Separable Equation:

Definition

A first order DE of the form $\dfrac{dy}{dx} = f(x, y)$ is said to be separable, or to have separable variables if we can rewrite $f(x, y)$ as $f(x, y) = M(x)N(y)$.

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Integrate:

$$-\int \left(x+1+\frac{1}{x-1}\right)dx = \int ydy$$

$$-\left(\frac{1}{2}x^2 + x + \ln|x-1|\right) + c_0 = \frac{1}{2}y^2$$

$$-x^2 - x - 2\ln|x-1| + c = y^2$$

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$$\frac{dy}{(1 + y)} = \frac{dx}{(2 + x)}$$

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Integrate: $\displaystyle\int \frac{dy}{(1+y)} = \int \frac{dx}{(2+x)}$

$$\ln\left|1+y\right| = \ln\left|2+x\right| + c_0$$