Theory of Structures

Analysis of Structures

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References

1- Elementary Theory of Structures, Yan-Yu Hsieh
2- Structural Analysis, RC. Hibbeler

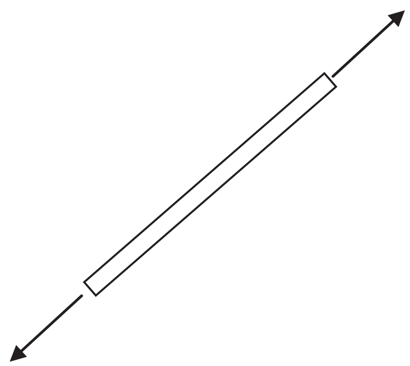
Chapter One

Introduction

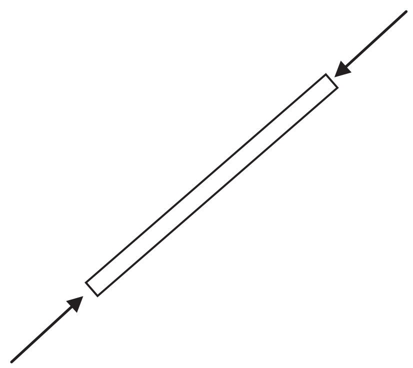
A "Structures" refers to a system of connected parts used to support a load. Important examples related to civil engineering include buildings, bridges, and towers. In other branches of engineering such as ships, aircraft frames, tanks and pressure vessels, mechanical systems, and electrical supporting structures are Important.

Types of structures

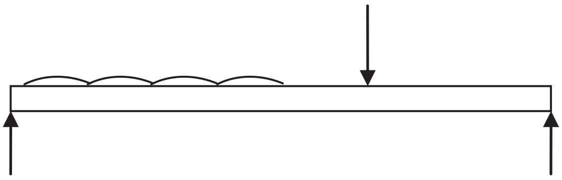
1- Ties: These are structural members that are subjected to axial tension only.



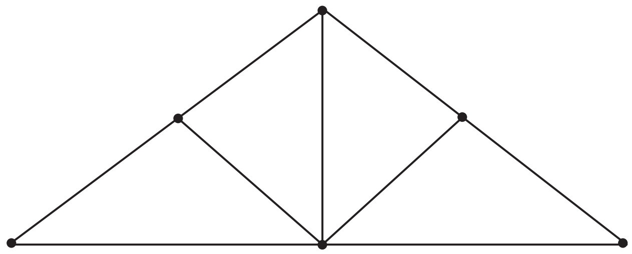
2- Struts (Columns): These are structural members that are subjected to axial compression only.



3- Beams: These are usually straight horizontal members subjected to transverse loading and hence to bending moment and shear force at each normal section.



4- Trusses: these are structures which consist of members which are pin-connected at each terminal. These members usually form one or more triangles in a single plan and are so arranged that the external loads at the joints and hence each member is subjected to direct force and is a tie or a strut.



5- Frames: these are structures which have moment-resisting joints. The members are rigidly connected at their ends so that no joint translation is possible (i. e. the members at a joint may rotate as a group but may not move with respect to each other). The members are subjected to axial and lateral loadings and hence to shear force, bending moments and axial load at each normal section.

Types of loads

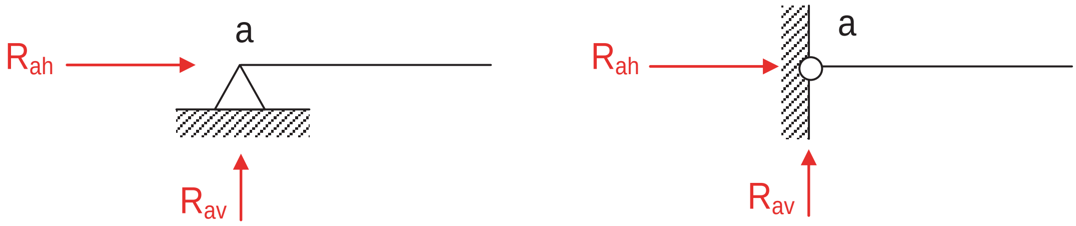
Loads can be classified as being "dead loads" and "live loads".

1- **Dead loads:** these are loads of constant magnitude that remain in one position. They consist of the structural frames own weight and other loads that are permanently attached to the frame. For a steel-frame building, some dead loads include the frame, walls, and floor.
2- Live loads: live loads are loads that may change in position and magnitude. Live loads that move under their own power are said to be "moving loads", such as tracks, people, and cranes whereas those loads that may be moved are movable loads such as furniture, goods, and snow. Examples of live loads to be considered include: traffic loads for bridges, **Impact loads**.

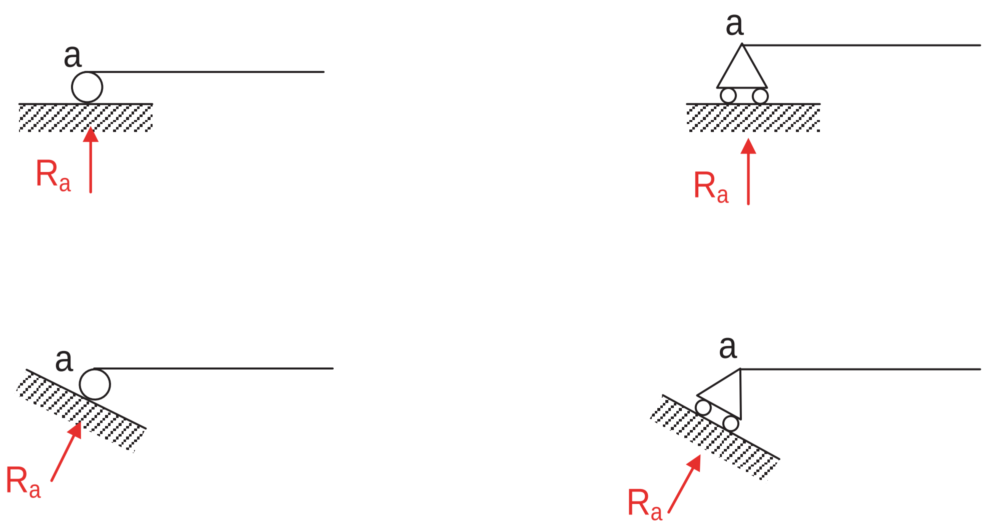
Types of support

Structures may be supported by hinges, rollers, fixed ends, or links;

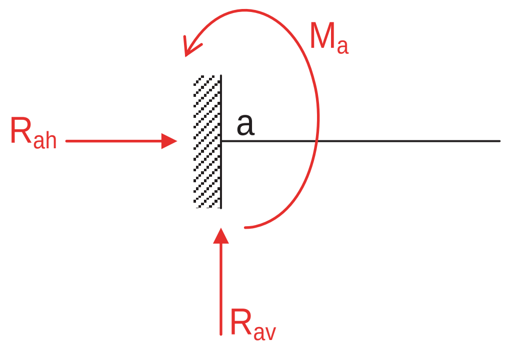
1- **A "hinge"** or pin-type support prevents movement in the horizontal and vertical direction but does not prevent rotation about the hinge. There are two unknown forces at a hinge.



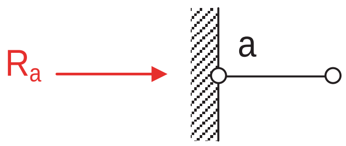
2- **A "roller"** type of support is assumed to offer resistance to movement only in a direction perpendicular to the supporting surface beneath the roller. There is no resistance to rotation about the roller or to movement parallel to the supporting surface. The magnitude of the force required to prevent movement perpendicular to the supporting surface is the one unknown.

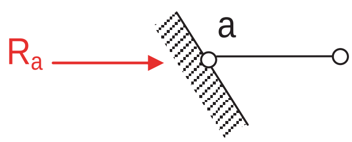


3- **A "fixed"** support is assumed to offer resistance to rotation about the support and to movement vertically and horizontally. There are three unknowns.



4- **A "link"** type of support is similar to the roller in its action. The line of action of the supporting force must be in the direction of the link and through the two pins. One unknown is present: the magnitude of the force in the direction of the link.





Equations of Equilibrium

The equations of equilibrium for a force system in the xy-plane are;

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0$$

The third equation is the algebraic sum of the moments of all the forces about z-axis and passes through some arbitrary point O. For complete equilibrium in two dimensions, all three of the independent equations must be satisfied.

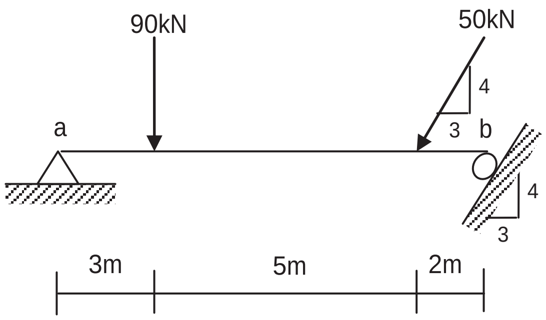
The equilibrium equations can also be expressed in two alternative forms;

$$\sum F_x = 0 \qquad \sum M_a = 0 \qquad \sum M_b = 0$$

$$\sum M_a = 0 \qquad \sum M_b = 0 \qquad \sum M_c = 0$$

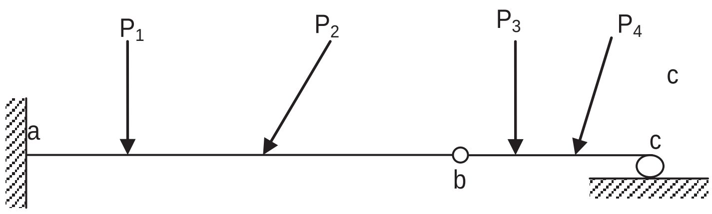
where the points a, b, and c are not lay on the same line

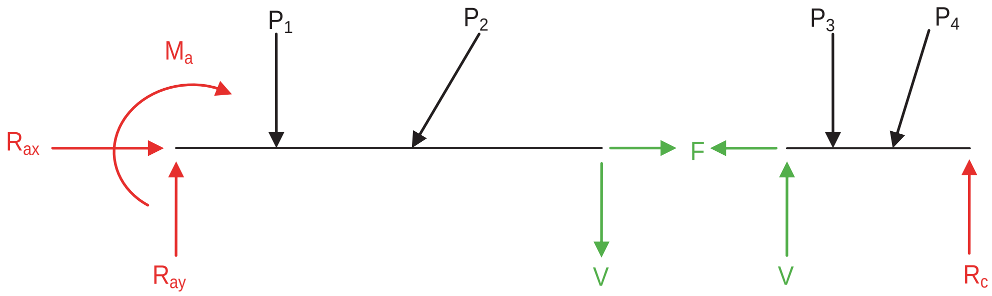
Example (1): Calculate the reactions for the beam shown.



Equations of Conditions

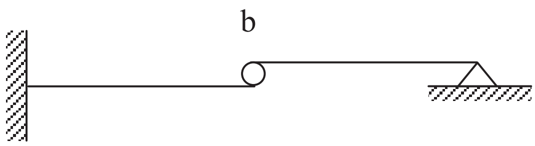
The beam shown in the figure below has an internal "hinge" built in it at point b.





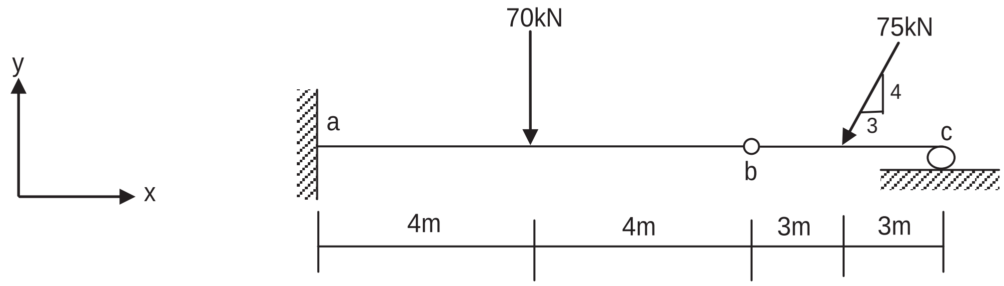
No bending moment can be transmitted through the beam at point b. From the free-body diagram for the two segments of the beam, it is shown that there are two internal components of force at point b, one parallel to the axis of the beam (F) and one there perpendicular to the axis (V). Since no moment is transmitted through the hinge, the equation $\sum M_b = 0$ can be imposed for the two individual free-body diagrams. The one independent equation introduced by the condition of construction is referred to as ***Equation of Condition***.

In the figure below, there are two equations of condition due to presence of roller at point b.

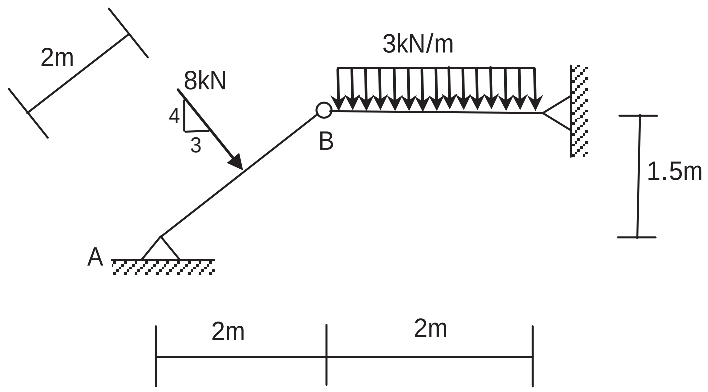


$$\sum F_x = 0$$
$$\sum M = 0$$

Example (1): Calculate the reactions for the beam illustrated.



Example (2): Determine the reactions for the two-member frame shown in the figure below.



Determinacy and Stability

Determinacy

The equilibrium equations provide both the "necessary and sufficient" conditions for equilibrium when all the forces in a structure can be determined from these equations, the structure is referred to as "***statically determinate***". Structures having more unknown forces than available equilibrium equations are called "***statically indeterminate***". For a coplanar structure there are at most "***three***" equilibrium equations for each part, so that if there is a total of " **n** " parts and " **r** " internal force and moment reactions components, we have;

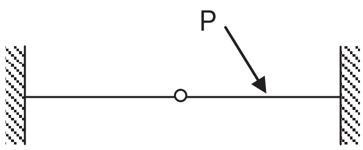
$$\begin{pmatrix} r = 3n \quad ,\text{statically determinate} \\ r > 3n \ ,\textit{statically indeterminate} \\ r < 3n \ , \ \textit{unstable} \end{pmatrix} \text{---------------------- Eq. (1)}$$

The above equation used for beams and frames.

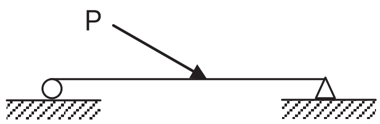
At the same time, we can use the equations of conditions to find the indeterminacy of ***beams as bellow***;

$$\begin{pmatrix} R = 3+c \quad , \ \text{statically determinate} \\ R > 3+c \ , \ \text{statically indeterminate} \\ R<3+c \ , \ \textit{unstable} \end{pmatrix} \text{---------------------- Eq. (2)}$$

where R: No. of reactions.
3: No. of equations of equilibrium.
c: No of equations of conditions.



By using Eq. 1
r = 8 , n = 2
8 ? 3(2)
8 > 6 *statically indeterminate to the second degree*
Or by using Eq. 2
R = 6 , c = 1
6 ? 3+1
6 > 4 *statically indeterminate to the second degree*



By using Eq. 1
r = 3 , n = 1
3 ? 3(1)
3 = 3(1)
statically determinate
Or by using Eq. 2
R = 3, c = 0
3 ? 3+0
3 = 3 *statically determinate*

In the presence of equations of condition in frames, we can use the Eq. (3) to fined the determinacy as bellow,

$$\begin{cases} 3m+R = 3j+c & ,\text{statically determinate} \\ 3m+R > 3j+c & ,\text{statically indeterminate} \\ 3m+R < 3j+c & ,unstable \end{cases} \text{ --------------------- Eq. (3)}$$

Where m: No. of members

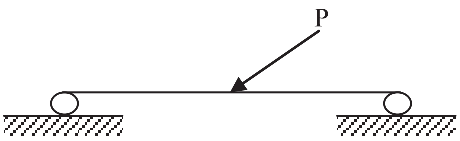
j: No. of joints

c: No of equations of conditions and equals to i-1, where i is the number of members meeting at that joint

In particular if a structure is statically indeterminate, additional equations needed to solve.

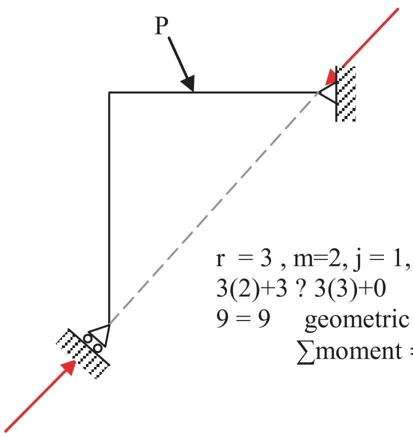
Stability

A structure will become "***unstable***"(i.e. it will move slightly or collapse) if there are fewer reactive forces than available equations (***Equations of equilibrium and conditions if any***).

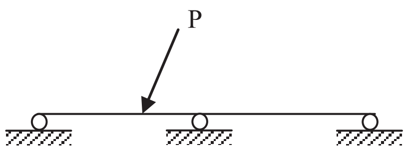


r = 2 , n = 1
2 < 3(1) Unstable

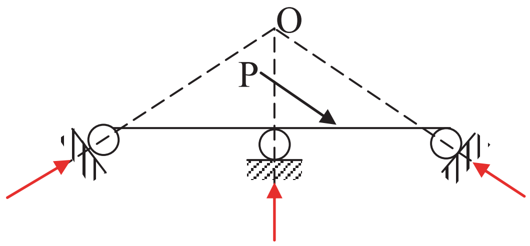
If there are enough reactions, instability will occur if the lines of action of the reactive forces intersect at a common point, or are parallel to one another (***Geometric instability***). The geometric instability may be occurred in the case of incorrect arrangement of members and supports.



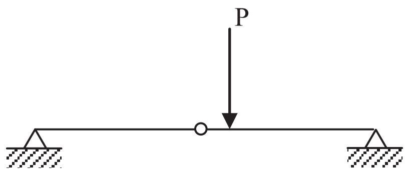
r = 3 , m=2, j = 1, c = 0, Eq. 2
3(2)+3 ? 3(3)+0
9 = 9 geometric unstable,
$\sum$moment $\neq 0$



r = 3 , n = 1, Eq. 1
3 ? 3(1)
3 = 3 geometric unstable due to parallel reaction



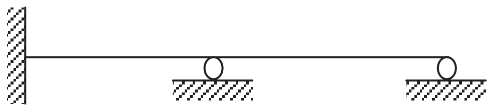
r = 3 , n = 1, Eq. 1
3 ? 3(1)
3 = 3 geometric unstable,
$\sum$moment $\neq 0$



r = 6 , n = 2, Eq. 1
6 ? 3(2)
6 = 6 unstable due to arrangement of support

Example (1): Classify each of the beams shown in figure as statically determinate or statically indeterminate.

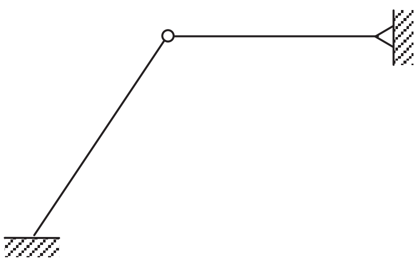
a

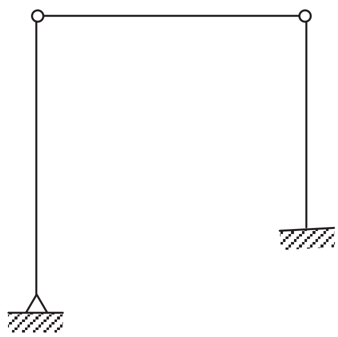


b

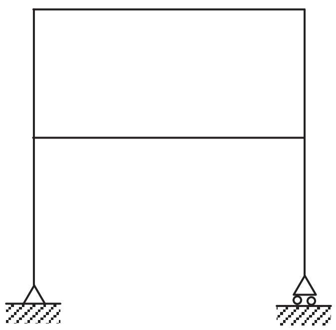
c

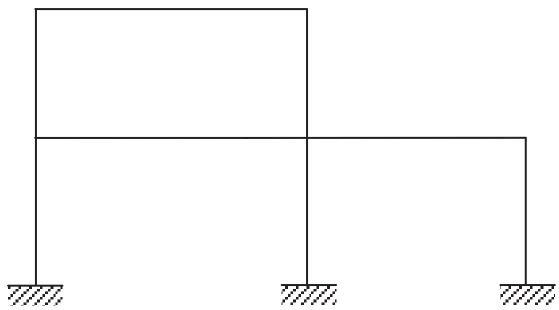
Example (2): Classify each of the pin-connected structures as statically determinate or statically indeterminate.

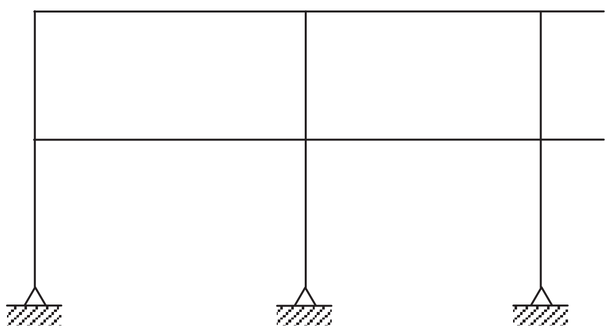




Example (3): Classify each of frames shown as statically determinate or statically indeterminate.







Chapter Two

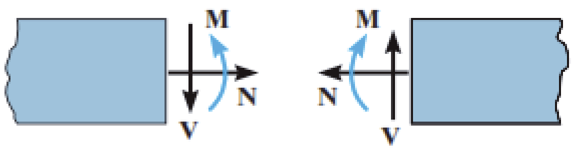
Internal Loadings Developed in Structural Members

The internal load at a specified point in a member can be determined by using the "***method of sections***". In general, this loading for a coplanar structure will consist of a normal force " ***N*** ", shear force " ***V*** ", and bending moment " ***M*** ". Once the resultant of internal loadings at any section are known, the magnitude of the induced stress on that section can be determined.

Sign Convention

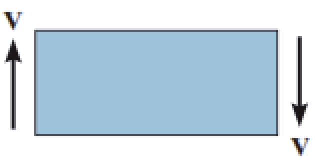
On the "left-hand face" of the cut member in Fig. (a), the normal force " N " acts to the right, the internal shear force " V " acts downward, and the moment " M " acts counterclockwise. In accordance with Newton's third law, an equal but opposite normal force, shear force, and bending moment must act on the right-hand face of the member at the section.

Isolate a small segment of the member; positive normal force tends to elongate the segment, Fig. (b); positive shear tends to rotate the segment clockwise, Fig. (c); and positive bending moment tends to bend the segment concave upward, Fig. (d).



(a)

(b)



(c)

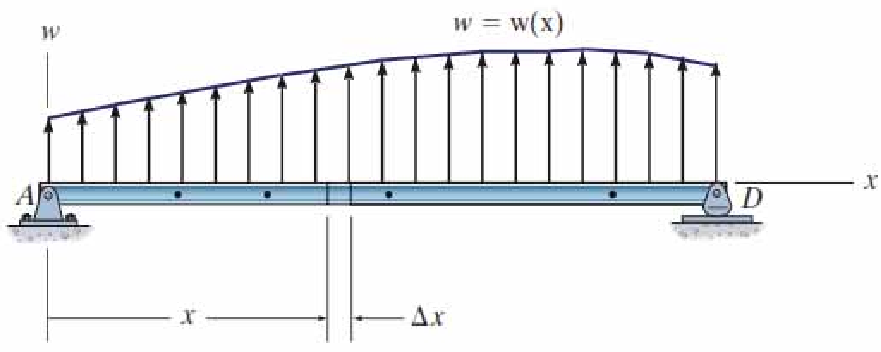
(d)

Shear Force and Bending Moment Diagrams for a Beam

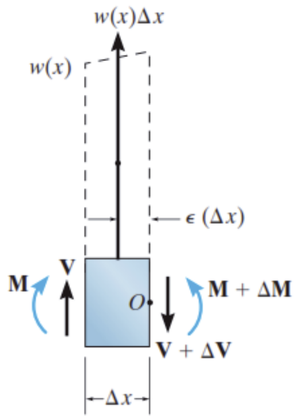
Plots showing the variations of V and M along the length of a beam are termed; ***Shear Forces Diagram (SFD) and Bending Moment Diagram (BMD)***, respectively.

Relationships between Load, Shear Force and Bending Moment

Consider the beam AD , shown in Fig. (a), which is subjected to an arbitrary distributed loading w = w (x). The distributed load is considered positive when the loading acts upward.



(a)



(b)

Applying the equations of equilibrium for the free-body diagram of a small segment of the beam having a length Δx.

$\sum F_y = 0; \quad V + w(x).\Delta x - (V + \Delta V) = 0$

$$\Delta V = w(x).\Delta x$$

$\sum M_O = 0; \quad -V.\Delta x - M - w(x).\frac{(\Delta x)^2}{2} + (M + \Delta M) = 0$

Since the term $w(x).\frac{(\Delta x)^2}{2}$ is very small and can be neglected;

$$\text{So, } \Delta M = V.\Delta x$$

Taking the limit as $\Delta x \rightarrow 0$;

$$\frac{dV}{dx} = w(x) \qquad \text{--------} \ (2.1)$$

$$\frac{dM}{dx} = V \qquad \text{--------} \ (2.2)$$

Equation (2.1) states that "the slope of the shear diagram at a point ($\frac{dV}{dx}$) is equal to the intensity of the distributed load w(x) at that point".

Likewise, Eq. (2.2) states that "the slope of the moment diagram ($\frac{dM}{dx}$) is equal to the intensity of the shear at that point".

From one point to another, in which case;

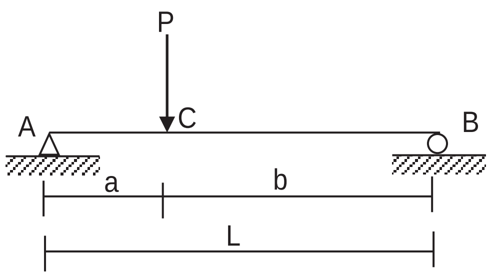
$$\Delta V = \int w(x).dx \qquad \text{-------} \ (2.3)$$

$$\Delta M = \int V(x).dx \qquad \text{-------} \ (2.4)$$

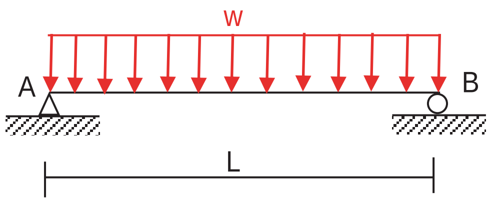
Equation (2.3) states that "the change in the shear between any two points on a beam equals the area under the distributed loading diagram between those two points".

Likewise, Eq. (2.4) states that "the change in the moment between any two points on a beam equals the area under the shear diagram between those two points".

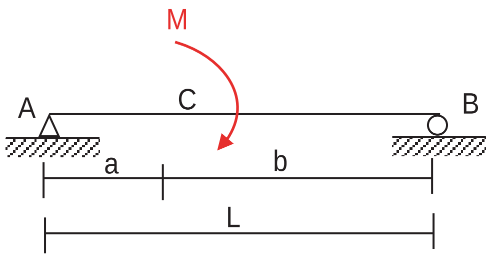
Example (1): Draw the shear force and bending moment diagrams for the simply supported beam subjected to a concentrated load as shown in the figure below.



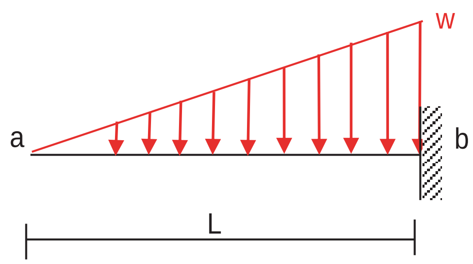
Example (2): Draw the shear force and bending moment diagrams for the simply supported beam subjected to a uniformly distributed load of intensity " w ", as shown in the figure below.



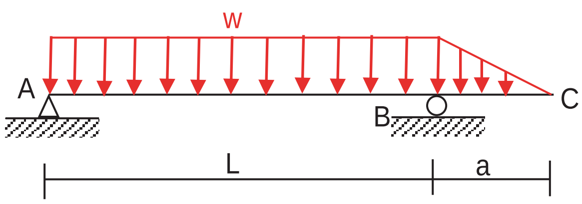
Example (3): Draw the shear force and bending moment diagrams for the simply supported beam subjected to a concentrated moment as shown in the figure below.



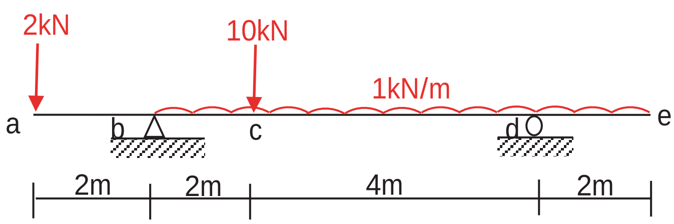
Example (4): Draw the shear force and bending moment diagrams for the simply supported beam subjected to a linearly varying load, as shown in the figure below.



Example (5): Draw the shear force and bending moment diagrams for the overhang beam subjected to a linearly varying load, as shown in the figure below.

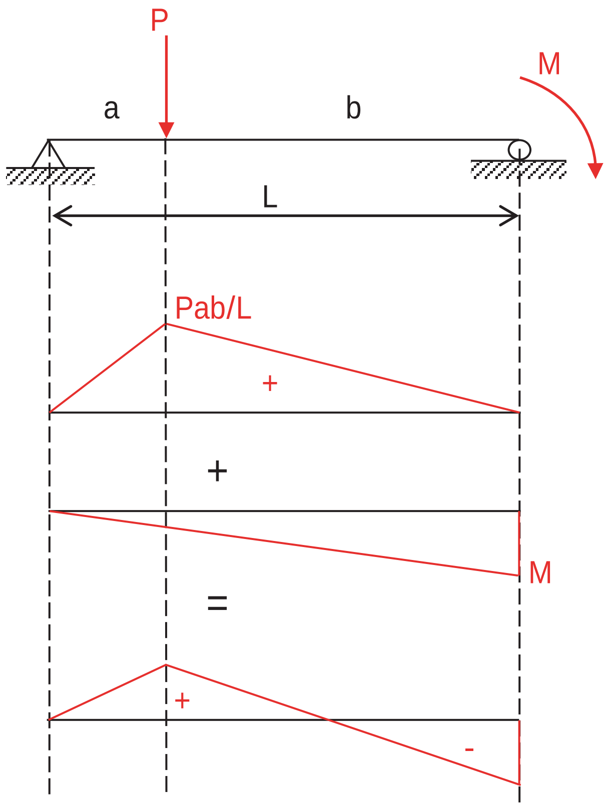


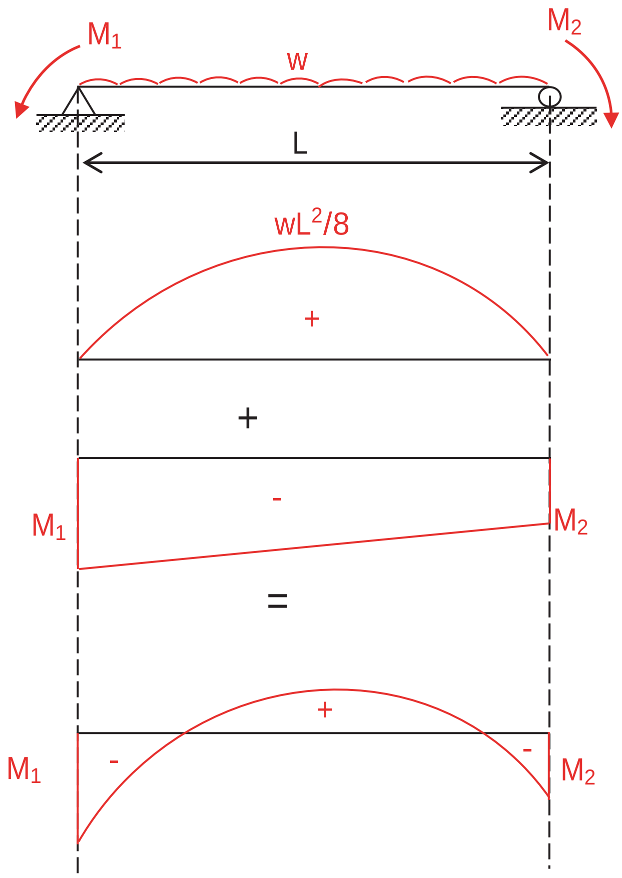
Example (6): Draw the shear force and bending moment diagrams for the double overhang beam subjected to a linearly varying load, as shown in the figure below.



Moment Diagrams by the Method of Superposition

Using the principle of superposition, each of the loads can be treated separately and the moment diagram can then be constructed in a series of parts rather than a single and sometimes complicated shape. This can be particularly useful when applying geometric deflection methods to determine both the deflection of abeam and the reactions on a statically indeterminate beams.





Shear and Moment Diagrams for a Frame

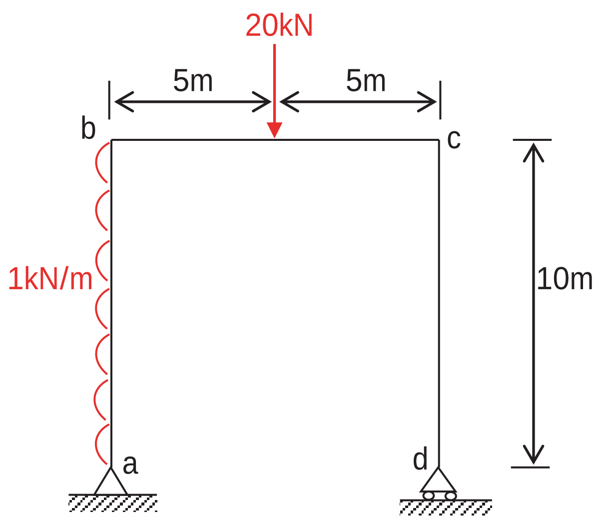
To draw the shear force and bending moment diagrams for a frame, it is first required to determine the reactions at the frame supports. Then, using the method of sections, we find the axial force, shear force, and moment acting at the ends of each member. All the loadings are resolved into components acting parallel nd perpendicular to the member's axis.

The sign convention followed will be to draw the bending moment diagram

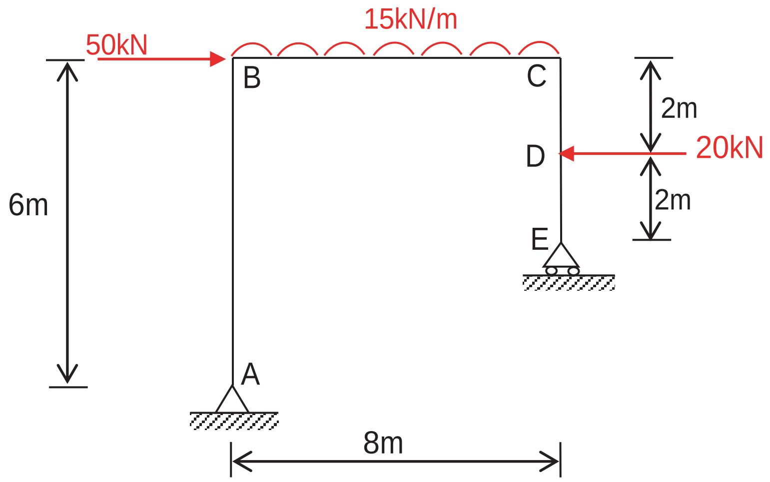
Example (1): The frame shown in the figure is pinned at a and supported on a roller at d. For the loading indicated:

i- Determine the support reactions.

ii- Draw the axial load, shear force, and bending moment diagrams.



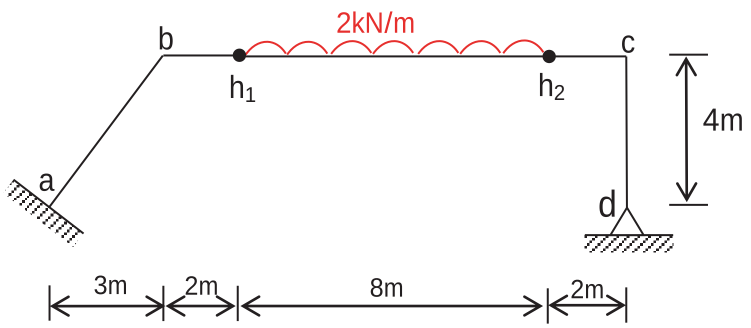
Example (2): Determine the support reactions and draw the axial force, shear force, and bending moment diagrams for the frame shown in the figure below.



Example (3): |The frame shown in the figure below is fixed at (a) and hinged at (d) and has two internal hinges (h_1) and (h_2). From the loading indicated:

i- Determine the support reactions.

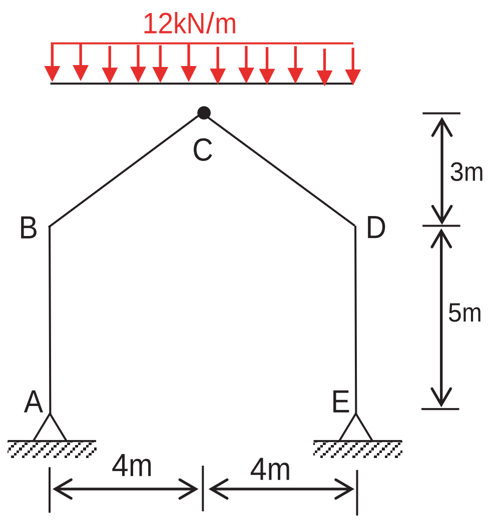
ii- Draw the axial force, shear force, and bending moment diagrams.



Example (4): |The frame shown in the figure below is subjected to a uniform vertical load of 12kN/m of the horizontal.

i- Determine the support reactions.

ii- Draw the axial force, shear force, and bending moment diagrams.

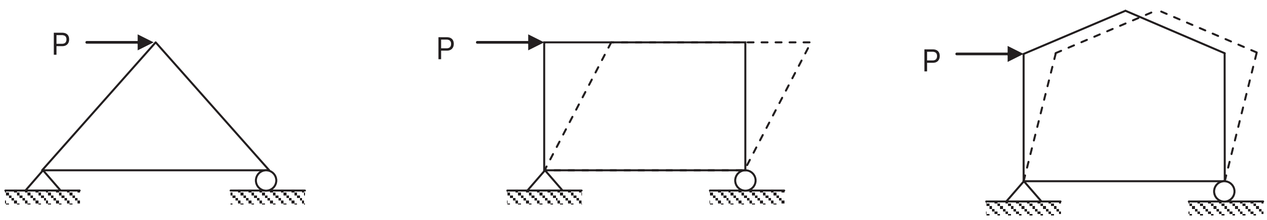


Chapter Three

Analysis of Statically Determinate Trusses

A truss is defined as a structure formed by group of members arranged in the shape of one or more triangles.

Because the members are assumed to be connected with frictionless pins, the triangle is the only stable shape. Figures of the four or more sides are not stable and may collapse under load.



Assumptions for Truss analysis:

1- Truss members are connected together with frictionless pins.
2- Truss members are straight.
3- The deformations of truss under load are of small magnitude and do not cause changes in the overall shape and dimensions of the truss.
4- Members are so arranged that the loads and reactions are applied only at the truss joints.

Determinacy and Stability of Trusses

For any problem in truss analysis, the total member of unknowns equals (b+r), where;

b: is the forces in the bars and

r: is number of external reactions.

Since the members are all straight axial force members lying in the same plane, the force system acting at each joint is "***Coplanar and concurrent***". Consequently, rotational or moment equilibrium is automatically satisfied at each joint and it is only necessary to satisfy $\sum F_x = 0$ and $\sum F_y = 0$ to insure translational or force equilibrium. Therefore, only two equations of equilibrium can be written for each joint, and if there are " j " numbers of joints, the total number of equations available for solution are " 2j ".

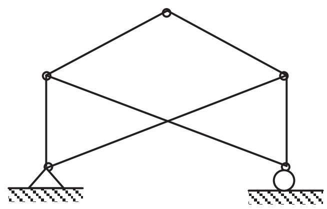
By comparing the total number unknowns (b + r) with the total number of available equilibrium equations, we have:

$b + r = 2j$ Statically determinate

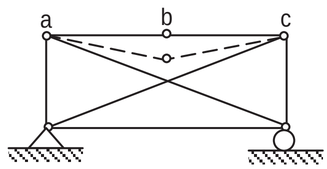
$b + r > 2j$ Statically indeterminate

$b + r < 2j$ Unstable {Truss will collapse, since there will be an insufficient number of bars or reactions to constrain all the joints}

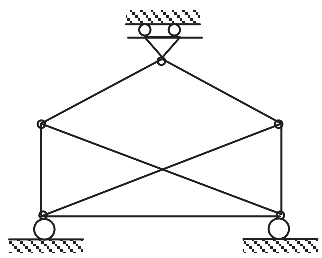
b + r ? 2j
6 + 3 ? 2 × 5
9 = 10 Unstable.

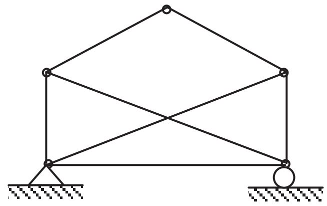


b + r ? 2j
7 + 3 ? 2 × 5
10 = 10 Unstable {points a, b, and c at the same line}



b + r ? 2j
7 + 3 ? 2 × 5
10 = 10 Unstable {parallel reactions}

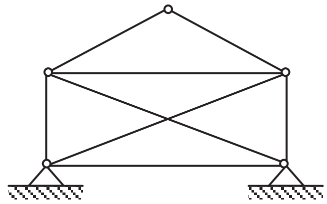




b + r ? 2j

7 + 3 ? 2 × 5

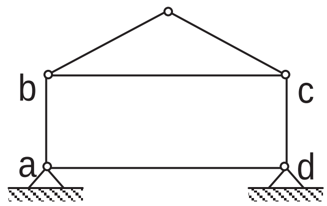
10 = 10   statically determinate.



m + r ? 2j

8 + 4? 2 × 5

12 > 10   statically indeterminate to the second degree.



m + r ? 2j

6 + 4? 2 × 5

10 > 10   Unstable (internal geometric instability due to the lack of lateral resistance in panel abcd)

The method of Joints

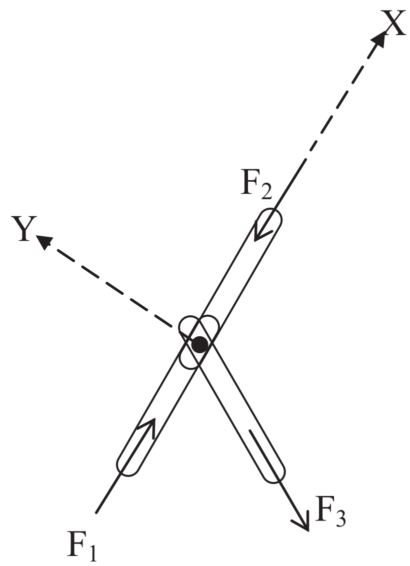
If a truss is in equilibrium, then each of its joints must also be in equilibrium. Hence, the method of joints consists of satisfying the equilibrium conditions $\sum F_x = 0$ and $\sum F_y = 0$ for the forces exerted on the pin at each joint of the truss.

Special Conditions

1- If in any truss, there be a joint at which only three bars meet and two of these bars lies along the same straight line, then the force in the third bar is zero, provided that there is no external force applied.

$\Sigma Y_i = 0 \Longrightarrow F_3 = 0$

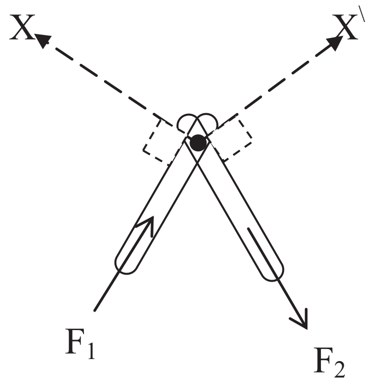
$\Sigma X_i = 0 \Longrightarrow F_1 = F_2$

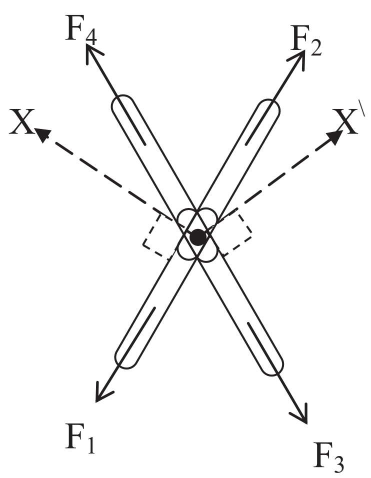


2- Since two forces can be in equilibrium only if they are equal, opposite, and collinear, we conclude that the forces in any two bars, their axes is not collinear, are equal to zero if there is no external force applied at their joint.

$\Sigma X_i = 0 \Longrightarrow F_2 = 0$

$\Sigma X^{\backslash}_i = 0 \Longrightarrow F_1 = 0$

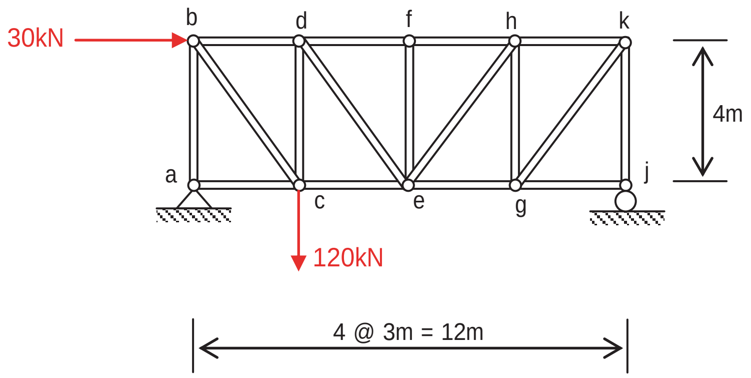


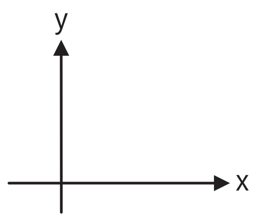


3- $\Sigma X_i = 0 \Longrightarrow F_3 = F_5$

$\Sigma X^{\backslash}_i = 0 \Longrightarrow F_1 = F_2$

Example (1): Calculate the member forces, F_{ab}, F_{ac}, F_{bd}, F_{cd}, F_{ce}, F_{de}, and F_{df} using the method of joints.



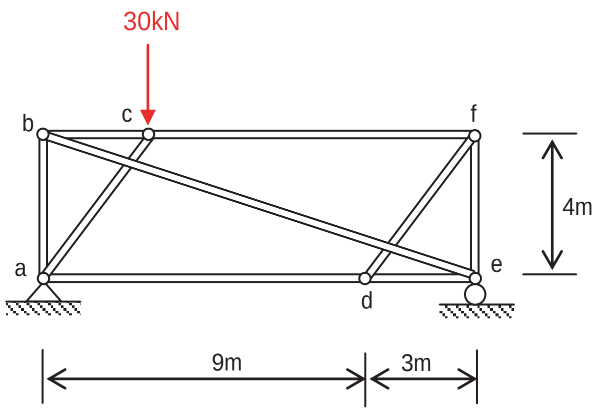


The method of Sections

If the forces in only a few members of a truss are to be found, the method of sections generally provides the most direct means of obtaining these forces. The "***method of sections***" consists of passing an "***imaginary section***" through the truss, thus cutting it into two parts. Provided the entire truss is in equilibrium, each of the two parts must also be in equilibrium; and as a result, the three equations of equilibrium may be applied to either one of these two parts to determine the member forces at the "***cut section***".

Example (2): Calculate the member forces, F_{df}, F_{de}, and F_{ce} for the truss of the previous example using the method of sections.

Example (3): Calculate all the member forces for the truss given in the figure below.

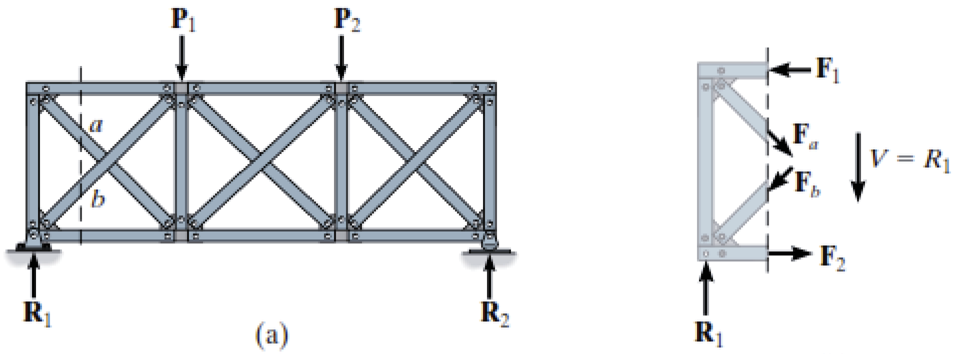


Chapter Four

Approximate Analysis of Statically Indeterminate Structures

Approximate methods of analysis are methods by which statically indeterminate structures are reduced into determinate structures, through the use of certain assumption. The determinate structure is then solved by equations of statics.

A- Trusses



(a) (b)

Consider the above truss which has two diagonals in each panel. The truss is statically indeterminate to the third degree. It can be noticed that if a diagonal is removed from each of the three panels, it will render the truss statically determinate

b = 16 , r = 3, and j =8; hence

b + r ? 2 j ; 16 + 3 > 16

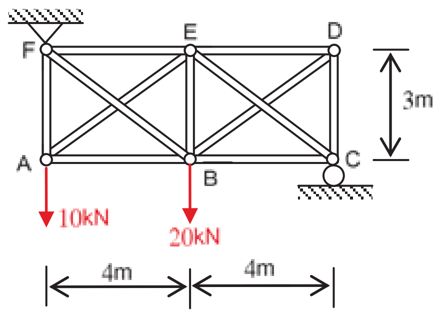
Therefore, we must make three assumptions regarding the bar forces in order to reduce the truss to one that is statically determinate. These assumptions can be made with regard to the cross-diagonals, realizing that when one diagonal in a panel is in tension the corresponding cross-diagonal will be in compression.

Two methods of analysis are generally acceptable;

Method (1): If the diagonals are intentionally designed to be long and slender, it is reasonable to assume that they cannot support a compressive force; otherwise, they may easily buckle. Hence the panel shear is resisted entirely by the tension diagonal, whereas the compressive diagonal is assumed to be ***a zero-force member***.

Method (2): If the diagonal members are intended to be constructed from large rolled sections such as angles or channels, they may be equally capable of supporting a tensile and compressive force. Here we will assume that the tension and compression diagonals each carry ***half the panel's shear***.

Example: Determine approximately the forces in the members of the truss shown in figure. (i) If the diagonals are constructed from large rolled sections to support both tensile and compressive forces. (ii) If the diagonals con not support compressive force.



Solution:

Since b = 11, r = 3, and j = 6

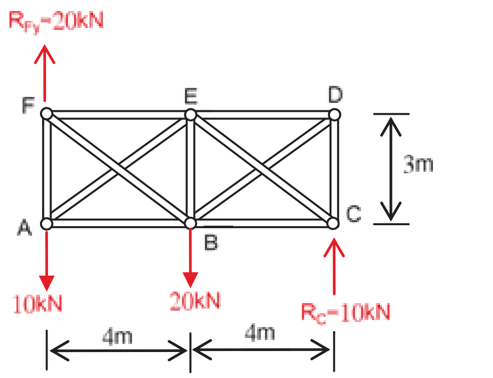
So, the truss is statically indeterminate to the second degree.

i) From the whole truss, using the Eqs. of equilibrium

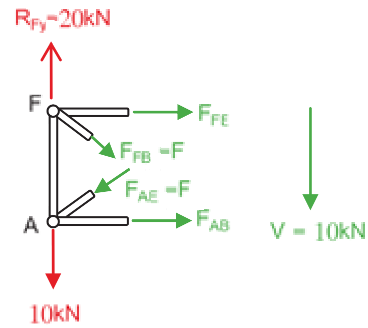
$\sum M_F = 0 \Rightarrow \quad R_c = 10kN$

$\sum F_Y = 0 \Rightarrow \quad R_{Fy} = 20kN$

$\sum F_X = 0 \Rightarrow \quad R_{FX} = 0$



The two assumptions require the tensile and compression diagonals to carry equal forces, i.e. $F_{FB} = F_{AE} = F$. For a vertical section through the left panel



$\sum F_Y = 0 \Rightarrow 20 - 10 - 2F\,(\frac{3}{5}) = 0$

$F = 8.33kN$, hence $F_{AE} = 8.33$ kN (C) and $F_{FB} = 8.33$kN (T)

$\sum M_F = 0 \Rightarrow F_{AB} \times 3 - F_{AE}\,(\frac{4}{5}) \times 3 = 0$

$F_{AB} \times 3 - 8.33 \times (\frac{4}{5}) \times 3 = 0$; $F_{AB} = 6.67$ kN (T)

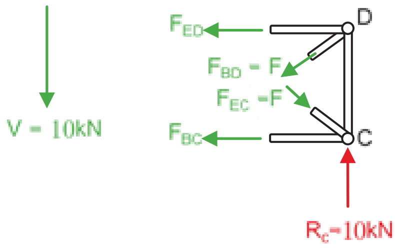
$\sum M_A = 0 \Rightarrow F_{FE} \times 3 + F_{FB}\,(\frac{4}{5}) \times 3 = 0$

$F_{FE} \times 3 + 8.33 \times (\frac{4}{5}) \times 3 = 0$; $F_{FE} = -\,6.67$ kN (C)

Assume a vertical section through the right panel

$\sum F_Y = 0 \Rightarrow 10 - 2F\,(\frac{3}{5}) = 0$; $F = 8.33kN$,

hence $F_{BD} = 8.33$ kN (T) and $F_{EC} = 8.33$kN (C)



$\sum M_D = 0 \Rightarrow F_{BC} \times 3 - F_{EC}\,(\frac{4}{5}) \times 3 = 0$

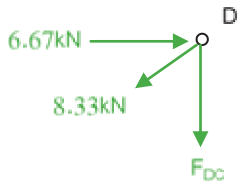
$F_{BC} \times 3 - 8.33 \times (\frac{4}{5}) \times 3 = 0$; $F_{BC} = 6.67$ kN (T)

$\sum M_C = 0 \Rightarrow F_{ED} \times 3 + F_{BD}\,(\frac{4}{5}) \times 3 = 0$

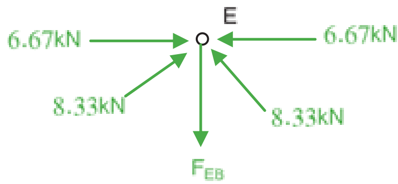
$F_{ED} \times 3 + 8.33 \times (\frac{4}{5}) \times 3 = 0$; $F_{ED} = -\,6.67$ kN (C)

Using F.B.D. of joints D, E, and F ;

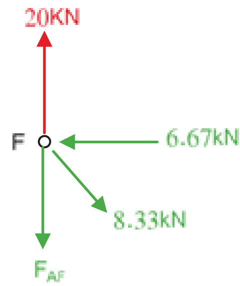
$\sum F_Y = 0 \Rightarrow F_{DC} + 8.33 \times (\frac{3}{5}) = 0$; $F_{DC} = -5$ kN (C)



$\sum F_Y = 0 \Rightarrow F_{EB} - 2 \times 8.33\,(\frac{3}{5}) = 0$; $F_{EB} = 10$ kN (T)



$\sum F_Y = 0 \Rightarrow 20 - F_{AF} - 8.33\,(\frac{3}{5}) = 0$; $F_{AF} = 15$ kN (T)



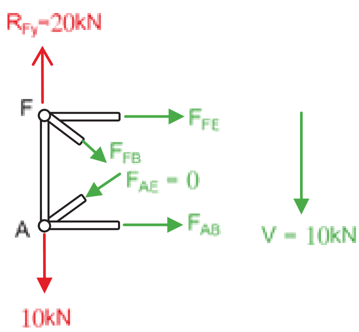
ii) If the diagonals cannot support a compressive force ;

Assume a vertical section through the left panel

$F_{AE} = 0$

$\sum F_Y = 0 \Rightarrow 20 - 10 - F_{FB}\,(\frac{3}{5}) = 0$

$F_{FB} = 16.67$kN (T)



$\sum M_F = 0 \Rightarrow F_{AB} \times 3 = 0$; $F_{AB} = 0$

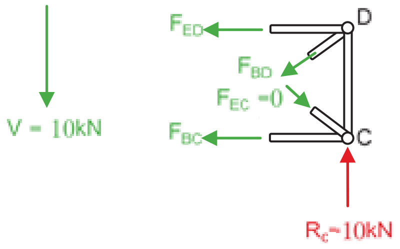
$$\sum M_A = 0 \Rightarrow F_{FE} \times 3 + F_{FB}\left(\frac{4}{5}\right) \times 3 = 0$$

$$F_{FE} \times 3 + 16.67\left(\frac{4}{5}\right) \times 3 = 0\ ;\ F_{FE} = -13.33\text{kN (C)}$$

Assume a vertical section through the right panel

$F_{EC} = 0$

$$\sum F_Y = 0 \Rightarrow 10 - F_{BD}\left(\frac{3}{5}\right) = 0\ ;\ F_{BD} = 16.67\text{ kN (T)}$$



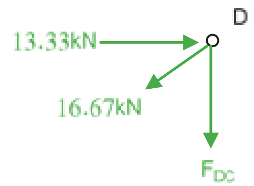
$$\sum M_D = 0 \Rightarrow F_{BC} \times 3 = 0\ ;\ F_{BC} = 0$$

$$\sum F_X = 0 \Rightarrow F_{ED} + F_{BD}\left(\frac{4}{5}\right) = 0$$

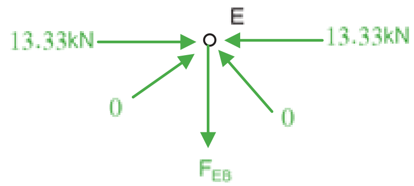
$$F_{ED} + 16.67 \times \left(\frac{4}{5}\right) = 0\ ;\ F_{ED} = -13.33\text{ kN (C)}$$

Using F.B.D. of joints D, E, and F ;

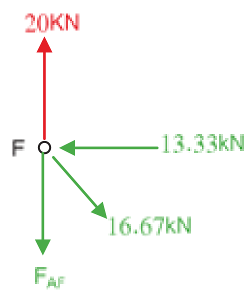
$$\sum F_Y = 0 \Rightarrow F_{DC} + 16.67 \times \left(\frac{3}{5}\right) = 0\ ;\ F_{DC} = -10\text{ kN (C)}$$



$$\sum F_Y = 0 \Rightarrow F_{EB} = 0$$



$$\sum F_Y = 0 \Rightarrow 20 - F_{AF} - 16.67\left(\frac{3}{5}\right) = 0\ ;\ F_{AF} = 10\text{ kN (T)}$$



B- Vertical Loads on Building Frames

Consider a typical girder located within a building, Fig. (1), bent and subjected to a uniform vertical load, as shown in Fig. (2). The column supports at A and B will each exert three reactions on the girder, and therefore the girder will be statically indeterminate to the third degree (6 reactions – 3 equations of equilibrium). To make the girder statically determinate, an approximate analysis will therefore require three assumptions. If the columns are extremely stiff, no rotation at A and B will occur, and the deflection curve for the girder will look like that shown in Fig. (3). For this case, the inflection points (Points of zero moments) occur at 0.21L from each support.

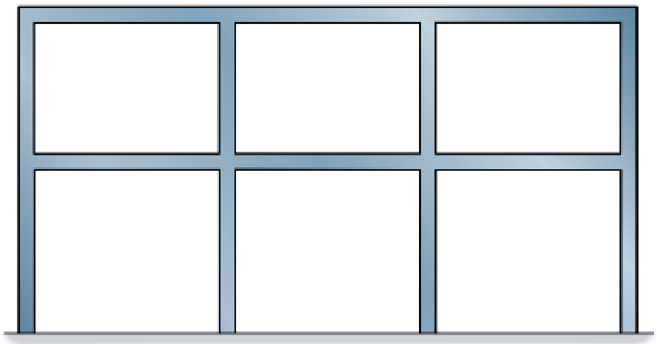


Fig. (1)

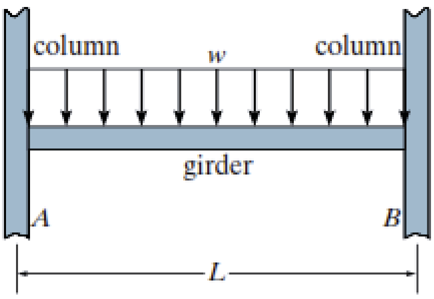


Fig. (2)

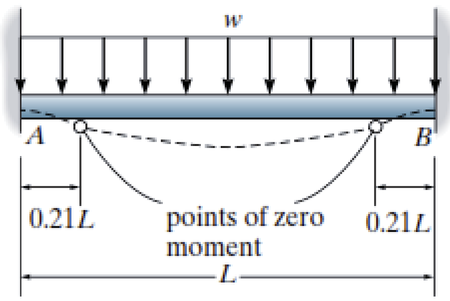


Fig. (3)

If, however, the column connections at A and B are very flexible, then like a simply supported beam, zero moment will occur at the supports, Fig. (4).

In reality, however, the columns will provide some flexibility at the supports, and therefore we will assume that zero moment occurs at the "average point" between the two extremes, $0.21L + 0)/2 \cong 0.1L$ from each support, Fig. (5).

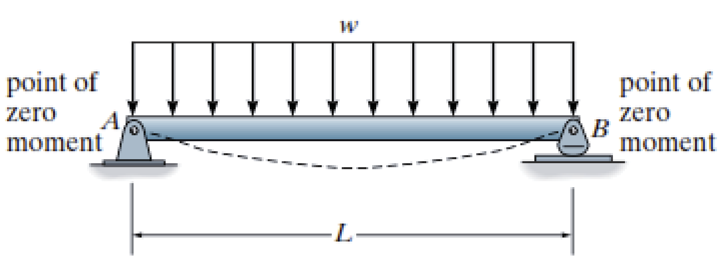


Fig. (4)

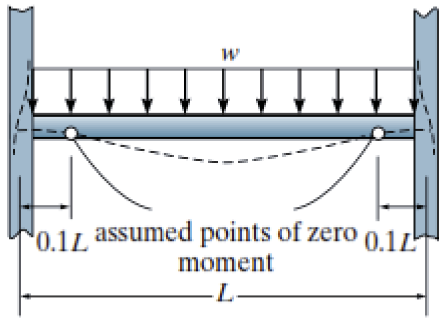


Fig. (5)

In summary then, each girder of length " L " may be modeled by a simply supported span of length 0.8L resting on two cantilevered ends, each having a length of (0.1L) , Fig. (6). The following three assumptions are incorporated in this model;

1- There is zero moment in the girder, 0.1L from the left support.

2- There is zero moment in the girder, 0.1L from the right support.

3- The girder does not support an axial force.

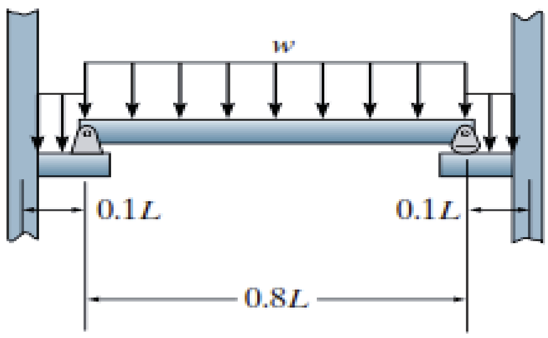
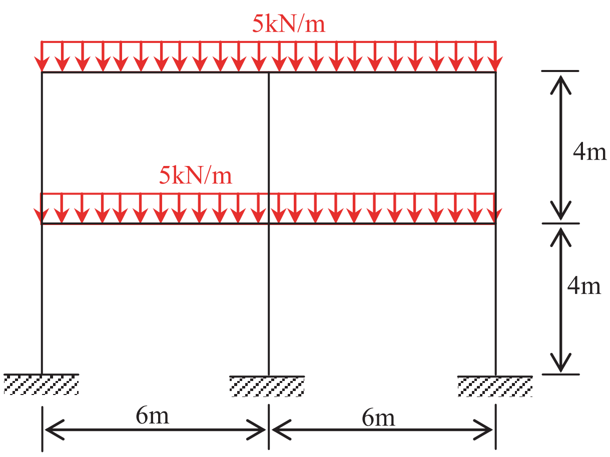


Fig. (6)

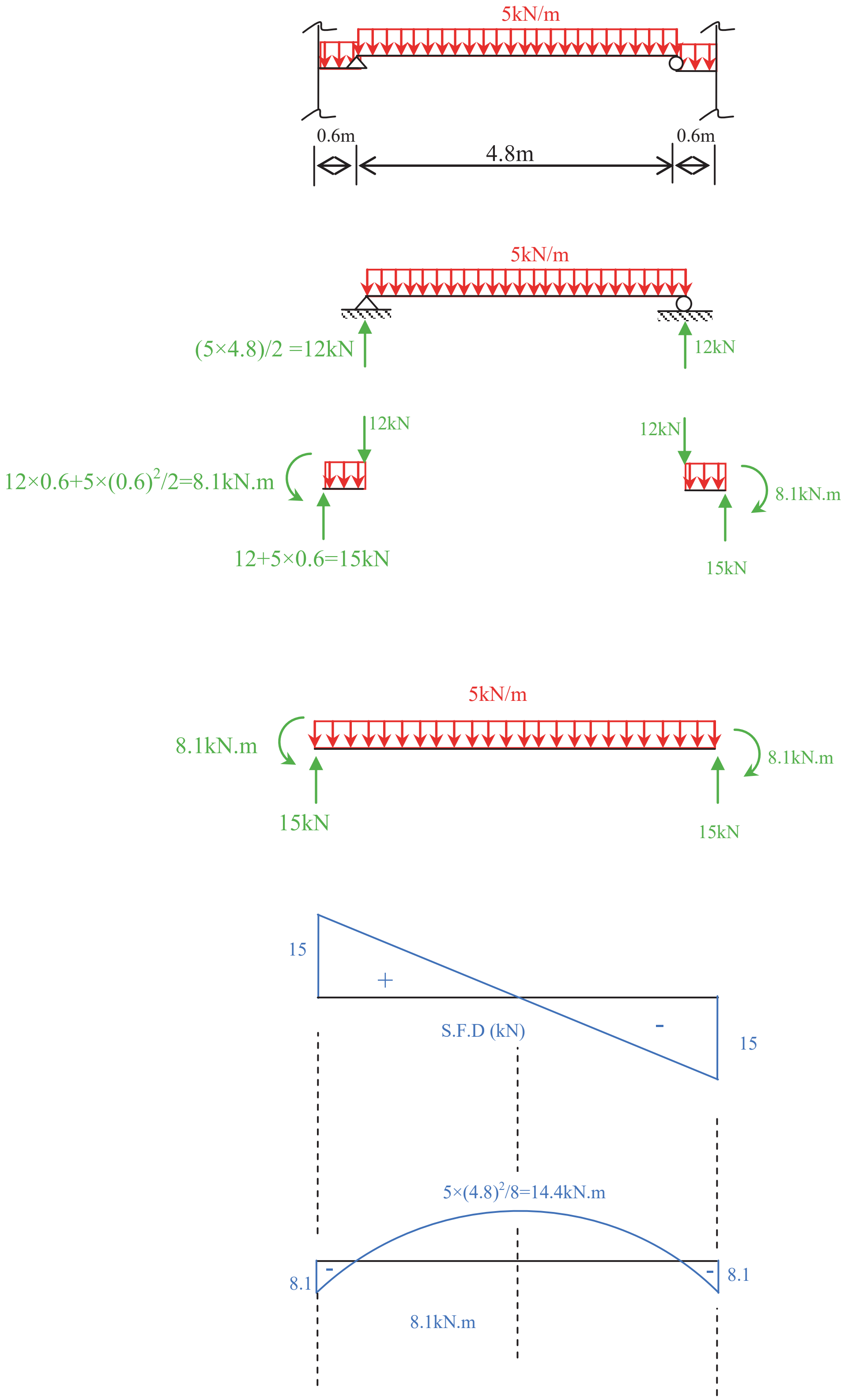
Example: Determine (approximately) the shear force and bending moments for the girders of the building frame shown in figure below.

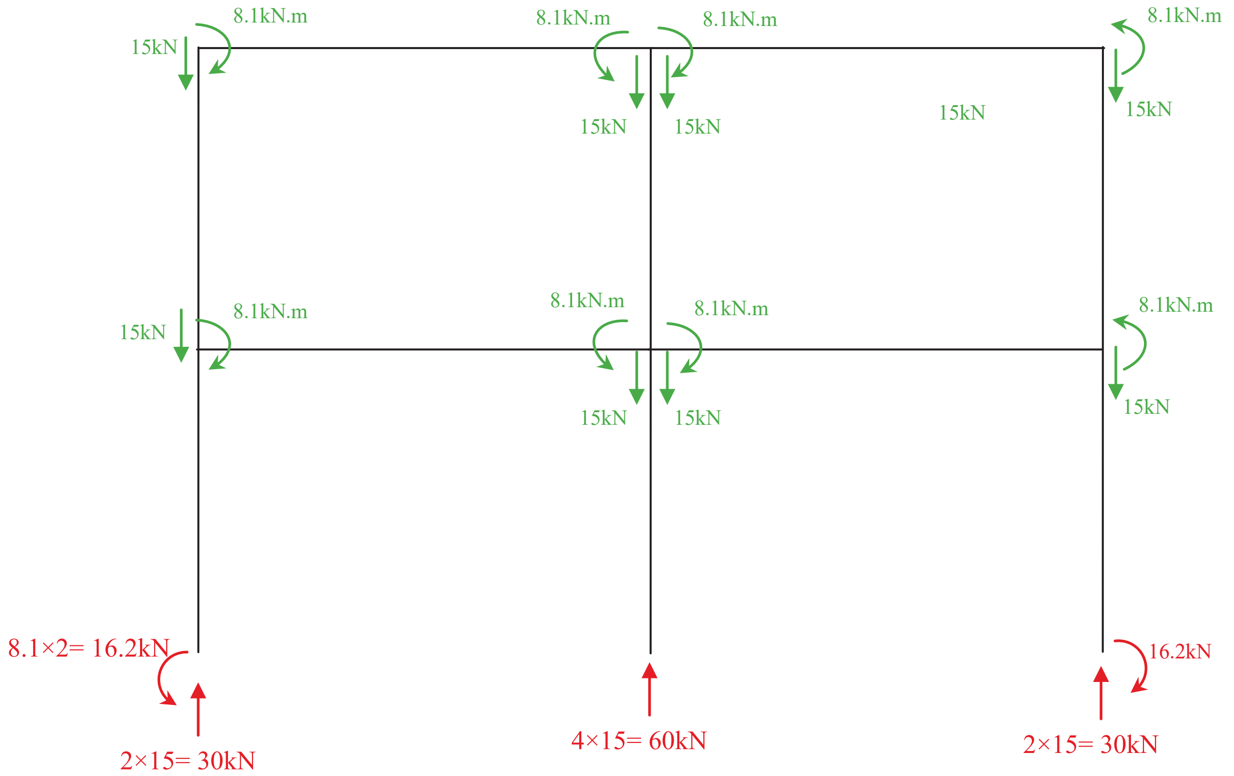


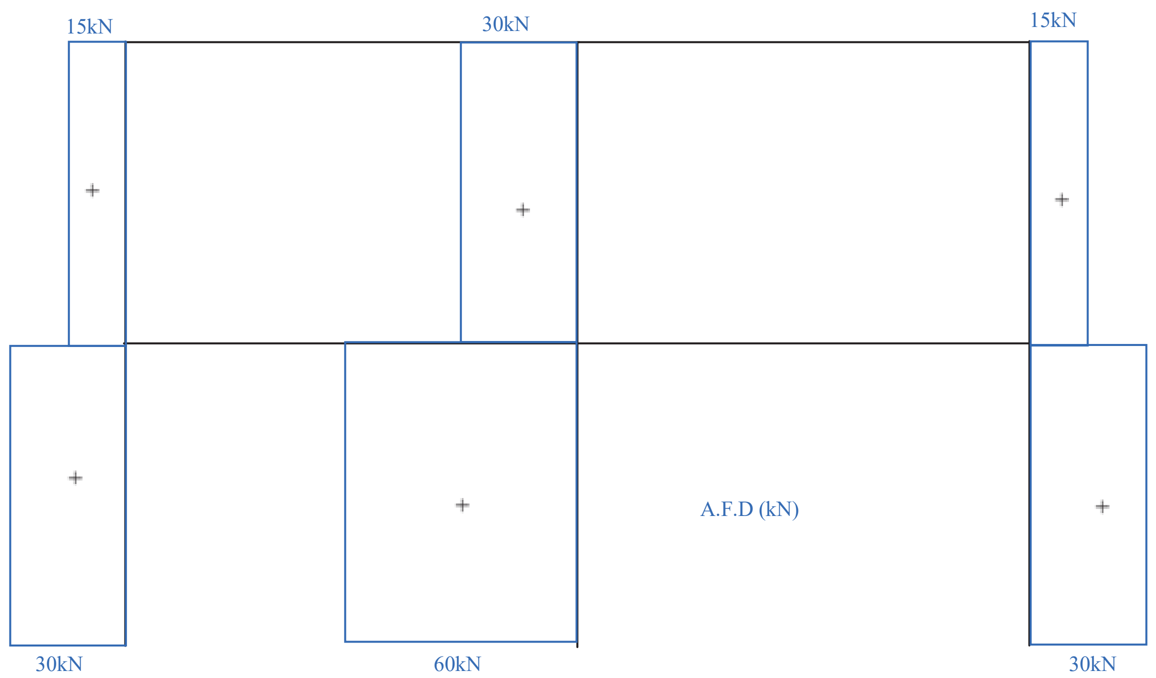
Solution:

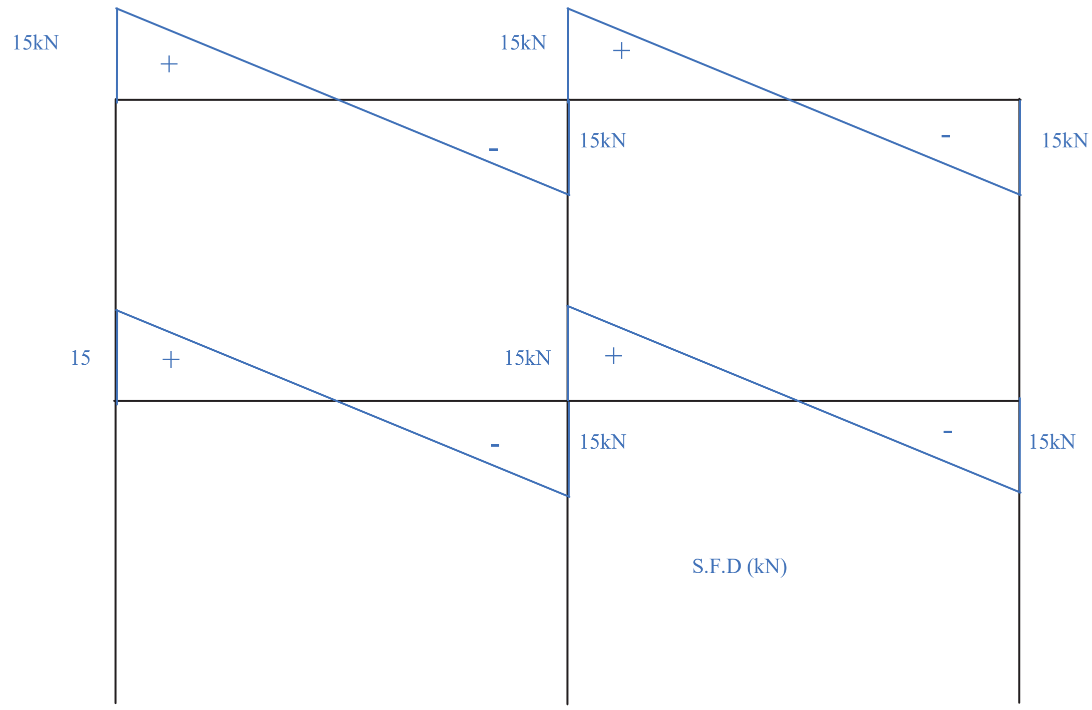
As the span lengths and loads for the four girders are the same, the approximate shear and bending moment diagrams for the girders will also be the same.

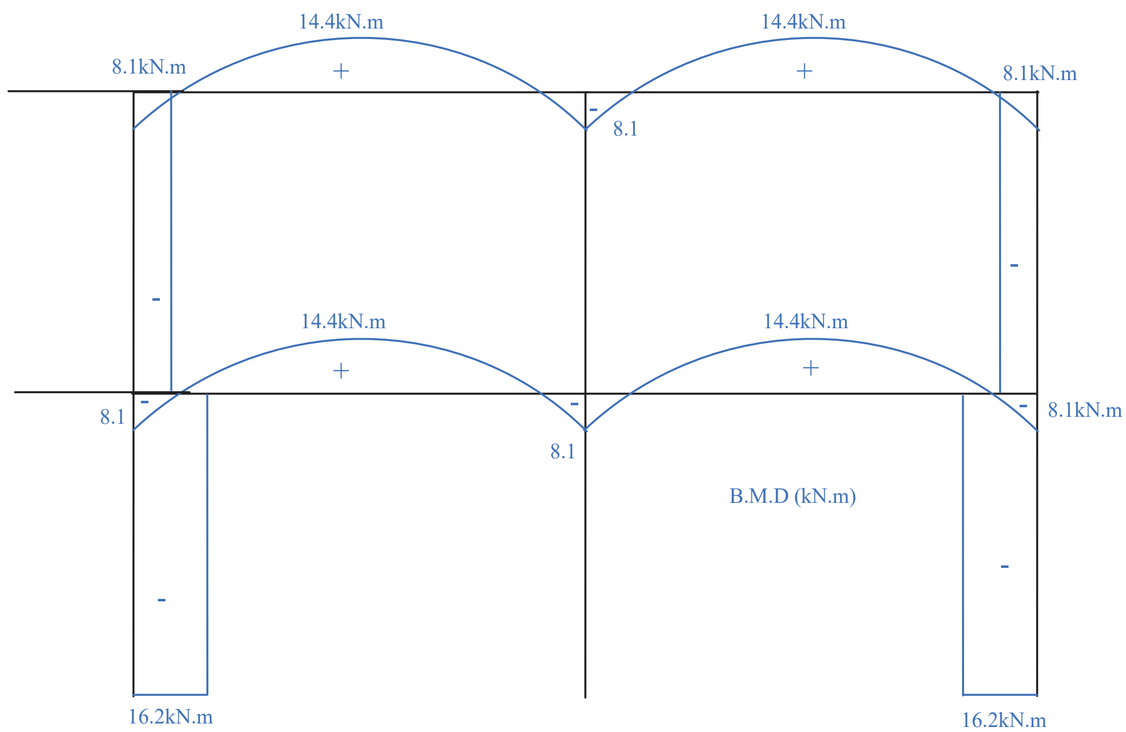
The inflection points are assumed to occur in the beam at (0.1L = 0.6m), the middle portion of the girder, which has a length of (0.8L = 4.8m), is simply supported on the two end portions, each of length 0.6m.




8.1kN.m
15kN
8.1kN.m
8.1kN.m
15kN
15kN
8.1kN.m
15kN
15kN
15kN
8.1kN.m
8.1kN.m
8.1kN.m
8.1kN.m
15kN
15kN
15kN
15kN
8.1×2= 16.2kN
2×15= 30kN
4×15= 60kN
16.2kN
2×15= 30kN


15kN
30kN
15kN
+
+
+
+
+
+
A.F.D (kN)
30kN
60kN
30kN


15kN
15kN
+
+
-
15kN
-
15kN
15
15kN
+
+
-
15kN
-
15kN
S.F.D (kN)


14.4kN.m
14.4kN.m
8.1kN.m
+
+
8.1kN.m
-
8.1
-
-
14.4kN.m
14.4kN.m
+
+
8.1
-
-
-
8.1kN.m
8.1
B.M.D (kN.m)
-
-
16.2kN.m
16.2kN.m

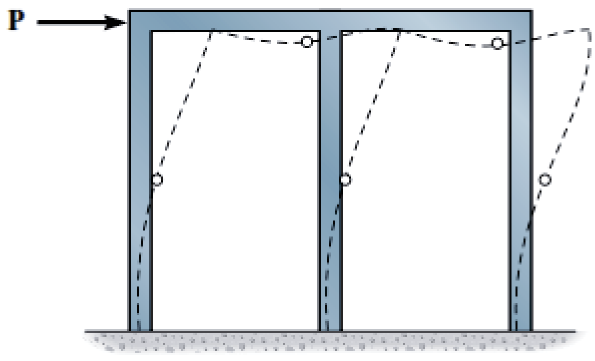
C- Lateral Loads on Building Frames

Portal Method:

The behavior of rectangular building frame is different under lateral (horizontal) loads than under vertical loads, so different assumptions must be used.

A method commonly used for the approximate analysis of relatively low building frames is the "Portal Method".

A building frame defects as shown in figure below,



o = inflection point

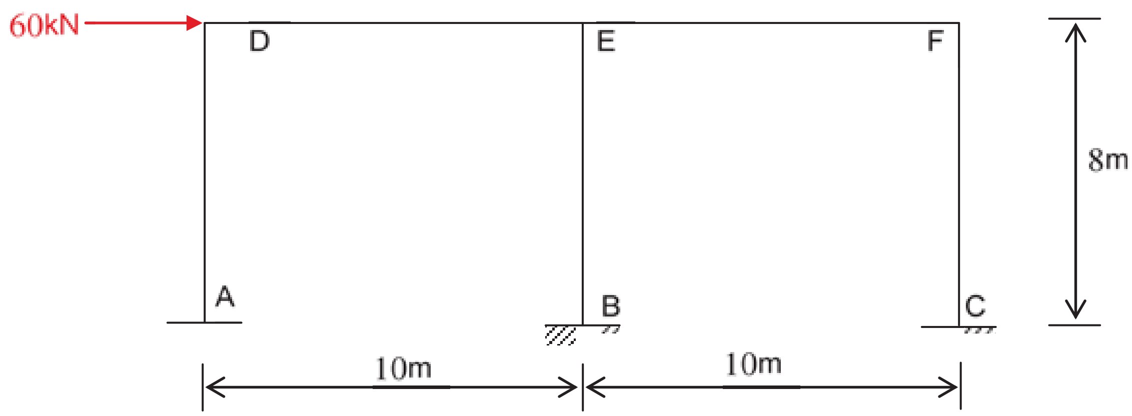
Therefore, it is appropriate to assume inflection points occur at the center of the columns and girders.

If we consider the frame to be composed of a series of portal, then as a further assumption, the interior columns would represent the effect of two portal columns and would therefore carry twice the shear (V) as the two exterior columns.

In summary, the portal method requires the following assumptions;

1- A hinge is placed at the center of each girder, since this is assumed to be a point of zero moment.
2- A hinge is placed at the center of each column, since this is assumed to be a point of zero moment.
3- At a given floor level, the shear at the interior columns is twice that at the exterior columns.

Example: Use the portal method to determine the external reactions, and draw the axial load, shear force, and bending moment diagrams for the frame shown in figure.

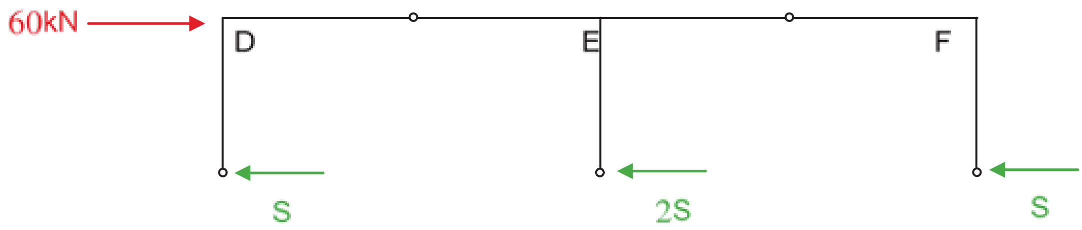


Solution:

i- Simplified frame: The simplified frame for approximate analysis is obtained by inserting internal hinges at the midpoints of all members of the given frame.

ii- Column shears: The shear in the interior column BE is assumed to be twice as much as in the exterior columns AD and CF.

By separating the frame into to two parts at the midpoint of the columns (upper and lower) where the hinges were assumed. From shear forces of the upper part

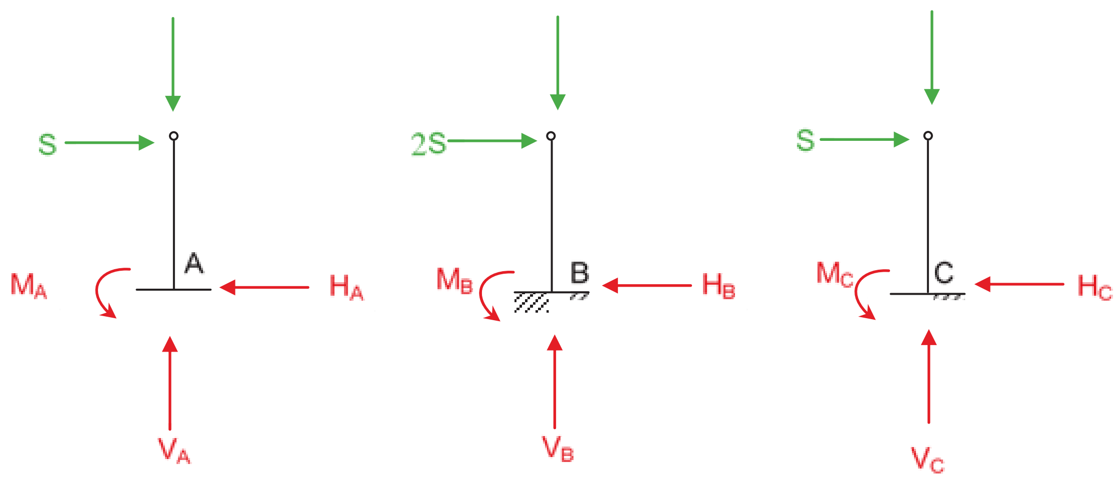


$\sum F_x = 0 \;\Rightarrow 60\text{-}S\text{-}2S\text{-}S\text{=}0 \;\Rightarrow S = 15kN$

Thus the shear forces (Horizontal reactions) at the lower ends of the columns are;

$H_A = H_C = S = 15kN \;(\longleftarrow)$

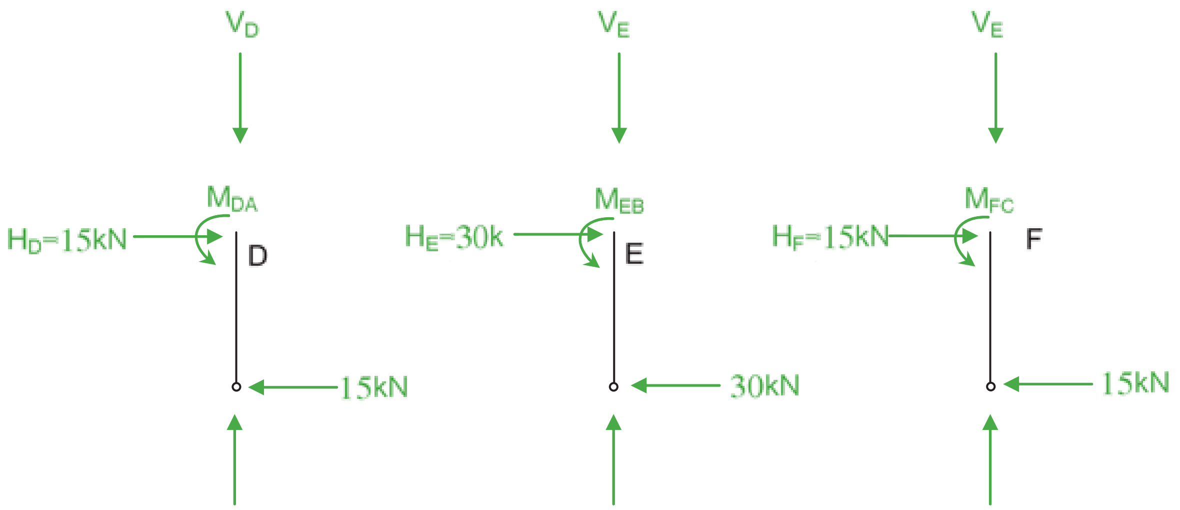
$H_B = 30kN \;(\longleftarrow)$



Shear forces at the upper ends of the columns are obtained by applying $\sum F_x = 0$ to the free body of each column,

$H_D = H_F = 15kN \;(\longrightarrow)$

$H_E = 30kN \;(\longrightarrow)$

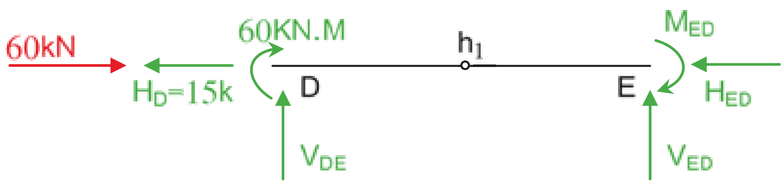


iii- Column moments: The column end moment moments can be computed using Eq. of $\sum M=0$ about lower and upper end of the columns,

$M_{AD} = M_{CF} = M_{DA} = M_{EC} = 15\times4 = 60kN.m$ (↶)

$M_{BE} = M_{EB} = 30\times4 = 120kN.m$ (↶)

iv- Girder axial forces, moments, and hear:



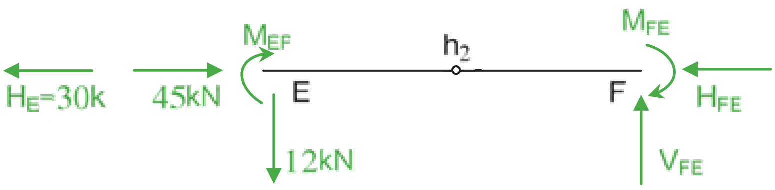
For Girder DE,

Using equation of $\sum F_x = 0 \Rightarrow 60\text{-}H_{ED}\text{-}15=0 \Rightarrow H_{ED} = 45kN\ (\longleftarrow)$

$\sum M_{h1} = 0$ (for left part) $\Rightarrow V_{DE} \times 5+60 =0 \Rightarrow V_{DE} = -12kN.m\ (\downarrow) = V_D$

$\sum F_Y = 0 \Rightarrow -12+V_{ED}=0 \Rightarrow V_{ED} = 12kN\ (\uparrow)$

$\sum M_{h1} = 0$ (for right part) $\Rightarrow 12 \times 5\text{-}\ M_{ED} =0 \Rightarrow M_{ED} = 60kN.m$ (⤾)



For Girder EF,

Using equation of $\sum F_x = 0 \Rightarrow 45\text{-}H_{FE}\text{-}30=0 \Rightarrow H_{FE} = 15kN\ (\longleftarrow) = H_F$

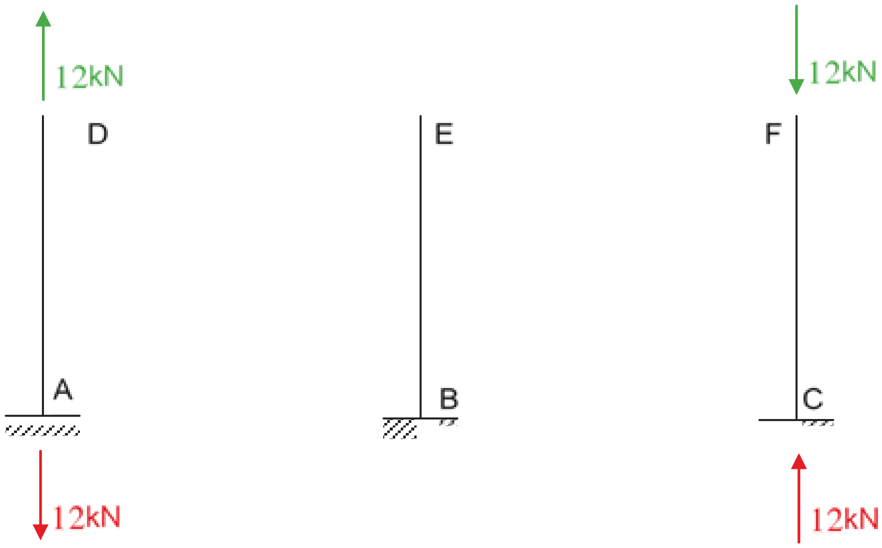
$\sum F_Y = 0 \Rightarrow -12+V_{FE}=0 \quad \Rightarrow \quad V_{FE} = 12kN\ (\uparrow) = V_F$
$\sum M_{h2} = 0$ (for left part) $\Rightarrow -12 \times 5+M_{EF} = 0 \quad \Rightarrow \quad M_{EF} = 60kN.m$ (↩)
$\sum M_{h2} = 0$ (for right part) $\Rightarrow 12 \times 5-M_{FE} = 0 \quad \Rightarrow \quad M_{FE} = 60kN.m$ (↩)

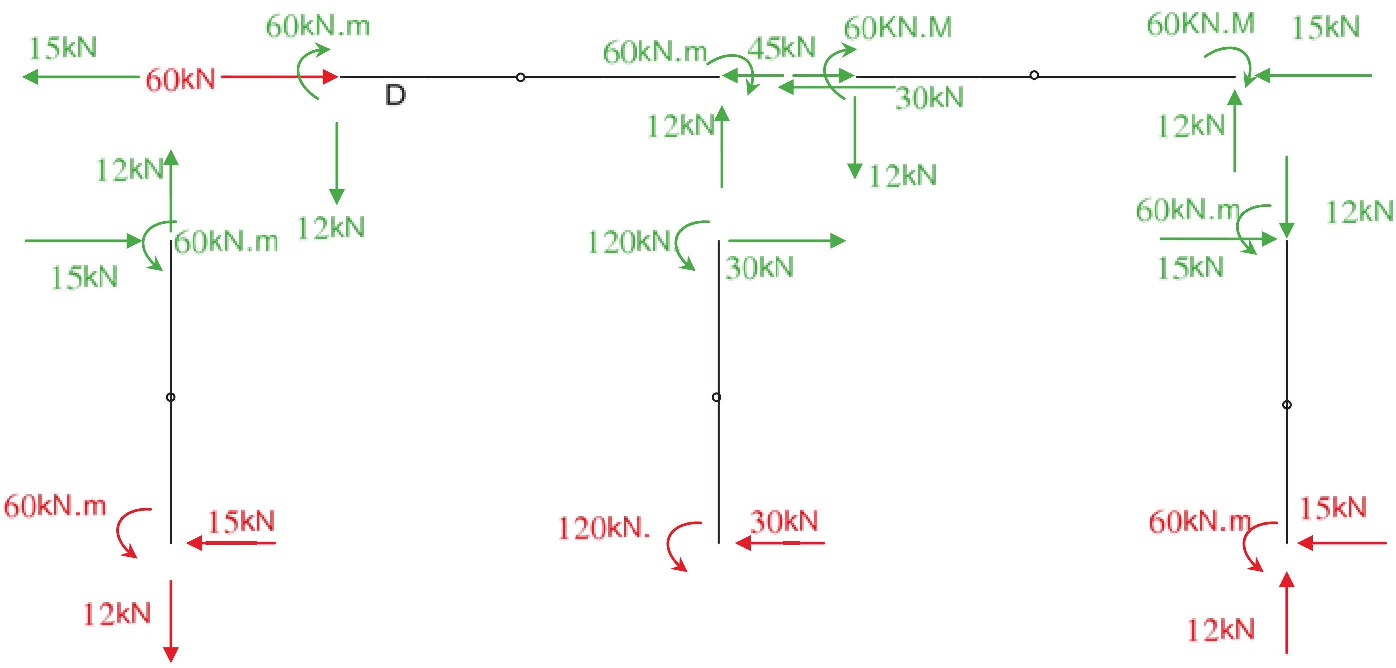
v- Column axis:

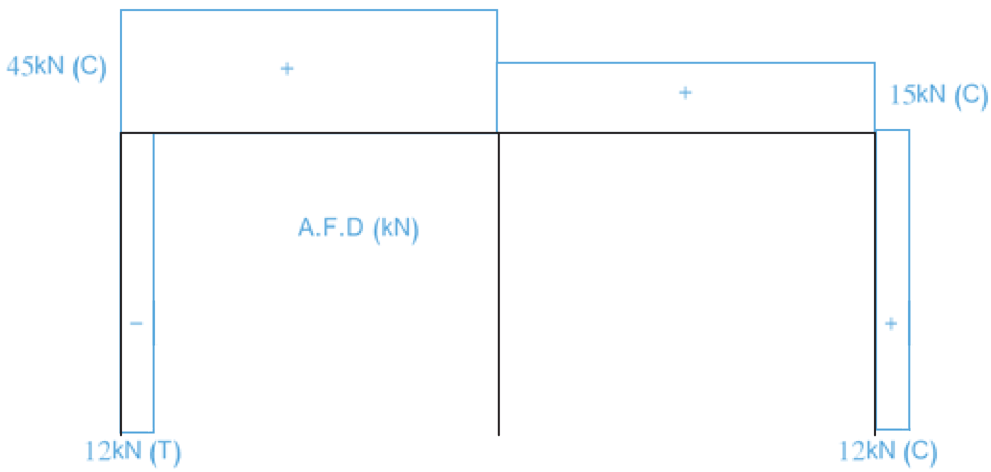
Using $\sum F_Y = 0$

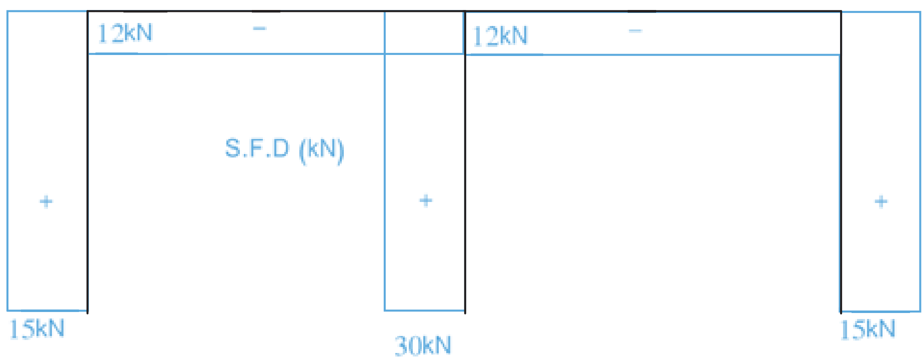
$V_A - 12 = 0$
$V_A = 12$ kN

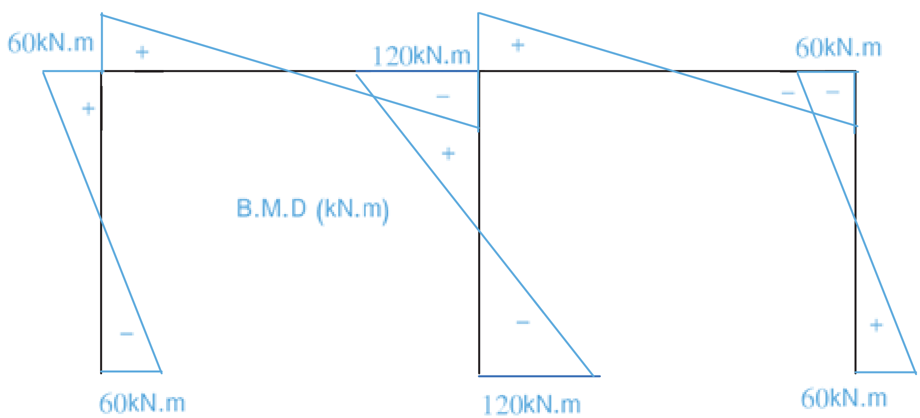
and , $V_c = 12$ kN






45kN (C)
+
+
15kN (C)
A.F.D (kN)
−
+
12kN (T)
12kN (C)


12kN
−
12kN
−
S.F.D (kN)
+
+
+
15kN
30kN
15kN


60kN.m
+
120kN.m
+
60kN.m
−
−
−
+
+
B.M.D (kN.m)
−
−
+
60kN.m
120kN.m
60kN.m

Chapter Five

Influence Lines for Statically Determinate Structures

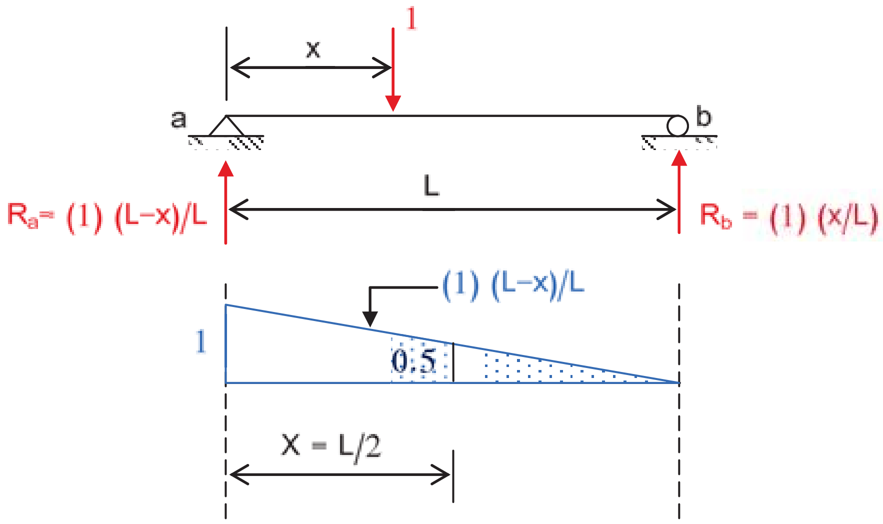
An "influence line" is a diagram showing the change in the values of a particular function (reaction, member axial force, internal shear, or bending moment) as a unit concentrated load moves across the structure.

Influence lines play an important role in the design of bridges, industrial crane, conveyors, and other structures where loads move across their span.

An influence line is constructed by placing a unit load at a 'variable position x" on the member and then computing the value of reactions, shear force, or bending moment at the point as a function of x.

In this manner, the equations of the various line segments composing the influence line can be determined and plotted.

Consider the simply supported beam shown in figure.



If the influence line for the reaction at point " a "is required, a single concentrated load is moved across the span from point " a " to " b ", and the reaction at point " a " is calculated. Placing the unit load at a typical position located at distance " x " from point " a " and summing moments about point " b " gives;

$$\sum Mb = R_a.(L)-(1)(L-x) = 0$$

$$R_a = \frac{(L-x)}{L}.(1) \qquad \text{"Straight line"}$$

When the load is positioned at the left reaction (x = 0), the value of R_a is a unity. As the load moves across the span and reaches mid-span (x = L/2), the diagram shows that R_a equals 0.5 . When the unit load is at the right support (x = L) R_a equals zero.

Influence Lines for Beams

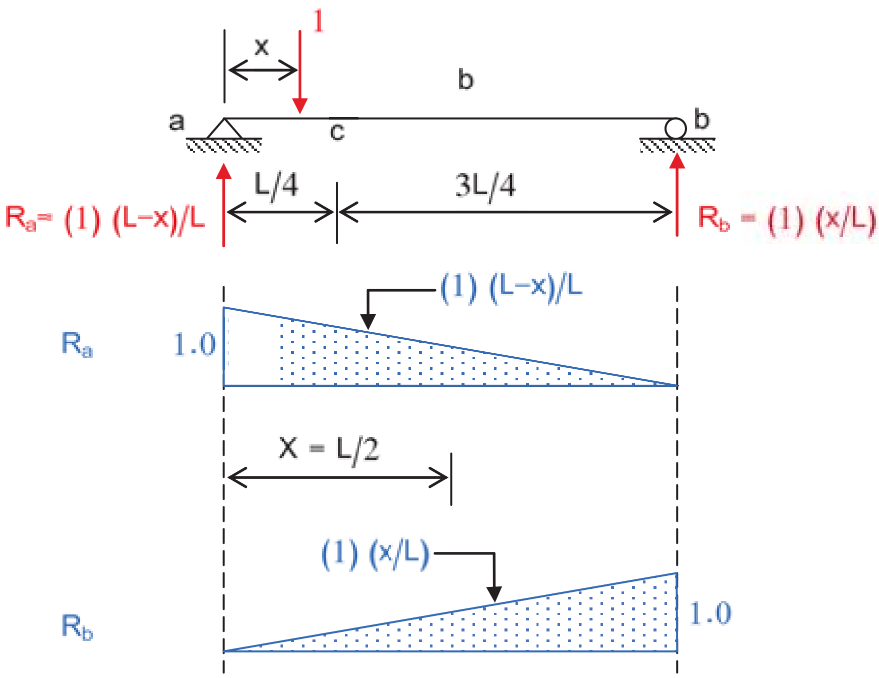
For beams, we are interested in the influence lines for the reactions, as well as the change in the internal quantities in the beams as the loading moves across the structure. Therefore, influence lines for the shear and moment at a specific cross-section must also be constructed for beam structures.

In order to do so, it is necessary to make an imaginary cut through the beam at the point of interest and then compute the value of the shear and moment at this cross-section as the unit concentrated load traverses the beam.

For the simply supported beam discussed in the previous section, the influence line for the reaction at point " b " can also be obtained by placing the unit load at a typical point on the beam and summing moments about point " a ", giving

$$\sum Ma = R_b.(L)-(1)(x) = 0$$

$$R_b = \frac{x}{L}.(1) \qquad \text{"Straight line"}$$



It is of interest to note that the sum of the influence ordinates for R_a and R_b is (1) for a given " x " value of their respective influences lines. Summation of forces in the vertical direction $R_a + R_b - 1 = 0$

Hence, $R_a + R_b = 1$

To obtain the influence line for shear and moment at point " c " as the unit load moves across the beam, the free-body diagrams are drawn for $0 \le x < L/4$ and $L/4 < x \le L$.

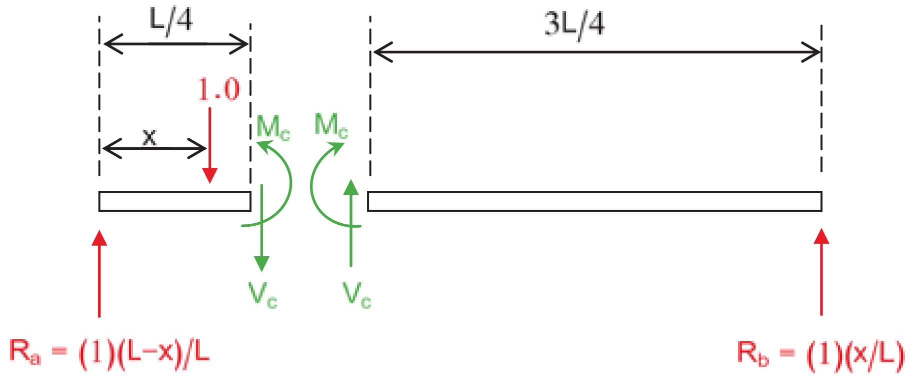


Fig. (1)
$0 \le x < L/4$

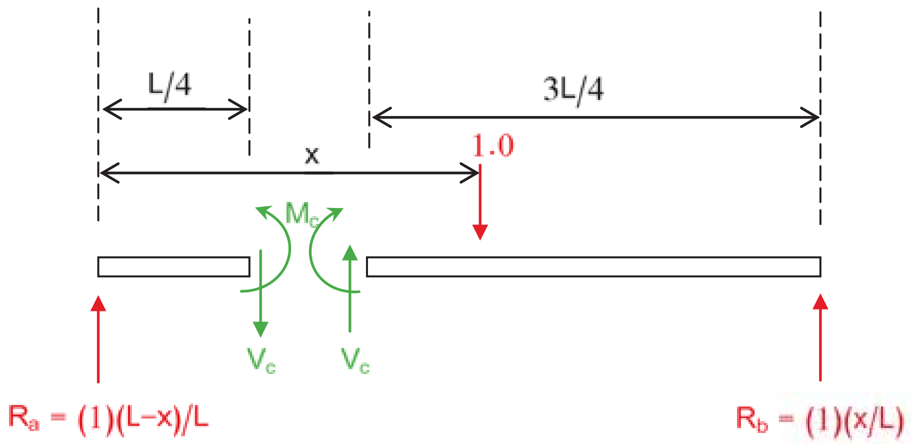


Fig. (2)
$L/4 < x \le L$

Figure (1) is correct if the unit load is located between points " a " and " c ", and Fig. (2) is valid for the load situated between points " c " and " b ".

From the left part of Fig. (1), the expression for shear force is given as;

$$V_c = -1 + R_a = -1 + \frac{(L-x)}{L} = -\frac{x}{L} \qquad 0 \le x < L/4 \qquad \text{------------- (5-1)}$$

Alternatively, the right hand part

$$V_c = -R_b = -\frac{x}{L} \qquad 0 \le x < L/4 \qquad \text{------------- (5-2)}$$

Either of the above equations can be used to construct the influence line for V_c for the segment from " a " to " c"

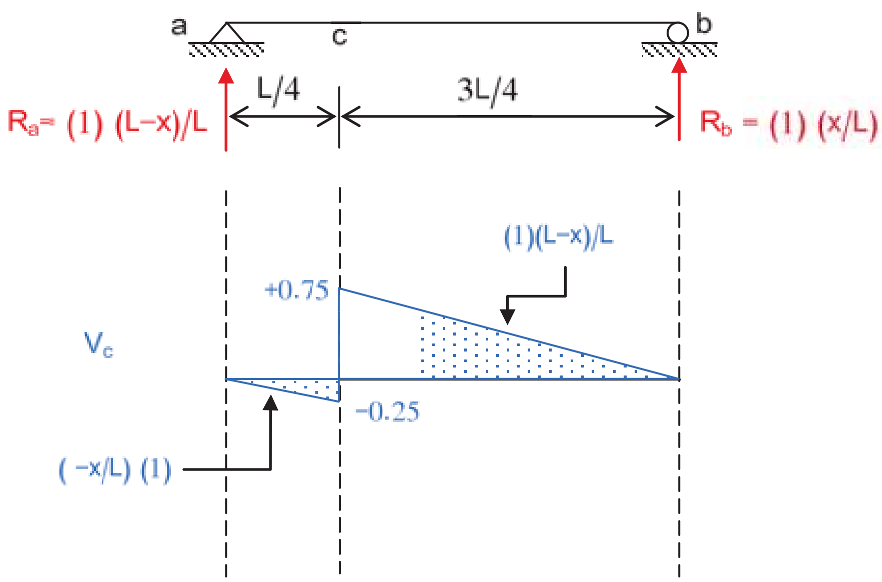
As the unit load traverses the segment from points " c " to " b ", Fig (2) is used to investigate the shear at section " c ".

Using the left part;

$$V_c = R_a = \frac{L-x}{L} \qquad L/4 < x \leq L \qquad \text{-------------} \ (5\text{-}3)$$

The right-hand part;

$$V_c = 1 - R_b = 1 - \frac{x}{L} = \frac{L-x}{L} \qquad L/4 < x \leq L \qquad \text{-------------} \ (5\text{-}4)$$



To obtain the moment influence line for the beam it is necessary to write expression for the moment at point " c " as the unit concentrated load is positioned at all locations on the span.

For the load positioned between points " a " and ' c " ;

$$M_c = R_a \left(\frac{L}{4}\right) - (1)\left(\frac{L}{4} - x\right)$$

$$= \left(\frac{L-x}{L}\right)\left(\frac{L}{4}\right) - \left(\frac{L}{4} - x\right) = \frac{L}{4} - x\frac{L}{4} - \frac{L}{4} + x = \frac{3x}{4} \qquad 0 \leq x \leq \frac{L}{4}$$

and

$M_c = R_b\,(\frac{3L}{4}) = (\frac{x}{L})\,(\frac{3L}{4}) = \frac{3x}{4}$ $\qquad\qquad 0 \le x \le \frac{L}{4}$

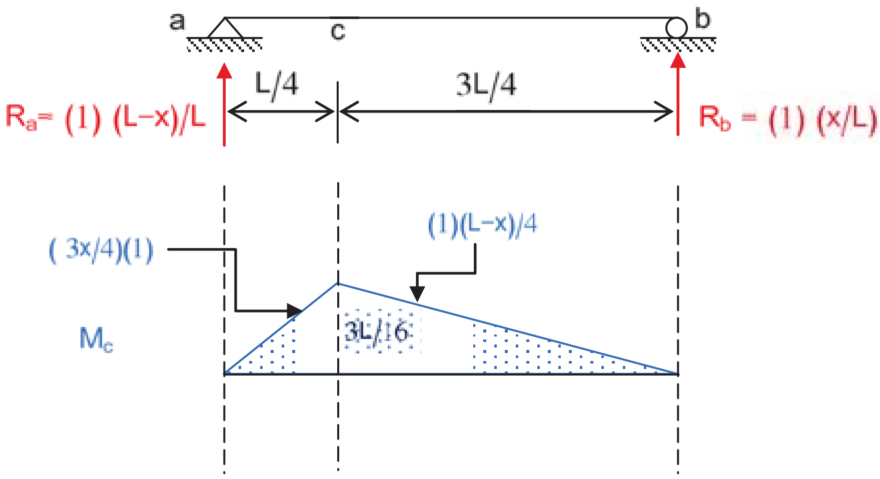
As the load goes from point " c " to " b " ;

$M_c = R_a\,(\frac{L}{4}) \;= (\frac{L-x}{L})(\frac{L}{4}) = \frac{L-x}{4}$ $\qquad\qquad \frac{L}{4} \le x \le L$

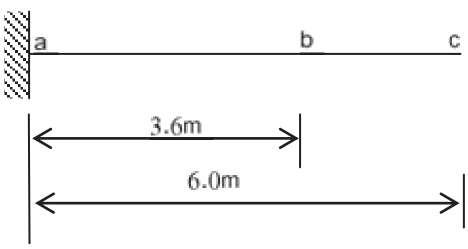
and

$M_c = R_b\,(\frac{3L}{4}) - (1)\,(x - \frac{L}{4}) = \frac{x}{L}.(\frac{3L}{4}) - (1)\,(x - \frac{L}{4})$

$= \frac{3x}{4} - x + \frac{L}{4} = \frac{L-x}{4}$ $\qquad\qquad \frac{L}{4} \le x \le L$



Example (1): Draw the influence lines for Ra, Ma, Vb, and Mb for the cantilever beam.



Solution

$\sum F_y = 0 \Rightarrow R_a = 1$

$\sum M_a = 0 \Rightarrow M_a = -1x$

when the load moves from " a " to " b "

$V_b = R_a - 1 = 1 - 1 = 0$

$M_b = 3.6R_a + M_a - 1(3.6 - x) = 3.6 - 1x - 3.6 + x =$

$= 0$

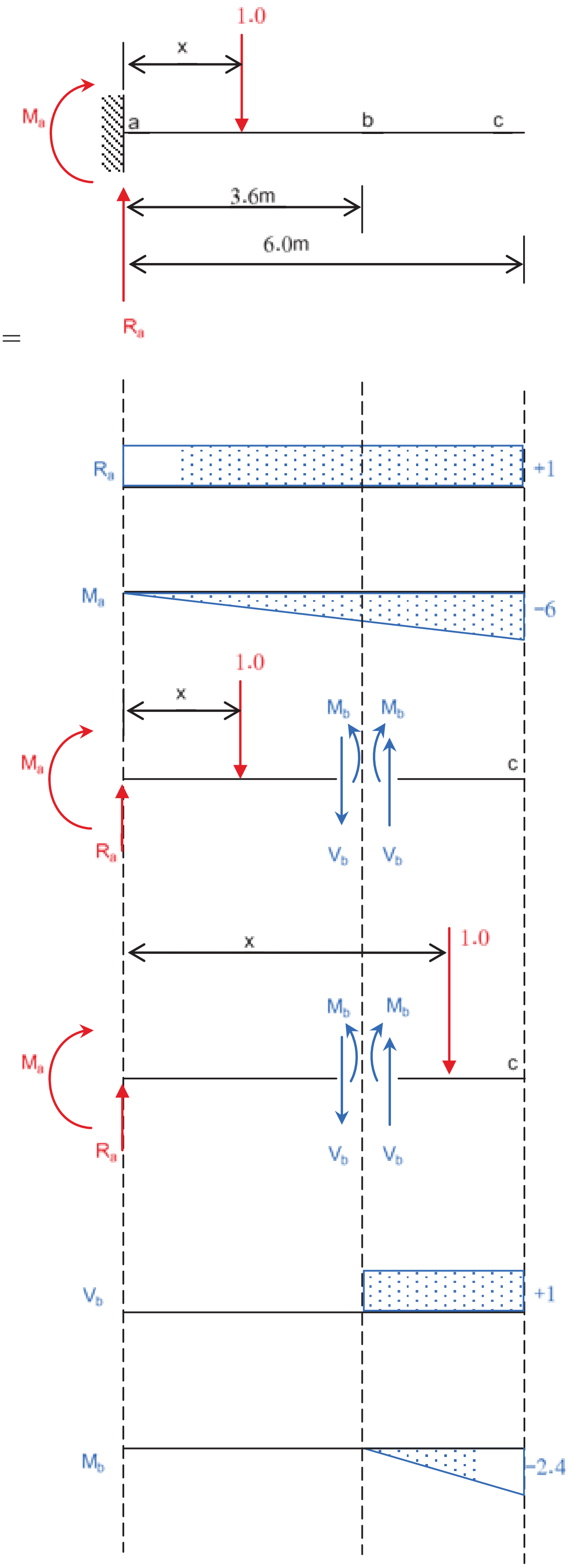
when the load moves from " b " to " c "

$V_b = R_a = 1$

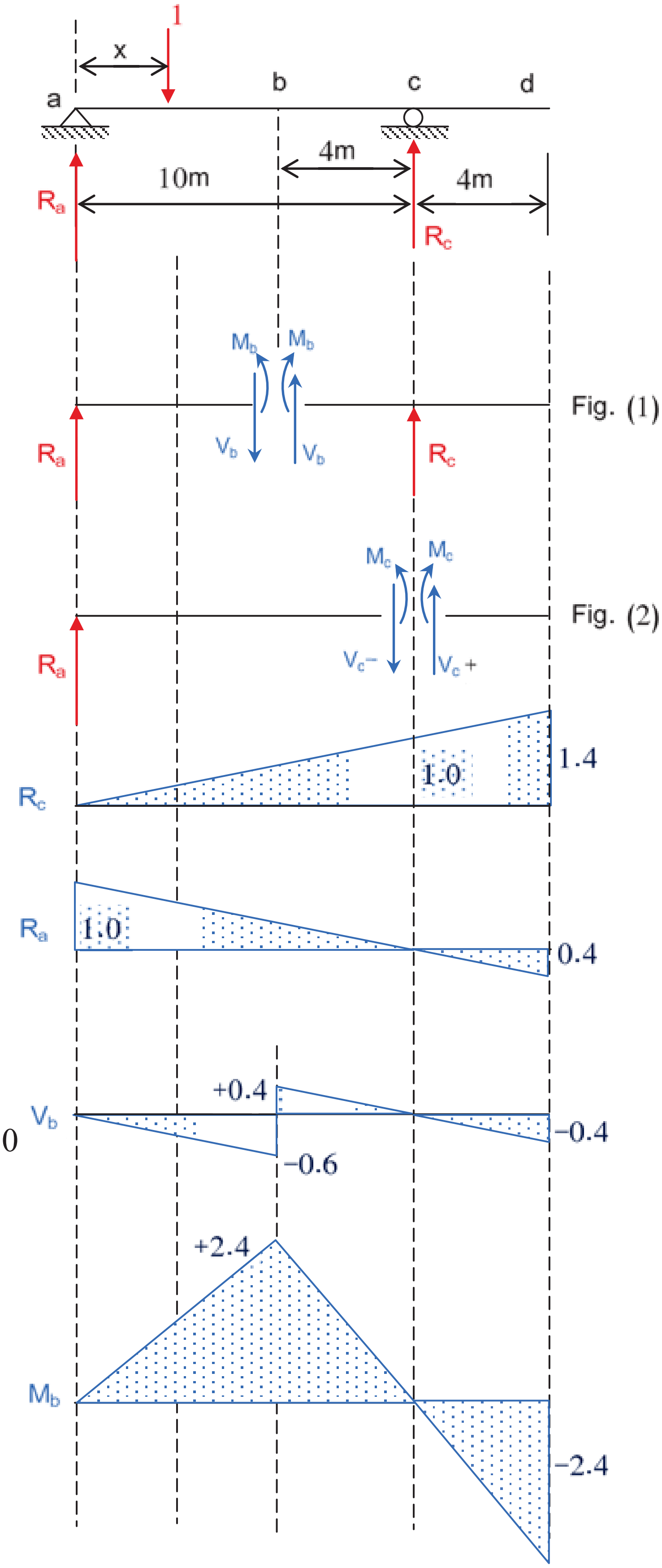
$M_b = 3.6R_a + M_a = 3.6\times 1 - x = 3.6 - x$

at $x = 3.6 \Rightarrow M_b = 3.6 - 3.6 = 0$

at $x = 6 \Rightarrow M_b = 3.6 - 6 = -2.4$



Example (2): Draw the influence lines for R_a, R_c, V_b, M_b, M_c, V_c-, V_c+ (the shear to the left and right of point " c " , respectively)



Solution

$$\sum M_a = 0 \;\Rightarrow\; R_c = \frac{x}{10}$$

$$\sum M_c = 0 \;\Rightarrow\; R_a = \frac{10-x}{10}$$

From Fig. (1);

For the load between " a " and " b "

$$V_b = R_a - 1 = -R_c = -\frac{x}{10}$$

$$M_b = 6R_a - 1(6 - x) = 4R_c = \frac{4x}{10}$$

For the load between " b " and " d "

$$V_b = R_a = 1 - R_c = \frac{10-x}{10}$$

$$M_b = 6R_a = 4R_c - (x - 6) = \frac{6(10-x)}{10}$$

From Fig. (2);

For the load between " a " and " c "

$$V_c - = R_a - 1 = -R_c = -\frac{x}{10}$$

$$V_c + = 0$$

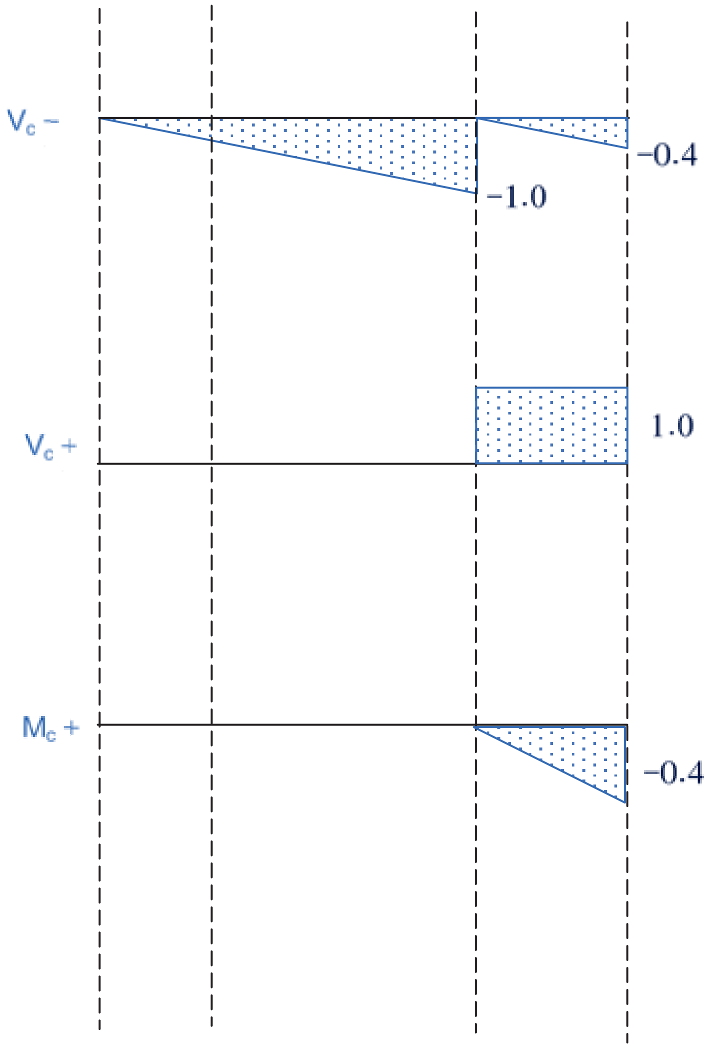
$$M_c = 10R_a - (10 - x) = 10 \times \frac{10-x}{10} - (10 - x) = 0$$

For the load between " c " and " d "

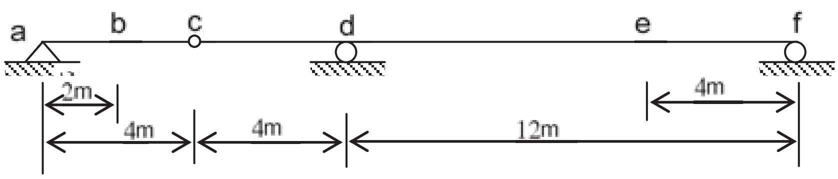
$$V_c - = R_a = \frac{10-x}{10}$$

$$V_c + = 1$$

$$M_c = 10R_a = 10 \times \frac{10-x}{10} = (10 - x)$$



Example (3): Draw the influence lines for R_a, R_d, R_f, V_b, M_b, V_e , M_e for the beam illustrated.



Solution

For the load between " a " and " c ";

$\sum M_c = 0$ (left part)

$4R_a - (4\text{-}x) = 0 \Rightarrow R_a = \frac{4-x}{4}$

`

$\sum M_f = 0$ (whole beam)

$20R_a - (20\text{-}x) + 12\, R_d = 0$

$20\left(\frac{4-x}{4}\right) - (20\text{-}x) + 12\, R_d = 0$

$$R_d = \frac{x}{3}$$

$\sum F_y = 0$ (whole beam)

$R_a + R_f + R_d - 1 = 0$

$\frac{4-x}{4} + R_f + \frac{x}{3} - 1 = 0$

$$R_f = -\frac{x}{12}$$

For the load between " c " and " f ";

$\sum M_c = 0$ (left part)

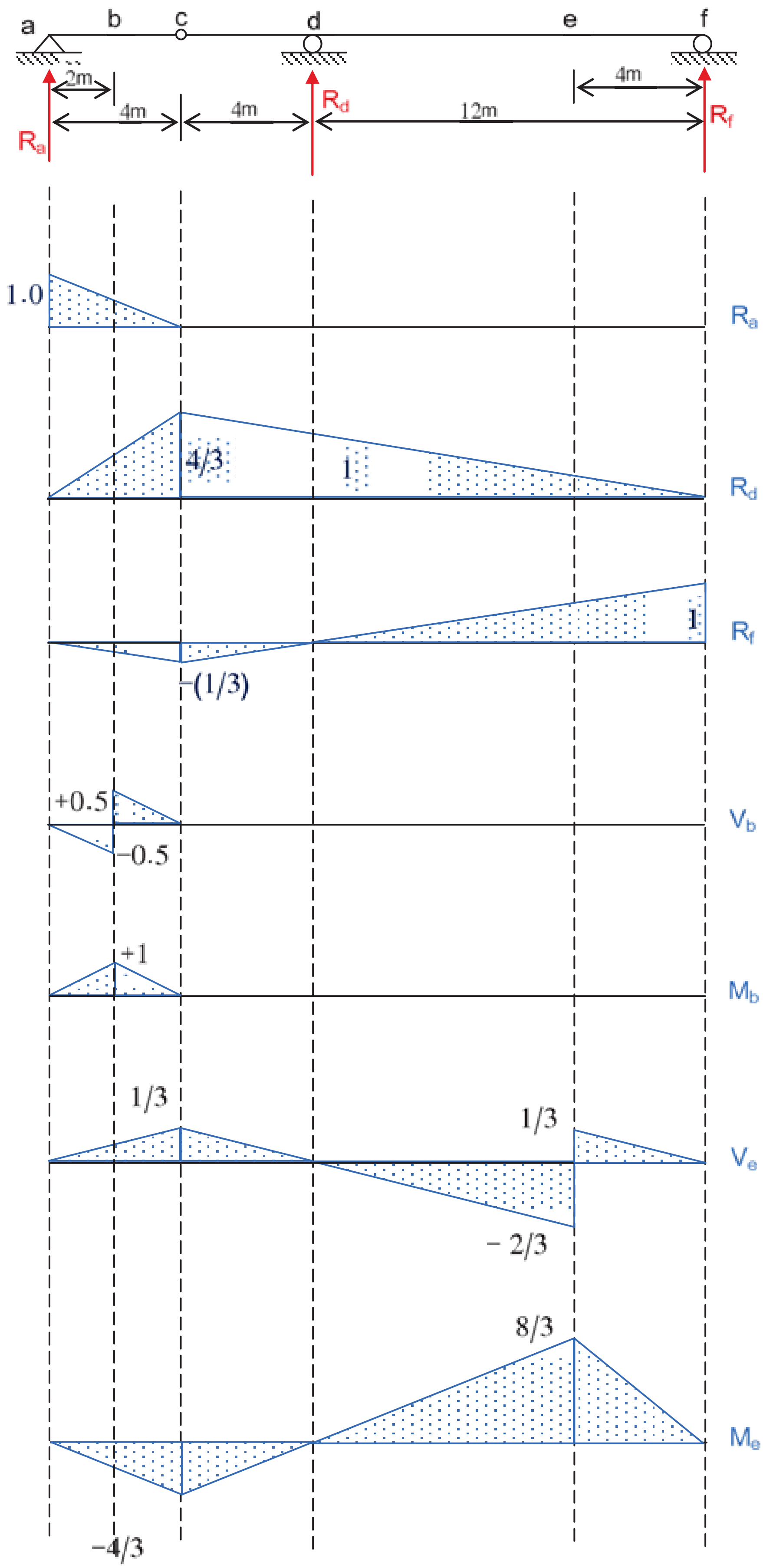
$$R_a = 0$$

$\sum M_f = 0$ (whole beam)

$20R_a - (20\text{-}x) + 12\, R_d = 0$

$0 - (20\text{-}x) + 12\, R_d = 0$

$$R_d = \frac{20-x}{12}$$



$\sum F_y = 0$ (whole beam)

$R_a + R_f + R_d - 1 = 0$

$0 + R_f + \frac{20-x}{12} - 1 = 0$

$$R_f = \frac{x-8}{12}$$

Influence lines for V_b and M_b

For the load between " a " and " b " ;

$V_b = R_a - 1 = \frac{4-x}{4} - 1$

$= -(R_d + R_f) = \frac{-x}{4}$

$M_b = 2R_a - 1(2-x) = (6 R_d + 18R_f)$

$= 2 \times \frac{4-x}{4} - (2-x) = \frac{x}{2}$

For the load between " b " and " c " ;

$V_b = R_a = 1 - (R_d + R_f) = \frac{4-x}{4}$

$M_b = 2R_a = (6 R_d + 18R_f) - (x-2) = \frac{4-x}{2}$

For the load between " c " and " f " ;

$V_b = R_a = 1 - (R_d + R_f) = 0$

$M_b = 2R_a = (6 R_d + 18R_f) - (x-2) = 0$

Influence lines for V_e and M_e

For the load between " a " and " c" ;

$V_e = R_a + R_d - 1 = -R_f = \frac{x}{12}$

$M_e = 16R_a + 8R_d - 1(16 - x) = 4\ R_f = -\ \frac{x}{3}$

For the load between " c " and " e " ;

$V_e = R_a + R_d - 1 = -\ R_f = -\ \frac{x-8}{12}$

$M_e = 16R_a + 8R_d - 1(16 - x) = 4\ R_f = \frac{x-8}{3}$

For the load between " e " and " f " ;

$V_e = R_a + R_d = 1 - R_f = \frac{20-x}{12}$

$M_e = 16R_a + 8R_d = 4\ R_f - 1\ (x - 16) = 8\ (\frac{20-x}{12})$

Relationship of Influence Lines and Structural Loading

Influence lines are used to investigate the effect of the actual load moving across the structure.

i- Concentrated Force:

If a single concentrated force of magnitude " P " moves across a beam, the effect of the load is obtained by simply placing it at a given location " x " , and multiplying the influence line ordinate IL (x_1)at that point by the magnitude of the load " P "

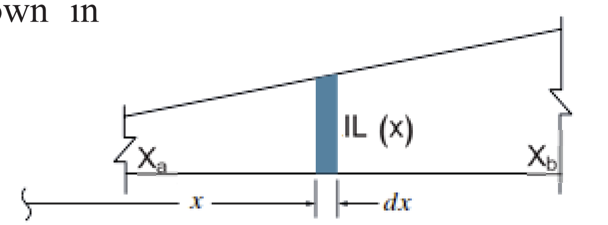
$$F = IL\,(x_1)\,P$$

Where " F " is the value of the function of interest-reaction, shear, bending moment , etc.

ii- Distributed load

If a distributed load q(x) is applied over a portion of a structure, its effect can also be calculated using the influence ordinates.

For a portion of the influence line shown in figure;



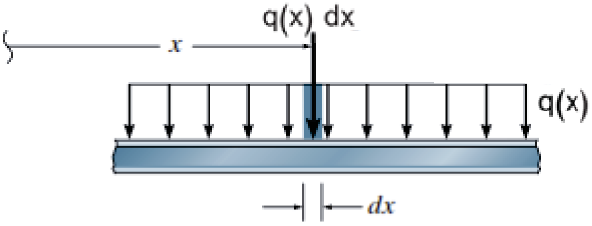
influence line for function

$$dF = IL\,(x)\,q(x)\,dx$$

Integrating

$$F = \int_{x_a}^{x_b} dF$$

$$= \int_{x_a}^{x_b} IL(x).\,q(x).\,dx$$



If the loading is uniformly distributed (q = const.), the value of the function is

$$F = q \int_{x_a}^{x_b} IL(x).\,dx$$

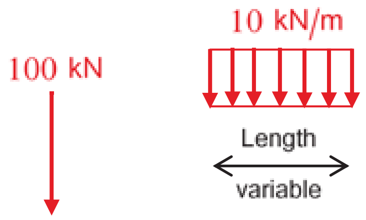
The integral in the above equation represents the area under the influence line between points x_a and x_b .

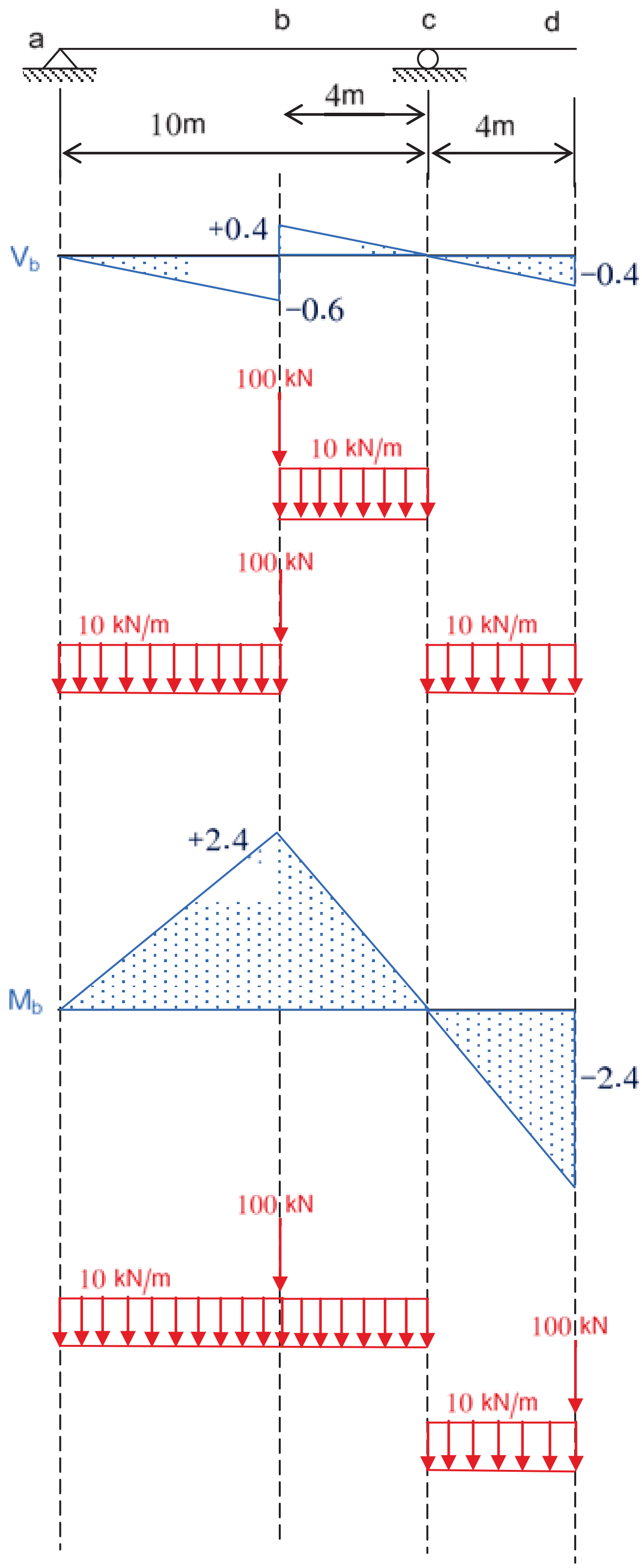
The following statements are made about the relationships between influence lines and structural loading:

1- The effect of concentrated load can be obtained by multiplying the value of the load by the influence ordinate where the load is positioned.
2- The greatest magnitude of a function, e.g. reaction, due to a concentrated load exists when the load is positioned on the structure where influence line has the largest ordinate.
3- The effect of uniformly distributed load is obtained by multiplying the area under the influence line (between the points where the load is distributed) by the values of the distributed loading.

4- The greatest magnitude of a function, e.g. reaction, due to uniformly distributed load of constant value and variable length is obtained by placing the loading over those portions of the influence line which have ordinates of the same sign.

Example (1): The beam in example (2) of the previous section has the illustrated loading applied to the structure. The uniformly distributed part of the load is a variable length. Calculate the largest positive and negative values of V_b and M_b due to this loading.





Solution

$$(V_b)^+_{Max} = 100(0.4) + \frac{1}{2}(0.4)\,(4)(10)$$

$$= 48\text{kN}$$

$$(V_b)^-_{Max} = 100(-0.6) + \frac{1}{2}(-0.6)\,(6)(10) + \frac{1}{2}(-0.4)\,(4)(10) = -86\text{ kN}$$

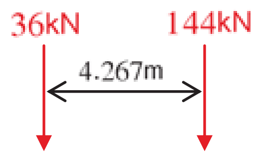
$$(M_b)^+_{Max} = 100(2.4) + \frac{1}{2}(2.4)\,(10)(10)$$

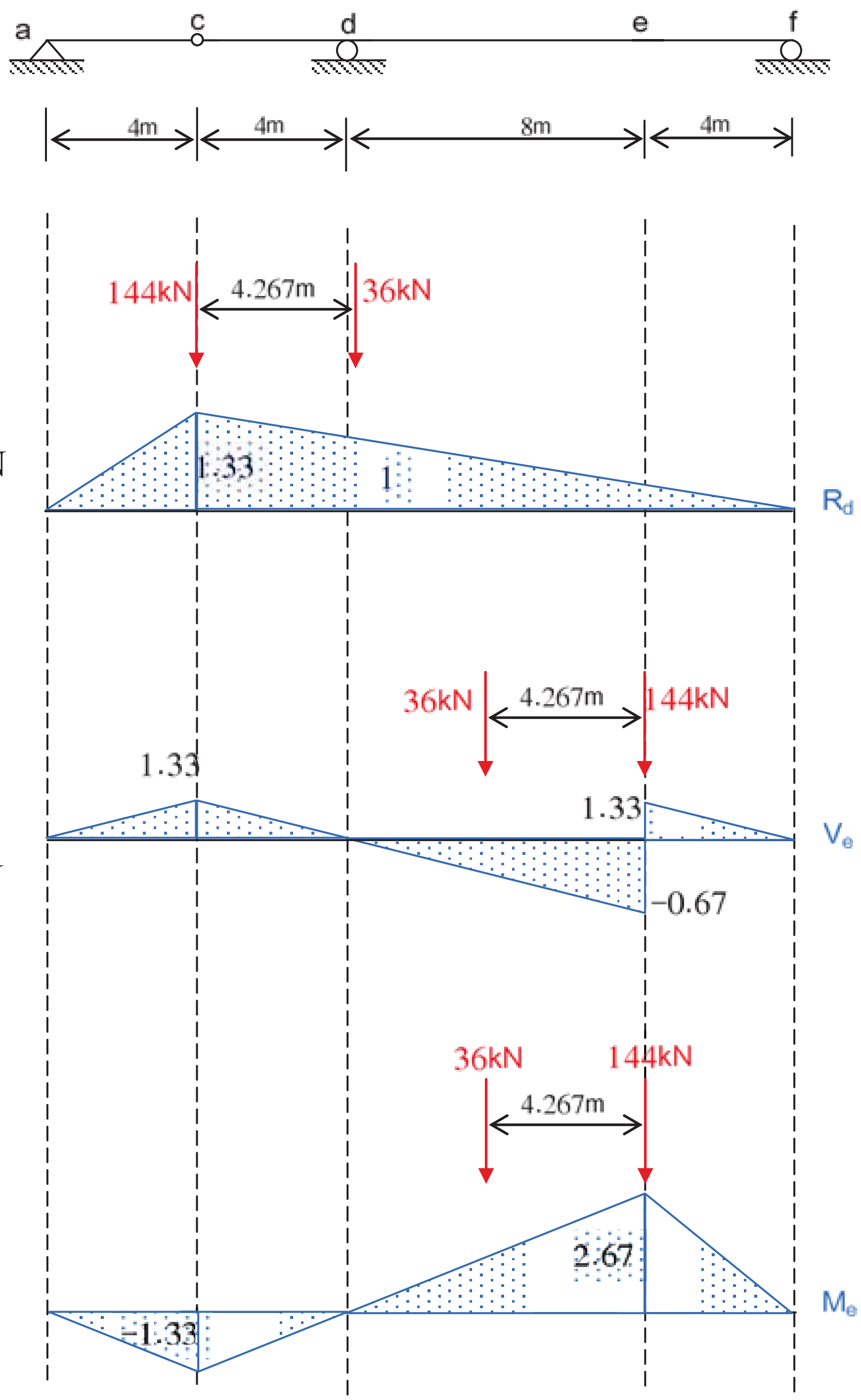
$$= 360\text{kN.m}$$

$$(M_b)^-_{Max} = 100(-2.4) + \frac{1}{2}(-2.4)\,(4)(10)$$

$$= -288\text{kN.m}$$

Example (2): The beam in example (3) of the previous section is loaded with a standard H20 (M18) high way wheel loading as shown. Using the influence lines developed previously, calculate the largest values of R_a, V_e, (negative), and M_e (positive).





solution

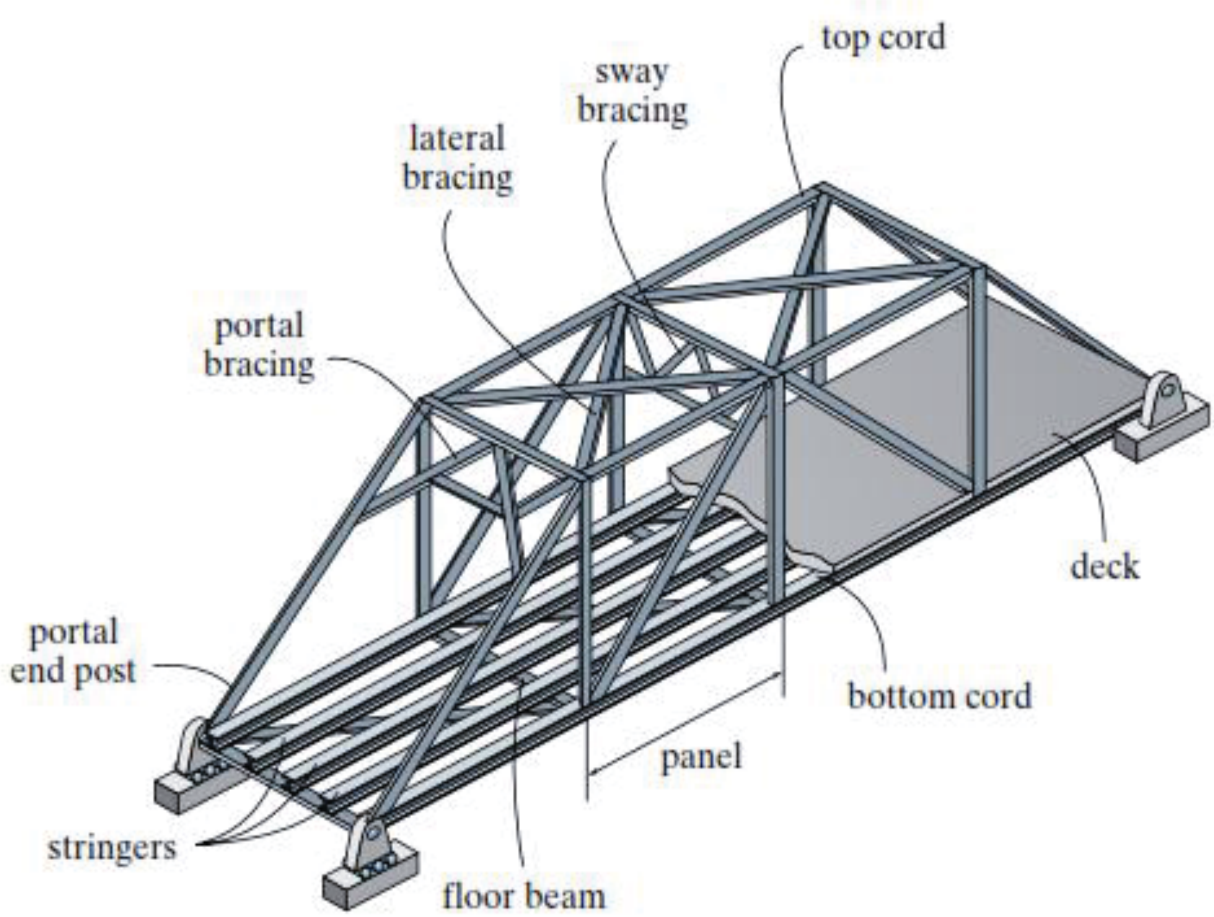
$$(R_d)_{Max} = 144(1.33) + 36\left[\frac{1.33}{16}(16-4.267)\right]$$
$$= 191.52 + 35.11 = 226.63\text{kN}$$

$$(V_e)^-_{\ Max} = 144(-0.67) + 36\left[\frac{-0.67}{8}(8-4.267)\right]$$
$$= -96.48 - 11.25 = -107.73\text{kN}$$

$$(M_e)^+_{\ Max} = 144(2.67) + 36\left[\frac{2.67}{8}(8-4.267)\right]$$
$$= 384.48 + 44.85$$
$$= 429.33\text{kN.m}$$

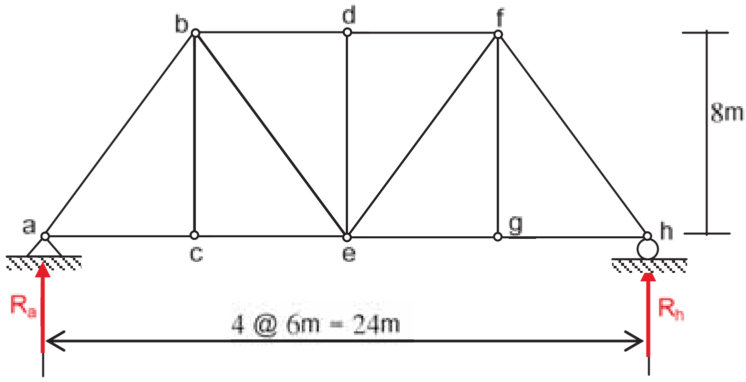
Influence Lines for Trusses

Trusses are frequently loaded with moving loads as in the case of bridges. In order to design individual truss members, it is necessary to know the largest tensile or compressive force they must sustain as the loading moves across the structure.



For the typical bridge truss shown in Figure above, the loading on the bridge deck is transmitted to stringers, which in turn transmit the loading to floor beams and then to the points along the bottom cord of the truss. Thus the trusses in this case will be loaded only at points where the floor beams attached to the bottom cord of the truss. These points are termed " joints " or " panel points " .

Example (1): Draw the influence lines for the members; ab, ac, bc, be, ce, and bd for the truss shown.



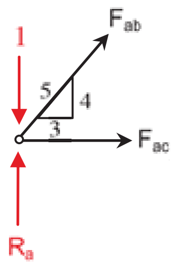
Solution

$\sum M_h = 0$ [for whole truss] $\Rightarrow R_a = \dfrac{24-x}{24}$

$\sum M_a = 0 \Rightarrow$ [for whole truss] $\Rightarrow R_h = \dfrac{x}{24}$

Influence lines for F_{ab} and F_{ac}

From F.B.D. for Joint " a "
when the load at joint " a "
$R_a = 1$;
Hence $F_{ab} = F_{ac} = 0$



when the load between " c " and " h ";

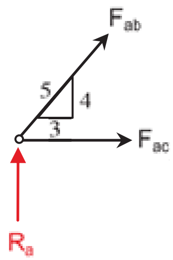
$\sum F_y = 0$

$F_{ab} \times \frac{4}{5} + R_a = 0$

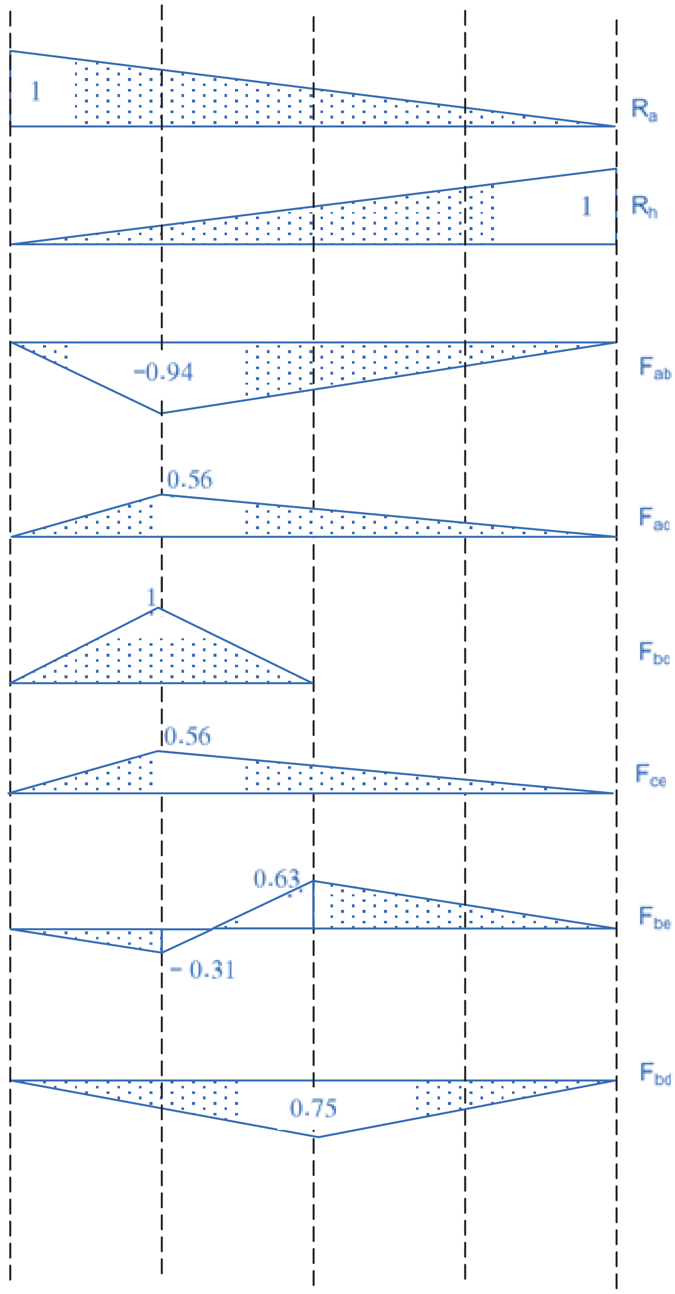
$\therefore F_{ab} = -\frac{5}{4} R_a$

$\sum F_x = 0 \Rightarrow \frac{3}{5} F_{ab} + F_{ac} = 0$

$F_{ac} = -\frac{3}{5} F_{ab}$

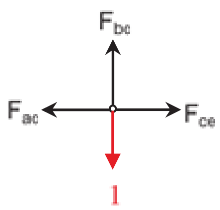


x	R_a	F_{ab}	F_{ac}
0	1	0	0
6	0.75	−0.94	0.56
12	0.50	−0.63	0.38
18	0.25	−0.31	0.19
24	0	0	0



Influence lines for F_{bc} and F_{ce}

From F.B.D. for Joint " c "
when the load at joint " c "
$\sum F_y = 0 \Rightarrow F_{bc} = 1$
when the load at any joint except " c "
$F_{bc} = 0$



when the load at any joint

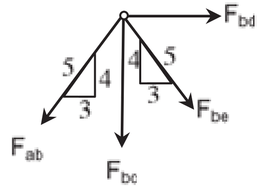
$\sum F_x = 0 \Rightarrow F_{ac} = F_{ce}$

Influence lines for F_{be} and F_{bd}

From F.B.D. for Joint " b "

when the load between joint " a " and " h "

x	F_{ab}	F_{bc}	F_{be}	F_{bd}
0	0	0	0	0
6	−0.94	1	−0.31	−0.37
12	−0.63	0	0.63	−0.75
18	−0.31	0	0.31	−0.37
24	0	0	0	0



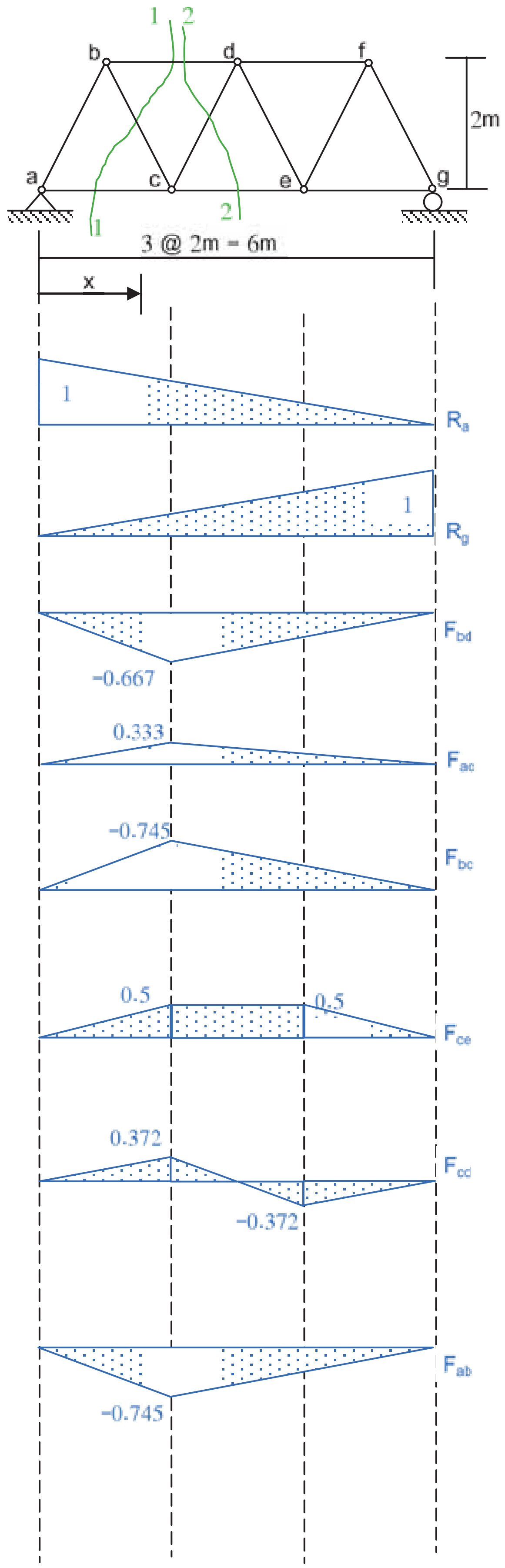
$$\sum F_y = 0 \Rightarrow \frac{4}{5}F_{ab} + F_{bc} + \frac{4}{5}F_{be} = 0$$

$$F_{be} = -\frac{5}{4}\left(\frac{4}{5}F_{ab} + F_{bc}\right)$$

$$\sum F_x = 0 \Rightarrow -\frac{3}{5}F_{ab} + \frac{3}{5}F_{be} + F_{bd} = 0$$

$$F_{bd} = \frac{3}{5}F_{ab} - \frac{3}{5}F_{be}$$

Example (2): The truss has the vehicle load applied to the bottom panel points. Draw the influence lines for reactions Ra, Rg, ab, ac, bc, bd, cd, and ce.

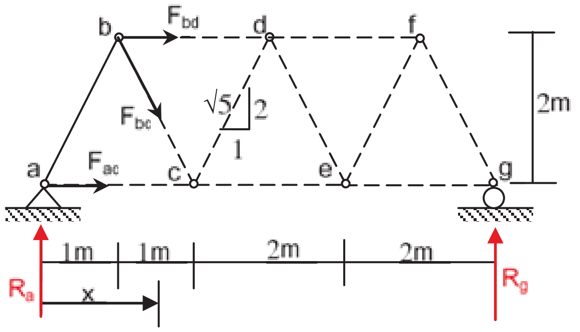


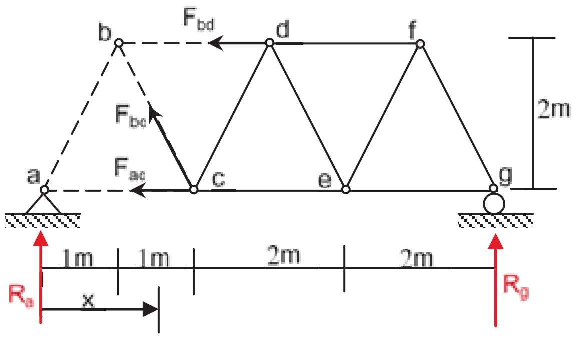
Solution

For the whole truss

$$\sum M_g = 0 \text{ [for whole truss] } \Rightarrow R_a = \frac{6-x}{6}$$

$$\sum M_a = 0 \Rightarrow \text{ [for whole truss] } \Rightarrow R_g = \frac{x}{6}$$





For section 1-1

when the load at joint " a "

$\sum M_c = 0$ [for right part]

$4Rg + 2F_{bd} = 0 \Rightarrow F_{bd} = -2Rg = 0$

$\sum M_b = 0$ [for right part]

$$5Rg - 2F_{ac} = 0 \Rightarrow F_{ac} = \frac{5}{2}Rg = 0$$

$\sum F_y = 0$ [for right part]

$$Rg + \frac{2}{\sqrt{5}}F_{bc} = 0 \Rightarrow F_{bc} = -\frac{\sqrt{5}}{2}Rg = 0$$

when the load between " c " and " g "

$\sum M_c = 0$ [for left part]

$$2Ra + 2F_{bd} = 0 \Rightarrow F_{bd} = -\ Ra\ = -\ \frac{6-x}{6}$$

$\sum M_b = 0$ [for left part]

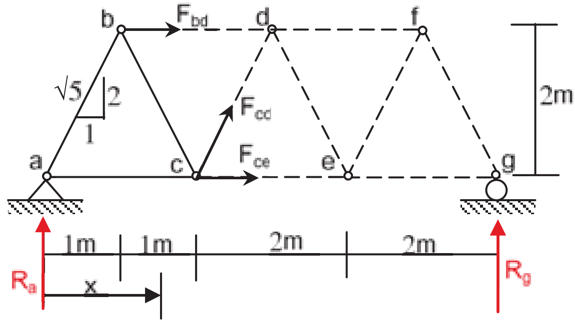
$$Ra - 2F_{ac} = 0 \Rightarrow F_{ac} = \frac{R_a}{2} = \frac{6-x}{12}$$

$\sum F_y = 0$ [for left part]

$$Ra - \frac{2}{\sqrt{5}}F_{bc} = 0 \Rightarrow F_{bc} = \frac{\sqrt{5}}{2}\ Ra = \frac{\sqrt{5}}{12}\ (6 - x)$$

For section 2-2

when the load between " a " and " c "



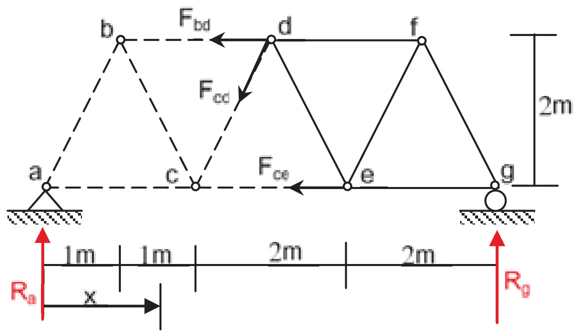
$\sum M_d = 0$ [for right part]

$$3Rg - 2F_{ce} = 0 \Rightarrow F_{ce} = \frac{3}{2}\ Rg = \frac{x}{4}$$

$\sum F_y = 0$ [for right part]

$$Rg - \frac{2}{\sqrt{5}}F_{cd} = 0 \Rightarrow F_{cd} = \frac{\sqrt{5}}{2}\ Rg = \frac{\sqrt{5}\,x}{12}$$

when the load between " e " and " g "



$\sum M_d = 0$ [for left part]

$$3Ra - 2F_{ce} = 0 \Rightarrow F_{ce} = \frac{3}{2}\ Ra = \frac{6-x}{4}$$

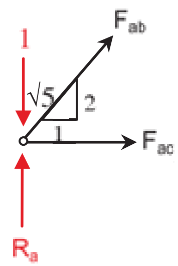
$\sum F_y = 0$ [for left part]

$$Ra + \frac{2}{\sqrt{5}}F_{cd} = 0 \Rightarrow F_{cd} = -\ \frac{\sqrt{5}}{2}\ Ra = -\ \frac{\sqrt{5}\,(6-x)}{12}$$

<u>Influence lines for F_{ab}</u>

From F.B.D. for Joint " a "

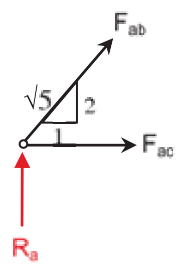
when the load at joint " a "



$R_a = 1$;

Hence $F_{ab} = F_{ac} = 0$

when the load between " c " and " g " ;

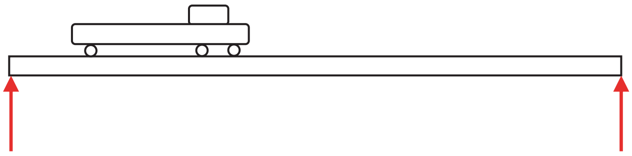


$\sum F_y = 0$

$$F_{ab} \times \frac{2}{\sqrt{5}} + R_a = 0 \quad \Rightarrow F_{ab} = -\ \frac{\sqrt{5}}{12}\,(6\text{-}x)$$

Moving Loads on Beams

Large vehicles, such as trucks or Lorries moving on a beam, impose a series of concentrated loads separated by fixed distances.

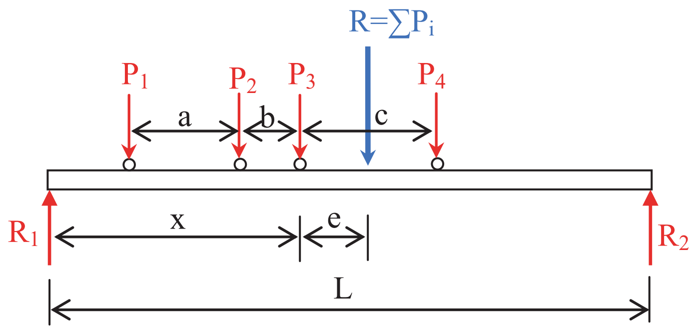


In order to design the beam, it is necessary to know the maximum shear and moment caused by the loads. This is possible only if it is known where the loading should be placed on the beam to cause maximum effect.

Absolute Maximum Moment in a Beam

For the beam subjected to a series of concentrated loads, the bending moment diagram consists of straight lines forming a polygon. Therefore, the section for maximum moment must be under one of the loads.

Consider a series of concentrated loads; P_1, P_2, P_3, and P_4 separated by fixed distances, moving on a beam as shown in the figure.



Suppose it is required to find the position of the section under the load P_3 in which maximum bending moment occurs.

Assuming a position of the loads such that the load under P_3 is at a distance " x " from R_1.

Let $R = \sum P_i$ be the resultant of the loads and " e " its distance from P_3, such that;

$$e = \frac{\sum P_i e_i}{R}$$

The bending moment at the section under P_3 is;

$M_3 = R_1.x - P_1 (a + b) - P_2.b$

From $\sum M = 0$ about R_2

$$R_1 = \frac{R}{L}(L - e - x)$$

Therefore,

$$M_3 = \frac{R}{L}(L - e - x).x - P_1 (a + b) - P_2.b$$

For maximum value of M_3;

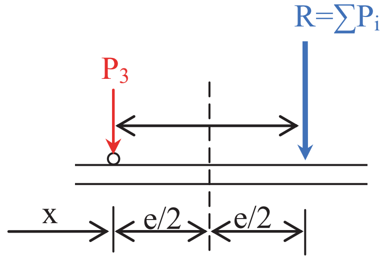
$$\frac{dM_3}{d_x} = \frac{R}{L}(L - e - 2x) = 0$$

$$(L - e - 2x) = 0$$

$$x = \frac{L}{2} - \frac{e}{2}$$

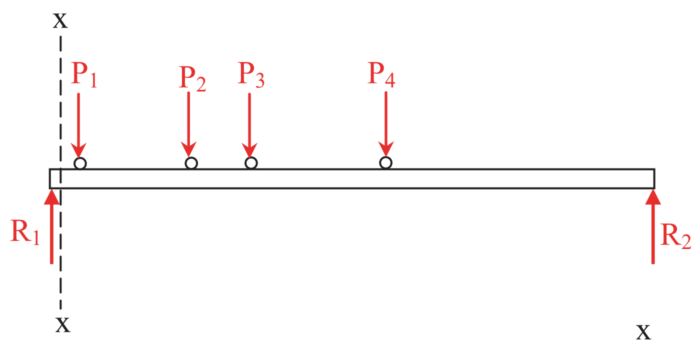
This means that the section for maximum bending under the load P_3 is when the loads are positioned such that the beam centerline is at the midpoint between P_3 and the resultant of the loads.

As a general rule, though, the absolute maximum moment often occurs under the largest force lying nearest the resultant force of the system.

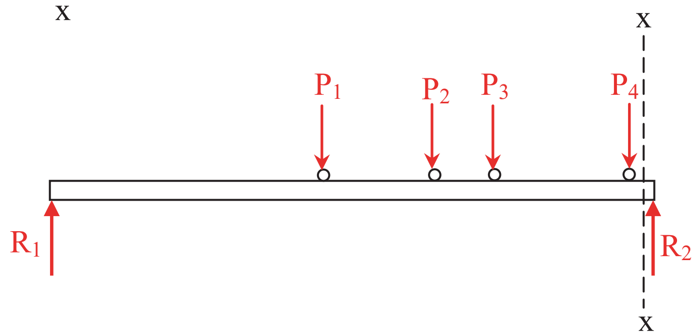


Absolute Maximum Shear

For a simply supported beam, the shear force is maximum at the ends (near the reactions). Therefore, it is necessary to maximize these reactions by positioning the loads as close as possible.

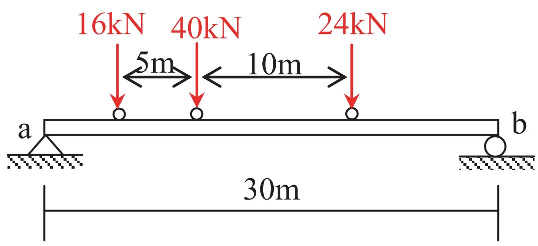


Loading position for max. R_1

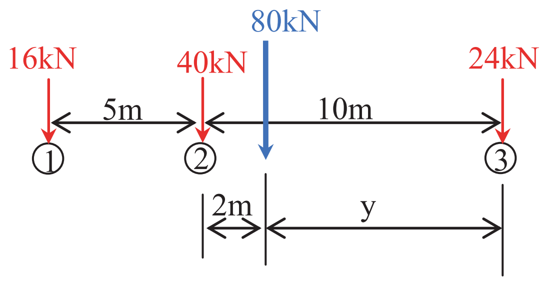


Loading position for max. R_2

Example: Three wheel loads move on a beam of span 30m as shown in figure. Find the absolute maximum moment and shear for the beam.



Solution



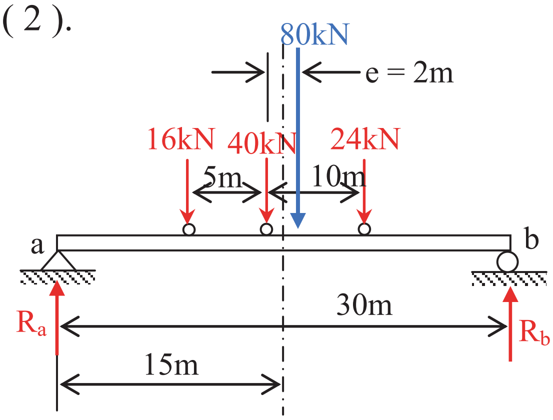
The resultant of the applied load is between wheel (2) and (3)

R = 16+40+24 = 80kN

To find the distance " y " from wheel (3) to the resultant, hence;

$$y = \frac{16\times15+40\times10}{80} = 8\text{m}$$

The maximum moment will occur under wheel (2).



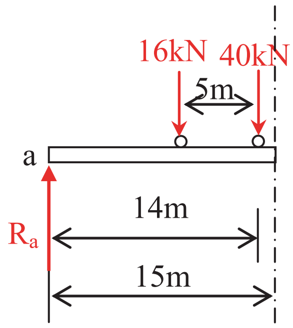
According to the criterion for absolute maximum moment, the wheel (2) and the resultant should be placed equidistant from the centerline of the beam.

$$R_a = \frac{80\times14}{30} = 37.33\text{kN}$$

R_b = 80-37.33 = 42.67kN

$M_{max.} = R_a \times 14 - 16 \times 5$

$= 37.33 \times 14 - 80 = 442.62$ kN.m



The maximum shear will occur near a reaction and is obtained by positioning the wheels as shown.

Thus with resultant as close as possible to one support and all wheels on the structure;

$$V_{max.} = R_a = \frac{80\times23}{30} = 61.33\text{kN}$$