Examples: using Boolean Algebra techniques , simplify the following expression:

$1 - X(\overline{X} + Y)$

$= X\overline{X} + XY$

$= XY$

$2 - AB(C + \overline{A}\,\overline{C})$

$= ABC + AB\overline{A}\,\overline{C}$

$= ABC + (A\overline{A})B\overline{C}$

$= ABC + 0.B\overline{C}$

$= ABC + 0$

$= ABC$
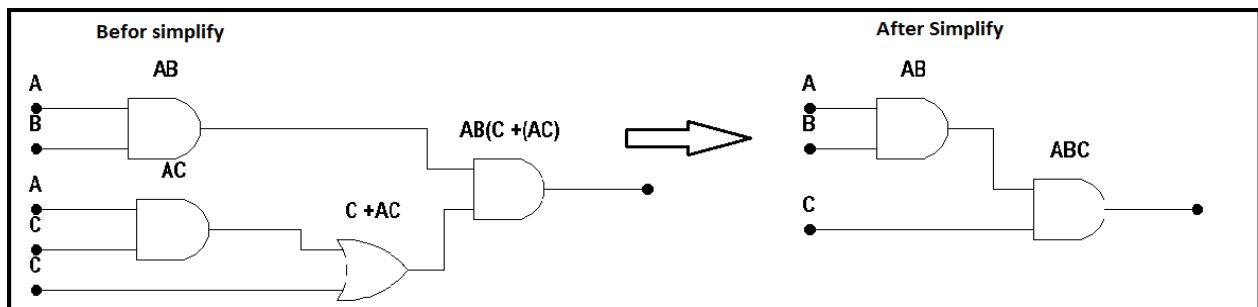
$3 - ABC + A\overline{B}C + AB\overline{C}$

$= AC(B + \overline{B}) + AB\overline{C}$

$= AC.1 + AB\overline{C}$

$= AC + AB\overline{C}$

$= A(C + B\overline{C})$

$= A(C + B)$

Befor simplify → **After Simplify**
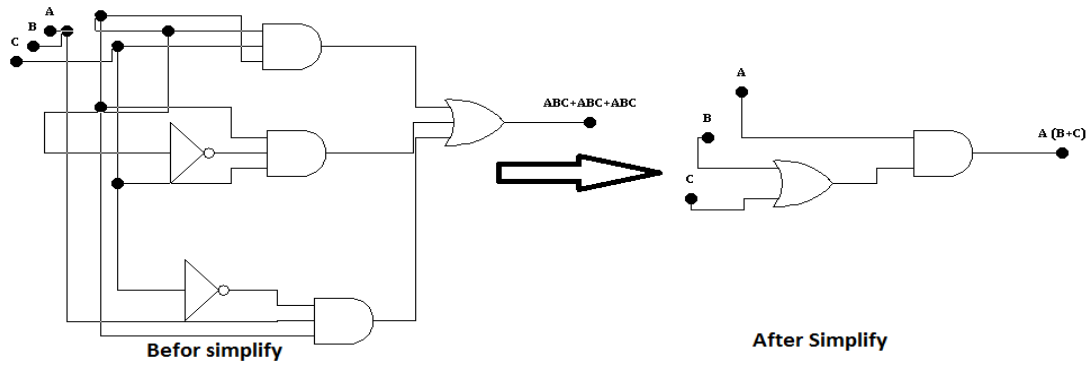
1- AB + A(B+C)+ B(B+C)

= AB +AB +AC +BB +BC   by applying distributive law

= AB +AB +AC +B +BC   by applying rule 7

= AB +AC +B +BC   by applying rule 5

=AB+AC+B   by applying rule 10

=B+AC   by applying rule 10

As this point the expression is simplified as much as possible.

✎ *Home work* : *Draw the logic circuit before and after simplification .*

$$4 - (C \oplus B) \oplus A$$

$$= \left(\overline{C} B + C \overline{B} \right) \oplus A$$

$$= \left(\overline{\overline{C} B + \overline{B} C} \right). A + \overline{A} \left(\overline{C} B + \overline{B} C \right)$$

$$= (\overline{\overline{C} B}. \overline{\overline{B} C}). A + \overline{A} \overline{C} B + \overline{A} \overline{B} C$$

$$= ((C + \overline{B})(B + \overline{C}). A + \overline{A} B \overline{C} + \overline{A} \overline{B} C$$

$$= (CB + C\overline{C} + \overline{B} B + \overline{B} \overline{C}). A + \overline{A} B \overline{C} + \overline{A} \overline{B} C$$

$$= ABC + A\overline{B}\overline{C} + \overline{A} B\overline{C} + \overline{A} \overline{B} C$$

✍ Home work

1 - Simplify the following expression and Draw the logic circuit before and after simplification

a) $A\overline{B} + A\left(\overline{A + C} \right) + B\left(\overline{B + C} \right)$

b) $\overline{A} BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B} C + ABC$

1-   Applying DeMorgan's theorem Draw the logic circuit before and

after simplification

$a)\quad \overline{(A+B+C)\,D}$

$b)\quad \overline{ABC+DEF}$

$c)\quad \overline{A\overline{B}+\overline{C}D+EF}$

1- Simplifying Logic Circuits

Two methods for simplifying

– Algebraic method (use Boolean algebra theorems)

– Karnaugh mapping method (systematic, step-by-step approach)

♦-Standard Form Of Boolean Expressions

All Boolean expressions , regardless of their form , can be converted into two standard forms:

- **The sum- of – products form.**

- **The product –of- sums form.**

Standardization makes the evaluation , simplification , and implementation of Boolean expressions much more systematic and sassier.

♦ The Sum – of – Products (SOP) form

When two or more product terms are summed by Boolean addition , the resulting expression is **a sum-of-products (sop).** Some examples are.

$$AB + ABC$$

$$ABC + CDE + \overline{B}\,C\overline{D}$$

$$\overline{AB} + \overline{AB}\,\overline{C} + AC$$

In an Pos expression , a single overbar cannot extend over more than one variable in a term can have an overbar . for example , an SOP expression can have the term $\overline{A}\,\overline{B}\,\overline{C}$ *but not* $\overline{ABC}$.

- ## Domain of a Boolean Expression

The **Domain** of a general Boolean expression is the set of variables contained in the expression in either complement or un complement form. For example , the domain of the expression $\overline{A}B + A\overline{B}C$ is the set of variables A,B ,C and the domain of the expression $AB\overline{C} + C\overline{D}E + \overline{B}C\overline{D}$ is the set of variables A , B , C , D , E.

- ## And / Or implementation of an SOP expression

Implementation an SOP expression is simply require

1- ORing the output of two or more AND gates. A product term is produced by an AND operation.

2- And the sum (addition) of two or more product terms is produced by an OR operation.

Note : Therefore, an **SOP** expression can be implemented by **AND-OR** logic in which the outputs of a number (equal to the number of product terms in the expression) of **AND** gates connect to the inputs of an **OR** gate , as show un the next figure , for the

expression **AB+BCD +AC** . the output **X** of the **OR** gate equals the **SOP** expression .
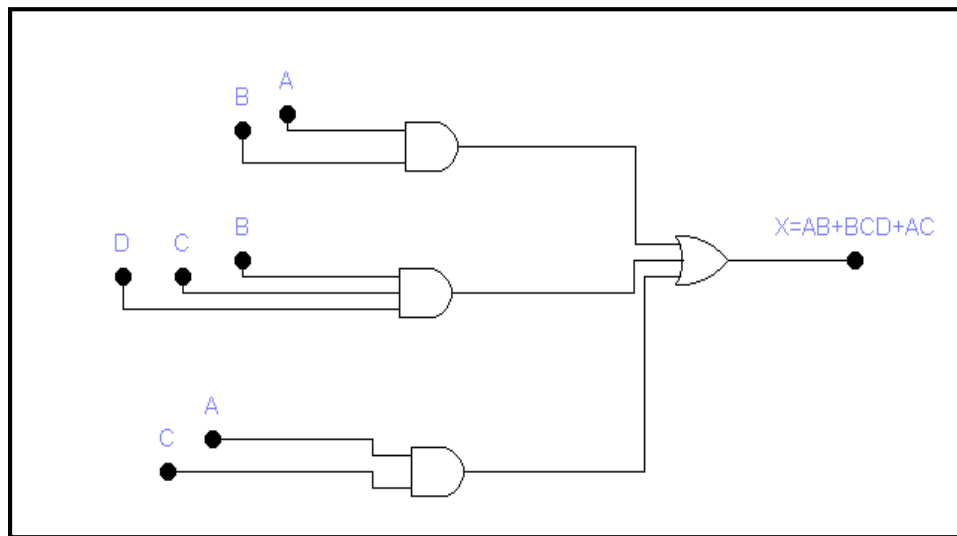
♦ Conversion of a General Expression to SOP Form

Any logic expression can be changed into **SOP** form by applying Boolean algrbra techniques. For example , the expression **A(B+CD)** can be converted to SOP form by applying the distributive law:

$$A (B +CD)= A +ACD$$

Example: convert each of the following Boolean expression to the SOP from:

(a)   AB + B(CD + EF)

Solution:

AB +BCD +BEF

$b)\ \overline{\overline{(A+B)}+C}$

Soluation :

$$=\overline{\overline{(A+B)}}.\overline{C}$$
$$=(A+B).\overline{C}$$
$$=A\overline{C}+B\overline{C}$$

✍ **Home work**

> **Convert** $\overline{A}B\overline{C}+(A+\overline{B})(B+\overline{C}+A\overline{B})$ **to SOP Form**

⬥ **The Standard SOP Form**

So far , you have been SOP expressions in which some of the product terms do not contain all of variables in the domain of the expression. For example, the expression $\overline{A}B\overline{C}+A\overline{B}D+\overline{A}B\overline{C}D$ variables has a domain made up of the A ,B ,C , and D . However ,

Note1: notice that the complete set of variables in the domain is not represented in the first two terms of the expression ; that is , $D\ OR\ \overline{D}$ is missing from the second term.

> A **Standard SOP expression** is one in which all the variables in the domain appear in each product term in the expression.

For example , $\overline{A}B\overline{C}D+\overline{A}\overline{B}CD+AB\overline{C}\overline{D}$ is a standard SOP expression . Standard SOP expressions are important in constructing truth tables, and in the **Karnaugh map** simplification method , which is covered in second section .

Note2: Any nonstandard SOP expression (referred to simply as SOP) can converted to the standard from using Boolean algebra.

- Converting Product Terms to Standard SOP

Each product term in an SOP expression that does not contain all the variables in the domain can be expanded to standard form to include all variables in the domain and their complements. As stated in the following steps, a nonstandard SOP expression is converted into standard from using Boolean algebra **Rule 6** $(A+\overline{A}=1)$ **: a variable added to its complement equals 1 .**

Step1: Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement .This results in two product terms. As you know , you can multiply anything by 1 without changing its value.

Step2: Repeat step1 until all resulting product terms contain all variables in the domain in either complemented or uncomplemented form. In converting a product term to standard form, the number of product terms is doubled for each missing variable as shows in following example:

Example: Convert the following expression into standard SOP form:

$$A\overline{B}C+\overline{A}\,\overline{B}+AB\overline{C}D$$

Solution: the domain of this SOP expression is A ,B,C,D . take one term at a time.

The first term $A\overline{B}C$*, is missing variable D or* $\overline{D}$ *,so multiply the first term by* $D + \overline{D}$ *as follows :*

$$A\overline{B}C =A\overline{B}C(D+\overline{D})=A\overline{B}CD+A\overline{B}C\overline{D}$$

In this case, two standard product terms are the result .

The second term $\overline{A}\,\overline{B}$ *, is missing variables C or* $\overline{C}$ *and D or* $\overline{D}$ *, so first multiply the second term by* $C+\overline{C}$ *as follows:*

$$\overline{A}\,\overline{B}=\overline{A}\,\overline{B}(C+\overline{C})=\overline{A}\,\overline{B}C+\overline{A}\,\overline{B}\,\overline{C}$$

The two resulting terms are missing variable D or $\overline{D}$, so multiply both terms by $D+\overline{D}$ as follows :

$$\overline{A}\overline{B}=\overline{A}\overline{B}C+\overline{A}\overline{B}\overline{C}=\overline{A}\overline{B}C(D+\overline{D})+\overline{A}\overline{B}\overline{C}(D+\overline{D})$$
$$=\overline{A}\overline{B}CD+\overline{A}\overline{B}C\overline{D}+\overline{A}\overline{B}\overline{C}D+\overline{A}\overline{B}\overline{C}\overline{D}$$

In this case , four standard product term are result .

The third term, $\overline{A}B\overline{C}\overline{D}$ is already in standard form. The complete standard SOP form of the original expression is as follows:

$$A\overline{B}C+\overline{A}\overline{B}+AB\overline{C}D=$$

$$A\overline{B}CD+A\overline{B}C\overline{D} +\overline{A}\overline{B}CD+\overline{A}\overline{B}C\overline{D}+\overline{A}\overline{B}\overline{C}D+\overline{A}\overline{B}\overline{C}\overline{D} +\overline{A}B\overline{C}\overline{D}$$

✎ **Home work**

> **Convert the expression** $W\overline{X}Y + \overline{X}Y\overline{Z} + WX\overline{Y}$ **to Standard SOP Form**

- **Binary Representation of a Standard Product Term**

A standard product term is equal to 1 for only on combination of variable values. For example , the product term $A\overline{B}C\overline{D}$ is equal to 1 when A=1 ,B=0 ,C=1 , D=0 , as shown below , and is 0 for all other complements of values for the variables.

$$A\overline{B}C\overline{D}=1.\overline{0}.1.\overline{0}=1.1.1.1=1$$

In this case , the product term has a binary of 1010 (decimal ten)

🔔 **Remember** , a product term is implement with an AND gate whose output is 1 only if each of its inputs is 1. inverters are used to produce the complements of the variables as required.

> **Notes :An SOP expression is equal to 1 only if one or more of the product terms in the expression is equal to 1**

Example : determine the binary value for which the following Standard SOP expression is equal 1:

$$ABCD + A\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D}$$

Solution :

The first term ABCD is equal to 1 when A=1 , B=1 ,C=1 and D=1 , so

$$ABCD = 1 . 1 . 1 . 1 = 1 . 1 . 1 . 1 = 1$$

The second term $A\overline{B}\overline{C}D$ is equal to 1 when A=1 , B=0, C=0, and D=1 , so

$$A\overline{B}\overline{C}D = 1 . \overline{0} . \overline{0} . 1 = 1 . 1 . 1 . 1 = 1$$

The third term $\overline{A}\overline{B}\overline{C}\overline{D}$ is equal to 1 when A=0 , B=0 , c=0 , and D=0 .

$$\overline{A}\overline{B}\overline{C}\overline{D} = \overline{0} . \overline{0} . \overline{0} . \overline{0} = 1 . 1 . 1 . 1 = 1$$

The Sop expression equals 1 when any or all of the three product terms is 1.

✐ Home work

> Determine the binary values for which the following SOP expression is equal to 1: $\overline{X}YZ + X\overline{Y}Z + XY\overline{Z} + \overline{X}Y\overline{Z} + XYZ$ is this a   Standard SOP expression?

♦ The Products –Of – Sum (POS) Form

When two or more sum terms are multiplied , the resulting expression is a Product –Of – Sum **(POS) ,** some examples are

$$(\overline{A}+B)(A+\overline{B}+C)$$

$$(\overline{A}+\overline{B}+\overline{C})(C+\overline{D}+E)(\overline{B}+C+D)$$

$$(A+B)(A+\overline{B}+C)(\overline{A}+C)$$

Notes:

1. A POS   expression can contain a single–variable term ,as $\overline{A}(A+\overline{B}+C)(\overline{B}+\overline{C}+D)$.

2. In a POS expression , a single overbar cannot extend over more than one variable; for example , a POS expression can have the term $\overline{A}+\overline{B}+\overline{C}$ *but not* $\overline{A+B+C}$.

- Implementation of a POS expression

Implementing a POS expression simply require ANDing the outputs of two or more OR gates. The next figure shows the expression (A+B)(B+C+D)(A+C). the output X of the AND gate equals the POS expression.
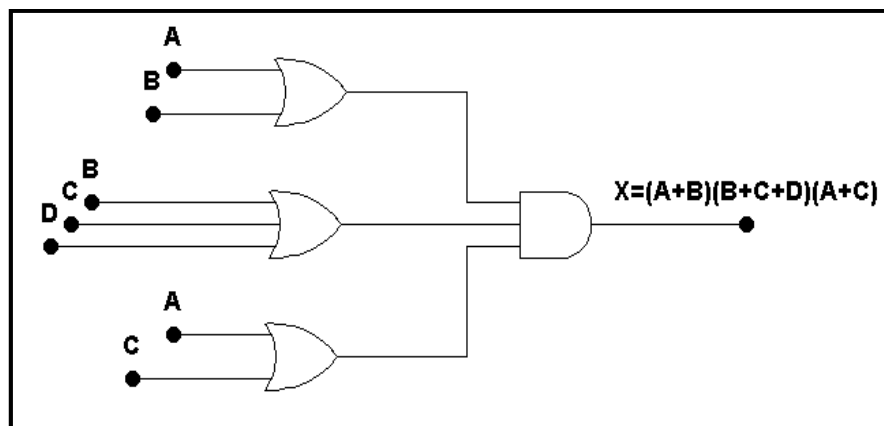
- The Standard POS Form

So far , you have seen POS expression in which some of the sum terms do not contain all of the variables in the domain of the expression. For example , the expression , $(A+\overline{B}+C)(A+B+\overline{D})(A+\overline{B}+\overline{C}+D)$ has the domain made up of the variables ,A ,B ,C and D .

Notes that the complete set of variables in the domain is not represented in the first two terms of the expression ; that is , D or $\overline{D}$ is missing from the first term and C or $\overline{C}$ is missing from the second term.

A standard POS expression is one in which all the variables in the domain appear in each sum term in the expression , for example ,

$$(\overline{A}+\overline{B}+\overline{C}+\overline{D})(A+\overline{B}+C+D)(A+B+\overline{C}+D)$$

Is a standard POS expression , Any nonstandard POS expression (referred to simply as POS) can be converted to the standard form using Boolean algebra .