## - Using Adders For Subtractor

With A few a little thinks we can use as adder to also do subtraction .there is a mathematical technique that helps us use an adder to do binary subtraction.

Exp: Decimal subtraction
10
1010
Binary Subtraction

1010
1010


In this special technique the steps are first to first wire the 1's complement of the number being subtracted (change all 1's to o's and all o'sto 1's ) and then add .

Now let us us adders to do binary subtraction in this example, the temporary answer to this addition is shown as (10011).next, the last carry on the left I carried around to the 1's place. This is called an end-around carry. When the end around carry is added to the rest of the number, the result is the difference between the original binary numbers , (1010) and (0110). The an swer to this problem is 0100.

## 1- Comparator

The comparison of two numbers is an operations that determines if one number is greater then, less than, or equal to other number.

A Magnitude comparator : is a combinational circuit that compares two numbers $A$ and $B$, and determines there relative . The outcome of the comparison is specified by three binary variables that indicate whether $A>B, A=B$ or $A<B$.

| A | B | $\mathrm{A}>\mathrm{B}$ | $\mathrm{A}<\mathrm{B}$ | $\mathrm{A}=\mathrm{B}$ | $\mathrm{A}>=\mathrm{B}$ | $\mathrm{A}<=\mathrm{B}$ | $\mathrm{A}<>\mathrm{B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O | O | O | O | 1 | 1 | 1 | 0 |
| O | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |

## -by us logic circuit

$$
1-\boldsymbol{A}>\boldsymbol{B} \Leftrightarrow \boldsymbol{F} 1=\boldsymbol{A} \overline{\boldsymbol{B}}
$$



$$
2-\boldsymbol{A}<\mathbf{B} \Rightarrow \boldsymbol{F} 2=\overline{\boldsymbol{A}} \boldsymbol{B}
$$


$\Longleftrightarrow$


1

$5-\boldsymbol{A}<=\boldsymbol{B} \Rightarrow \boldsymbol{F} 5=\overline{\boldsymbol{A}}+\boldsymbol{B}$


This is a comparison of two binary variables each has one bit only.

## \& ote : Now that about comparing two numbers with n-

 bits?Let $A=a_{n-1} a_{n-2} \ldots \ldots . . . . a_{1} a_{0}$
Let $B=b_{n-1} b_{n-2} \ldots \ldots . . . . b_{1} b_{0}$
where $\boldsymbol{a}_{0}, \boldsymbol{b}_{0}$ is LSB while $\boldsymbol{a}_{n-1}, \boldsymbol{b}_{n-2}$ is MSB of two number

## - Equality Relation

We say that if the number A equal to B we are implement an XNOR gate to do this state, but with one bit number, then now about numbers with n - bits?

Note: the two numbers are equal if all pairs of significant digit are equal:

$$
\xrightarrow{a_{n-1}=b_{n-1}, \ldots \ldots \ldots \ldots \ldots, a_{0}=b_{0}}
$$

That means for the equality condition being true (equal 1 ) if all equality relation of each pair must equal to 1, this dictates AND gate to combine the
outputs to gather to get the final output 1 .
Therefore Equality Relation:-
$\boldsymbol{A}=\boldsymbol{B}$ if $\left(\overline{a_{n-1} \oplus b_{n-1}}\right) \cdot\left(\overline{a_{n-2} \oplus b_{n-2}}\right) \ldots \ldots \ldots . . . . . . .,\left(\overline{a \oplus b_{0}}\right)=1$


## - Greater than relation

If the corresponding digit of is (1) and that of $B$ is ( 0 ) , we conclude that $\mathrm{A}>\mathrm{B}$.

Then How can we implement a comparator circuit to do this comparison:

$$
1-\text { if } N=2
$$

$$
\begin{aligned}
& \text { Let } A=\boldsymbol{a}_{1} a_{0} \\
& \text { Let } B=b_{1} b_{0} \\
& \text { to prof that } A>B \text { if } \boldsymbol{a}_{1} \boldsymbol{a}_{0}>\boldsymbol{b}_{1} \boldsymbol{b}_{0} \\
& \quad=\left(\boldsymbol{a}_{1}>\boldsymbol{b}_{1}\right)+\left(\boldsymbol{a}_{1}=\boldsymbol{b}_{1}\right) \cdot\left(\boldsymbol{a}_{0}>\boldsymbol{b}_{0}\right)
\end{aligned}
$$

and from truthtable:
$\left(a_{1}>b_{1}\right)=a_{1} \bar{b}_{1}$
$\left(\boldsymbol{a}_{1}=\boldsymbol{b}_{1}\right)=\left(\overline{\boldsymbol{a}_{1} \oplus \boldsymbol{b}_{1}}\right)$
$\left(\boldsymbol{a}_{\mathrm{o}}>\boldsymbol{b}_{\mathrm{o}}\right)=\boldsymbol{a}_{\mathrm{o}} \overline{\boldsymbol{b}_{\mathrm{o}}}$
then $\boldsymbol{F}(A>B)=a_{1} \overline{b_{1}}+\left(\overline{a_{1} \oplus b_{1}}\right) \cdot a_{0} \overline{b_{0}}$


2- if $\mathbf{N}=3$

Let $A=a_{2} a_{1} a_{0}$
Let $\quad B=b_{2} b_{1} b_{0}$
then

$$
\boldsymbol{F}(\boldsymbol{A}>\boldsymbol{B})=\left(\boldsymbol{a}_{2}>\boldsymbol{b}_{2}\right)+\left(\boldsymbol{a}_{2}=\boldsymbol{b}_{2}\right) \cdot\left(\boldsymbol{a}_{1}>\boldsymbol{b}_{1}\right)+\left(\boldsymbol{a}_{2}=\boldsymbol{b}_{2}\right) \cdot\left(\boldsymbol{a}_{1}=\boldsymbol{b}_{1}\right) \cdot\left(\boldsymbol{a}_{0}>\boldsymbol{b}_{0}\right)
$$

$$
\begin{aligned}
& \because\left(\boldsymbol{a}_{2}>\boldsymbol{b}_{2}\right)=\boldsymbol{a}_{2} \overline{\boldsymbol{b}_{2}} \\
&\left(\boldsymbol{a}_{2}=\boldsymbol{b}_{2}\right)=\left(\overline{\boldsymbol{a}_{2} \oplus \boldsymbol{b}_{2}}\right) \\
&\left(\boldsymbol{a}_{1}>\boldsymbol{b}_{1}\right)=\boldsymbol{a}_{1} \overline{\boldsymbol{b}_{1}} \\
&\left(\boldsymbol{a}_{1}=\boldsymbol{b}_{1}\right)=\left(\overline{\boldsymbol{a}_{1} \oplus \boldsymbol{b}_{1}}\right) \\
&\left(\boldsymbol{a}_{0}>\boldsymbol{b}_{0}\right)=\boldsymbol{a}_{0} \overline{\boldsymbol{b}_{0}}
\end{aligned}
$$

$$
\therefore A>B=\left(a_{2} \overline{b_{2}}\right)+\left(\overline{a_{2} \oplus b_{2}}\right) \cdot\left(a_{1} \overline{b_{1}}\right)+\left(\overline{a_{2} \oplus b_{2}}\right) \cdot\left(\overline{a_{1} \oplus b_{1}}\right) \cdot\left(a_{0} \overline{b_{0}}\right)
$$

Note : in the same way of comparison we can find

- If $\mathrm{N}=2$

Then $A<B=\overline{a_{1}} \boldsymbol{b}_{1}+\left(\overline{a_{1} \oplus b_{1}}\right) \cdot \overline{a_{0}} \boldsymbol{b}_{0}$

- If $\mathbf{N}=\mathbf{3}$

Then $A<B=\left(\overline{a_{2}} b_{2}\right)+\left(\overline{a_{2} \oplus b_{2}}\right) \cdot\left(\overline{a_{1}} b_{1}\right)+\left(\overline{a_{2} \oplus b_{2}}\right) \cdot\left(\overline{a_{1} \oplus b_{1}}\right) \cdot\left(\overline{a_{0}} b_{0}\right)$

## 2- Decoders

A decoder is a digital circuit that detects the presence of a specified combination of bits (code)on its inputs and indicates the presence of that code by a specified output level .

In its generalform : a decoder has $n$ input lines to handle $n$ bits and from one to $\mathbf{2}^{\mathbf{n}}$ output lines to indicate the presence of one or more nbit combinations.

## - The Basic Binary Decoder

Suppose you need to determine when a binary 1001 occurs on the inputs of a digital circuit. An AND gate can be used as the basic decoding element because it produces a HIGH or (1) output only when all of its inputs are HIGH (1).

Therefore, you must make sure that all of the inputs to the AND gate are HIGH when the binary number 1001 occurs; this can be done by inverting the two middle nits (the os), as shown in the next figure .

you should verify that the output is ( 0 ) except when $\mathrm{Ao}=1, \mathrm{~A} 1=0, \mathrm{~A} 2=0$, AND $\mathrm{A} 3=1$ are applied to the inputs. Ao is the LSB and A3 is the MSB.

Note: If a NAND gate is used in place of the AND gate in pervious figure, a LOW output will indicate the presence of the proper binary code, which is 1001 in this case.

Note :Then the decoders presented here are called $\mathbf{n}$ to $\mathbf{m}$ line decoders, where $m<=\mathbf{2}^{\mathbf{n}}$. the purpose is to generate the $\mathbf{2}^{\mathbf{n}}$ ( or fewer) minterms of $n$ input variable. The name decoder is also used in conjunction with other code converters such as BCD-to-Seven segment decoder.

## 2 to 4 line decoder

We can design 2 to 4 line decoder by using the following truth table:

| Inputs |  | Outputs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | Fo | F1 | F2 | F3 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 |



3 to 8 line decoder

We can design 3 to 8 line decoder by using the following truth table:

| A | B | C | Fo | F1 | F2 | F3 | F4 | F5 | F6 | F7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 |  | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

The three inputs are decoded into eight outputs each representing one of the mineterms of the three input variables.

The three inputs are decoded into eight outputs each representing one of the mineterms of the three input variables.

The three inverters provides the complement of the inputs, and each of the eight AND gates generates one of the mineterms. A particular application of this decoder is Binary -To -Octal conversion.


The input variables represent a binary number, and the outputs represent the eight digits in the octal number system.

However, a (3-to-8) line decoder can be used for decoding any 3bit code to provide eight output, one for each element of the code.

The operation of the decoder may be clarified by the truth table. For each possible input combination there are seven outputs that are equal to (o) and only one that is equal to (1) .

The output whose value is equal to (1) represents the minterms equivalent of binary number a variable in the input lines.

Notes :Decoders as we say came in several varieties in typical decoder , the inputs may be: 8421 BCD , axcess- 3 , or gray code .


## A typical decoder block diagram

## BCD-to-Seven segment decoder

The decoders are translating the ( 8421 BCD ) code to ( a seven segement display)


Suppose you wanted the decimal (4) to light on the display board, the BCD umber (0100) at the input of the BCE-to-Seven-segment decoder.

The Decoder activates outputs( $\mathrm{a}, \mathrm{c}, \mathrm{d}, \mathrm{f}$ and g ) to light segments.


## Segment identification

|  |  |  | Digit Shown | Illuminated Segment ( $1=$ illumination) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | a | b | c | d | e | $f$ | $g$ |
|  |  |  | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
|  |  |  | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
|  |  |  | 2 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
|  |  |  | 3 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
|  |  |  | 4 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
|  |  |  | 5 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
|  |  |  | 6 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
|  |  |  | 7 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
|  |  |  | 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  |  |  | 9 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

Decimal numbers on typical seven-segment display

| Decimal | Inputs |  |  |  | Outputs |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{D}$ | $\mathbf{C}$ | $\mathbf{B}$ | $\mathbf{A}$ | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{c}$ | $\mathbf{d}$ | $\mathbf{E}$ | $\mathbf{f}$ | $\mathbf{g}$ |
| $\mathbf{0}$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| $\mathbf{1}$ | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\mathbf{2}$ | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| $\mathbf{3}$ | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| $\mathbf{4}$ | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| $\mathbf{5}$ | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| $\mathbf{6}$ | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| $\mathbf{9}$ | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\mathbf{9}$ | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Truth Table of Typical BCE-Seven-Segment Decoder
© The 74139 decoder


## 2-line-to 4 Decoder IC. No(74139)

This type of decoders (which we used in laboratory) with an enable input to control the circuit operation, as we see in previous figure. the decoder is enabled when (enable )is equal to (o).

The circuit operates with complement outputs and complement input , when ( E ) is equal to ( O ) only one output can be equal to ( O ) at any given time all other outputs are equal to (1) .

The output whose value is equal to (o) represents the minterm selected by inputs (A) and (B).

The circuit is disabled when ( E ) is equal to (1). When the circuit is disabled, none of the outputs are equal to ( O ) and none of the minterms.

## Example : Design Half-adder using (74139) (2-to-4) Decoder .

As indicated by the truth table, only two output can be equal to (1) at (S) output, while (Co) has only one output equal to (o).

| Input |  | Outputs |  |
| :---: | :---: | :---: | :---: |
| $A$ | B | $\sum(s)$ | Co |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| Binary digits <br> to be added | Sum | Carry <br> out |  |

The (74139) decoder provide a complement outputs as is shown in IC figure, therefore we must connect not gate for each active output.


