Reduction to Separation of Variables:

A DE of the form

$$y' = f(Ax + By + C)$$

can always be reduced to a separable equation by means of substitution

$$u = Ax + By + C.$$

Example 1

Solve $dy/dx = (-2x + y)^2 - 7$, $y(0) = 0$.

Solution:

Let $u = -2x + y$, then $du/dx = -2 + dy/dx$,

$$du/dx + 2 = u^2 - 7 \quad \text{or} \quad du/dx = u^2 - 9$$

This is separable. Using partial fractions,

$$\frac{du}{(u-3)(u+3)} = dx$$

or

$$\frac{1}{6}\left[\frac{1}{u-3} - \frac{1}{u+3}\right]du = dx$$

then we have

Example 1

$$\frac{1}{6}\ln\left|\frac{u-3}{u+3}\right| = x + c_1$$

$$or \quad y = 2x + \frac{3(1+ce^{6x})}{1-ce^{6x}}$$

Solving the equation for *u* and the solution is

$$u = \frac{3(1+ce^{6x})}{1-ce^{6x}}$$

Applying $y(0) = 0$ gives $c = -1$.

$$y = 2x + u = 2x + \frac{3(1-e^{6x})}{1+e^{6x}}$$