## **Definition: Homogeneous Function**

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#### Example

 $f(x, y) = x^3 + y^3$  is homogeneous of degree 3,  $f(\lambda x, \lambda y) = (\lambda x)^3 + (\lambda y)^3 = \lambda^3 f(x, y)$ 

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### **Definition: Homogeneous Equation**

A first-order DE:

M(x, y) dx + N(x, y) dy = 0

is said to be homogeneous, if both *M* and *N* are homogeneous of the same degree, that is, if

 $M(\lambda x, \lambda y) = \lambda^{\alpha} M(x, y), N(\lambda x, \lambda y) = \lambda^{\alpha} N(x, y)$ 

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#### Examples

 $(x^2 - 3xy)dx + (7x^2 - y^2)dy = 0$ 

 $(x+5y)dx - (x^2+4y^2)dy = 0$ 

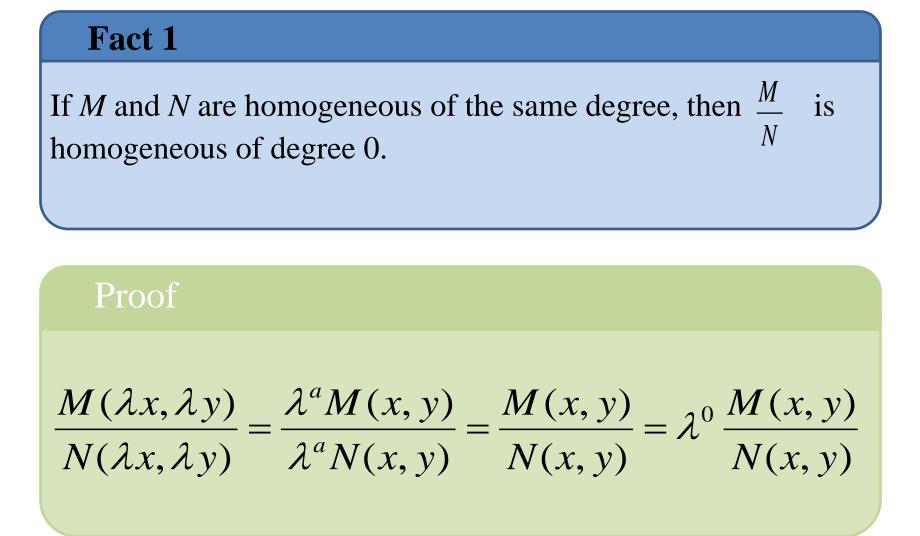
### homogeneous DE of order 2

not a homogeneous DE

#### Fact 1

If *M* and *N* are homogeneous of the same degree, then  $\frac{M}{N}$  is homogeneous of degree 0.

### Facts about homogeneous equation



#### Facts about homogeneous equation

#### Fact 2

If f is homogeneous of degree 0, then f can be expressed as a function of  $\frac{y}{x}$ .

$$f(x, y) = \phi(v), \qquad v = \frac{y}{x}$$

### Solve the DE $(x^2 + y^2) dx + (x^2 - xy) dy = 0$ .

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We have  $M = x^2 + y^2$ ,  $N = x^2 - xy$  are homogeneous of degree 2. Let y = ux, dy = u dx + x du, then  $(x^2 + u^2x^2) dx + (x^2 - ux^2)(u dx + x du) = 0$ 

$$\frac{1-u}{1+u}du + \frac{dx}{x} = 0$$

$$\left[-1 + \frac{2}{1+u}\right]du + \frac{dx}{x} = 0$$

Then

$$-u + 2\ln|1 + u| + \ln|x| = \ln|c|$$
$$-\frac{y}{x} + 2\ln\left|1 + \frac{y}{x}\right| + \ln|x| = \ln|c|$$

#### Then

$$-u + 2\ln|1 + u| + \ln|x| = \ln|c|$$
$$-\frac{y}{x} + 2\ln|1 + \frac{y}{x}| + \ln|x| = \ln|c|$$

#### Simplify to get

$$\ln \left| \frac{(x+y)^2}{cx} \right| = \frac{y}{x} \quad \text{or} \quad (x+y)^2 = cxe^{y/x}$$

\* Note: We may also try x = vy.