

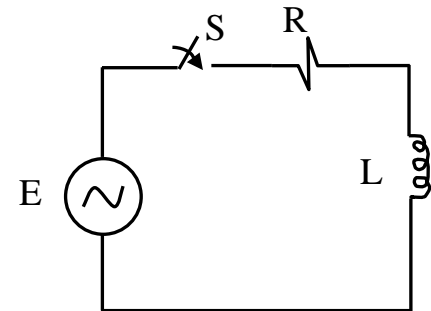
**Chapter Two**  
**Three-Phase Balanced Faults Analysis**

**2-1 Introduction:-**

$$E = Ri(t) + L \frac{di(t)}{dt} = V_m \sin(\omega t + \alpha)$$

$$i(t) = \frac{V_m}{|Z|} \left[ \sin(\omega t + \alpha - \theta) - e^{-\frac{R}{L}t} \sin(\alpha - \theta) \right]$$

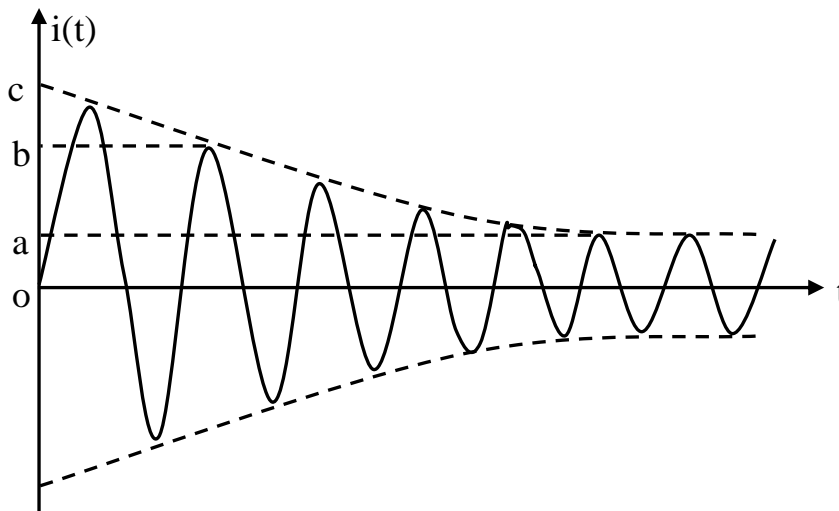
$$|Z| = \sqrt{(\omega L)^2 + R^2} \quad , \quad \theta = \tan^{-1} \left( \frac{\omega L}{R} \right)$$



The power system faults may be divided as one of four types:-

- Three-phase symmetrical faults (5%)
- Single line to ground faults (65%)
- Line to line faults (double line faults) (20%)
- Line to line to ground faults (double line to ground faults) (10%)

**2-2 Fault Current on a Synchronous Machine:-**



The fault current flowing in a synchronous machine is:-

- Sub-transient Current:- which flowing immediately after the occurrence of fault

$$I'' = \frac{oc}{\sqrt{2}} = \frac{E}{X_d''}$$

Where:  $I''$  = sub-transient current,  $E$  = internal e.m.f. of the machine,  $X_d''$  = sub-transient reactance of the machine.

- Transient Current:- which flowing a few cycles later

$$I' = \frac{ob}{\sqrt{2}} = \frac{E}{X_d}$$

Where:  $I'$  = transient current,  $X_d'$  = transient reactance of the machine.

- Steady state Current:-

$$I = \frac{oa}{\sqrt{2}} = \frac{E}{X_d}$$

Where:  $I$  = steady state current,  $X_d$  = steady state reactance of the machine.

These currents are differ considerably because of the effect of the armature current on the flux that generated the voltage in the machine.

### **2-3 Steps for Calculating the Symmetrical 3-Phase Short-Circuit Currents:-**

- Immediately during the fault, the system is on no-load at rated voltage and frequency. So all the currents are neglected except fault current.
- All generators operating in parallel.
- System resistances are neglected.
- Pre-fault currents are neglected.

\* To calculate the symmetrical 3-ph fault current:-

- Draw the one line diagram of the network.
- Draw the reactance diagram.
- Calculate the total reactance to the fault point.
- Calculate the fault current and fault kVA.

**Short Circuit Capacity (SCC): (Fault Level)**

It is defined as the product of magnitudes of pre-fault voltage and post-fault current.

$$SCC = |V_{prefault}| * |I_{postfault}| \text{ p.u.}$$

$$SCC = \sqrt{3} |V_{prefault} (kV)| * |I_{postfault} (A)| * 10^{-3} \text{ MVA}$$

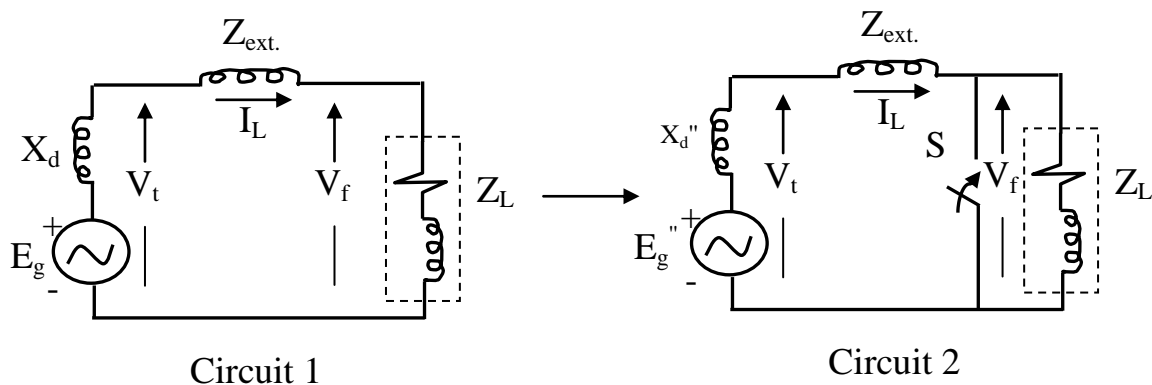
Since the voltage is about one per unit, the SCC is equal to

$$SCC = |I_{postfault} (A)| \text{ p.u.}$$

$$\because |I| = \frac{|V|}{|Z|} \rightarrow Z \text{ p.u.} = \frac{1}{|SCC| \text{ p.u.}}$$

**2-4 Internal Voltage of Loaded Machine under Transient**

**Condition:-**



$I_L$  = Load current before fault,  $V_f$  = load voltage,  $V_t$  = generator terminal voltage,  $Z_{ext}$  = external impedance,  $E_g$  and  $X_d$  synchronous e.m.f. and reactance of machine.

When a 3-ph short circuit occurs, the circuit 1 converted to circuit 2; and:

For generator:

$$E_g'' = V_t + jI_L X_d''$$

Similarly, at transient conditions:  $E_g' = V_t + jI_L X_d'$

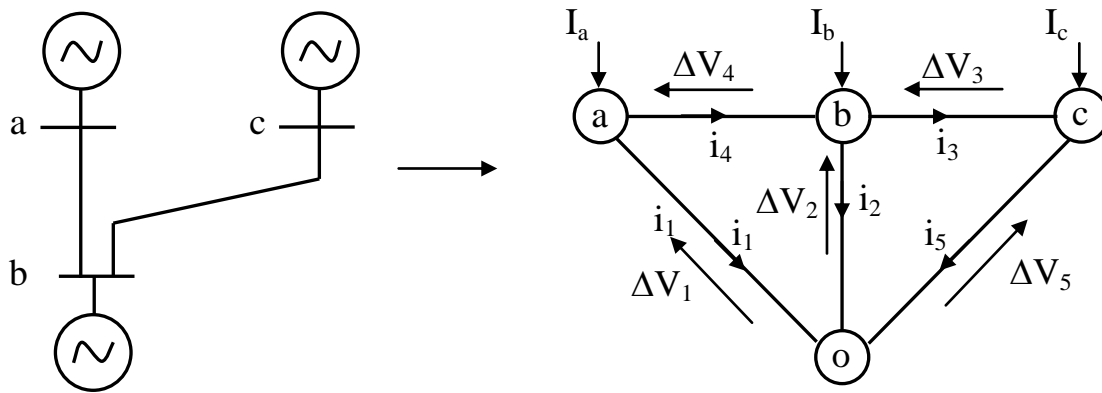
For motor (SM):

$$E_m'' = V_t - jI_L X_d'' \quad , \quad E_m' = V_t - jI_L X_d'$$

At these conditions:-

- $E_g$  remains constant while  $E_g''$  and  $E_g'$  changes with  $I_L$ .
- For the synchronous motor (SM)  $X_d''$  or  $X_d'$  is used depending on the accuracy of the current wanted. ( $X_d' = 1.5 * X_d''$ )
- Induction motors (IM) effects less than 50hp are neglected.
- $E_g''$  is called voltage behind sub-transient reactance.

**2-5 Fault Calculation Using Digital Computers:-**



Bus Impedance Matrix in Fault Calculation

For the nodal formation of network equations, there are two quantities associated with each branch ( the through quantities (i)) corresponding to the flow in the branch and the ( across quantities (Δv)) corresponding to the potential difference across the branches.

$$\Delta v = z * i \quad \dots (1)$$

Or the matrix

$$\Delta V = ZI \quad \dots (2)$$

Where:

$$Z = \begin{bmatrix} z_1 & \dots & \dots \\ \vdots & \ddots & \vdots \\ \vdots & \dots & z_b \end{bmatrix}, \Delta V = \begin{bmatrix} \Delta v_1 \\ \vdots \\ \Delta v_b \end{bmatrix}, I = \begin{bmatrix} i_1 \\ \vdots \\ i_b \end{bmatrix}$$

Where b is the total number of branches in the network

$$\Delta V_j = V_j(s) - V_j(r) \quad \dots (3)$$

s: sending, r: receiving, j: corresponding branch

$$\Delta V_j = \sum_{i=1}^n C_{ij} V_i \quad \dots (4), \quad j = 1, 2, 3, \dots, b$$

n: total no. of nodes – reference node)

$$C_{ij} = \begin{cases} +1, & \text{if the branch } j \text{ is directed away from node } i \\ -1, & \text{if the branch } j \text{ is directed towards the node } i \\ 0, & \text{if the branch } j \text{ is not connected to the node } i \end{cases}$$

$$\Delta V = C^T V \quad \dots (5), \quad C^T: \text{transposed matrix}$$

\* state that algebraic sum of the entering and leaving any given node is equal to zero.

$$\sum_{j=1}^b C_{ij} I_j = I_i \quad \dots (6), \quad i = 1, 2, \dots, n$$

$$C_i = I_i \quad \dots (7), \quad C \text{ is the branch nodal incidence matrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\Delta V_1 = V_a, \quad \Delta V_2 = V_b, \quad \Delta V_3 = V_b - V_c, \quad \Delta V_4 = V_a - V_b, \quad \Delta V_5 = V_c$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \\ \Delta V_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$i_1 + i_4 = I_a, \quad i_2 + i_3 - i_4 = I_b, \quad -i_3 + i_5 = I_c$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

substituting I from equ. (2) into equ. (7) gives:

$$CZ^{-1}\Delta V = I \quad \dots (8)$$

and substituting  $\Delta V$  from equ. (5) into equ. (8) gives:

$$CZ^{-1}C^T V = I \quad \text{or} \quad YV = I$$

$\therefore Y = CZ^{-1}C^T$ , Y is the admittance matrix for the network above

$$Y = \begin{bmatrix} \left(\frac{1}{z_1} + \frac{1}{z_4}\right) & -\frac{1}{z_4} & 0 \\ -\frac{1}{z_4} & \left(\frac{1}{z_1} + \frac{1}{z_3} + \frac{1}{z_4}\right) & -\frac{1}{z_3} \\ 0 & -\frac{1}{z_3} & \left(\frac{1}{z_3} + \frac{1}{z_5}\right) \end{bmatrix}$$

$$I_f'' = \frac{V_f}{Z_{kk}} \quad (k \text{ is the faulted bus}), \quad V_n = V_f - \frac{Z_{nk}}{Z_{kk}} * V_f = V_f - I_f'' Z_{nk}$$

The current can be determined in any part as:

$$I_{f \ i-j}'' = \frac{V_i - V_j}{Z_{ij}}$$

Generator current is:

$$I_g'' = \frac{E_g'' - V_i}{X_{gi}}$$

$$I_{gA}'' = \frac{E_{gA}'' - V_1}{X_{oi}} \quad [\text{when generator A is connected between reference o and node 1}]$$

## **2-6 Selection of Circuit Breakers:-**

A part from normal load switching, circuit breakers are required in power system to give rapid fault clearance, in order to avoid over current damage to equipments and loss of system stability. The important of operating times in circuit breaker are:-

**Clearance Time:** relay and protective system time + breaker interrupting time.

**Relay and Protective System Time:** time from fault inception to the closing of the breaker trip coil circuit.

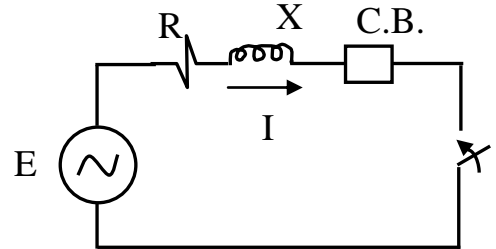
**Breaker Interrupting Time:** time for the current in the trip coil to rise to the value at which the moving contact is released + contact operating time + arcing time.

\* For electromechanical relays, the relay time is from 20 msec. to 2 sec. Static relays can be give very fast operation.

\* The arcing time is of one-half to two-half cycles. Contact opening may be occupy between about one and three cycles.

\* In modern systems, the fault clearance can be reach as a little as about three cycles.

\* A circuit breaker will be able to make and brake currents between 10 to 100 times the normal current which is the load current can carry continuously.



$$I_{ac} = \frac{E}{\sqrt{R^2+X^2}}, I_{max} = \sqrt{2} I_{ac}$$

$$DC \text{ offset possible} = I_{dc \text{ max}} = I_{max} = \sqrt{2} I_{ac}$$

DC offset exponentially decay with time constant  $\tau = \frac{L}{R} = \frac{X}{\omega R}$

$$I_{dc}(t) = I_{dc \text{ max}} e^{-t/\tau} = \sqrt{2} I_{ac} e^{-t/\tau}$$

The transient r.m.s. current accounting for both ac and dc terms is

$$I(t) = \sqrt{I_{ac}^2 + I_{dc}^2} = I_{ac} \sqrt{1 + 2e^{-2t/\tau}}$$

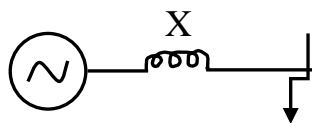
$$\therefore K_i = \frac{I_{top}}{I_{ac}} = \sqrt{1 + 2e^{-2t/\tau}} \quad (\text{max. } \sqrt{3})$$

Where: top: operating time of C.B.,  $K_i$ : multiplying factor

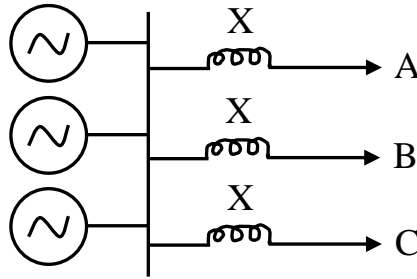
**2-7 Current Limiting Reactors:-**

Short circuit current can be limited by increasing inductive reactor

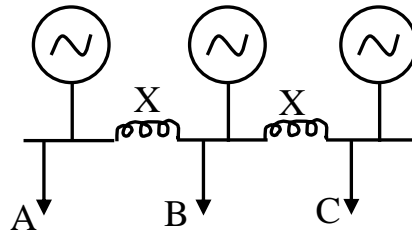
- Series Reactors



- Feeder Reactors



- Bus Bar Reactors (Ring System)



- Bus Bar Reactors (Tie-Bar System)

