Bernoulli's Equation:

Definition:

The DE: $dy/dx + P(x)y = f(x)y^n$ *(1)*

where n is any real number, is called ***Bernoulli's Equation***.

Note:

Note for $n = 0$ and $n = 1$, (1) is linear, otherwise, let

$$u = y^{1-n}$$

to reduce (1) to a linear equation.

Example 1

Solve $x\,dy/dx + y = x^2y^2$.

Solution:

Rewrite the DE as

$$dy/dx + (1/x)y = xy^2$$

With $n = 2$, then $y = u^{-1}$, and

$$dy/dx = -u^{-2}(du/dx)$$

From the substitution and simplification,

$$du/dx - (1/x)u = -x$$

The integrating factor on $(0, \infty)$ is

$$e^{-\int dx/x} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1}$$

Example 1

Integrating

$$\frac{d}{dx}\left[x^{-1}u\right] = -1$$

gives $x^{-1}u = -x + c$, or $u = -x^2 + cx$.

Since $u = y^{-1}$, we have $y = 1/u$ and a solution of the DE is

$$y = 1/(-x^2 + cx).$$