Chapter one

Electromagnetic Theory

Time Varying Electromagnetic Field and Maxwell's Equation:-

Electrostatic and Magnetostatic (steady Magnetic) filed snare uncoupled. Maxwell's Equation describing such fields are:-

Integral form	Differential (point form)
$\oint \bar{E} \cdot d\bar{L} = 0$	$\bar{\nabla} \times \bar{E} = 0$
$\oint \bar{H} \cdot d\bar{L} = I = \int \bar{J}.ds$	$\bar{\nabla} \times \bar{H} = \bar{J}$
$\oint \bar{D} \cdot d\bar{s} = Q = \int \rho_\vartheta ds$	$\bar{\nabla} \cdot \bar{D} = \rho_\vartheta$
$\oint \bar{B} \cdot d\bar{s} = 0$	$\bar{\nabla} \cdot \bar{B} = 0$

Time varying Electromagnetic fields are coupled. The electric field produced by a changing magnetic field, and the magnetic field produced by a changing electric field.

Integral form	Differential (point form)
$\oint \bar{E} \cdot d\bar{L} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$	$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$
$\oint \bar{H} \cdot d\bar{L} = I + \int \frac{\partial \bar{D}}{\partial t} \cdot d\bar{s}$	$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$
$\oint \bar{D} \cdot d\bar{s} = \int \rho_\vartheta ds$	$\bar{\nabla} \cdot \bar{D} = \rho_\vartheta$
$\oint \bar{B} \cdot d\bar{s} = 0$	$\bar{\nabla} \cdot \bar{B} = 0$

The four equations form the basic of electromagnetic theory. In point form (widly used) they are partial differential equation and relate the electric and magnetic fields to each other and to their source (charges and current densities).

The satutions and units of measurements are:-

Electric field intensity	$\bar{E}$	volt/meter
Electric flux density	$\overline{D}$	coulomb/m^2
Magnetic field intensity	$\overline{H}$	Ampere/m
Magnetic flux density	$\bar{B}$	weber/m^2 (Tesla)
Charge density	ρ_ϑ	coulomb/m^3
Current density	$\bar{J}$	Ampere/m^2

Auxiliary equations relating the fields and densities to the properties of the medium:-

$$\overline{D} = \bar{\varepsilon}\,\bar{E}$$

$$\bar{B} = \mu\overline{H}$$

$$\bar{J} = \sigma\bar{E} \qquad \text{(conduction current)}$$

$$\bar{J} = \rho\overline{U} \qquad \text{(convection current)}$$

$$\overline{J_d} = \frac{\partial\overline{D}}{\partial t} \qquad \text{(displacement current)}$$

$\varepsilon =$ permittivity (Farad/m)

$\mu =$ permeability (Henry/m)

In some materials involving polarization and magnetization where the following detailed relationships:-

$$\overline{D} = \bar{\varepsilon}\,\bar{E} + \bar{P}$$

$$\bar{B} = \mu(\overline{H} + \overline{M})$$

Where:-

$\bar{P}$**:** polarization ; $\overline{M}$**:** magnetization

Some Remarks on the Solution

In most general form, all components of the field will be functions of three space dimensions and time.

Solutions are obtained for free space conditions:

For perfect dielectric,

for lossy dielectric or poor conductor,

for good conductors

free space is a perfect dielectric.

Note:-

The free3 space is considered a special case of a perfect dielectric. The free space is defined by its material properties $\varepsilon_\circ, \mu_\circ$.

Note:-

We shall consider the general case of perfect dielectric (i.e. $\sigma = 0$, and it has ε, μ) with no stored charges (i.e. $\rho = 0$, $I = 0$) in the perfect dielectric, also $\varepsilon \ and \ \mu$ are considered to be scalar constant.

- The time dependence of any signal of interest is considered to be sinusoidal (co-sinusoidal) if the angular frequency ω, all fields will have a time dependence of $e^{j\omega t}$, this is understandable from Euler's identity

$$e^{j\omega t} = cos\omega t + jsin\omega t$$

The partial differentiation $(\frac{\partial}{\partial t})$ is equivalent to multiplication by $j\omega$.

The Maxwell's equations for sinusoidal or exponential signal become:-

$$\overline{\nabla} \cdot \overline{D} = 0 \quad \rightarrow \quad \overline{\nabla} \cdot \overline{E} = 0 \tag{1}$$

$$\overline{\nabla} \cdot \overline{B} = 0 \quad \rightarrow \quad \overline{\nabla} \cdot \overline{H} = 0 \tag{2}$$

$$\overline{\nabla} \times \overline{E} = -j\omega\mu\overline{H} \tag{3}$$

$$\overline{\nabla} \times \overline{H} = j\omega\varepsilon\overline{E} \tag{4}$$

<u>Note:-</u>

The characteristics solution to these equations can be formulated by eliminating one of the two field vectors:-

To eliminate $\overline{H}$ and find expression for $\overline{E}$ and similar expression for $\overline{H}$ if we eliminate $\overline{E}$.

Take the curl of both side of equation (3)

$$\overline{\nabla} \times \overline{\nabla} \times \overline{E} = -j\omega\mu\overline{\nabla} \times \overline{H}$$

Substituting from equation 4

$$\overline{\nabla} \times \overline{\nabla} \times \overline{E} = \omega^2\mu\varepsilon\overline{E}$$

<u>Note:-</u>

We have the vector identity

$$\overline{\nabla} \times \overline{\nabla} \times \overline{E} = \overline{\nabla}(\overline{\nabla} \cdot \overline{E}) - \overline{\nabla}^2\overline{E}$$

$$= -\overline{\nabla}^2\overline{E}$$

From equation (1) the ist term of the R.H side of this identity equal to zero (there is no charge density as assumed).

$$\overline{\nabla}^2\overline{E} + \omega^2\mu\varepsilon\overline{E} = 0 \tag{5}$$

Similarly:-

$$\overline{\nabla}^2\overline{H} + \omega^2\mu\varepsilon\overline{H} = 0 \tag{6}$$

Equations (5) and (6) are general solution of Maxwell's equation in term of the material constants and the angular-frequency of the electromagnetic signals (within the assumption) of the problem initially considered $\rho = 0$, $J = 0$, $\sigma = 0$.

From these two equations (5,6) a solution can be obtained for the field quantities in terms of particular system of space coordinates and the physical constraints. For example in rectangular coordinates equation (5) becomes after using the expression for $\overline{\nabla}^2$:-

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} = -\omega^2 \mu \varepsilon \bar{E} \tag{7}$$

(where $\bar{E}$ is the resultant field). It can be written in terms of its components:-

$$\bar{E} = \bar{E}_x \bar{a}_x + \bar{E}_y \bar{a}_y + \bar{E}_z \bar{a}_z$$

This equation (7) is separable into its individual component parts such as:-

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu \varepsilon E_x \tag{8.a}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = -\omega^2 \mu \varepsilon E_y \tag{8.b}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z \tag{8.c}$$

Note:-

Equation (5) and (6) are phaser-vector equations. They are known as the vector Helmholts equations. Three scalar equations result from equation (7) as described by equation (8.a,b, and c).

Plane Wave Solution

A solution to equation (7) is attempted by assuming a variation of field quantities in only one direction. Let the direction of the propagation is the Z-direction

$$\frac{\partial(\ )}{\partial x} = 0\ ,\frac{\partial(\ )}{\partial y} = 0$$

Then equation (7) becomes

$$\frac{\partial^2 \bar{E}}{\partial z^2} = -\omega^2 \mu\varepsilon \bar{E}$$

$$\bar{E}(z) = E(z)\bar{a}_x + E(z)\bar{a}_y + E(z)\bar{a}_z$$

This leads to three components of the electric field, they are :-

$$\frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu\varepsilon E_x \quad \rightarrow \quad \frac{d^2 E_x}{\partial z^2} = -\omega^2 \mu\varepsilon E_x \qquad (9.a)$$

$$\frac{\partial^2 E_y}{\partial z^2} = -\omega^2 \mu\varepsilon E_y \quad \rightarrow \quad \frac{d^2 E_y}{\partial z^2} = -\omega^2 \mu\varepsilon E_y \qquad (9.b)$$

$$\frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu\varepsilon E_z \quad \rightarrow \quad \frac{d^2 E_z}{\partial z^2} = -\omega^2 \mu\varepsilon E_z \qquad (9.c)$$

The solution of ordinary differential equation (9.a,b,c) is in the form (take 9.a as a typical example) :-

$$E_x = A.exp^{(-\gamma z)} + B.exp^{(\gamma z)} \qquad (10)$$

Where $$\gamma^2 = -\omega^2 \mu\varepsilon \qquad (11)$$

Reinserting the factor $e^{j\omega t}$ and reduce equation (10) to trigonometric form by taking the real part, and replacing the arbitrary amplitude constant by E_{ox} (which is the value of E_x of z=0, t=0). We have

$$E_x = E_{ox}.exp^{(j\omega t - \gamma z)}$$

$$= E_{ox} \cdot \cos[\omega(t - z\sqrt{\mu\varepsilon})] \qquad (12)$$

γ is called propagation constant and generally a complex quantity.

$$\gamma = \alpha + j\beta$$

But we have assumed originally that $\sigma = 0$ and ε, μ are constant and real

$$\gamma = j\beta \qquad (13)$$

And $$\beta = \omega\sqrt{\mu\varepsilon} \qquad (14)$$

In perfect dielectric medium.

Note:-

In a lossy medium $\sigma \neq 0$, γ will have a real part as well

$$\gamma = \alpha + j\beta \qquad (15)$$

$\alpha =$ attenuation constant

$\beta =$ phase constant

Note:-

If $\gamma = j\beta$ (γ imajinary, $\alpha = 0$, there is not loss, no attenuation).

The field is travelling without changing its phase velocity, the phase velocity :-

$$\nu = \frac{\omega}{\beta}$$

$$= \frac{1}{\sqrt{\mu\varepsilon}}$$

Note:-

The field quantities of all electromagnetic waves will be modified by three terms:-

$exp^{j\omega t}$ denote a sinusoidal oscillation with regard to time.

$exp^{-j\beta z}$ denote a sinusoidal oscillation with regard to distance.

$exp^{-\alpha z}$ denote an exponential decay with distance.

Remarks concerning the plane wave:-

Properties of plane waves:-

1- No field components acting in the direction of propagation.
2- No variation of field strength in the plane perpendicular to the direction of propagation.
3- An electric field is normal to the magnetic field.
4- Both fields $(\bar{E} and\ \bar{H})$ are acting in direction perpendicular to the direction of propagation (they are both in the plane perpendicular to the direction of propagation).
5- The electric field and magnetic field are in time phase.

Note:-

A plane wave (TEM) (Transverse ElectroMagnetic wave) that propagate in unbounded medium (free space included).

A plane wave cannot exist when any boundary condition have to be considered.

Field Components of a Plane Wave

Assume perfect dielectric medium (free space is included).

One of Maxwell's curl equation 1

$$\bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H}$$

Can be separated into three orthogonal components

$$(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}) = -j\omega\mu H_x$$

$$(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}) = -j\omega\mu H_y$$

$$(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}) = -j\omega\mu H_z$$

Similarly from the second curl equation

$$\bar{\nabla} \times \bar{H} = j\omega\varepsilon\bar{E}$$

$$(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}) = j\omega\varepsilon E_x$$

$$(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}) = j\omega\varepsilon E_y$$

$$(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) = j\omega\varepsilon E_z$$

Remember the condition for plane waves that there is a variation of fields in one dimension only and no variation in the other two dimensions.

The variation with one dimension considered z-dimension according to $exp^{(-\gamma z)}$this gives:-

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0\,, \qquad \frac{\partial}{\partial z} = -\gamma$$

These condition are substituted

$$\gamma E_y = -j\omega\mu H_x$$

$$-\gamma E_x = -j\omega\mu H_y$$

$$0 = -j\omega\mu H_z$$

$$\gamma H_y = j\omega\varepsilon E_x$$

$$-\gamma H_z = j\omega\varepsilon E_y$$

$$0 = j\omega\varepsilon E_z$$

We have $\gamma = j\omega\sqrt{\mu\epsilon}$ (for lossless medium)

Define $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ (this is the intrinsic impedance of the medium through which the electromagnetic field is propagating).

Note:-

We can simplify the above six equations describing the field components in a perfect dielectric medium by using η to be:-

$$E_x = \eta H_y$$

$$E_y = -\eta H_x$$

$$E_z = H_z = 0$$

For plane wave (TEM-wave) propagating in perfect dielectric of $\varepsilon \; and \; \mu, \; \sigma = 0$.

Note:-

In the perfect dielectric $\gamma = j\beta$ and if we denote the wavelength in the medium by λ

We have $\lambda = \frac{2\pi}{\beta}$ for the sinusoidal waves

The wave length in a free space is λ_o

$$\lambda_o = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu_o \varepsilon_o}}$$

$\frac{1}{\sqrt{\mu_o \varepsilon_o}} =$ velocity of EM wave in free space

$C = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 3 \times 10^8 \; m/sec$ (speed of light in free space)

Note:-

Free space (perfect dielectric) has $\varepsilon = \varepsilon_o$, $\mu = \mu_o$

- The EM wave propagate in free space as a plane wave (TEM).
- The free space is considered as unbounded medium

$\lambda_o = \frac{2\pi C}{\omega} = \frac{C}{f}$ (f is the frequency of the wave in Hertz and C in meter/sec.)

General Development of the Wave Equation

The more general development of the wave equation is given starting from Maxwell's Equations:-

$$\overline{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t} \qquad (1)$$

$$\overline{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t} \qquad (2)$$

Taking the curl of both sides of equation (2) and substitute the value of $\overline{\nabla} \times \bar{H}$ from equation (1) given:-

$$\overline{\nabla} \times \overline{\nabla} \times \bar{E} = -\mu \frac{\partial [\overline{\nabla} \times \bar{H}]}{\partial t} = -\mu \frac{\partial \left[\sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}\right]}{\partial t} \qquad (3)$$

By using the vector identity

$$\overline{\nabla} \times (\overline{\nabla} \times \bar{E}) = \overline{\nabla}(\overline{\nabla} \cdot \bar{E}) - \overline{\nabla}^2 \bar{E} \qquad (4)$$

Equating equation (4) and (3) and noting that in space have no free charge

$$\overline{\nabla} \cdot \bar{E} = 0 \qquad , we\ got$$

$$\overline{\nabla}^2 \bar{E} = \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} + \mu\sigma \frac{\partial \bar{E}}{\partial t} \qquad (5)$$

With the assumption of harmonic variation (sinusoidal with time) of the field equation reduced to:-

$$\overline{\nabla}^2 \bar{E} = (-\omega^2 \mu\varepsilon + j\omega\mu\sigma)\bar{E} \qquad (6)$$

$$\overline{\nabla}^2 \bar{E} - \gamma^2 \bar{E} = 0 \qquad (7)$$

From equation (4) we can also write

$$\overline{\nabla} \times \overline{\nabla} \times \bar{E} + \gamma^2 \bar{E} = 0 \qquad (8)$$

Equations (5) and (8) are vector wave equations

Note:-

In equation (5) time is explicit while in the other three following it is implicit, since harmonic variation (sinusoidal variation) with time has been assumed.

Note:-

These wave equations in corporate all four of Maxwell's two curl equations are the starting point for the wave equations.

Equation (5) and (8) satisfying Maxwell's equation. Maxwell's two divergence equation are also satisfied.

For plane wave, travelling in the z-direction with E assumed in the x-direction $\bar{E} = E_x \bar{a}_x$ (it could have three components i.e. $\bar{E} = E_x \bar{a}_x + E_y \bar{a}_y + E_z \bar{a}_z$) equation reduce to

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} + \mu\sigma \frac{\partial E_x}{\partial t} \tag{9}$$

$$\frac{\partial^2 E_y}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} + \mu\sigma \frac{\partial E_y}{\partial t}$$

If $\sigma = 0$ $\rightarrow$ $\frac{\partial^2 E_x}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2}$ (10)

Equations (9) and (10) are scalar wave equations.

Note:-

It $\bar{E}$ does not change with time (i.e. $\bar{E}$ is static field)

$$\bar{\nabla}^2 \bar{E} = 0 \tag{11}$$

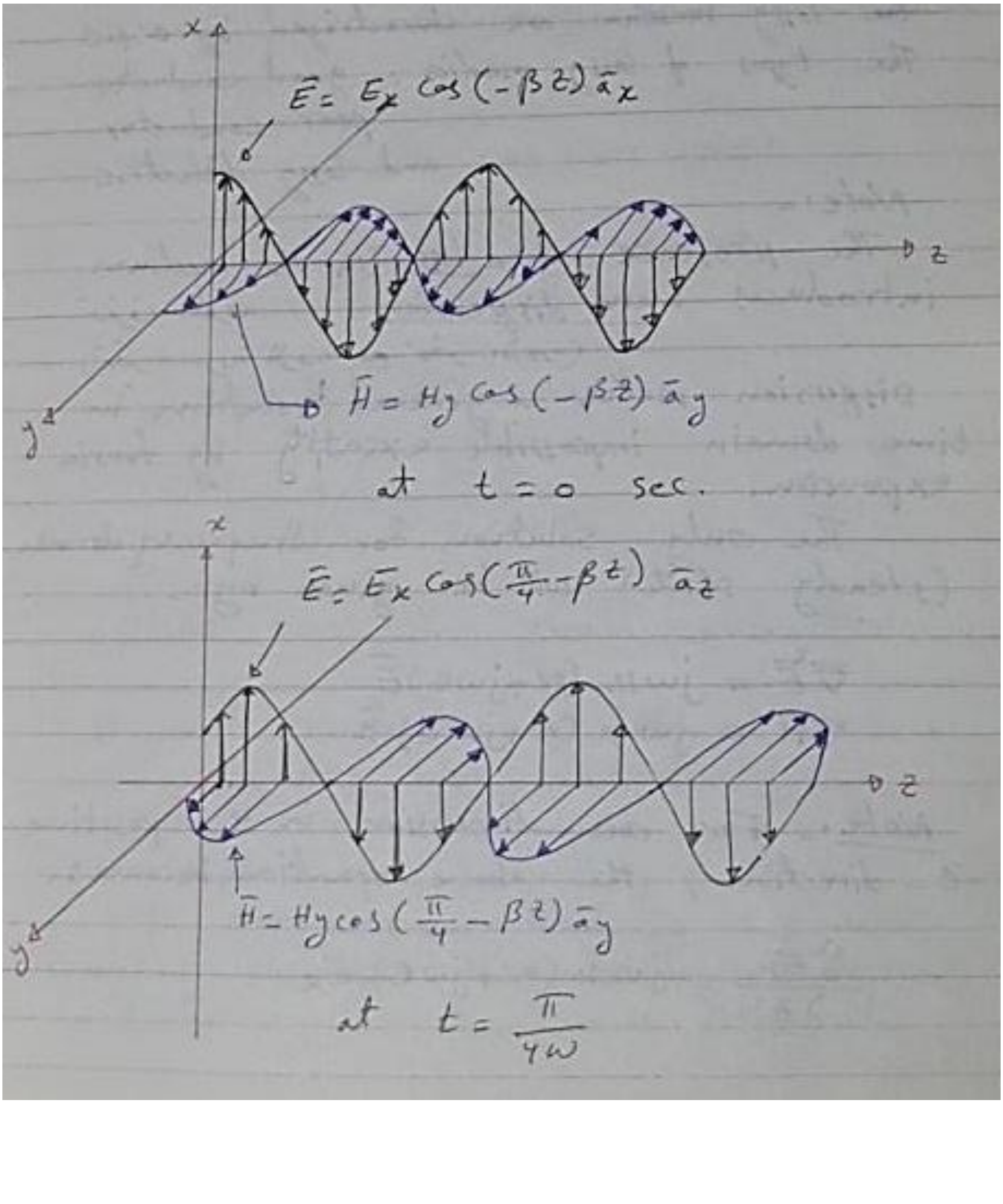
And when $\bar{E}$ is a harmonic function of time, equation (9) becomes:-

$$\frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu\varepsilon E_x + j\omega\mu\sigma E_x \tag{12}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x \qquad \rightarrow \qquad \frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$$

Where $\gamma^2 = -\omega^2 \mu\varepsilon + j\omega\mu\sigma$

Graphical Representation of Phase waves



Plane Wave Propagating in Lossy Medium

The lossy medium are characterized by $\sigma \neq 0$. The types of lossy media: good conductor, poor conductor, and lossy dielectric.

Note:-

The presence of a loss in the medium introduces **wave dispersion**. **Dispersion** makes a general solution in time domain impossible except by Fourier expansion. The only solution for frequency domain (steady state) will be given by:-

$$\overline{\nabla}^2 \overline{E} = j\omega\mu(\sigma + j\omega\varepsilon)\overline{E}$$

$$\overline{\nabla}^2 \overline{H} = j\omega\mu(\sigma + j\omega\varepsilon)\overline{H}$$

Note:-

For the dimension in the positive z-direction, the above equation becomes:-

$$\frac{\partial^2 E_x}{\partial z^2} = j\omega\mu(\sigma + j\omega\varepsilon)E_x$$

$$\frac{\partial^2 H_y}{\partial z^2} = j\omega\mu(\sigma + j\omega\varepsilon)H_y$$

The complex frequency solution would be given by:-

$$E_x = E_o e^{-\alpha z}\cos(\omega t - \beta z)$$

$$H_y = \frac{E_o}{\eta} e^{-\alpha z}\cos(\omega t - \beta z)$$

Where

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Plane Wave in Good Conductor:-

A good conductor is defined as one having a very high conductivity (i.e. conduction current is much larger than the displacement current). The energy transmitted by the wave travelling through the medium will decrease continuously as the wave propagates because the ohmic losses are present mathematically, a good conductor requires criterion:-

$$\sigma \gg \omega\varepsilon$$

The propagation constant γ is expressed

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$= j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$$

$$\cong j\omega\sqrt{\mu\varepsilon}\sqrt{-j\frac{\sigma}{\omega\varepsilon}} \qquad for \quad \frac{\sigma}{\omega\varepsilon} \gg 1$$

$$\cong j\sqrt{\omega\mu\sigma}\sqrt{-j}$$

$$\cong j\sqrt{\omega\mu\sigma}[\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}]$$

$$\cong (1 + j)\sqrt{\pi f\mu\sigma}$$

Hence $$|\alpha| = |\beta| = \sqrt{\pi f\mu\sigma}$$

Note:-

Define $e^{-1} = \frac{1}{e} = 0.368$

We have $e^{-\alpha z}$

$$-\alpha z = -1$$

$$\alpha = \sqrt{\pi f\mu\sigma}$$

$$z = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

This depth (distance) z is called the **skin depth** and denoted by δ (z is replaced by δ)

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha} = \frac{1}{\beta} \qquad \text{for conductors}$$

- For conductors at microwave frequencies the skin depth is extremely short.
- The intrinsic impedance of a good conductor is given as:-

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} \qquad for \quad \sigma \gg \omega\varepsilon$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{-i45^o} = (1+j)R_s$$

Where $$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} \qquad \Omega$$

R_s is known as the **skin effect** and it is the conductor surface resistance.

- The average power density for a good conductor is given by:-

$$P = \frac{1}{2}|H|^2 R_s \qquad w/m^2$$

- The phase velocity within a good conductor :-

$$\upsilon = \omega \cdot \delta$$

Plane Wave in Poor Conductor

Some conducting materials with low conductivity normally cannot be considered either good conductor or good dielectric. The propagation constant and the intrinsic impedance for a poor conductor are given :-

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Plane Wave in Lossy Dielectric

All dielectric materials have some conductivity, but the conductivity is very small

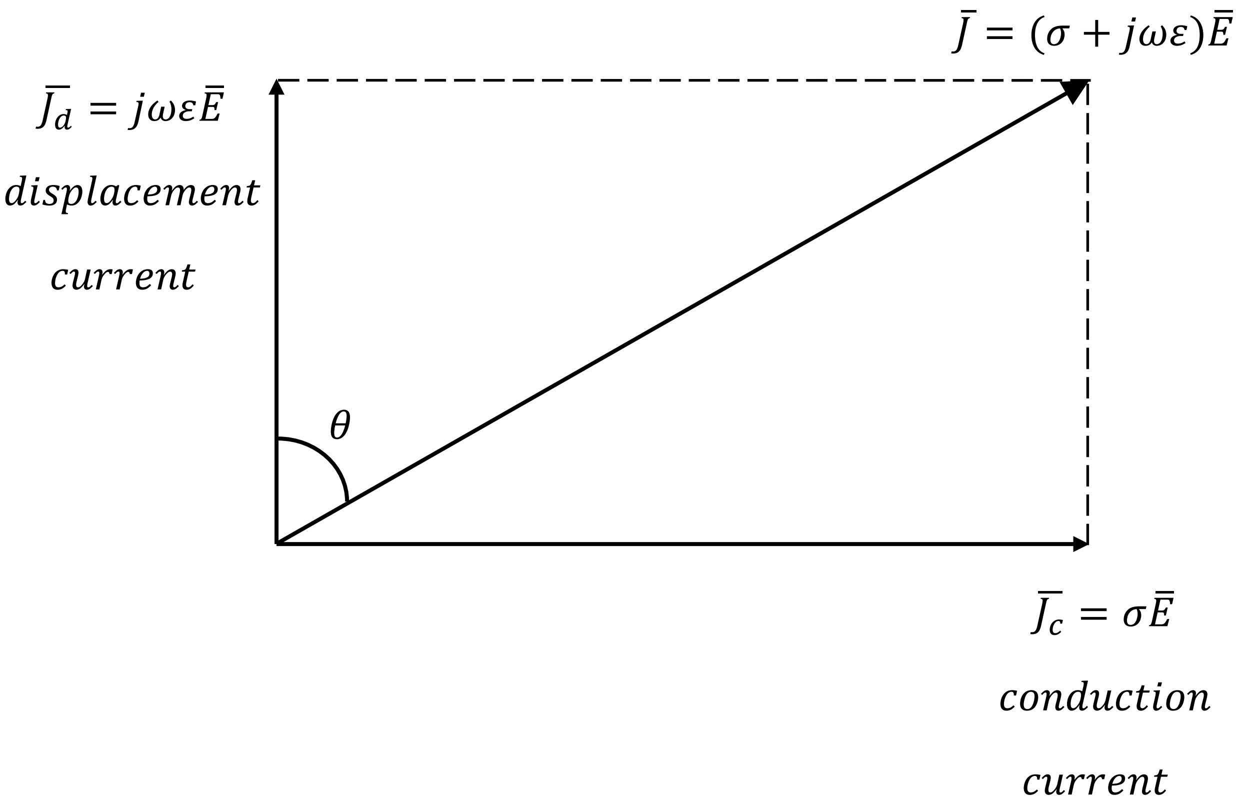
$$\sigma \ll \omega\varepsilon \qquad\qquad \left(\frac{\sigma}{\omega\varepsilon} \ll 1\right)$$

When the conductivity cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. This fact can be seen from the intrinsic impedance of the dielectric:-

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}\,(1 - j\,\frac{\sigma}{\omega\varepsilon})^{\frac{-1}{2}}$$

The term $\frac{\sigma}{\omega\varepsilon}$ is called **<u>loss tangent</u>** and define by $tan\theta = \frac{\sigma}{\omega\varepsilon}$

$$\overline{J_d} = j\omega\varepsilon\bar{E}$$



The loss tangent for dielectric

$$tan\theta = \frac{J_c}{J_d}$$

The relationship is the result of displacement current density leading conduction current density by 90^o , as shown in the diagram.

If the loss tangent is very small, that is $\frac{\sigma}{\omega\varepsilon} \ll 1$

The propagation constant and intrinsic impedance can be calculated approximately by binomial expansion.

Since :-

$$\gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$$

Then

$$\gamma = j\omega\sqrt{\mu\varepsilon}[1 - j\frac{\sigma}{2\omega\varepsilon} + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + \cdots]^{\frac{1}{2}}$$

Hence $$\alpha = j\omega\sqrt{\mu\varepsilon}[-j\frac{\sigma}{2\omega\varepsilon}]$$

$$\alpha \cong \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon}[1 + \frac{1}{8}(\frac{\sigma}{\omega\varepsilon})^2]$$

$$\cong \omega\sqrt{\mu\varepsilon}$$

Similarly:-

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left[1 + j\frac{\sigma}{2\omega\varepsilon} - \frac{3}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + \cdots\right]$$

Or

$$\eta \cong \sqrt{\frac{\mu}{\varepsilon}}\left[1 + j\frac{\sigma}{2\omega\varepsilon}\right]$$