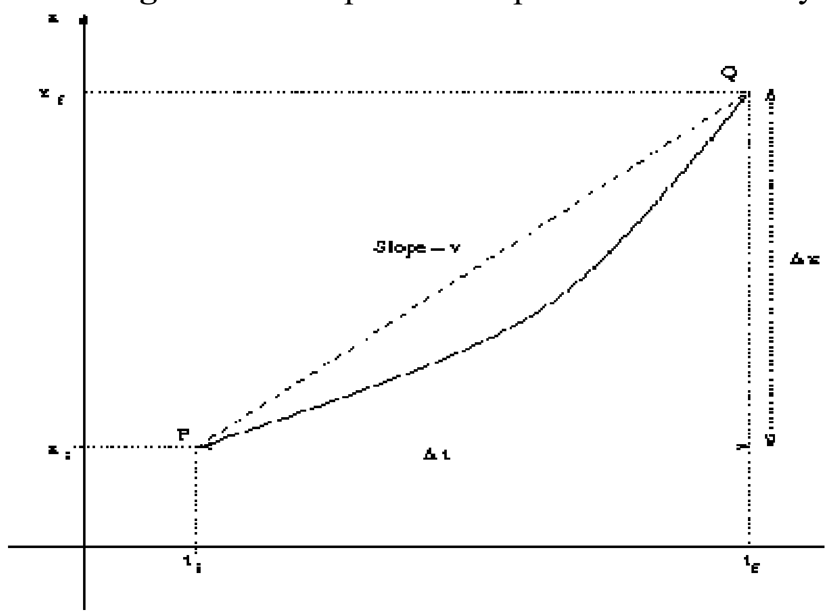
$$\bar{v} \equiv \frac{\Delta x}{\Delta t} \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{displace}{elapsed}$$

Note:

- the overbar is frequently used to denote an average quantity
- Δt is always > 0 so the sign of $\bar{v}$ depends only on the sign of Δx .

Graphical interpretation of velocity: Consider 1-d motion from point P (with coordinates x_i , t_i) to point Q (at x_f , t_f). We can plot the trajectory on a graph (see Figure 2.1).

Figure 2.1: Graphical interpretation of velocity



Then $\bar{v}$ from Eq.(2.1) is just the slope of the line joining P and Q.

Instantaneous Velocity

Definition: **Instantaneous velocity** is defined mathematically:

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Example: Table 2.1 gives data on the position of a runner on a track at various times.

Table 2.1: Position and time for a ru

t(s)	x(m)
1.00	1.00
1.01	1.02
1.10	1.21
1.20	1.44
1.50	2.25
2.00	4.00
3.00	9.00

Find the runner's instantaneous velocity at $t = 1.00$ s. As a first estimate, find the average velocity for the total observed part of the run. We have,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{9.00\ \mathrm{m} - 1.00\ \mathrm{m}}{3.00\ \mathrm{s} - 1.00\ \mathrm{s}} = 4\ m/s.$$

From the definition of instantaneous velocity Eq.(2.2), we can get a better approximation by taking a shorter time interval. The best approximation we can get from this data gives,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{1.02\ \mathrm{m} - 1.00\ \mathrm{m}}{1.01\ \mathrm{s} - 1.00\ \mathrm{s}} = 2\ m/s.$$

We can interpret the instantaneous velocity graphically as follows. Recall that the average velocity is the slope of the line joining P and Q (from Figure 2.1). To get the instantaneous velocity we need to take $\Delta t \to 0$, or P $\to$ Q. When P $\to$ Q, the line joining P and Q approaches the tangent to the curve at P (or Q). Thus the slope of the tangent at P is the instantaneous velocity at P. Note that if the trajectory were a straight line, we would get $v = \bar{v}$, the same for all t .

Note:

- Instantaneous velocity gives more information than average velocity.
- The magnitude of the velocity (either average or instantaneous) is referred to as the **speed**.

Acceleration

Definition: **Average acceleration** is the change in velocity over the change in time:

$$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad (5)$$

Definition: **Instantaneous acceleration** is calculated by taking shorter and shorter time intervals, i.e. taking $\Delta t \to 0$:

$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad (6)$$

Note:

- Acceleration is the rate of change of velocity.
- When velocity and acceleration are in the same direction, speed increases with time. When velocity and acceleration are in opposite directions, speed decreases with time.
- Graphical interpretation of acceleration: On a graph of v versus t , the average acceleration between P and Q is the slope of the line between P and Q, and the instantaneous aceleration at P is the tangent to the curve at P.
- From now on ``velocity'' and ``acceleration'' will refer to the instantaneous quantities.

One Dimensional Motion with Constant Acceleration

Constant acceleration means velocity increases or decreases at the same rate throughout the motion. Example: an object falling near the earth's surface (neglecting air resistance).

- Derivation of Kinematic Equations of Motion
- Freely Falling Bodies

Derivation of Kinematic Equations of Motion

Choose $t_i \equiv 0$, $x_i \equiv 0$, $v_i \equiv v_0$, and write $x_f \equiv x$, $v_f \equiv v$ and $t_f \equiv t$.

$a = constant \Rightarrow a = \bar{a}$. Then Eq.(2.5) $\Rightarrow a = \frac{v - v_0}{t}$ or

$$v = v_0 + at \qquad (7)$$

$a = constant \Rightarrow v$ changes uniformly $\Rightarrow \bar{v} = \frac{1}{2}(v_0 + v)$. From Eq.(2.1) $\bar{v} = x/t$. Combining: $x = \bar{v}t = \frac{1}{2}(v_0 + v)t$. Using Eq.(2.7) we get:

$$x = v_0 t + \frac{1}{2} at^2 \qquad (8)$$

Eq.(2.7) $\Rightarrow t = (v - v_0)/a$. Substitute into Eq.(2.8) $\Rightarrow x = (v + v_0)(v - v_0)/(2a)$ or,

$$v^2 = v_0^2 + 2ax \qquad (9)$$

- Note that only two of these equations are independent.

Freely Falling Bodies

A **freely falling object** is an object that moves under the influence of gravity only. Neglecting air resistance, all objects in free fall in the earth's gravitational field have a constant acceleration that is directed towards the earth's center, or perpendicular to the earth's surface, and of magnitude $|\vec{a}| \equiv g = 9.8\ m/s^2$. If motion is straight up and down and we choose a coordinate system with the positive y-axis pointing up and perpendicular to the earth's surface, we describe the motion with Eq.(2.7), Eq.(2.8), Eq.(2.9) with $a \rightarrow -g$, $x \rightarrow y$.

Equations of Motion for the 1-d vertical motion of an object in free fall:

- $v = v_0 - gt$
- $y = v_0 t - \frac{1}{2} g t^2$
- $v^2 = v_0^2 - 2gy$

Note: Since the acceleration due to gravity is the same for any object, a heavy object does not fall faster than a light object.

Problems

PROBLEM 2.1

A car travelling at a constant speed of 30 m/s passes a police car at rest. The policeman starts to move at the moment the speeder passes his car and accelerates at a constant rate of 3.0 m/ s^2 until he pulls even with the speeding car. Find a) the time required for the policeman to catch the speeder and b) the distance travelled during the chase.

Solution:

We are given, for the speeder:

$$v_0^s = 30\ \ m/s \quad = \quad v^s$$
$$a^s = 0$$

and for the policeman:

$$v_0^p \quad = $$
$$a^p \quad = \quad 3.0\ \ m/s^2.$$

a)

Distance travelled by the speeder $x^s = v^s t = (30)t$. Distance travelled by policeman $x^p = v_0^p + \frac{1}{2} a^p t^2 = \frac{1}{2}(3.0)t^2$. When the policeman catches the speeder $x^s = x^p$ or,

$$30t = \frac{1}{2}(3.0)t^2.$$

Solving for t we have $t = 0$ or $t = \frac{2}{3}(30) = 20\ s$. The first solution tells us that the speeder and the policeman started at the same point at $t = 0$, and the second one tells us that it takes 20 s for the policeman to catch up to the speeder.

b)

Substituting back in above we find,

$$x^{s} = 30(20) = 600\ m$$

and,

$$x^{p} = \frac{1}{2}(3.0)(20)^2 = 600\ m = x^{s}.$$

PROBLEM 2.2

A car decelerates at 2.0 m/s^2 and comes to a stop after travelling 25 m. Find a) the speed of the car at the start of the deceleration and b) the time required to come to a stop.

Solution:
We are given:

$$a = -2.0\ m/s^2$$
$$v =$$
$$x = 25\ m$$

a)

From $v^2 = v_0^2 + 2ax$ we have $v_0^2 = v^2 - 2ax = -2(-2.0)(25) = 100\ m^2/s^2$ or $v_0 = 10\ m/s$.

b)

From $v = v_0 + at$ we have $t = \frac{1}{a}(v - v_0) = \frac{1}{-2.0}(-10) = 5\ s$.

PROBLEM 2.3

A stone is thrown vertically upward from the edge of a building 19.6 m high with initial velocity 14.7 m/s. The stone just misses the building on the way down. Find a) the time of flight and b) the velocity of the stone just before it hits the ground.

Solution:
We are given,

$$v_0 = 14.7 \ m/s$$
$$a = -9.8 \ m/s^2$$

At the time the stone hits the ground,

$$x = -19.6 \ m$$

a)

From $x = v_0 t + \frac{1}{2}at^2$ we have,

$$t = \frac{1}{a}\left(-v_0 \pm \sqrt{v_0^2 - 4(-x)\frac{1}{2}a}\right)$$

$$= \frac{1}{-9.8}\left(-14.7 \pm \sqrt{(14.7)^2 - 4(19.6)\frac{1}{2}(-9.8)}\right)$$

$$= \frac{1}{-9.8}(-14.7 \pm 24.5).$$

The two solutions are $t = 4$ s and $t = -1$ s. The second (negative) solution gives the time the stone would have left the ground, and is unphysical in this case. The solution we want is the first one.

b)

We substitute to find $v = v_0 + at = 14.7 - 9.8(4) = -24.5$ m/s. Note that the negative velocity correctly shows that the stone is moving down.

PROBLEM 2.4

A rocket moves upward, starting from rest with an acceleration of 29.4 m/s^2 for 4 s. At this time, it runs out of fuel and continues to move upward. How high does it go?

Solution:
For the first stage of the flight we are given:

$$v_0 =$$
$$a = 29.4\ m/s^2$$
$$t = 4\ s$$

This gives, for the velocity and position at the end of the first stage of the flight: $v_1 = v_0 + at = 29.4\ m/s^2(4\ s) = 117.6\ m/s$ and $x_1 = v_0t + \frac{1}{2}at^2 = \frac{1}{2}(29.4)(4)^2 = 235.2\ m$.

For the second stage of the flight we start with,

$$v_1 = 117.6\ m/s$$
$$a = -9.8\ m/s^2$$

and end up with $v_2 = 0$. We want to find the distance travelled in the second stage $(x_2 - x_1)$. We have,

$$\begin{aligned} v_2^2 - v_1^2 &= 2a(x_2 - x_1) \\ \Rightarrow (x_2 - x_1) &= \frac{1}{2a}(v_2^2 - v_1^2) \\ &= \frac{1}{2(-9.8)}(-(117.6)^2) \\ &= 705.6\ m. \end{aligned}$$

Therefore $x_2 = x_1 + 705.6 = 235.2 + 705.6 = 940.8\ m$.

Lectures 3

Motion in Two Dimensions

In two dimensions, it is necessary to use vector notation to describe physical quantities with both magnitude and direction. In this chapter, we define displacement, velocity and acceleration as vectors in two dimensions. We also discuss the solution of projectile motion problems in two dimensions.

Scalars and Vectors

Scalars have *magnitude* only. Temperature, speed, mass, and volume are examples of scalars.

Vectors have *magnitude* and *direction*. The magnitude of $\vec{v}$ is written $|\vec{v}| \equiv v$. Position, displacement, velocity, acceleration and force are examples of vector quantities. Vectors have the following properties:

1. Vectors are equal if they have the same *magnitude* and *direction*.
2. Vectors must have the same units in order for them to be added or subtracted.
3. The negative of a vector has the same magnitude but opposite direction.
4. Subtraction of a vector is defined by adding a negative vector:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

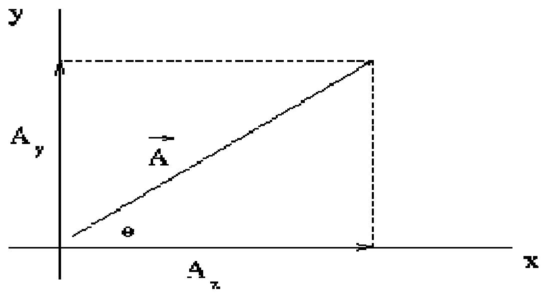
5. Multiplication or division of a vector by a scalar results in a vector for which

(a) only the magnitude changes if the scalar is positive

(b) the magnitude changes and the direction is reversed if the scalar is negative.

6. The projections of a vector along the axes of a rectangular co-ordinate system are called the **components** of the vector. The components of a vector completely define the vector.

Figure 3.1: Projections of a vector in 2-D.



$$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta$$

$$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta$$

We can invert these equations to find A and θ as functions of A_x and A_y By Pythagoras we have,

$$A = \sqrt{A_x^2 + A_y^2}$$

and from the diagram,

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}.$$

7. To add vectors by components: $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \ldots$

(a) Find the components of all vectors to be added.

(b) Add all x components to get $R_x = A_x + B_x + C_x + \ldots$

Add all y components to get $R_y = A_y + B_y + C_y + \ldots$

(c) Then

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x}$$

Displacement, Velocity and Acceleration in 2-Dimensions

Recall: In 1-dimension, the vector nature of velocity and acceleration is taken into account by the sign (positive or negative) of the quantity. In 2-dimensions we must use 2 components to specify a velocity or acceleration vector.

- Displacement
- Average Velocity
- Instantaneous Velocity
- Average Acceleration
- Instantaneous Acceleration

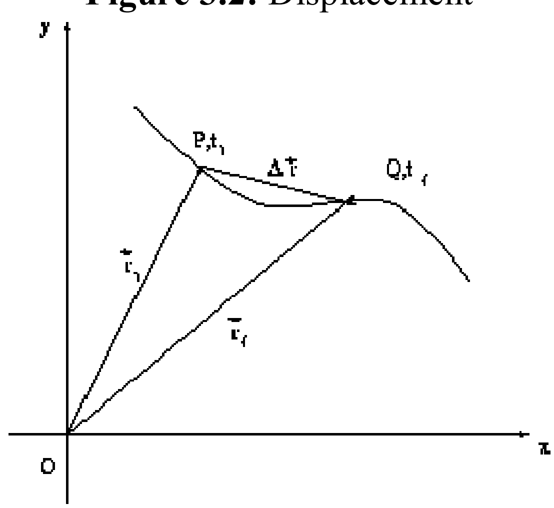
Displacement

In Figure 3.2, an object is at position $\vec{r}_i$ at time t_i (point P). Some time later, t_f, the object is at position $\vec{r}_f$ (point Q). The displacement vector of the object is given by:

(1)

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

Figure 3.2: Displacement



Average Velocity

Definition:

$$\vec{\bar{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \qquad (2)$$

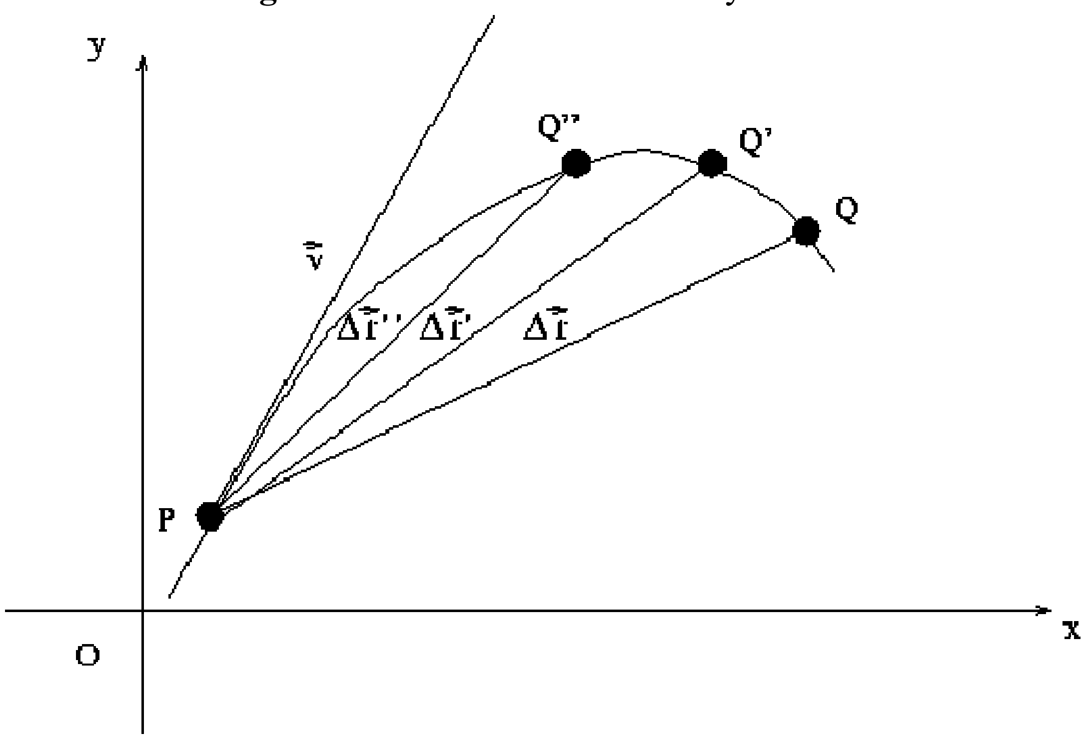
As with the 1 dimensional definition, $\vec{\bar{v}}$ is independent of the path between the end points.

Instantaneous Velocity

Definition:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

Figure 3.3: Instantaneous velocity in 2-D.



Interpretation: When $\Delta t \to 0$, the point Q in the figure gets closer and closer to the point P and the direction of $\Delta\vec{r}$ approaches the direction of a tangent to the curve at point P. Thus the instantaneous velocity $\vec{v}$ is parallel to the tangent and in the same direction as the motion.

Average Acceleration

Definition:

$$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous Acceleration

Definition:

$$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\vec{v}}{\Delta t}$$

Note: a particle can accelerate in different ways:

1. The magnitude of $\vec{v}$ can change in time, while the direction of motion stays the same.
2. The magnitude of $\vec{v}$, $|\vec{v}|$, can stay constant, while the direction of motion changes. This only happens in more than one dimension. We will discuss this further in Chapter 7.
3. Both $|\vec{v}|$ and the direction of $\vec{v}$ can change.

Projectile Motion

Projectile motion is a particular kind of 2 dimensional motion. We make the following assumptions:

- The only force present is the force due to gravity.
- The magnitude of the acceleration due to gravity is $|\vec{a}| = g = 9.8\ m/s^2$ We choose a coordinate system in which the positive y-axis points up perpendicular to the earth's surface. This definition gives, $a_y = -g$ and $a_x = 0$.
- The rotation of the earth does not affect the motion.

Initial Conditions:

We choose the coordinate system so that the particle leaves the origin ($x_0 = 0$, $y_0 = 0$) at time $t_0 = 0$ with an initial velocity of $\vec{v}_0$.

Procedure for Solving Projectile Motion Problems

1. Separate the motion into the x (horizontal) part and y (vertical) part.
2. Consider each part separately using the appropriate equations. The equations of motion become,

(a) x motion ($a_x = 0$):

$$v_x = v_{x0} = constant$$
$$x = v_{x0} t$$

(b) y motion ($a_y = -g$):

$$v_y = v_{y0} - gt$$
$$y = v_{y0} t - \frac{1}{2} a t^2$$
$$v_y^2 = v_{y0}^2 - 2gy.$$

3. Solve the resulting system of equations for the unknown quantities.

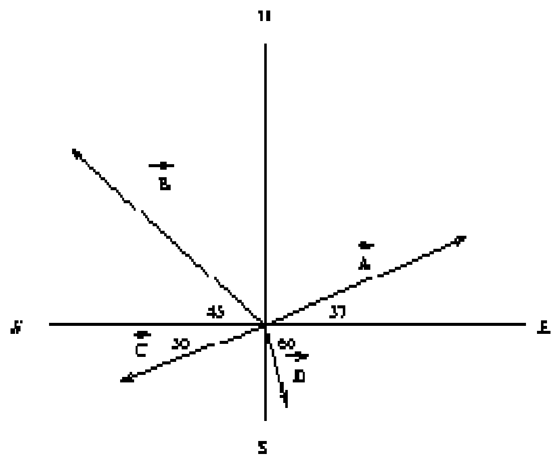
Worked Problems

PROBLEM 3.1

Find the sum of the following displacement vectors:

$$\vec{A} = 5.0 \ m \ at \ 37^\circ N \ of \ E$$
$$\vec{B} = 6.0 \ m \ at \ 45^\circ N \ of \ W$$
$$\vec{C} = 4.0 \ m \ at \ 30^\circ S \ of \ W$$
$$\vec{D} = 3.0 \ m \ at \ 60^\circ S \ of \ E$$

Figure 3.4: (Problem 3.1)



Solution:

a)

Resolve each vector into components:

$$A_x = 5\cos 37^{\circ} = 3.99\ m$$
$$A_y = 5\sin 37^{\circ} = 3.01\ m$$
$$B_x = -6\cos 45^{\circ} = -4.24\ m$$
$$B_y = 6\sin 45^{\circ} = 4.24\ m$$
$$C_x = -4\cos 30^{\circ} = -3.46\ m$$
$$C_y = -4\sin 30^{\circ} = -2.00\ m$$
$$D_x = 3\cos 60^{\circ} = 1.50\ m$$
$$D_y = -3\sin 60^{\circ} = -2.60\ m$$

b)

Add up all the x-components and all the y-components:

$$R_x = A_x + B_x + C_x + D_x = -2.21\ m$$
$$R_y = A_y + B_y + C_y + D_y = 2.65\ m$$

c)

Find $|\vec{R}|$ and θ:

$$R = \sqrt{R_x^2 + R_y^2} = 3.45\ m$$

$$\theta = \tan^{-1}\frac{R_y}{|R_x|} = 50^{\circ}\ N\ of\ W$$

PROBLEM 3.2

The current in a river is 1.0 m/s. A woman swims 300 m downstream and then back to her starting point without stopping. If she can swim 2.0 m/s in still water, find the time of the round trip.

Solution:

We need to find the velocity of the woman relative to the shore for each part of the swim. Let downstream be the positive direction and let v_w be the velocity of the water. $v_{w/w}$ is the velocity of the woman relative to the water and $v_{w/s}$ is the velocity of the woman relative to the shore. Then: (i) going downstream $v_{w/s} = v_w + v_{w/w} = 1.0 + 2.0 = 3.0\ m/s$ (ii) going upstream $v_{w/s} = 1.0 - 2.0 = -1.0\ m/s$.

To find the time to go 300 m in each direction use $x = v_0 t + \frac{1}{2} a t^2$. With $a = 0$ we have $t = x/v_0$.

This gives (i) downstream:

$$t_d = \frac{300\ \mathrm{m}}{3\ \mathrm{m/s}} = 100\ s$$

(ii) upstream:

$$t_u = \frac{-300\ \mathrm{m}}{-1\ \mathrm{m/s}} = 300\ s$$

The total time of the swim is $t_t = 100\ s + 300\ s = 400\ s$.

PROBLEM 3.3

The woman in the previous problem swims accross the river to the opposite bank and back. The river is 300 m wide and she swims perpendicular to the current so she ends up downstream from where she started. Find the time for the return trip.

Solution:

Since the woman swims perpendicular to the current we can define the y-axis as parallel to the river and treat the x and y motion independently. We are only interested in the motion in the x-direction. For the first half of her swim we have:

$$\begin{aligned} a_x &= \\ v_{x0} &= v_x = 2.0\ m/s \\ x &= 300\ m. \end{aligned}$$

To find the time to cross the river we use $x = v_{x0} t + \frac{1}{2} a_x t^2$ which gives,

$$t = \frac{x}{v_{x0}} = \frac{300\ \text{m}}{2.0\ \text{m/s}} = 150\ s. \qquad (3)$$

Since the motion is symetric, the time to return is the same as the time to cross. The total time is $t_t = 2(150\ s) = 300\ s$.

PROBLEM 3.4

(Example 3.4) A plane drops a package of emergency rations to a stranded party of explorers. The plane is travelling horizontally at 40.0 m/s at 100 m above the ground. Find a) where the package strikes the ground relative to the spot it was dropped and b) the velocity of the package just before it hits the ground.

Solution:

Set up the coordinate system as in the Figure 3.5. Consider the x and y components separately. We are given:

Table 3.1: (Problem 3.4)

x-motion	y-motion
$x = ?$	$y = -100\ m$
$v_{0x} = 40\ m/s$	$v_{0y} = 0$
$a_x = 0$	$a_y = -9.8\ m/s^2$

a)

First we find the time of flight from the y-motion.

$$\begin{aligned} y &= v_{oy}t + \tfrac{1}{2}a_y t^2 \\ -100 &= \tfrac{1}{2}(-9.8)t^2 \\ t &= 4.5\ s. \end{aligned} \qquad (4)$$

Then we can find x from,

$$\begin{aligned} x &= v_{x0}t + \tfrac{1}{2}a_x t^2 \\ x &= 40(4.5) + 0 \\ &= 180\ m. \end{aligned} \qquad (5)$$

b)

We find v_y from,

$$v_y = v_{y0} + at$$

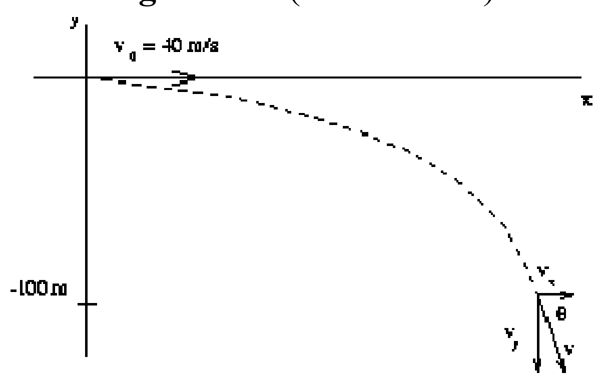
$= 0 - 9.8(4.5) = - 44.1 \; m/s.$

Note that the negative sign correctly indicates that the package falls downward. Since $a_x = 0$, we have $v_x = v_{0x} = 40 \; m/s$. We can combine the two velocity components to obtain,

$$v = \sqrt{v_x^2 + v_y^2} = 59.5 \; m/s$$

$$\theta = \tan^{-1}\left(\frac{44.1}{40}\right) = 48^{\circ}. \qquad (6)$$

Figure 3.5: (Problem 3.4)



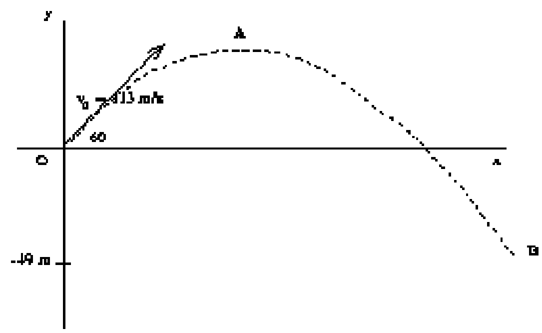
PROBLEM 3.5

A projectile is fired with an initial speed of 113 m/s at an angle of 60° above the horizontal from the top of a cliff 49 m high (see Figure 3.6). Find a) the time to reach the maximum height, b) the maximum height, c) the total time in the air, d) the horizontal range and e) the components of the final velocity just before the projectile hits the ground.

Solution:

Set up the coordinate system.

Figure 3.6: (Problem 3.5)



Consider the x- and y-motion separately. We are given:

Table 3.2: (Problem 3.5)

x-motion	y-motion
x =?	$y_B = -49\ m$
$v_{0x} = 113\cos 60^o$	$v_{0y} = 113\sin 60^o$
$v_{0x} = v_{Ax} = v_{Bx}$	$v_{Ay} = 0$
$a_x = 0$	$a_y = -9.8\ m/s^2$

a)
Find the time to reach the maximum height:

$$v_{Ay} = v_{0y} + a_y t_A$$

$$\Rightarrow t_A = \frac{v_{Ay} - v_{0y}}{a_y} = \frac{-113\sin 60^\circ}{-9.8} = 9.99\ s. \quad (7)$$

b)
Find the maximum height:

$$y_A = v_{0y}t_A + \frac{1}{2}a_y t_A^2 = 113\sin 60(9.99) - \frac{1}{2}(9.8)(9.99)^2 = 489\ m. \quad (8)$$

c)
Find the total time t_B in the air:

$$y_B = v_{0y}t_B + \frac{1}{2}a_y t_B^2$$
$$-49\ m = (113\sin 60)t_B - \frac{1}{2}(9.8)t^2_B$$
$$\Rightarrow 0 = 4.9t_B^2 - 97.9t_B - 49. \qquad (9)$$

Solving the quadratic we obtain,

$$t_B = \frac{1}{9.8}\left(97.9 \pm \sqrt{(97.9)^2 + 4(4.9)(49)}\right)$$

which gives, $t_B = 20.5\ s$ or $t_B = -0.49\ s$. We reject the second solution (it gives the time the projectile would have left the ground, if it had been thrown from there).

d)

Find the horizontal range (x_B):

$$x_B = v_{0x}t_B + \frac{1}{2}a_x t_B^2$$
$$= (113\cos 60)(20.5) = 1158\ m. \qquad (10)$$

e)

Find the components of the final velocity (v_{Bx} , v_{By}):

$$v_{Bx} = v_{0x} = 113\cos 60 = 56.5\ m/s$$
$$v_{By} = v_{0y} + a_y t_B$$
$$= 113\sin 60 - 9.8(20.5) = -103\ m/s. \qquad (11)$$

Note that the negative value of v_{By} correctly gives the direction as down.

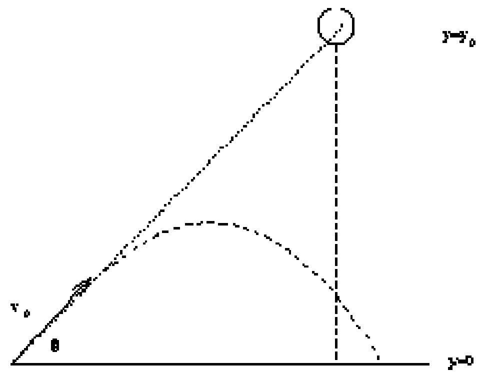
PROBLEM 3.6

(Problem 3.50) A projectile is fired at a falling target. The projectile leaves the gun at the same instant that the target falls from rest. Assuming that the gun is initially aimed at the target, show that the bullet will hit the target.

Solution:

Let x_B and y_B be the x and y positions of the bullet, and let x_T and y_T be the x and y positions of the target. We need to show that, $y_B = y_T$ and $x_B = x_T$ at some common time t_c , the time at which the bullet will hit the target.

Figure 3.7: (Problem 3.6)



The motion of the bullet is described by the two equations:

$$x_B = v_0 \cos\theta \; t$$
$$y_B = v_0 \sin\theta \; t - \frac{1}{2} g t^2.$$

It will take the bullet some time, t_c , to arrive at the x position of the target, x_T . At that time, the bullet and target will be at the same x postion (the target's x postion does not change since it falls straight down). Thus, we have at time t_c :

$$x_T = x_B = v_0 \cos\theta \; t_c$$

at time t_c .
In order for the bullet to hit the target, the y positions must be equal at t_c .
The y position of the target is given by

$$y_T = y_0 - \frac{1}{2} g t_c^2$$

and for the bullet

$$y_B = v_0 \sin\theta \; t_c - \frac{1}{2} g t_c^2.$$

We can see from the above equations that in order for $y_B = y_T$ at t_c , we need to have $y_0 = v_0 \sin\theta \; t_c$.
To show this, consider Figure 3.7:

$$\Rightarrow \tan\theta = \frac{y_0}{x_T} = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow y_0 = x_T \frac{\sin\theta}{\cos\theta}.$$

Using the previous result, $x_T = v_0 \cos\theta \; t_c$ or $\cos\theta = (x_T)/(v_0 t_c)$ gives:

$$y_0 = \frac{x_T \sin\theta}{x_T / v_0 t_c} = v_0 \sin\theta \; t_c.$$

Thus we have show that at time t_c , $x_T = x_B$ and $y_T = y_B$, and therefore that the bullet will hit the target.

Lectures 4

Force

In this section we introduce the concept of force. We discuss Newton's laws, which describe the way a body responds to a net force. We discuss frictional forces and the way they can be mathematically represented. We study several applications of Newton's laws.

Idea: Force is the cause of motion in classical mechanics. Classical mechanics deals with systems of size $\gg 10^{-10}$ m (atomic dimensions) and velocity $\ll 3.0 \times 10^8$ m/s (the speed of light).

Note:

- Force is a vector.
- There are two kinds of forces:
    - **Contact Forces** - involve physical contact between objects. Examples: the force involved in kicking a ball, pulling a wagon, compressing a spring, etc.
    - **Field forces** - don't involve physical contact between objects. Examples: the gravitational force and the electromagnetic force.
- Newton's First Law
- Newton's Second Law
- Newton's Third Law
- Applications of Newton's Laws
- Friction
    - Experimental facts about friction
- Problem Solving Strategy
- Problems

Newton's First Law

Newton's First Law States: an object at rest stays at rest, an object in motion stays in motion with a constant velocity, if there is no net external force between the object and the environment. In equation form we can write:

$$\sum \vec{F} = 0 \Rightarrow \vec{a} = 0. \qquad (1)$$

If $\sum \vec{F} \neq 0$ (i.e. there is a net external force acting on an object) then,

$$\sum \vec{F} = m\vec{a}. \qquad (2)$$

Definition: **Inertia** is the tendancy of an object to resist any attempt to change its state of motion. **Mass** is the force required per unit of acceleration produced and is a measure of inertia. Mass is a scalar and has SI units of kilograms (kg). Example: If a bowling ball and a golf ball are hit with a bat, the bowling ball would be much harder to get moving since it has greater mass and thus greater inertia.

Note:

- $\vec{a}$ is inversely proportional to m . This means that, for the same force, a smaller mass will have a larger acceleration.
- Newton's second law is a vector equation which contains three scalar equations (in three dimensions): $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$, $\Sigma F_z = ma_z$.
- The first law is a special case of the second law.
- The SI unit of force is the **Newton** (N). **Definition**: **1 Newton** is the force that produces an acceleration of 1 m/s ^{2} when acting on a 1 kg mass. In the cgs system: 1 dyne = 1 g cm/ s^2= 10^{-5}N. In the British engineering system: 1 pound (lb) = 4.448 N.

Definition: **Weight** ($\vec{w}$) is the force exerted on an object by a gravitational field. From Newton's second law,

$$w = mg. \qquad (3)$$

Note:

- Weight is a vector with direction towards the earth's center, or perpendicular to the earth's surface.
- The weight of an object is different on the earth and on the moon since the strength of the gravitational field is different ($g_{\text{earth}} \neq g_{\text{moon}}$).
- The value of g varies with distance from the center of the earth (more on this in chapter 7). As a consequence:
  - Since the earth isn't a perfect sphere, the weight of an object varies slightly from place to place on the earth's surface.

    - The weight of an object varies slightly with altitude above the earth's surface.
- In comparison, **mass** is a scalar with a value independent of location. Notice however that, in the approximation that g is constant, mass is proportional to the magnitude of the weight and the two quantities can be used interchangeably. This is called the equivalence principle.

Newton's Third Law

Idea: Forces in nature always exist in pairs. **Newton's third law** states: For every action, there is an equal and opposite reaction. When two bodies interact:

$$\vec{F}_{2\text{ on }1} = -\vec{F}_{1\text{ on }2}. \qquad (4)$$

Where $\vec{F}_{2\text{ on }1}$ is the force exerted on body 1 by body 2 and $\vec{F}_{1\text{ on }2}$ is the force exerted on body 2 by body 1.

For Example: When an object falls towards the earth, the earth exerts a force on it that causes it to accelerate towards the earth. According to Newton's third law, the object exerts a force on the earth as well, and the earth accelerates towards the object. Why don't we feel the earth accelerate?

Solution:

$$\begin{aligned}
2nd\ Law &\rightarrow m_e a_e = \vec{F}_{\text{obj on earth}} \\
3rd\ Law &\rightarrow \vec{F}_{\text{obj on earth}} = -\vec{F}_{\text{earth on obj}} \equiv -\vec{w} \\
&\Rightarrow \vec{a}_e = -\vec{w}/m_e \\
&\Rightarrow |\vec{a}_e| = \left(\frac{m_{\text{obj}}}{m_e}\right) g \ll g.
\end{aligned}$$

Conclusion: the acceleration of the earth is too small to detect because the mass of the earth is much larger than the mass of the object.

Applications of Newton's Laws

Assumptions:

- We treat objects as **point particles** (no rotational motion - more on this in chapter 8).
- We neglect masses of ropes and springs. One consequence of this assumption is that the force exerted along a rope is the same at all points in the rope.

Note: In problems with several bodies, apply Newton's 2nd law to **one body at a time**.

Friction

Friction originates from forces between atoms and molecules when surfaces are in contact. For example, friction occurs when a body moves on a rough surface or through a fluid medium (water, air, etc.). There are two types of friction:

1. The **Static force of friction** (f_s) is the force of friction between two objects when there is no motion.

2. The **Kinetic force of friction** (f_k) is the force of friction between two objects when there is motion.

Consider a block on a rough surface. Apply an external force F_{ext} to the block.

- if $F_{ext} < f_{s(max)}$ the block won't move
- as F_{ext} increases, f_s will increase until it reaches its maximum value. When $F_{ext} = f_{s(max)}$ the block will start to move (this is called the **point of slipping**).
- Once the block starts to move, the force of friction is given by f_k .

Experimental facts about friction

1. $f_s \leq \mu_s N$ where μ_s is the **coefficient of static friction** and N is the magnitude of the **normal force**. Equality holds when the object is on the point of slipping: $f_{s(max)} = \mu_s N$.

2. $f_k = \mu_k N$ where μ_k is the **coefficient of kinetic friction** and is approximately constant for any given pair of materials.

3. Values of μ_s and μ_k depend on the nature of the surfaces that are in contact. Usually $\mu_k < \mu_s$. Examples: rubber on concrete $\mu_s = 1.0$, $\mu_k = 0.8$; waxed wood on wet snow $\mu_s = 0.14$, $\mu_k = 0.10$.
4. The direction of the force of friction is opposite to the direction the object wants to move.
5. μ_k and μ_s are nearly independent of the area of contact between the two surfaces.
6. μ_k is nearly independent of the velocity of the object under consideration.

Problem Solving Strategy

- Draw a picture of the situation and a force diagram of all the forces for each body (a **free body diagram**).
  - In the force diagram for each object, include only the forces acting on that object.
  - The force exerted by a rope is called the tension and usually denoted $\vec{T}$.
  - The contact force exerted by a surface is called the **normal force** and always acts perpendicular to the surface.
- Set up a coordinate system and apply Newton's second law:

$$\Sigma F_x = ma_x, \quad \Sigma F_y = ma_y.$$

- If necessary, use the kinematic equations of motion to solve for the desired quantities.

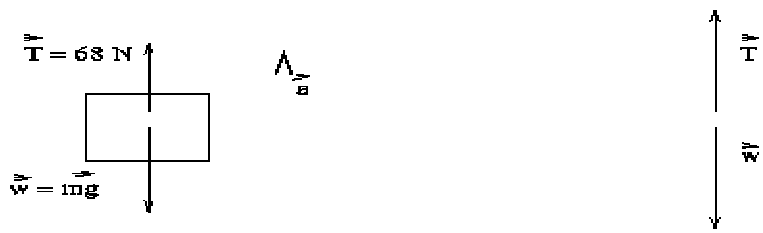
Problems

PROBLEM 4.1

A box of mass 5.0 kg is pulled vertically upwards by a force of 68 N applied to a rope attached to the box. Find a) the acceleration of the box and b) the vertical velocity of the box after 2 seconds.

Solution:

Figure 4.1: Problem 4.1



a) From the 2nd Law:

$$ma = T - mg$$

$$\Rightarrow a = \frac{T}{m} - g$$

$$= \frac{68\ \text{N}}{5\ \text{kg}} - 9.8\ m/s^2 = 3.8\ m/s^2 \qquad (5)$$

b) Since a is constant

$$v = v_0 + at$$

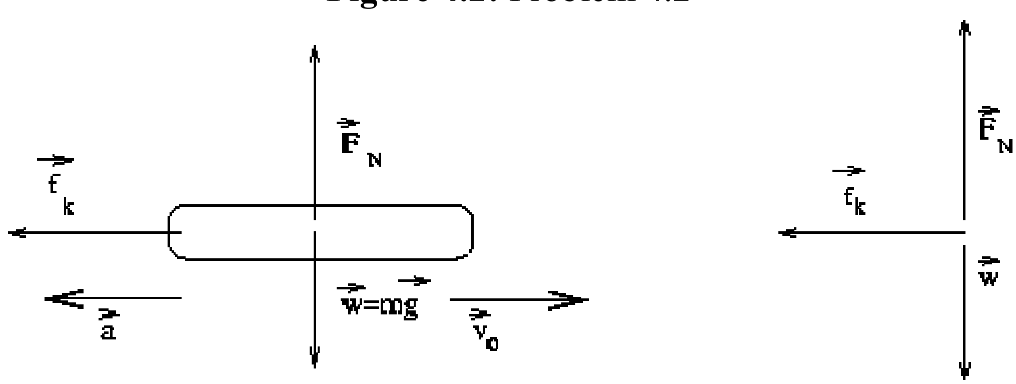
$$= 0 + 3.8(2) = 7.6\ \ m/s. \qquad (6)$$

PROBLEM 4.2

A hockey puck of mass .5 kg travelling at 10 m/s slows to 2.0 m/s over a distance of 80 m. Find a) the frictional force acting on the puck and b) the coefficient of kinetic friction between the puck and the surface.

Solution:

Figure 4.2: Problem 4.2



a)

First we find the acceleration of the puck from the kinematic equations of motion. We have, $v_0 = 10\ \ m/s$, $v = 2\ \ m/s$ and $x = 80\ \ m$. The third equation of motion gives,

$$v^2 = v_0^2 + 2ax$$

$$\Rightarrow a = \frac{v^2 - v_0^2}{2x} = \frac{4 - 100}{2(80)} = -0.6 \ m/s^2 \qquad (7)$$

From the Second Law:
In the x-direction,

$$f_k = ma = .5(-0.6) = -0.3 \ N. \qquad (8)$$

b)

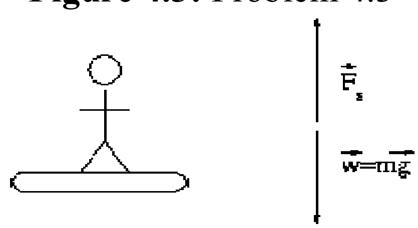
Use $f_k = -\mu_k N$. From the y-component of the 2nd Law: $N - mg = 0$. Combining,

$$-\mu_k mg = ma$$
$$\Rightarrow \mu_k = -a/g = 0.061. \qquad (9)$$

PROBLEM 4.3

A student of mass 50 kg tests Newton's laws by standing on a bathroom scale in an elevator. Assume that the scale reads in newtons. Find the scale reading when the elevator is a) accelerating upward at .5 m/s ^{2}, b) going up at a constant speed of 3.0 m/s and c) going up but decelerating at 1.0 m/s ^{2}.
Solution:

Figure 4.3: Problem 4.3



From the 2nd Law:

$$F_s - mg = ma$$
$$\Rightarrow F_s = m(g + a). \qquad (10)$$

This gives:

a) $F_s = 50(9.8 + 0.5) = 515 \ N$
b) $F_s = 50(9.8 + 0) = 490 \ N$

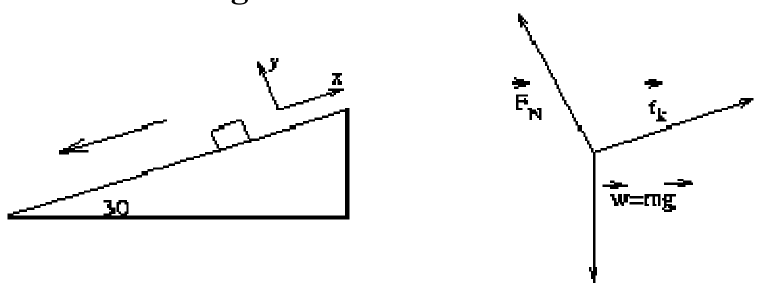
c) $F_s = 50(9.8 - 1.0) = 440\ N$

PROBLEM 4.4

A wooden plank is raised at one end to an angle of 30 °. A 2.0 kg box is placed on the incline 1.0 m from the lower end and given a slight tap to overcome static friction. The coefficient of kinetic friction between the box and the plank is $\mu_k = 0.20$. Find a) the rate of acceleration of the box and b) the speed of the box at the bottom. Assume that the initial speed of the box is zero.

Solution:

Figure 4.4: Problem 4.4



a)

Find the components of the weight of the object:

$$w_x = -mg\sin\theta$$
$$w_y = -mg\cos\theta.$$

Write out the two components of Newton's 2nd Law:

$$x:\ -mg\sin\theta + f_k = -ma$$
$$y:\ N - mg\cos\theta = 0. \qquad (11)$$

Using $f_k = \mu_k N$ we get,

$$ma = -\mu_k(mg\cos\theta) + mg\sin\theta$$
$$= g(\sin\theta - \mu_k\cos\theta)$$
$$\Rightarrow a = 9.8(\sin 30 - 0.2\cos 30) = 3.20\ m/s^2 \qquad (12)$$

b)

Since a is constant and $v^2 = v_0^2 + 2ax$. With $x = 1\ m$, $v_0 = 0$ we have

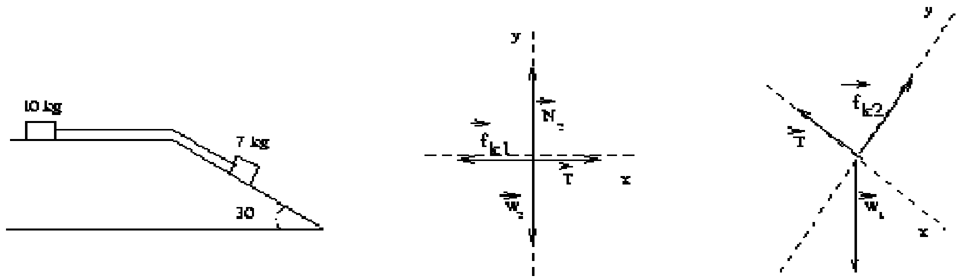
$$v = \sqrt{2(3.2)(1)} = 2.53\ m/s\ .$$

PROBLEM 4.5

A 10 kg box is attached to a 7 kg box which rests on a 30 ^{o} incline. The coefficient of kinetic friction between each box and the surface is $\mu_k = .1$. Find a) the rate of acceleration of the system and b) the tension in the rope.

Solution:

Figure 4.5: Problem 4.5



We apply the 2nd law separately to each box.

For the 10 kg box:

y direction:

$$N_2 - m_2 g =$$
$$N_2 = m_2 g,$$

x direction:

$$T - f_{k\,2} = m_2 a$$
$$T - \mu_k N_2 = m_2 a$$
$$= m_2 a. \qquad (13)$$
$$\Rightarrow \quad T - \mu_k m_2 g$$

For the 7 kg box:

y direction:

$$N_1 = m_1 g\cos\theta,$$

x direction:

$$m_1 g\sin\theta - T - f_{k\,1} = m_1 a$$
$$m_1 g\sin\theta - T - \mu_k N_1 = m_1 a$$
$$= m_1 a. \qquad (14)$$
$$\Rightarrow \quad m_1 g\sin\theta - T - \mu_k m_1 g\cos\theta$$

We have a system of two equations and two unknowns: *a* and *T* . We can solve as follows.

a)

From equation (4.1), $T = m_2a + \mu_k m_2g$. Substituting into equation (4.2) gives,

$$m_1a = m_1g\sin\theta - \mu_k m_1g\cos\theta - m_2a - \mu_k m_2g$$

$$\Rightarrow m_1a + m_2a = m_1g\sin\theta - \mu_k m_2g - \mu_k m_1g\cos\theta \quad (15)$$

$$\Rightarrow a = \frac{1}{m_1+m_2}[m_1g\sin\theta - \mu_k m_2g - \mu_k m_1g\cos\theta]g^{\mu_k}]$$

$$= \frac{1}{17}[7(9.8)\sin 30 - (0.1)(10)(9.8) - (0.1)(7)(9.8)\cos 30)] = 1.1\ m/s^2 \quad (16)$$

b)

Then substituting into the first equation gives,

$$T = m_2(a + g\mu_k)$$

$$= 10(9.8(.1) + 1.1) = 20.8\ N. \quad (17)$$

Lectures 5

Work and Energy

In this chapter we introduce the concepts of work, energy and power. We define kinetic energy, gravitational potential energy, and the potential energy stored in a compressed or stretched spring. If all forces are conservative, the mechanical energy of an isolated system is constant. If non-conservative forces are present, we use the work-energy theorem to equate the work done by the non-conservative forces and the change in mechanical energy.

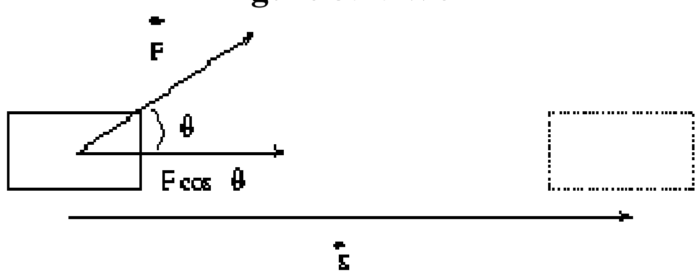
- Work
- Kinetic Energy and the Work Energy Theorem
- Gravitational Potential Energy
- Potential Energy Stored in a Spring

Work

Definition: The **work** done by an agent exerting a constant force ($\vec{F}$) and causing a displacement ($\vec{s}$) equals the magnitude of the displacement, *s*, times the component of $\vec{F}$ along the direction of $\vec{s}$. In Figure 5.1, the work done by $\vec{F}$ is:

$$W = s\, F\cos\theta.$$

Figure 5.1: Work



Note:

- If $\vec{s} = 0 \Rightarrow W = 0$. (ie: no work is done when holding a heavy box, or pushing against a wall).
- $W = 0$ if $\vec{F} \perp \vec{s}$(ie: no work is done by carrying a bucket of water horizontally).
- The sign of W depends on the direction of $\vec{F}$ relative to $\vec{s}$: W > 0 when component of $\vec{F}$ along $\vec{s}$ is in the same direction as $\vec{s}$, and W < 0 when it is in the opposite direction. This sign is given automatically if we write θ as the angle between $\vec{F}$ and $\vec{s}$ and write $W = Fs\cos\theta$.
- If $\vec{F}$ acts along the direction of $\vec{s}$ then $W = Fs$, since $\cos\theta = \cos 0 = 1$.
- Work is a scalar.
- The SI units of work are **Joules (J)** (1 Joule = 1 Newton $\cdot$ meter). In cgs units: 1 erg = 1 dyne $\cdot$ cm.

Kinetic Energy and the Work Energy Theorem

Idea: Force is a vector, work and energy are scalars. Thus, it is often easier to solve problems using energy considerations instead of using Newton's laws (i.e. it is easier to work with scalars than vectors).
Definition: The **kinetic energy (** ***KE*** **)** of an object of mass *m* that is moving with velocity *v* is:

$$KE = \frac{1}{2}mv^2. \qquad (1)$$

Note:

- Kinetic energy is a scalar.
- The units are the same as for work (i.e. Joules, J).

Relation bewteen KE and W: The work done on an object by a net force equals the change in kinetic energy of the object:

$$W = KE_f - KE_i. \qquad (2)$$

This relationship is called the work-energy theorem.
Proof (for $\vec{F}$ parallel to $\vec{s}$):
1. $W = Fs \Rightarrow W = (ma)s$ (by Newton's second law).
2. From the third equation of motion: $as = (v^2 - v_0^2)/2 \Rightarrow W = 1/2m(v^2 - v_0^2) = KE_f - KE_i$.
Note:
If the speed of an object increases ($v_f > v_i$) $\Rightarrow$ W > 0.

- If W < 0 then the object is doing work on the agent exerting the net force.
- Interpretation of Eq.(5.2): We can think of *KE* as the work an object can do in coming to rest.

Gravitational Potential Energy

Definition: **Gravitational Potential Energy** (PE_g) is given by:

$$PE_g = mgy, \qquad (3)$$

where *m* is the mass of an object, *g* is the acceleration due to gravity, and *y* is the distance the object is above some reference level.
The term ``energy'' is motivated by the fact that potential energy and kinetic has lost all of its initial PE_g but gained an equal amount of *KE*.
Proof: Find the work done by the force of gravity when an object falls from rest at position y_i to $y_f = 0$. We have $W = Fs$, $F = |m\vec{a}| = mg$ and $s = (y_i - y_f) = y_i$. This gives, $W = mgy_i$.
Combining with Eq.(5.2) gives $1/2m(v_f^2 - 0) = mgy_i$ or $PE_i = KE_f$.

Potential Energy Stored in a Spring

Definition: The **spring constant**, *k* , is a measure of the stiffness of a spring (large $k \rightarrow$ stiff spring, small $k \rightarrow$ soft spring).

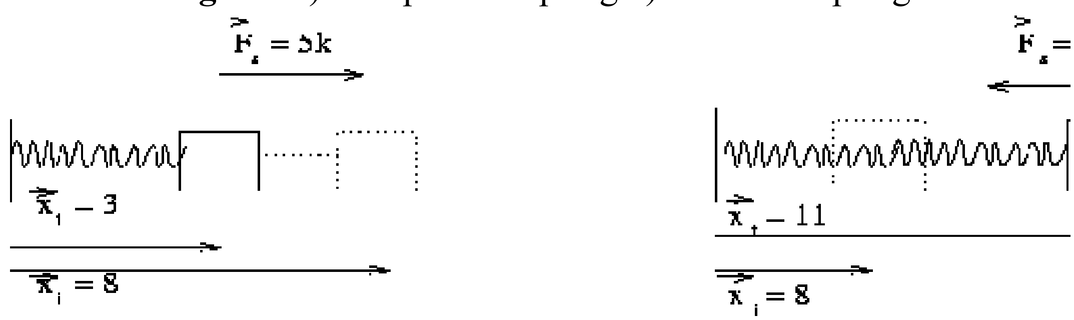
To compress a spring by a distance Δx we must apply a force $F_{ext} = k\Delta x$. By Newton's 3rd law, if we hold a spring in a compressed position, the spring exerts a force $F_s = - k\Delta x$. This is called a **linear restoring force** because the force is always in the *opposite direction from the displacement*.

Note:

- The sign of F_s shows that the spring resists attempts to compress or stretch it; therefore F_s is a **restoring force**.

For Example: In Figure (5.2a) $\Delta x = x_f - x_i = - 5$ which gives $F_s = - k(- 5) = 5k$. This force is positive and therefore directed to the right. This means that the spring resists the compression. In Figure (5.2b) $\Delta x = x_f - x_i = 3$ which gives $F_s = - 3k$. The negative sign indicates that the force is to the left and that the spring resists the stretching.

Figure: a) Compressed spring b) Stretched spring



- The farther we compress or stretch the spring, the greater the restoring force.
- We usually define $x_i = 0$ and $x_f = x$ which gives $F_s = - kx$. This is called **Hooke's law**.

To find the potential energy stored in a compressed (or stretched) spring, we calculate the work to compress (or stretch) the spring: the force to compress a spring varies from $F_{ext} = F_0 = 0$ (at $x_i = 0$), to $F_{ext} = F_x = kx$ (at $x_f = x$). Since force increases linearly with x , the average force that must be applied is

$$\bar{F}_{ext} = \frac{1}{2}(F_0 + F_x) = \frac{1}{2}kx$$

The work done by $\bar{F}_{ext}$ is $W = \bar{F}_{ext}x = \frac{1}{2}kx^2$. This work is stored in the spring as potential energy:

$$PE_s = \frac{1}{2}kx^2. \qquad (4)$$

Note:

- $PE_s = 0$ when $x = 0$ (at equilibrium).

- PE_s always > 0 when the spring is not in equilibrium.
- PE_s is the same if $x = \pm x_f$ (same PE_s for equal expansion or compression).

Choosing a Coordinate System

Idea: When solving problems with potential energy, choosing the origin of the coordinate system is equivalent to choosing the place where the potential energy is zero. We know that *the physics must be independent of the choice of coordinate system* → the value of the potential energy at any given place has no physical significance. The quantity that does have physical significance is the *change in PE* from one position to another.
Usual Choices:
1. For PE_g we usually choose the origin of the coordinate system at the point where the motion originates. **For Example:** If a stone is thrown upward from the surface of the earth, we choose $y = 0$ at the surface of the earth which means $PE_g = 0$ at the surface of the earth. If the stone is thrown from the top of a building, we choose $y = 0$ at the top of the building. Then the stone has some negative value of PE_g when it reaches the ground.

2. For PE_s we usually choose $x = 0$ at the equilibrium position of the spring (where the spring is neither stretched or compressed).

Conservation Laws

There are many forms of energy - mechanical, chemical, electrostatic, heat, nuclear. In any isolated system, energy can be transformed from one kind to another, but the total amount of energy is constant (conserved). Example: a battery contains chemical energy and can be used to produce mechanical energy. Example: when a block slides over a rough surface, the force of friction gives rise to the heating of block and surface. As a result, mechanical energy is transformed into heat energy, but total energy is constant. In this chapter, we are interested in two kinds of mechanical energy:

- Kinetic Energy (*KE*) (energy of motion)
- Potential Energy (*PE*) (energy of position)

- Conservative and Non-Conservative Forces
- The Conservation of Mechanical Energy
- Non-Conservative Forces and the Work-Energy Theorem

Conservative and Non-Conservative Forces

Idea: It is not always true that the work done by an external force is stored as some form of potential energy. This is only true if the force is **conservative**:

$$W_c = F_c^{ext}(x_B - x_A) = PE_A - PE_B. \qquad (5)$$

Definition: The work a **conservative force** does on an object in moving it from A to B is path independent - it depends only on the end points of the motion. Examples: the force of gravity and the spring force are conservative forces. For a **non-conservative** (or dissipative) force, the work done in going from A to B depends on the path taken. Examples: friction and air resistance.

The Conservation of Mechanical Energy

Definition: **Mechanical energy** is the kinetic energy plus all of the kinds of potential energy that are present. In the absence of non-conservative forces, mechanical energy is conserved. Example: if gravitational and spring forces are present:

$$KE_i + PE_{gi} + PE_{si} = KE_f + PE_{gf} + PE_{sf}. \qquad (6)$$

Notice that while the total amount of energy is conserved, the distribution of energy may change. For example, there may be more *KE* in the initial state and more *PE* in the final state (or the other way around).

Non-Conservative Forces and the Work-Energy Theorem

Idea: If there are **non-conservative forces** then mechanical energy is not conserved. We write,

$$W = W_{nc} + W_c = KE_f - KE_i \qquad (7)$$

where W_c is the work done by conservative forces and W_{nc} is the work done by non-conservative forces. Since $W_c = PE_i - PE_f$ we have,

$$W_{nc} = (KE_f - KE_i) + (PE_f - PE_i). \qquad (8)$$

The work done by non-conservative forces is equal to the change in mechanical energy.

Power

Definition: **Power** is the time rate of doing work or, the amount of work done per second.
Average Power:

$$\bar{P} = \frac{W}{\Delta t} = F\frac{\Delta s}{\Delta t} = F\bar{v} \qquad (9)$$

where Δt is the time interval in which the work is done.
Instantaneous Power:

$$P = Fv.$$

Note:

- Power is a scalar.
- SI Units: 1 Watt (W) = 1 $Joule/sec$ = 1 $kg\ m^2/\ s^3$
- British Engineering Units: 1 horsepower (hp) = 746 W .

Problems

PROBLEM 5.1

a) A 2000 kg car is travelling 50 miles per hour. Find the kinetic energy in Joules. b) The same car is lifted vertically upward and then dropped from rest. Find the height from which it is dropped if it strikes the ground at 50 miles per hour (neglect air resistance).
Solution:
a)

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(2 \times 10^3 \ kg)\left[\left(50\frac{\text{mi}}{\text{hr}}\right)\frac{1609 \ \text{m}}{1 \ \text{mi}}\frac{1 \ \text{hr}}{3600 \ \text{s}}\right]^2$$

$$= 4.99 \times 10^5 \ J \qquad (10)$$

b)

$$PE_i = KE_f$$

$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow h = \frac{v^2}{2g}$$

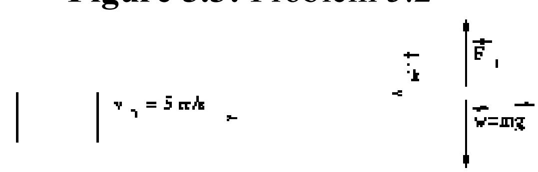
$$= \left[\frac{50(1609)}{3600}\right]^2 \frac{1}{2(9.8)} = 25.5 \ m \qquad (11)$$

PROBLEM 5.2

An object of mass 1 kg travelling at 5.0 m/s enters a region of ice where the coefficient of kinetic friction is .10. Use the work energy theorem to find the distance the object travels before coming to rest.

Solution:

Figure 5.3: Problem 5.2



The work energy theorem gives $W = \Delta KE$. We have $W = -f_k d = -\mu_k N d = -\mu_k mgd$ and $\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\frac{1}{2}mv_i^2$. Combining,

$$-\mu mgd = -\frac{1}{2}mv_i^2$$

$$\rightarrow d = \frac{1}{2\mu g}v_i^2$$

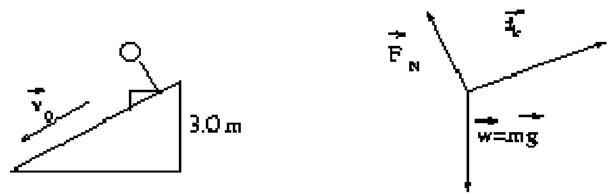
$$= \frac{25}{2(0.1)(9.8)} = 13 \ \ m. \tag{12}$$

PROBLEM 5.3

A 30 kg child enters the final section of a waterslide travelling at 2.0 m/s. The final section is 5.0 m long and has a vertical drop of 3.0 m. The force of friction opposing the child's motion is 50 N. Find a) the loss of potential energy, b) the work done by friction in the final section and c) the child's velocity at the end of the section (using energy considerations).

Solution:

Figure 5.4: Problem 5.3



a)

$$\begin{aligned} \Delta PE &= mg(h_f - h_i) \\ &= 30(9.8)(0 - 3) = - 882 \ \ J \end{aligned} \tag{13}$$

b)

$$W = - f_k x = - 50(5) = - 250 \ \ J \tag{14}$$

c)

$$W_{nc} = \Delta KE + \Delta PE$$

$$\rightarrow - 250 = \frac{1}{2}(30)(v_f^2) - \frac{1}{2}(30)(2.0)^2 - 882$$

$$\rightarrow v_f^2 = \frac{2}{30}\left[-250 + 882 + \frac{1}{2}(30)(2.0^2)\right]$$

$$\rightarrow v_f = 6.8 \ \ m/s \tag{15}$$

PROBLEM 5.4

A 2.0 kg wood block is on a level board and held against a spring of spring constant k=100 N/m which has been compressed .1 m. The block is released and pushed horizontally across the board. The coefficient of friction between the block and the board is μ_k = .20. Find a) the velocity of the block just as it leaves the spring and b) the distance the block travels after it leaves the spring.

Solution:
a)
The work energy theorem gives:

$$W_{nc} = \Delta KE + \Delta PE$$

$$-f_k x = \left(\frac{1}{2} m v_f^2 - 0\right) + \left(0 - \frac{1}{2} k x^2\right)$$

$$-\mu_k mgx = \frac{1}{2} m v_f^2 - \frac{1}{2} k x^2$$

$$\rightarrow v_f^2 = \frac{2\left[\frac{1}{2}kx^2 - \mu_k mgx\right]}{m} =$$

$$\rightarrow v_f = 0.33 \ m/s. \qquad (16)$$

b)
The work energy theorem gives,

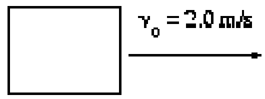
$$-\mu_k mgd = 0 - \frac{1}{2} m v_i^2$$

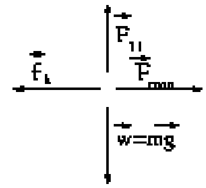
$$\rightarrow d = \frac{1}{2\mu_k g} v_i^2 = \frac{(0.33)^2}{2(.2)(9.8)} = 0.028 \ m. \qquad (17)$$

PROBLEM 5.5

A man pushes a 100 kg box across a level floor at a constant speed of 2.0 m/s for 10 s. If the coefficient of friction between the box and the floor is $\mu_k = 0.20$, find the average power output by the man.
Solution:

Figure 5.5: Problem 5.5





Since the acceleration of the box is zero the force exerted by the man is obtained from $F_{\text{man}} - f_k = 0 \Rightarrow F_{\text{man}} = f_k = \mu_k mg$. Then

$$P = \frac{W}{\Delta t} = F\bar{v} = \mu_k mg\bar{v} = .2(100)(9.8)(2) = 392 \;\; W. \qquad (18)$$