$$X_A = x_A \cos\theta_1 + y_A \sin\theta_1 = -60.26(0.9151) + 39.74(0.4032) = -39.12 \text{ mm}$$

$$Y_A = y_A \cos\theta_1 - x_A \sin\theta_1 = 39.74(0.9151) - (-60.26)(0.4032) = 60.66 \text{ mm}$$

and

$$X_B = 19.74(0.9151) - 80.26(0.4032) = -14.30 \text{ mm}$$

$$Y_B = -80.26(0.9151) - 19.74(0.4032) = -81.41 \text{ mm}$$

The moment components are

$$M_x = M \sin\phi = 4.80 \times 10^3(0.9941) = 4.772 \text{ kN} \cdot \text{m}$$

$$M_Y = -M \cos\phi = -4.80 \times 10^3(-0.1084) = 520 \text{ N} \cdot \text{m}$$

The stresses at A and B, calculated using Eq. 7.4, are

$$\begin{aligned} \sigma &= \frac{M_x Y_A}{I_X} - \frac{M_Y X_A}{I_Y} \\ &= \frac{4.772 \times 10^6(60.66)}{3.212 \times 10^6} - \frac{0.520 \times 10^6(-39.12)}{0.574 \times 10^6} \\ &= 125.6 \text{ MPa} \\ \sigma_B &= \frac{M_x Y_B}{I_X} - \frac{M_Y X_B}{I_Y} \\ &= \frac{4.772 \times 10^6(-81.41)}{3.212 \times 10^6} - \frac{0.520 \times 10^6(-14.30)}{0.574 \times 10^6} \\ &= -108.0 \text{ MPa} \end{aligned}$$

These values for σ_A and σ_B agree with the values calculated in part (a).

EXAMPLE 7.5 *T-Beam*

A T-shaped cantilever beam of structural steel is subjected to a transverse load P at its free end (Figure E7.5). The beam is 6.1 m long. According to the Tresca yield criterion, the material yields when the maximum shear stress reaches 165 MPa. Determine the maximum load P.

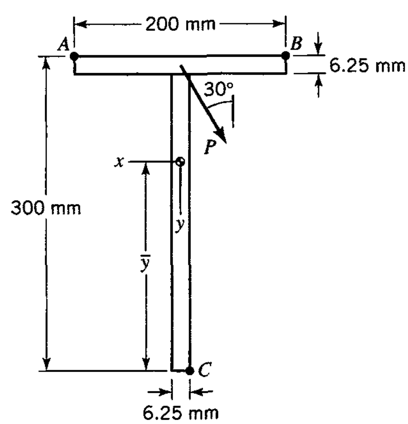

FIGURE E7.5

Solution First calculate the location of the centroid $\bar{y}$ and the moments of inertia I_x and I_y. The centroid is found relative to the bottom of the stem. Since the calculations are routine, only the results are given:

$$\begin{aligned} A &= 3085.9\ \text{mm}^2, \quad I_x = 29.94 \times 10^{-6}\ \text{m}^4 \\ \bar{y} &= 207.64\ \text{mm}, \quad I_y = 4.167 \times 10^{-6}\ \text{m}^4 \\ & \qquad I_{xy} = 0 \end{aligned} \tag{a}$$

Also, by Figure E7.5,

$$\begin{aligned} M_x &= -6.1\left(\frac{\sqrt{3}}{2}P\right) = -5.283P \\ M_y &= -6.1\left(\frac{1}{2}P\right) = -3.05P \end{aligned} \tag{b}$$

Hence, by Eqs. (a), (b), and 7.4,

$$\sigma_{zz} = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = -\frac{5.283Py}{29.94 \times 10^{-6}} + \frac{3.05Px}{4.167 \times 10^{-6}}$$

The critical points in the cross section are points A, B, and C in Figure E7.5. At these locations, the flexural stress is the maximum (unordered) principal stress.

At A, $\quad x = +100\ \text{mm}, \quad y = -92.36\ \text{mm}, \quad \sigma_{zz} = 89.49 \times 10^3 P$

At B, $\quad x = -100\ \text{mm}, \quad y = -92.36\ \text{mm}, \quad \sigma_{zz} = -56.90 \times 10^3 P$

At C, $\quad x = -3.125\ \text{mm}, \quad y = 207.64\ \text{mm}, \quad \sigma_{zz} = -38.93 \times 10^3 P$

The minimum principal stress is zero at A, B, and C. Hence, yielding will occur at A when

$$\tau_{max} = 165 \times 10^6\ \text{Pa} = \left|\frac{\sigma_{max}(A) - \sigma_{min}(A)}{2}\right| = \frac{89.49 \times 10^3}{2}P$$

or

$$P = 3688\ \text{N} = 3.69\ \text{kN}$$

7.3 DEFLECTIONS OF STRAIGHT BEAMS SUBJECTED TO NONSYMMETRICAL BENDING

Consider a straight beam subjected to transverse shear loads and moments, such that the transverse shear loads lie in a plane and the moment vectors are normal to that plane. The neutral axes of all cross sections of the beam have the same orientation as long as the beam material remains linearly elastic. The deflections of the beam will be in a direction perpendicular to the neutral axis. It is relatively simple to determine the component of the deflection parallel to an axis, say, the y axis. The total deflection is easily determined once one component has been determined.

Consider the intersection of the y–z plane with the beam in Figure 7.9. A side view of this section of the deformed beam is shown in Figure 7.10. Before deformation, the lines FG and HJ were parallel and distance Δz apart. In the deformed beam, the two

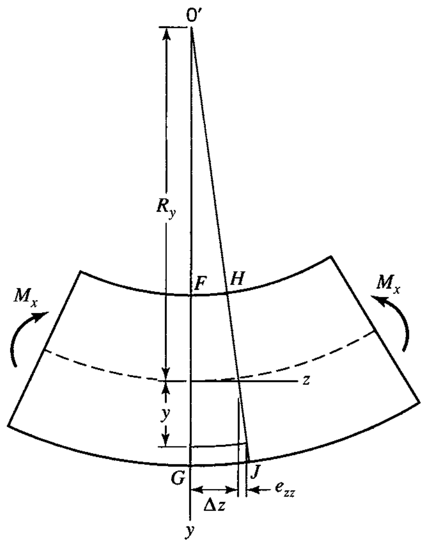

FIGURE 7.10 Deflection of a nonsymmetrically loaded beam.

straight lines FG and HJ represent the intersection of the y–z plane with two planes perpendicular to the axis of the beam, a distance Δz apart at the neutral surface. Since plane sections remain plane and normal to the axis of the beam, the extensions of FG and HJ meet at the center of curvature 0′. The distance from 0′ to the neutral surface is the radius of curvature R_y of the beam in the y–z plane. Since the center of curvature lies on the negative side of the y axis, R_y is negative. We assume that the deflections are small so that $1/R_y \approx d^2v/dz^2$, where v is the y component of displacement. Under deformation of the beam, a fiber at distance y below the neutral surface elongates an amount $e_{zz} = (\Delta z)\epsilon_{zz}$. Initially, the length of the fiber is Δz. By the geometry of similar triangles,

$$-\frac{\Delta z}{R_y} = \frac{(\Delta z)\epsilon_{zz}}{y}$$

Dividing by Δz, we obtain

$$-\frac{1}{R_y} = \frac{\epsilon_{zz}}{y} \quad \text{where} \quad \frac{1}{R_y} \approx \frac{d^2v}{dz^2} \tag{7.19}$$

For linearly elastic behavior, Eqs. 7.18 and 7.12, with $x = 0$, and Eq. 7.7 yield

$$\frac{\epsilon_{zz}}{y} = \frac{M_x}{E(I_x - I_{xy}\tan\alpha)} = -\frac{M_x I_y + M_y I_{xy}}{E(I_x I_y - I_{xy}^2)}$$

which, with Eq. 7.19, yields

$$\frac{d^2v}{dz^2} = -\frac{M_x}{E(I_x - I_{xy}\tan\alpha)} = -\frac{M_x I_y + M_y I_{xy}}{E(I_x I_y - I_{xy}^2)} \tag{7.20}$$

Note the similarity of Eq. 7.20 to the elastic curve equation for symmetrical bending. The only difference is that the term I has been replaced by $(I_x - I_{xy}\tan\alpha)$. The solution of the differential relation Eq. 7.20 gives the y component v of the total deflection at any

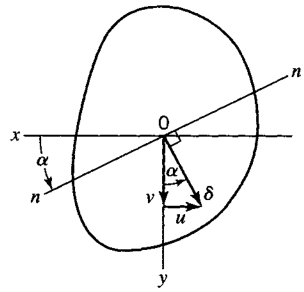

FIGURE 7.11 Components of deflection of a nonsymmetrically loaded beam.

section of the beam. As is indicated in Figure 7.11, the total deflection δ of the centroid at any section of the beam is perpendicular to the neutral axis. Therefore, the x component of δ is

$$u = -v \tan\alpha \tag{7.21}$$

and the total displacement is

$$\delta = \sqrt{u^2 + v^2} = \frac{v}{\cos\alpha} \tag{7.22}$$

EXAMPLE 7.6
Channel Section Simple Beam

Let the channel section beam in Figure E7.3 be loaded as a simple beam with a concentrated load P = 35.0 kN acting at the center of the beam. Determine the maximum tensile and compressive stresses in the beam if $\phi = 5\pi/9$. If the beam is made of an aluminum alloy (E = 72.0 GPa), determine the maximum deflection of the beam.

Solution

Analogous to the solution of Example 7.3, we have

$$\tan\alpha = -\frac{I_x}{I_y}\cot\phi = -\frac{39{,}690{,}000}{30{,}730{,}000}\cot\frac{5\pi}{9} = 0.2277$$

$$\alpha = 0.2239 \text{ rad}$$

$$M = \frac{PL}{4} = \frac{35.0(3.00)}{4} = 26.25 \text{ kN} \cdot \text{m}$$

$$M_x = M\sin\phi = 25.85 \text{ kN} \cdot \text{m}$$

$$\sigma_{\text{tension}} = \frac{25{,}850{,}000[82 - (-70)(0.2277)]}{39{,}690{,}000} = 63.8 \text{ MPa}$$

$$\sigma_{\text{compression}} = \frac{25{,}850{,}000[-118 - 70(0.2277)]}{39{,}690{,}000} = -87.2 \text{ MPa}$$

Since the deflection of the center of a simple beam subjected to a concentrated load in the center is given by the relation $PL^3/48EI$, the y component of the deflection of the center of the beam is

$$v = \frac{PL^3\sin\phi}{48EI_x} = \frac{35{,}000(3000)^3\sin 5\pi/9}{48(72{,}000)(39{,}690{,}000)} = 6.78 \text{ mm}$$

The lateral deflection is

$$u = -v\tan\alpha = -6.78(0.2277) = -1.54 \text{ mm}$$

Finally, the total deflection is

$$\delta = \sqrt{u^2 + v^2} = 6.95 \text{ mm}$$

EXAMPLE 7.7 *Cantilever I-Beam*

A cantilever beam has a length of 3 m with cross section indicated in Figure E7.7. The beam is constructed by welding two 40 mm by 40 mm steel (E = 200 GPa) bars longitudinally to the S-200 × 27 steel I-beam ($I_x = 24 \times 10^6$ mm^4 and $I_y = 1.55 \times 10^6$ mm^4). The bars and I-beam have the same yield stress, Y = 300 MPa. The beam is subjected to a concentrated load P at the free end at an angle $\phi = \pi/3$ with the x axis. Determine the magnitude of P necessary to initiate yielding in the beam and the resulting deflection of the free end of the beam.

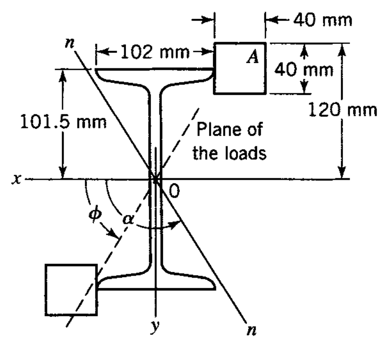

FIGURE E7.7

Solution

Values of I_x, I_y, and I_{xy} for the composite cross section can be obtained using the procedure outlined in Appendix B, which gives

$$I_x = 56.43 \times 10^6 \text{ mm}^4, \quad I_y = 18.11 \times 10^6 \text{ mm}^4$$

$$I_{xy} = 22.72 \times 10^6 \text{ mm}^4$$

The orientation of the neutral axis for the beam is given by Eq. 7.17. We find

$$\tan\alpha = \frac{I_{xy} - I_x \cot\phi}{I_y - I_{xy}\cot\phi} = \frac{22.72 \times 10^6 - 56.43 \times 10^6(0.5774)}{18.11 \times 10^6 - 22.72 \times 10^6(0.5774)} = -1.9759$$

$$\alpha = 2.039 \text{ rad} = 116.8°$$

The orientation of the neutral axis n–n is indicated in Figure E7.7. The maximum tensile stress occurs at point A; the magnitude of the stress is obtained using Eq. 7.18.

$$M = -3P$$

$$M_x = M\sin\phi = -2.598P$$

$$\sigma_A = Y = \frac{M_x(y_A - x_A\tan\alpha)}{I_x - I_{xy}\tan\alpha}$$

$$P = \frac{Y(I_x - I_{xy}\tan\alpha)}{(-2.598 \times 10^3)(y_A - x_A\tan\alpha)}$$

$$= \frac{300[56.43 \times 10^6 - 22.72 \times 10^6(-1.9759)]}{-2.598 \times 10^3[-120 - (-91)(-1.9759)]} = 39.03 \text{ kN}$$

Since the deflection of the free end of a cantilever beam subjected to symmetrical bending is given by the relation $P_y L^3/3EI$, the y component of the deflection of the free end of the beam is

$$v = \frac{PL^3 \sin\phi}{3E(I_x - I_{xy}\tan\alpha)}$$

$$= \frac{39.03 \times 10^3 (3 \times 10^3)(0.8660)}{3(200 \times 10^3)[56.43 \times 10^6 - 22.72 \times 10^6 (-1.9759)]} = 17.33 \text{ mm}$$

Hence,

$$u = -v \tan\alpha = 34.25 \text{ mm}$$

and the total displacement of the free end of cantilever beam is

$$\delta = \sqrt{u^2 + v^2} = 38.39 \text{ mm}$$

7.4 EFFECT OF INCLINED LOADS

Some common rolled sections such as I-beams and channels are designed so that I_x is many times greater than I_y, with $I_{xy} = 0$. Equation 7.17 indicates that the angle α may be large even though ϕ is nearly equal to $\pi/2$. Thus, the neutral axis of such I-beams and channels is steeply inclined to the horizontal axis (the x axis) of symmetry when the plane of the loads deviates only slightly from the vertical plane of symmetry. As a consequence, the maximum flexure stress and maximum deflection may be quite large. These rolled sections should not be used as beams unless the lateral deflection is prevented. If lateral deflection of the beam is prevented, nonsymmetrical bending cannot occur.

In general, however, I-beams and channels make poor long-span cantilever beams. The following example illustrates this fact.

EXAMPLE 7.8 ***An Unsuitable Beam***

An S-610 × 134 I-beam ($I_x = 937 \times 10^6$ mm^4 and $I_y = 18.7 \times 10^6$ mm^4) is subjected to a bending moment M in a plane with angle $\phi = 1.5533$ rad; the plane of the loads is 1° ($\pi/180$ rad) clockwise from the (y, z) plane of symmetry. Determine the neutral axis orientation and the ratio of the maximum tensile stress in the beam to the maximum tensile stress for symmetrical bending.

Solution

The cross section of the I-beam with the plane of the loads is indicated in Figure E7.8. The orientation of the neutral axis for the beam is given by Eq. 7.17:

$$\tan\alpha = \frac{-I_x \cot\phi}{I_y} = -\frac{937 \times 10^6 (0.01746)}{18.7 \times 10^6} = -0.8749$$

$$\alpha = 2.423 \text{ rad}$$

The orientation of the neutral axis is indicated in Figure E7.8. If the beam is subjected to a positive bending moment, the maximum tensile stress is located at point A. By Eqs. 7.13 and 7.18,

$$M_x = M \sin\phi = 0.9998M$$

$$\sigma_A = \frac{0.9998M[305 - 90.5(-0.8749)]}{937 \times 10^6} = 4.099 \times 10^{-7} M \quad \text{(a)}$$

When the plane of the loads coincides with the y axis (Figure E7.8), the beam is subjected to symmetrical bending and the maximum bending stress is

$$\sigma_A = \frac{My}{I_x} = \frac{305M}{937 \times 10^6} = 3.225 \times 10^{-7} M \quad \text{(b)}$$

The ratio of the stress σ_A given by Eq. (a) to that given by Eq. (b) is 1.259. Hence, the maximum stress in the I-beam is increased 25.9% when the plane of the loads is merely 1° from the symmetrical vertical plane.

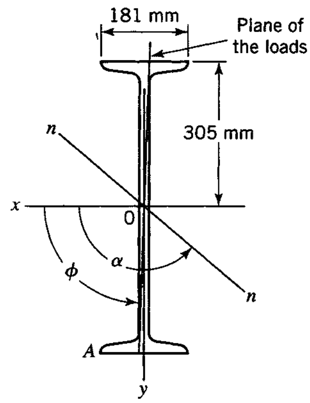

FIGURE E7.8

7.5 FULLY PLASTIC LOAD FOR NONSYMMETRICAL BENDING

A beam of general cross section (Figure 7.12) is subjected to pure bending. The material in the beam has a flat-top stress–strain diagram with yield point Y in both tension and compression (Figure 4.4a). At the fully plastic load, the deformations of the beam are unchecked and continue.

In contrast to the direct calculation of fully plastic load in symmetrical bending (Section 4.6), an inverse method is required to determine the fully plastic load for a beam subjected to nonsymmetrical bending. Although the plane of the loads is generally specified for a given beam, the orientation and location of the neutral axis, when the fully plastic moment is developed at a given section of the beam, must be determined by trial and error. The analysis is begun by assuming a value for the angle α (Figure 7.12). The

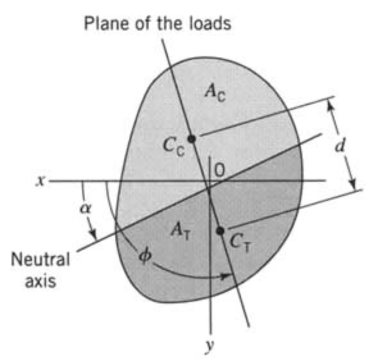

FIGURE 7.12 Location of a neutral axis for fully plastic bending of a nonsymmetrically loaded beam.

neutral axis is inclined to the x axis by the angle α, but it does not necessarily pass through the centroid as in the case of linearly elastic conditions. The location of the neutral axis is determined by the condition that it must divide the cross-sectional area into two equal areas. This follows because the yield point stress is the same for tension and compression, and so the area A_T that has yielded in tension must be equal to the area A_C that has yielded in compression. In other words, the net resultant tension force on the section must be equal to the net resultant compression force.

The yield stress Y is uniform over the area A_T that has yielded in tension; the resultant tensile force $P_T = YA_T$ is located at the centroid C_T of A_T. Similarly, the resultant compressive force $P_C = YA_C$ is located at the centroid C_C of A_C. The fully plastic moment M_P is given by

$$M_P = YA_T d = \frac{YAd}{2} \tag{7.23}$$

where A is the total cross-sectional area and d is the distance between the centroids C_T and C_C as indicated in Figure 7.12. A plane through the centroids C_T and C_C is the plane of the loads for the beam. In case the calculated angle ϕ (Figure 7.12) does not correspond to the plane of the applied loads, a new value is assumed for α and the calculations are repeated. Once the angle ϕ (Figure 7.12) corresponds to the plane of the applied loads, the magnitude of the fully plastic load is calculated by setting the moment caused by the applied loads equal to M_P given by Eq. 7.23.

EXAMPLE 7.9 Fully Plastic Moment for Nonsymmetrical Bending

A steel beam has the cross section shown in Figure E7.9. The beam is made of a steel having a yield point stress $Y = 280$ MPa. Determine the fully plastic moment for the condition that the neutral axis passes through point B. Determine the orientation of the neutral axis and the plane of the loads.

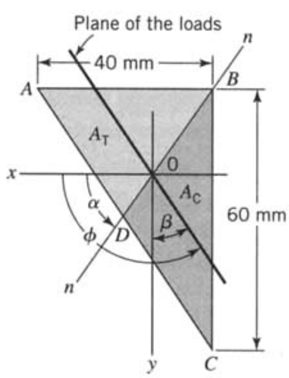

FIGURE E7.9

Solution

The neutral axis must divide the cross section into two equal areas since the area that has yielded in tension A_T must equal the area that has yielded in compression A_C. The neutral axis bisects edge AC. Therefore,

$$\tan\alpha = \frac{30}{20} = 1.5$$

$$\alpha = 0.9828 \text{ rad}$$

The plane of the loads passes through the centroids of area ABD and BCD. The centroids of these areas are located at $\left(\frac{20}{3}, -10\right)$ for ABD and $\left(-\frac{20}{3}, 10\right)$ for BCD. Therefore,

$$\tan\beta = \frac{\frac{20}{3} - \left(-\frac{20}{3}\right)}{10-(-10)} = 0.6667$$

$$\beta = 0.5880 \text{ rad}$$

$$\phi = \frac{\pi}{2} + \beta = 2.1588 \text{ rad}$$

The fully plastic moment M_P is equal to the product of the force on either of the two areas (A_T or A_C) and the distance d between the two centroids:

$$d = \sqrt{\left(20 - \frac{20}{3}\right)^2 + (30-10)^2} = 24.04 \text{ mm}$$

$$M_P = A_T Y d = \frac{1}{2}(40)(30)(280)(24.04) = 4.039 \times 10^6 \text{ N} \bullet \text{m}$$
$$= 4.039 \text{ kN} \bullet \text{m}$$

Since the orientation of the neutral axis is known *a priori*, iteration is not necessary in this example.

PROBLEMS

Section 7.1

7.1. A box beam has the cross section shown in Figure P7.1. The allowable flexural stress is $\sigma_{zz} = 75$ MPa. Determine the maximum allowable bending moment M_x.

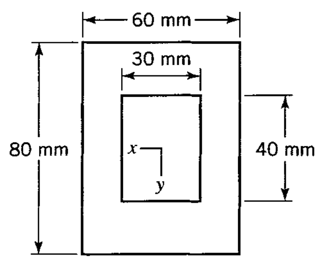

FIGURE P7.1

7.2. The T-section beam shown in Figure P7.2 is subjected to a positive bending moment $M_x = 5.0$ kN • m. Determine the maximum tensile and compressive flexural stresses for the member and the total tensile force acting on the cross section.

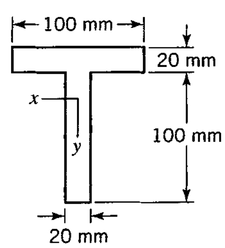

FIGURE P7.2

7.3. The cross section of a modified I-section beam is shown in Figure P7.3. A positive bending moment causes a maximum compressive flexural stress in the beam of magnitude 50 MPa. Determine the magnitude of M_x and the maximum tensile flexural stress for the cross section.

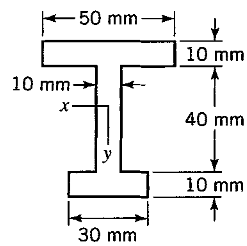

FIGURE P7.3

7.4. The cross section of a modified box-section beam is shown in Figure P7.4. A positive bending moment $M_x = 80$ kN • m is applied to the beam cross section. Determine the magnitude of the maximum flexural stress for the section.

7.5. An aircraft wing strut is made of an aluminum alloy (E = 72 GPa and Y = 300 MPa) and has the extruded cross section shown in Figure P7.5. Strain gages, located at 55 mm and 75 mm below the top of the beam, measure axial strains of −0.00012 and 0.00080, respectively, at a section where the positive moment is $M_x = 6.0$ kN • m. Determine the maximum tensile and compressive flexural stresses in the beam cross section, the location of the neutral axis, and the moment of inertia of the cross section.

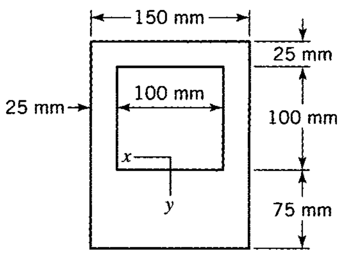

FIGURE P7.4

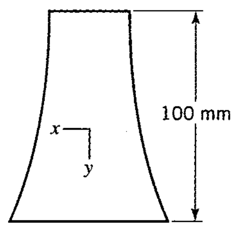

FIGURE P7.5

7.6. The simple beam, loaded as shown in Figure P7.6, is made of yellow pine. The cross section is 150 mm wide and 300 mm deep. The uniformly distributed load w includes the weight of the beam. Determine the maximum tensile and compressive flexural stresses for the section of the beam located 1.0 m from the right end.

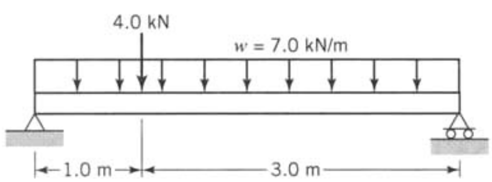

FIGURE P7.6

7.7. A cantilever beam is made by nailing together three boards of cross section 50 mm by 150 mm (Figure P7.7). It is subjected to an end load $P = 8.0$ kN. Determine the magnitude and location of the maximum tensile flexural stress for the beam cross section and the maximum tensile flexural stress in the center board.

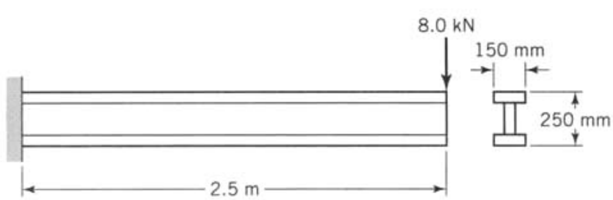

FIGURE P7.7

7.8. For the beam of Problem 7.7, what is the maximum tensile load in the top board?

7.9. The simple T-section beam in Figure P7.9 is subjected to a uniform load $w = 60$ kN/m, including the weight of the beam. Determine the magnitude of the maximum flexural stress at the top of the beam, at the junction between the web and flange, and at the bottom of the beam.

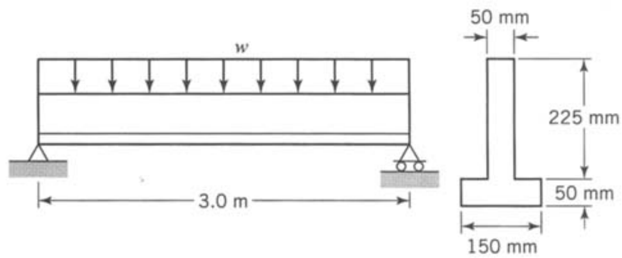

FIGURE P7.9

7.10. The simple beam shown in Figure P7.10 has a circular cross section of diameter $d = 150$ mm. Determine the magnitude of the maximum flexural stress in the beam.

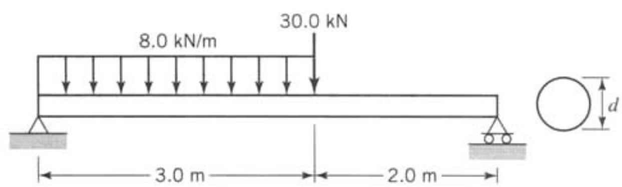

FIGURE P7.10

7.11. For the overhang beam shown in Figure P7.11, determine the maximum tensile flexural stress on sections just to the left and to the right of the section on which the 16.0 kN • m couple acts and their locations in the cross section. The flanges and the web of the cross section are all 20 mm thick.

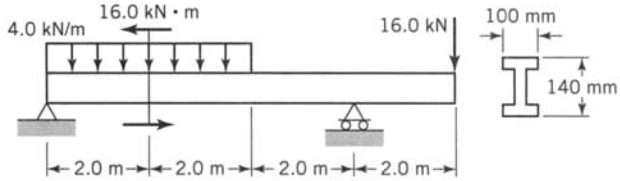

FIGURE P7.11

7.12. A double overhang beam has a cross section as shown in Figure P7.12. The vertical stems of the section are 10 mm thick and the horizontal flange is 20 mm thick. For the loads shown, determine the maximum values for the tensile and compressive stresses in the beam and their locations.

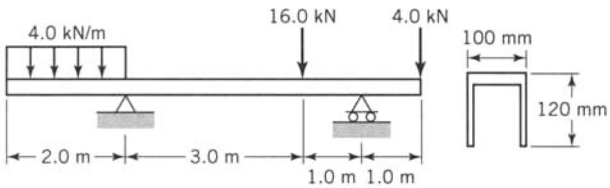

FIGURE P7.12

7.13. A beam is built up by welding four L64 × 64 × 9.5 angles to a plate having cross-sectional dimensions 200 mm by 10 mm (Figure P7.13). Properties for the angle section are given in Table A.5 of Appendix A. The allowable flexural stress is 60 MPa. Determine the magnitude of the allowable bending moment M_x.

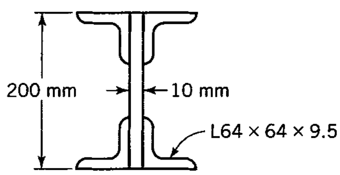

FIGURE P7.13

7.14. A beam is built up by welding a C150 × 19 channel to a W310 × 33 wide-flange section as shown in Figure P7.14. Properties for the wide-flange section and channel are given in Tables C.1 and C.2 of Appendix C. The maximum allowable moment is M_x = 20 kN • m. Determine the maximum tensile and compressive flexural stress for the cross section.

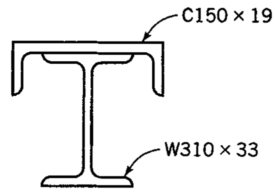

FIGURE P7.14

7.15. The beam in Figure P7.15 has the cross section shown in Figure P7.13. Determine the maximum flexural stresses at the left support and at the center of the beam. Neglect the weight of the beam.

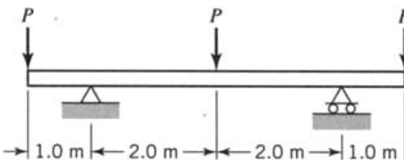

FIGURE P7.15

7.16. The beam in Figure P7.16 has the cross section shown in Figure P7.14. A load P = 20 kN is applied at its center and it is supported by distributed loads of magnitude w. Determine the maximum tensile and compressive flexural stresses that act on the section 1.5 m from the left end and at midspan of the beam.

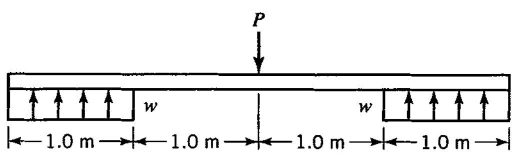

FIGURE P7.16

Section 7.2

7.17. A steel (Y = 250 MPa) cantilever beam has a design length of 2 m. A cable hoist hangs from the free end of the beam and is designed to lift a maximum load of 4.0 kN. In application, it is estimated that the cable may swing from the vertical by as much as 15°. Two cross sections are considered for the design (Figure P7.17). One cross section is that of a W130 × 24 wide flange with area 3020 mm^2 and cross-sectional dimensions shown (fillets are ignored). The other cross section is a rectangular tube of constant thickness and the same cross-sectional area as the wide-flange section. Determine the factor of safety against yielding for each beam. Consider flexural stresses only and neglect fillets and any dynamic effects resulting from swinging of the load.

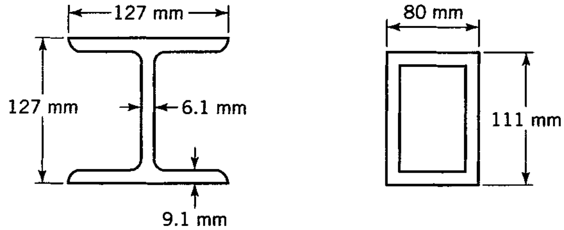

FIGURE P7.17

7.18. A timber member 250 mm wide by 300 mm deep by 4.2 m long is used as a simple beam on a span of 4 m. It is subjected to a concentrated load P at midspan. The plane of the loads makes an angle $\phi = 5\pi/9$ with the horizontal x axis. The beam is made of yellow pine with a yield stress Y = 25.0 MPa. The beam has been designed with a factor of safety SF = 2.50

against initiation of yielding. Determine the magnitude of P and the orientation of the neutral axis.

7.19. The plane of the loads for the rectangular section beam in Figure P7.19 coincides with a diagonal of the rectangle. Show that the neutral axis for the beam cross section coincides with the other diagonal.

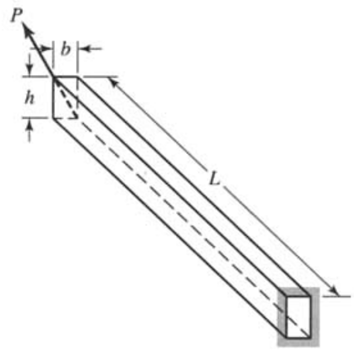

FIGURE P7.19

7.20. In Figure P7.20 let b = 300 mm, h = 300 mm, t = 25.0 mm, L = 2.50 m, and P = 16.0 kN. Calculate the maximum tensile and compressive stresses in the beam and determine the orientation of the neutral axis.

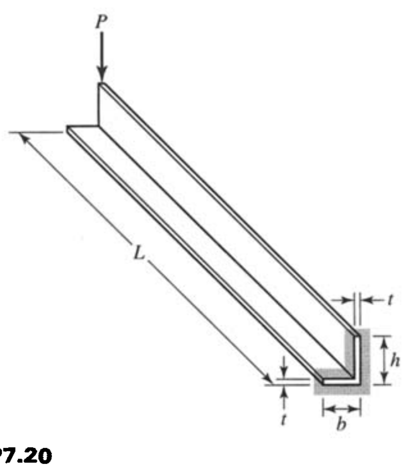

FIGURE P7.20

7.21. In Figure P7.20 let b = 200 mm, h = 300 mm, t = 25.0 mm, L = 2.50 m, and P = 16.0 kN. Calculate the maximum tensile and compressive stresses in the beam and determine the orientation of the neutral axis.

7.22. In Figure P7.22 let b = 150 mm, t = 50.0 mm, h = 150 mm, and L = 2.00 m. The beam is made of a steel that has a yield stress Y = 240 MPa. Using a factor of safety of SF = 2.00, determine the magnitude of P if $\phi = 2\pi/9$ from the horizontal x axis.

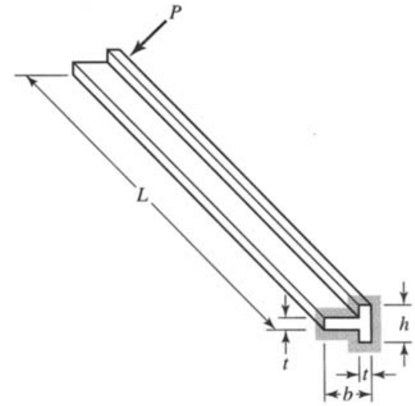

FIGURE P7.22

7.23. A simple beam is subjected to a concentrated load P = 4.00 kN at the midlength of a span of 2.00 m. The beam cross section is formed by nailing together two 50 mm by 150 mm boards as indicated in Figure P7.23. The plane of the loads passes through the centroid of the cross section as indicated. Determine the maximum flexure stress in the beam and the orientation of the neutral axis.

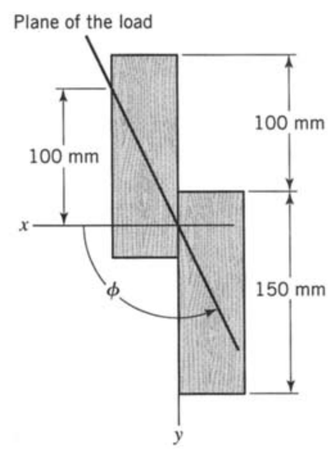

FIGURE P7.23

7.24. Solve Problem 7.23 if ϕ = 1.90 rad.

7.25. A C-180 × 15 rolled steel channel ($I_x = 8.87 \times 10^6$ mm^4, depth = 178 mm, width = 53 mm, and x_B = 13.7 mm) is used as a simply supported beam as, for example, a purlin in a roof (Figure P7.25). If the slope of the roof is $\frac{1}{2}$ and the span of the purlin is 4 m, determine the maximum tensile and compressive stresses in the beam caused by a uniformly distributed vertical load of 1.00 kN/m.

7.26. Two rolled steel angles ($I_{x1} = 391 \times 10^3$ mm^4, I_{y1} = 912×10^3 mm^4, $I_{x1y1} = 349 \times 10^3$ mm^4, and A = 1148 mm^2)

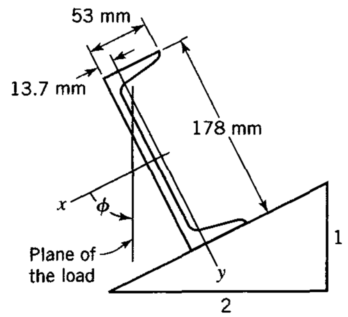

FIGURE P7.25

are welded to a 200 mm by 10 mm steel plate to form a composite Z-bar (Figure P7.26). The Z-bar is a simply supported beam used as a purlin in a roof of slope $\frac{1}{2}$. The beam has a span of 4.00 m. The yield stress of the steel in the plate and angles is Y = 300 MPa. The beam has been designed using a factor of safety of SF = 2.50 against initiation of yielding. If the plane of the loads is vertical, determine the magnitude of the maximum distributed load that can be applied to the beam.

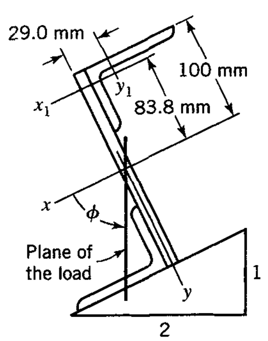

FIGURE P7.26

7.27. A steel Z-bar is used as a cantilever beam having a length of 2.00 m. When viewed from the free end toward the fixed end of the beam, the cross section has the orientation and dimensions shown in Figure P7.27. A concentrated load P = 14.0 kN acts at the free end of the beam at an angle ϕ = 1.25 rad. Determine the maximum flexure stress in the beam.

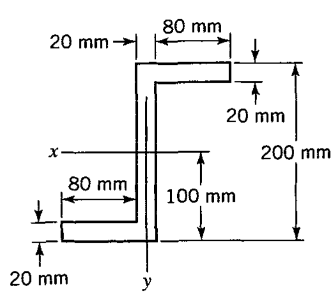

FIGURE P7.27

7.28. An extruded bar of aluminum alloy has the cross section shown in Figure P7.28. A 1.00-m length of this bar is used as a cantilever beam. A concentrated load P = 1.25 kN is applied at the free end and makes an angle of $\phi = 5\pi/9$ with the x axis. The view in Figure P7.28 is from the free end toward the fixed end of the beam. Determine the maximum tensile and compressive stresses in the beam.

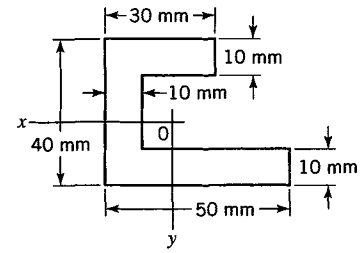

FIGURE P7.28

7.29. An extruded bar of aluminum alloy has the cross section shown in Figure P7.29. A 2.10-m length of this bar is used as a simple beam on a span of 2.00 m. A concentrated load P = 5.00 kN is applied at midlength of the span and makes an angle of ϕ = 1.40 rad with the x axis. Determine the maximum tensile and compressive stresses in the beam.

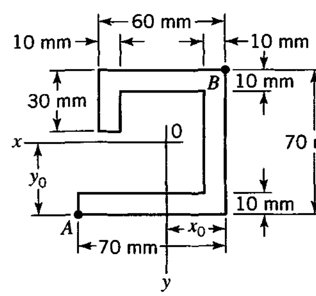

FIGURE P7.29

7.30. A cantilever beam has a right triangular cross section and is loaded by a concentrated load P at the free end (Figure P7.30). Solve for the stresses at points A and C at the fixed end if P = 4.00 kN, h = 120 mm, b = 75.0 mm, and L = 1.25 m.

7.31. A girder that supports a brick wall is built up of an S-310 × 47 I-beam (A_1 = 6030 mm^2, I_{x1} = 90.7 × 10^6 mm^4, and I_{y1} = 3.90 × 10^6 mm^4), a C-310 × 31 channel (A_2 = 3930 mm^2, I_{x2} = 53.7 × 10^6 mm^4, and I_{y2} = 1.61 × 10^6 mm^4), and a cover plate 300 mm by 10 mm riveted together (Figure P7.31). The girder is 6.00 m long and is simply supported at its ends. The load is uniformly distributed such that w = 20.0 kN/m. Determine the orientation of the neutral axis and the maximum tensile and compressive stresses.

7.32. A load P = 50 kN is applied to a rolled steel angle (I_x = I_y = 570 × 10^3 mm^4, I_{xy} = −332.5 × 10^3 mm^4, and A =

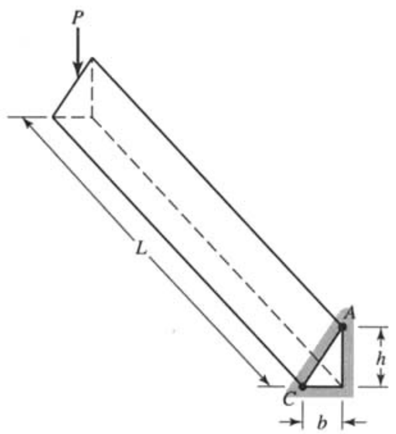

FIGURE P7.30

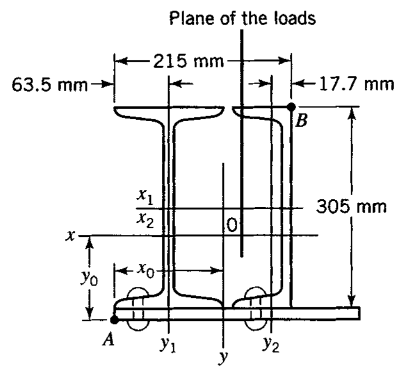

FIGURE P7.31

1148 mm^2) by means of a 76 mm by 6 mm plate riveted to the angle (Figure P7.32). The action line of load P coincides with the centroidal axis of the plate. Determine the maximum stress at a section, such as AA, of the angle. *Hint*: Resolve the load P into a load (equal to P) at the centroid of the angle and a bending couple.

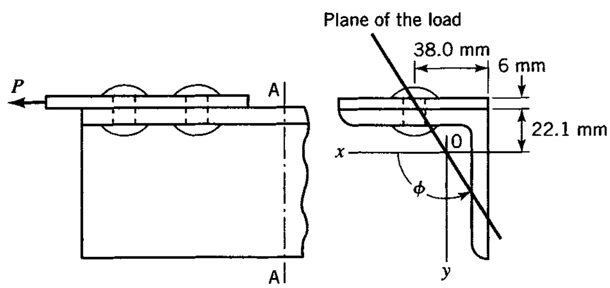

FIGURE P7.32

7.33. The beam shown in Figure P7.33 has a cross section of depth 60 mm and width 30 mm. The load P and reactions R_1 and R_2 all lie in a plane that forms an angle of 20° counterclockwise from the y axis. Determine the point in the beam at which the maximum tensile flexural stress acts and the magnitude of that stress.

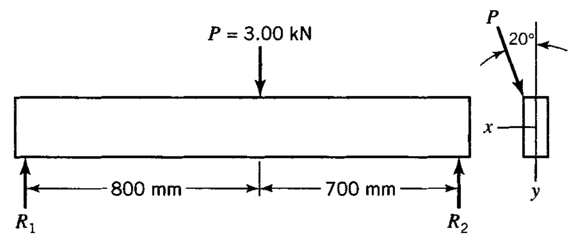

FIGURE P7.33

7.34. A beam has a square cross section (Figure P7.34).

a. Determine an expression for σ_{max} in terms of M, h, and ψ.

b. Compare values of σ_{max} for $\psi = 0$, 15, and 45°.

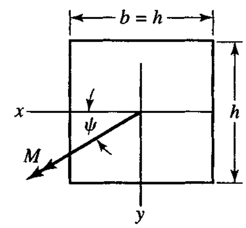

FIGURE P7.34

7.35. Consider the beam shown in Figure P7.35.

a. Derive an expression for σ_{max} in terms of M, h, and ψ.

b. Compare values of σ_{max} for $\psi = 0$, 30, 45, 60, and 90°.

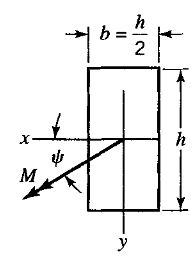

FIGURE P7.35

7.36. An I-beam has the cross section shown in Figure P7.36. The design flexural stress is limited to 120 MPa. Determine the allowable bending moment M.

7.37. A T-beam has the cross section shown in Figure P7.37. The design flexural stress is limited to 150 MPa. Determine the allowable bending moment M.

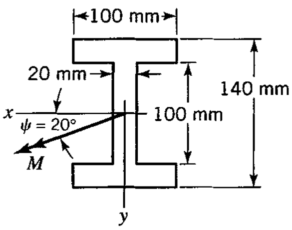

FIGURE P7.36

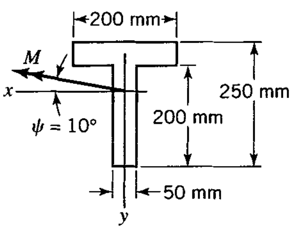

FIGURE P7.37

7.38. A beam has an isosceles triangular cross section (Figure P7.38). The maximum flexural stress is limited to 90 MPa. Determine the magnitude of the allowable bending moment M.

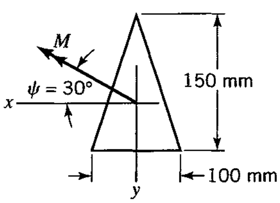

FIGURE P7.38

7.39. A circular cross section shaft is mounted in bearings that develop shear reactions only (Figure P7.39). Determine the location and magnitude of the maximum flexural stress in the beam.

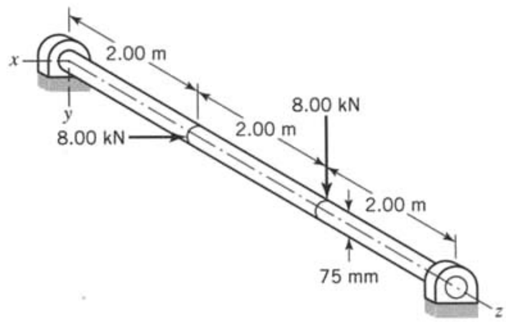

FIGURE P7.39

7.40. A wood beam of rectangular cross section 200 mm by 100 mm is simply supported at its ends (Figure P7.40). Determine the location and magnitude of the maximum flexural stress in the beam.

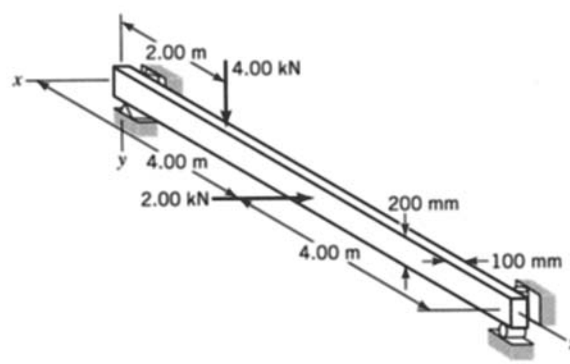

FIGURE P7.40

Section 7.3

7.41. Determine the deflection of the beam in Problem 7.18. $E = 12.0$ GPa for the yellow pine.

7.42. The beam in Problem 7.20 is made of 7075-T6 aluminum alloy for which $E = 71.7$ GPa. Determine the deflection of the free end of the beam.

7.43. The beam in Problem 7.21 is made of 7075-T6 aluminum alloy for which $E = 71.7$ GPa. Determine the deflection of the free end of the beam.

7.44. The beam in Problem 7.23 is made of yellow pine for which $E = 12.0$ GPa. Determine the deflection at the center of the beam.

7.45. Determine the deflection of the center of the beam in Problem 7.25. $E = 200$ GPa.

7.46. The beam in Problem 7.26 is subjected to a distributed load of $w = 6.5$ kN/m. Determine the deflection at the center of the beam. $E = 200$ GPa.

7.47. Determine the deflection of the beam in Problem 7.27. $E = 200$ GPa.

7.48. Determine the deflection of the free end of the beam in Problem 7.28. $E = 72.0$ GPa.

7.49. Determine the deflection of the midspan of the beam in Problem 7.29. $E = 72.0$ GPa.

7.50. Determine the deflection of the free end of the beam in Problem 7.30. $E = 200$ GPa.

7.51. Determine the deflection at midspan of the beam of Problem 7.33. $E = 200$ GPa.

7.52. A cantilever beam of length L has the cross section shown in Figure P7.34 and is subjected to moment M at its free end. Determine the deflection of the free end in terms of E, L, M, ψ, and h. Consider only the case for which $0 \leq \psi \leq 90°$.

7.53. A structural steel cantilever beam ($E = 200$ GPa) of length $L = 3.0$ m has the cross section shown in Figure P7.36 and is subjected to a moment $M = 5$ kN • m at its free end. Determine the deflection of the free end.

Section 7.5

7.54. The cantilever beam in Problem 7.28 is made of a low-carbon steel that has a yield stress $Y = 200$ MPa.

a. Determine the fully plastic load P_P for the beam for the condition that $\alpha = 0$.

b. Determine the fully plastic load P_P for the beam for the condition that $\alpha = \pi/6$.

7.55. The cantilever beam in Problem 7.30 is made of a mild steel that has a yield stress $Y = 240$ MPa. Determine the fully plastic load P_P for the condition that $\alpha = 0$.

REFERENCE

BORESI, A. P., and CHONG, K. P. (2000). *Elasticity in Engineering Mechanics*, 2nd ed. New York: Wiley-Interscience.