$$q_P = \frac{D}{K} = \frac{4288}{110} = 38.98 \text{ mm}$$

$$M_{\max} = 3lA + 2lB + lC = 4.58 \times 10^6 [\text{N} \bullet \text{mm}]$$

$$\sigma_{\max} = \frac{M_{\max}c}{I} = 93.5 \text{ MPa}$$

Except for the simplifying assumptions that the shear introduced negligible error in the flexure formula and contributed negligible strain energy, this solution is exact. A simple approximate solution of the same problem is presented in Chapter 10.

PROBLEMS

Sections 5.1–5.4

5.1. A third spring of constant k_3 and a weight W_3 are added to the system of Example 5.3. Determine the displacements q_1, q_2, and q_3 of weights W_1, W_2, and W_3.

5.2. In the system of Example 5.3, additional springs and weights are added so that there is a string of n springs and n weights. Determine the n displacements $q_1, q_2, \ldots, q_n$ of the n weights $W_1, W_2, \ldots, W_n$.

5.3. Assume that the force–elongation relation for the springs of Example 5.3 is of the form

$$F = kx^n$$

where n is a constant different from -1. Determine the displacements q_1 and q_2 of the weights W_1 and W_2 as functions of W_1, W_2, k_1, k_2, and n.

5.4. The shear stress distribution in a beam of rectangular cross section (Figure P5.4) subjected to shear force V_y is given by

$$\sigma_{zy} = \frac{V_y Q}{I_x b}$$

Show that the shear stress is, as a function of y,

$$\sigma_{zy} = \tau_{\max}\left(1 - \frac{4y^2}{h^2}\right)$$

where $\tau_{\max} = 3V_y/2A$ and A is the cross-sectional area. Hence, show that the shear stress varies parabolically and is a maximum at $y = 0$.

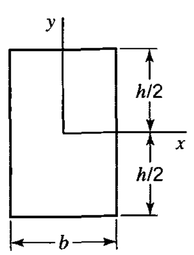

FIGURE P5.4

5.5. By means of Eqs. 5.6 and 5.7 and the formula for shear stress in a beam of rectangular cross section given in Problem 5.4, show that the correction coefficient k (see Eq. 5.14) is 1.20.

5.6. The state of stress in a beam subjected to pure bending is uniaxial with the nonzero stress given by

$$\sigma_{zz} = \frac{M_x y}{I_x} \qquad \text{(a)}$$

By Eqs. 5.6, 5.7, and (a), show that the strain energy U_M of bending is given by

$$U_M = \int \frac{M_x^2}{2EI_x} dx$$

5.7. The distribution of shear stress σ_{xy} in a torsion member of circular cross section is given by

$$\sigma_{xy} = \frac{T\rho}{J} \qquad \text{(a)}$$

where T denotes torque, J is the polar moment of inertia of the cross section, and ρ is the radial coordinate from the center O of the cross section to an interior point in the cross section. By Eqs. 5.6, 5.7, and (a), show that the strain energy U_T of torsion is

$$U_T = \int \frac{T^2}{2GJ} dx$$

5.8. Two tension members have the same length ($L = 1.5$ m), the same diameter ($D = 100$ mm), and the same proportional limit ($\sigma_{PL} = 320$ MPa). One is made of steel ($E_s = 200$ GPa) and the other is made of aluminum ($E_a = 72.0$ GPa). Which member is capable of absorbing the most energy without exceeding the proportional limit? Explain why.

5.9. A tension member of length $L = 2.0$ m is made of brass ($Y = 210$ MPa and $E = 82.7$ GPa). The member is required to absorb a design energy of 20 kJ with a safety factor of 2.5 compared to the energy absorbed at yield.

a. For this design energy, what must the cross-sectional area of the member be?

b. What is the axial stress σ in the member at the design energy?

c. Determine the ratio Y/σ. Why is this ratio not equal to the safety factor of 2.5?

5.10. The two tension members shown in Figure P5.10 are made of the same material with yield strength Y. Which member A or B can absorb the greater energy up to initiation of yield? Explain your answer. Ignore the stress concentration at the change in cross section of member B.

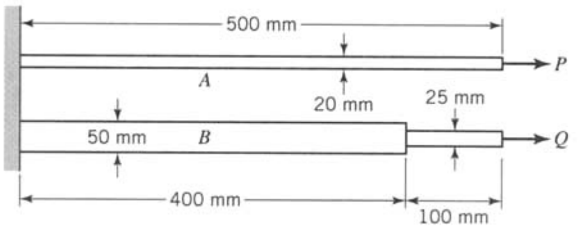

FIGURE P5.10

Remark regarding Problems 5.11, 5.12, and 5.13: *The relative importance of the strain energies U_M in bending and U_S in shear loading of beams is influenced by the ratio of the length L of the beam to its depth h, by the shape of the cross section, and by the type of loading. The relative magnitudes of U_M and U_S are examined in Problems 5.11, 5.12, and 5.13, for given ratios of L/h and given cross sections.*

5.11. The simple beam in Figure P5.11 is made of steel (E = 200 GPa and G = 77.5 GPa) and has a rectangular cross section. Show that

a. the length L must be greater than $17.6h$ for $U_S < 0.01U_M$,

b. the length L must be greater than $7.9h$ for $U_S < 0.05U_M$.

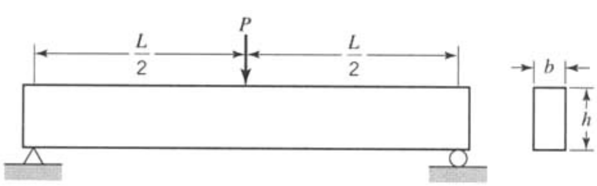

FIGURE P5.11

5.12. The simple beam shown in Figure P5.12 is made of steel (E = 200 GPa and G = 77.5 GPa) and has a circular cross section. Show that $U_S < 0.01U_M$ for $L = 5.0h$.

5.13. The simple beam with I-shaped cross section (S250 × 38, $I_x = 51.61 \times 10^6$ mm^4, h = 254 mm, b = 7.9 mm; for the area, see footnote c of Table 5.1) shown in Figure P5.13 is made of steel (E = 200 GPa and G = 77.5 GPa). Show that

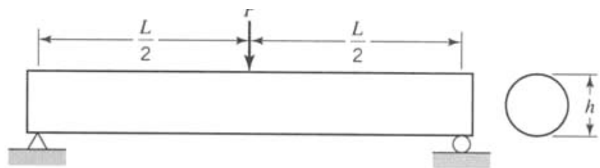

FIGURE P5.12

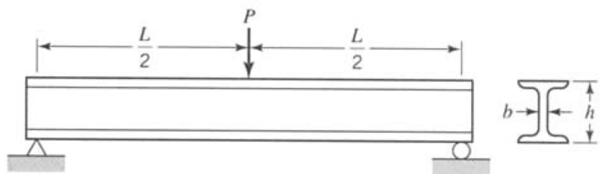

FIGURE P5.13

a. the length L must be greater than $35.1h$ for $U_S < 0.01U_M$,

b. the length L must be greater than $15.7h$ for $U_S < 0.05U_M$.

5.14. The three torsion members shown in Figure P5.14 are made of an aluminum alloy (G = 27 GPa). Each section of each member has the same cross-sectional area. Member A has a solid circular cross section of diameter 50 mm. The hollow portion of member B and the hollow member C have inside diameters of 50 mm. Determine the torsional strain energy for each member for an allowable maximum shear strain of 60 MPa.

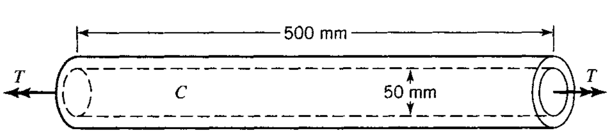

FIGURE P5.14

5.15. For the hanger shown in Figure P5.15, determine the vertical deflection of point A, assuming that bending effects dominate. Express the results in terms of P, E, I, L, and R.

5.16. For the hanger shown in Figure P5.15, determine the change in slope of the hanger at point A, assuming that bending effects dominate. Express the results in terms of P, E, I, L, and R.

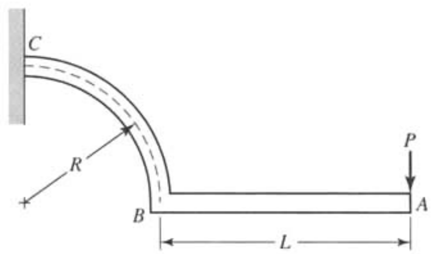

FIGURE P5.15

5.17. For the hanger shown in Figure P5.15, determine the change in slope of the hanger at point B, assuming that bending effects dominate. Express the results in terms of P, E, I, L, and R.

5.18. For the hanger shown in Figure P5.15, determine the vertical deflection of point B, assuming that bending effects dominate. Express the results in terms of P, E, I, L, and R.

5.19. For the hanger shown in Figure P5.15, the vertical load is removed and a horizontal load to the left of magnitude P is applied. Determine the horizontal deflection of the hanger at point A, assuming that bending effects dominate. Express the results in terms of P, E, I, L, and R.

5.20. For the hanger shown in Figure P5.15, the vertical load is removed and a horizontal load to the left of magnitude P is applied. Determine the rotation of the hanger at point A, assuming that bending effects dominate. Express the results in terms of P, E, I, L, and R.

5.21. Solve Example 5.14 including the effects of shear force and normal force in the wire. Plot the relationship between spring stiffness and pitch angle β to demonstrate the relative contributions of torsion, bending, shear, and normal force for a spring with a single coil ($n_c = 1$), coil diameter $D = 100$ mm, wire diameter $d = 10$ mm, modulus of elasticity $E = 200$ GPa, and Poisson's ratio $\nu = 0.3$. Consider $0 \le \beta \le 30°$.

5.22. A spring is formed by bending a wire into a flat spiral as shown in Figure P5.22. The wire has diameter d and the radial spacing between loops is $5d$. A load P, which is normal to the plane of the spring and concentric with the center of the spiral, is applied slowly to deform the spring. Determine the stiffness of the spring in its undeformed configuration.

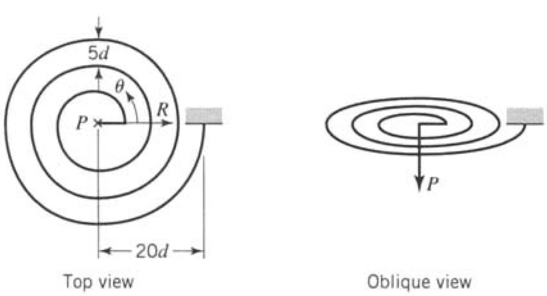

FIGURE P5.22

5.23. Determine the horizontal component of deflection of the free end of the curved beam described in Example 5.9. Assume that U_N and U_S are so small that they can be neglected.

5.24. For the pin-connected truss in Figure E5.12, determine the component of the deflection of point E in the direction of force P.

5.25. Find the vertical deflection of point C in the truss shown in Figure P5.25. All members have the same cross section and are made of the same material.

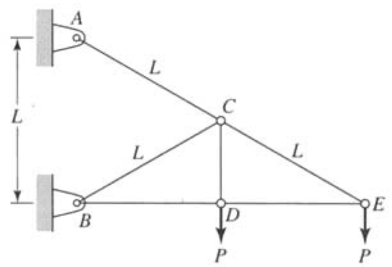

FIGURE P5.25

5.26. The beam in Figure P5.26 has its central half enlarged so that the moment of inertia I is twice the value for each end section. Determine the deflection at the center of the beam.

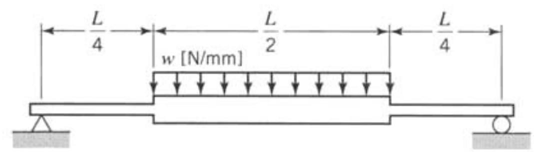

FIGURE P5.26

5.27. Member ABC in Figure P5.27 has a uniform symmetrical cross section and depth that is small compared to L and R. Determine the component of the deflection of point C in the direction of load P.

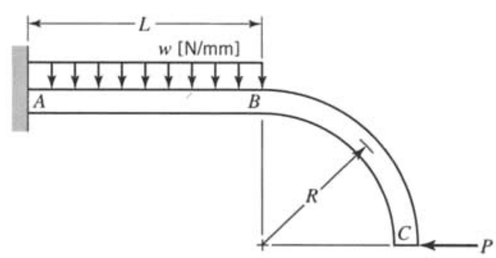

FIGURE P5.27

5.28. Member $0AB$ in Figure P5.28 lies in one plane and has the shape of two quadrants of a circle. Assuming that U_S and U_N

can be neglected, determine the vertical component of the deflection of point B.

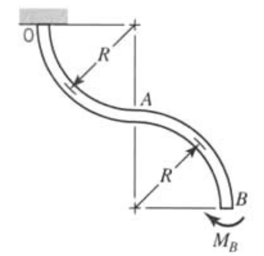

FIGURE P5.28

5.29. Determine the horizontal deflection of point B for the member in Figure P5.28.

5.30. Determine the change in slope of the cross section at point B for the member in Figure P5.28.

5.31. Determine the x and y components of the deflection of point B of the semicircular beam in Figure P5.31. The depth of the beam is small compared with R.

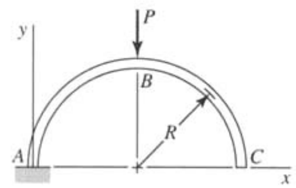

FIGURE P5.31

5.32. Determine the vertical component of the deflection of point C for the semicircular beam in Problem 5.31.

5.33. The structure in Figure P5.33 is made up of a cantilever beam AB (E_1, I_1, A_1) and two identical rods BC and CD (E_2, A_2). Let A_1 be large compared with A_2 and L_1 be large compared with the beam depth.

a. Determine the component of the deflection of point C in the direction of load P.

b. If $E_1 = E_2 = E$, the beam and rods have solid circular cross sections with radii r_1 and r_2, respectively, and $L_1 = L_2 = 25r_1$, determine the ratio of r_1 to r_2 such that the beam and rods contribute equally to q_P.

5.34. Beam ABC in Figure P5.34 is simply supported and subjected to a linearly varying distributed load as shown. Determine the deflection of the center of the beam.

5.35. Member ABC in Figure P5.35 has a circular cross section with radius r. It has a right angle bend at B and is loaded by a load P perpendicular to the plane of ABC. Determine the component of deflection of point C in the direction of P. Assume that L_1 and L_2 are each large compared to r.

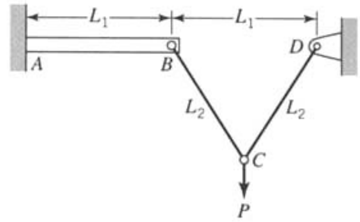

FIGURE P5.33

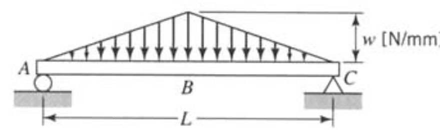

FIGURE P5.34

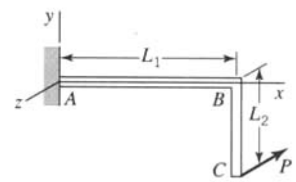

FIGURE P5.35

5.36. Member ABC in Figure P5.36 lies in the plane of the paper, has a uniform circular cross section, and is subjected to torque T_0, also in the plane of the paper, as shown. Determine the displacement of point C perpendicular to the plane of ABC. $G = E/[2(1 + \nu)]$.

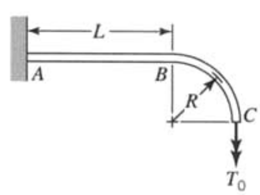

FIGURE P5.36

5.37. For the member in Problem 5.36, determine the rotation of the section at C in the direction of T_0.

5.38. Member ABC in Figure P5.38. lies in the plane of the paper, has a uniform circular cross section, and is subjected to a uniform load w [N/mm] that acts perpendicular to the plane of ABC. Determine the deflection of point C perpendicular to ABC, if length L is large compared with the diameter of the member. $G = E/[2(1 + \nu)]$.

5.39. Member ABC in Figure P5.39 lies in the (x, y) plane, has a uniform circular cross section, and is subjected to loads P per-

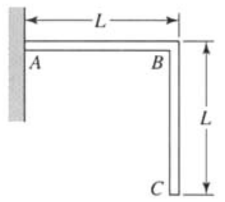

FIGURE P5.38

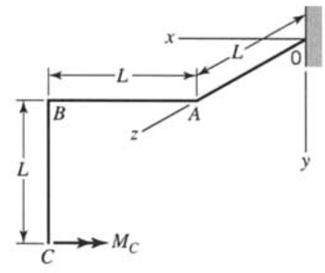

FIGURE P5.42

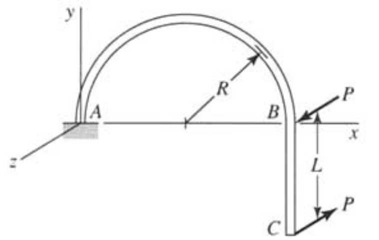

FIGURE P5.39

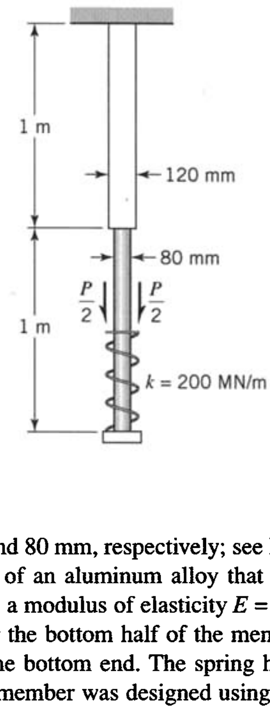

FIGURE P5.43

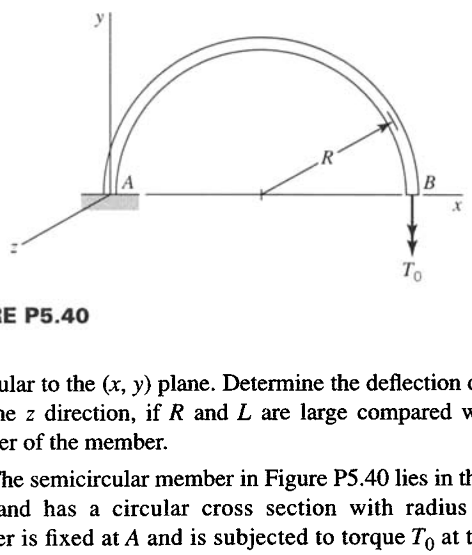

FIGURE P5.40

pendicular to the (x, y) plane. Determine the deflection of point C in the z direction, if R and L are large compared with the diameter of the member.

5.40. The semicircular member in Figure P5.40 lies in the (x, y) plane and has a circular cross section with radius r. The member is fixed at A and is subjected to torque T_0 at the free end at B. Determine the angle of twist of the cross section at B. $G = E/[2(1 + \nu)]$.

5.41. For the semicircular member in Problem 5.40, determine the x, y, and z components of the deflection of point B.

5.42. A bar having a circular cross section is fixed at the origin 0 as shown in Figure P5.42 and has right angle bends at points A and B. Length $0A$ lies along the z axis; length AB is parallel to the x axis; length BC is parallel to the y axis. Determine the x, y, and z components of the deflection of point C. Moment M_C is a couple lying parallel to the x axis. $G = E/[2(1 + \nu)]$.

5.43. A stepped tension member has two sections of length 1.00 m, each section being circular in cross section with diameters of 120 mm and 80 mm, respectively; see Figure P5.43. The member is made of an aluminum alloy that has a yield stress $Y = 330$ MPa and a modulus of elasticity $E = 72$ GPa. A spring slides freely over the bottom half of the member and bears on an end plate at the bottom end. The spring has a constant $k = 200$ MN/m. The member was designed using a safety factor of 1.80 for general yielding. Determine the deflection of the free end of the spring caused by the maximum allowable load P_{max}.

5.44. The beam ABC in Figure P5.44 is made of steel ($E = 200$ GPa) and has a rectangular cross section, 70 mm by 50 mm. Member BD is made of an aluminum alloy ($E = 72$ GPa) and has a circular cross section of diameter 10 mm. Determine the vertical deflection under the load $Q = 8.50$ kN.

5.45. The beam ABC in Figure P5.45 is made of steel ($E = 200$ GPa). It has a hollow circular section, with outer diameter 180 mm and inner diameter 150 mm. The spring has a constant $k = 2$ MN/m. A moment $M_0 = 40$ kN • m is applied at C. Determine the rotation of the cross section at C.

5.46. The beam in Figure P5.46 is made of brass ($E = 83$ GPa) and has a square cross section with a dimension of 10 mm. The identical coil springs have constant $k = 30$ kN/m. A load $Q = 250$ N is applied at midspan ($a = 100$ mm). Determine the deflection at midspan.

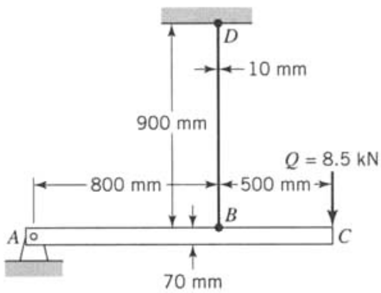

FIGURE P5.44

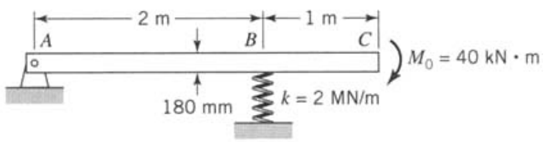

FIGURE P5.45

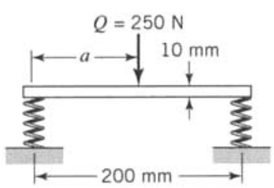

FIGURE P5.46

5.47. Let the location of the load be a = 150 mm in Problem 5.46. Determine the deflection under the load Q.

5.48. The beam in Figure P5.48 is made of an aluminum alloy (E = 72 GPa) and has a rectangular cross section with external dimensions 80 mm by 100 mm and a wall thickness of 10 mm. The identical springs have constant k = 300 kN/m. A couple M_0 is applied at distance a from the left end. Let M_0 = 15 kN • m and a = 1.50 m. Determine the rotation of the section where M_0 is applied.

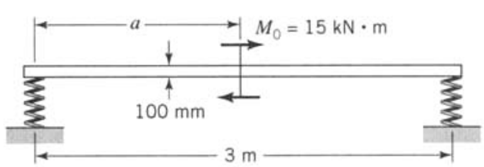

FIGURE P5.48

5.49. In Problem 5.48, let a = 2.5 m. Determine the rotation at the section where M_0 is applied.

5.50. A structure is fabricated by welding together three lengths of I-shape members (Y = 250 MPa, E = 200 GPa, and G = 77.5 GPa), as shown in Figure P5.50. The members have cross-section properties $I_x = 695 \times 10^6$ mm^4, $I_y = 20.9 \times 10^6$ mm^4,

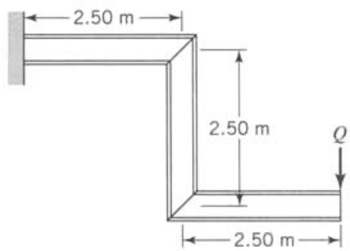

FIGURE P5.50

$S_x = 2705 \times 10^3$ mm^3, $S_y = 228 \times 10^3$ mm^3, depth = 515.6 mm, and area = 18,190 mm^2. The structure was designed with a safety factor of 2.00 for general yielding.

a. Determine the maximum allowable load Q.

b. For this load, what is the deflection at the point where Q is applied?

c. Determine the error in neglecting the strain energy resulting from axial load or from shear.

5.51. A structure (Figure P5.51) is made by welding a circular cross section steel shaft (E = 200 GPa and G = 77.5 GPa), of length 1.2 m and diameter 60 mm, to a rectangular cross section steel beam of length 1.5 m and cross-section dimensions 70 mm by 30 mm. A torque T_0 = 2.50 kN • m is applied to the free end of the shaft as shown. Determine the rotation of the free end of the shaft.

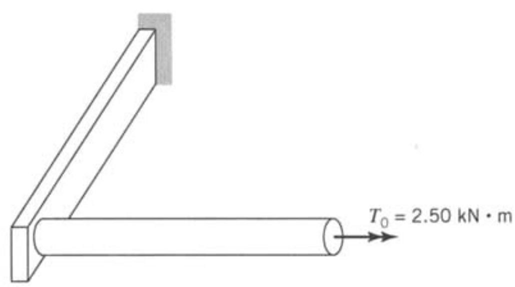

FIGURE P5.51

5.52. A circular cross section shaft AB, with diameter 80 mm and length 1.0 m, is made of an aluminum alloy (G = 27 GPa); see Figure P5.52. It is attached at point A to a torsional spring with stiffness β = 200 kN • m per rad. A torque T_0 = 4 kN • m is applied at the free end B. Determine the rotation of the shaft at B.

5.53. A rectangular box-section beam is welded to a 180-mm diameter shaft (Figure P5.53). The box-section has external dimensions 100 mm by 180 mm and a wall thickness of 20 mm. Both members are made of an aluminum alloy (E = 72 GPa and G = 27 GPa). For a load Q = 16 kN, determine the vertical deflections at the free ends of the beam and shaft.

5.54. For the beam shown in Figure P5.54, determine the vertical deflection at midspan of the beam in terms of M_0, L, E, and I.

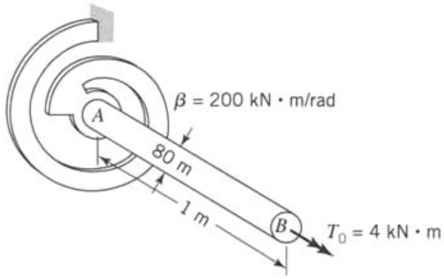

FIGURE P5.52

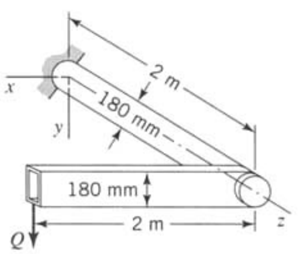

FIGURE P5.53

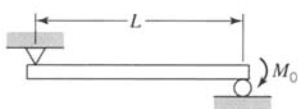

FIGURE P5.54

5.55. For the structure shown in Figure P5.55, determine the horizontal and vertical displacement components of point C in terms of Q, w, L_1, L_2, E, and I.

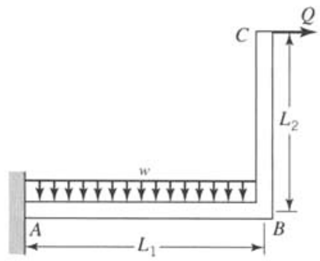

FIGURE P5.55

5.56. The circular curved beam AB in Figure P5.56 has a radius of curvature R and circular cross section of diameter d. Determine the horizontal and vertical displacement components of point B in terms of E, R, d, and load Q. Neglect the strain energy resulting from axial load and shear.

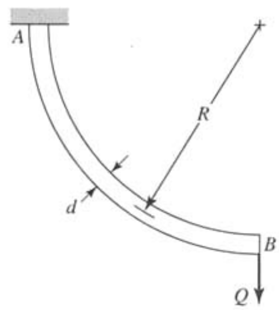

FIGURE P5.56

5.57. Determine the rotation of the section at B in Problem 5.56.

Section 5.5

5.58. Arm $ABCD$ (Figure P5.58) has a constant symmetrical cross section. For the case $Q = P$, determine the support reaction at C and the deflection at B. The length L is very large compared to the cross-sectional dimensions of the arm.

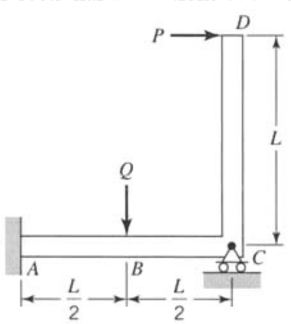

FIGURE P5.58

5.59. A propped cantilever beam is subjected to a midspan concentrated load P (Figure P5.59). Select the reaction at the right end as the redundant.

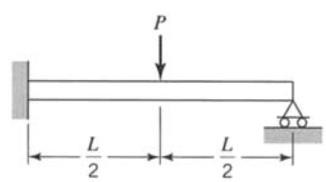

FIGURE P5.59

a. Determine the magnitude of the reaction at the right end.

b. Determine the deflection of the beam under the load P.

c. If the support at the right end settles a vertical distance $PL^3/32EI$, determine the magnitude of the reaction at the right end.

5.60. Let tension member EF be added to the structure in Figure E5.12 as indicated in Figure P5.60. All members are aluminum, for which E = 72 GPa. Members BC, CD, and DE each have cross-sectional area of 900 mm^2. The remaining members have cross-sectional area of 150 mm^2. Load magnitudes are P = 10 kN and Q = 5 kN. Determine the axial force in member EF and deflection of point E in the direction of force P.

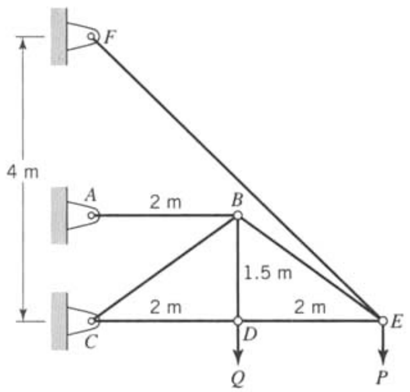

FIGURE P5.60

5.61. The beam in Figure P5.61 is fixed at the right end and simply supported at the left end. Determine the reaction R at the left end, assuming that the length L of the beam is large compared with its depth.

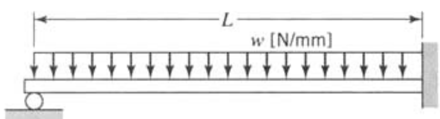

FIGURE P5.61

5.62. The beam in Problem 5.61 has a circular cross section with a diameter of 40 mm, has a length of 2.00 m, and is made of a steel (E = 200 GPa) having a working stress limit of 140 MPa.

a. Determine the magnitude of w that will produce this limiting stress.

b. How much would the stress in the beam be increased for the same value of w if the left end of the beam deflects 5.00 mm before making contact with the support?

5.63. The beam in Figure P5.63 is subjected to two loads P and is supported at three locations A, B, and C as shown. Determine the reaction at B, assuming that the beam length is large compared to its depth.

5.64. The beam in Figure P5.64 is fixed at the left end and is supported on a roller at its center B. Assuming that the beam length is large compared to its depth, determine the reaction at B and slope of the beam over the support at B.

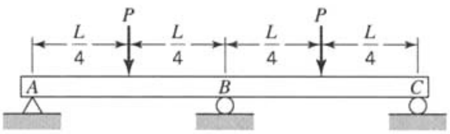

FIGURE P5.63

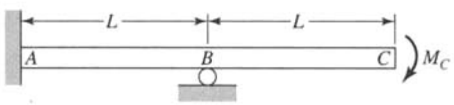

FIGURE P5.64

5.65. Member ABC in Figure P5.65 has a constant cross section. Assuming that length R is large compared to the depth of the member, determine the horizontal H and vertical V components of the pin reaction at C.

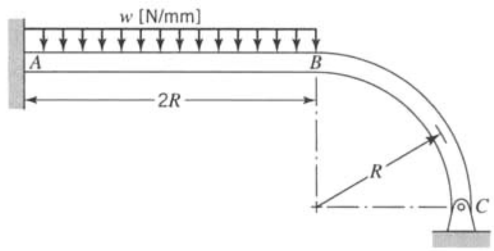

FIGURE P5.65

5.66. The beam in Figure P5.66 is fixed at the right end and rests on a coil spring with spring constant k at the left end. Assuming that the beam length is large compared to its depth, determine the force R in the spring.

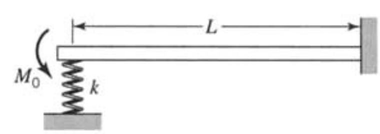

FIGURE P5.66

5.67. The structure in Figure P5.67 is constructed of two steel columns AB and CD with moment of inertia I_1 and steel beam BC with moment of inertia I_2. Assuming that lengths H and L are large compared with the depths of the members, determine the horizontal component of the pin reaction at D.

5.68. Dimensions R and L are large compared with the depth of the member. Determine the maximum moment for the chain link shown in Figure P5.68.

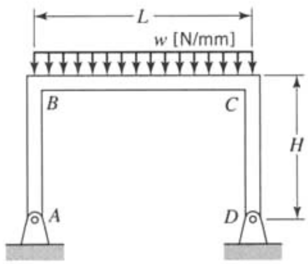

FIGURE P5.67

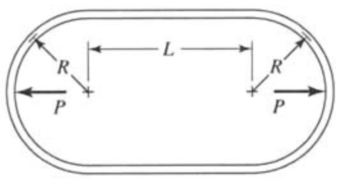

FIGURE P5.68

5.69. Member *ABCD* in Figure P5.69 lies in the plane of the paper. If length L is large compared with the depth of the member, determine the pin reaction at D and the horizontal displacement of the pin at D.

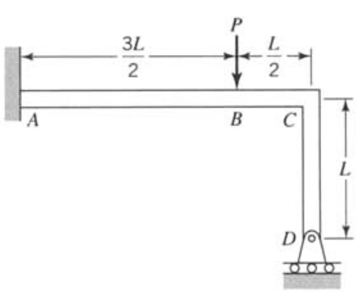

FIGURE P5.69

5.70. Let the pin at D for member *ABCD* in Problem 5.69 be prevented from displacing horizontally as load P is applied. Determine horizontal and vertical pin reactions at D.

5.71. The structure in Figure P5.71 is made up of a steel (E = 200 GPa) rectangular beam *ABC* with depth h = 40.0 mm and width b = 30.0 mm and two wood (E = 10.0 GPa) pin-connected members *BD* and *CD* with 25.0-mm square cross sections. If load P = 9.00 kN is applied to the beam at C, determine the reaction at support D and the maximum stresses in the steel beam and wood members.

5.72. Member *ABC* in Figure P5.72 has a uniform circular cross section with radius r that is small compared with R. Determine the pin reaction at C and horizontal component of the displacement of point B.

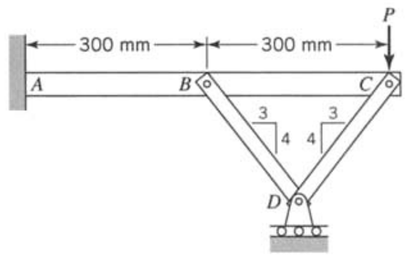

FIGURE P5.71

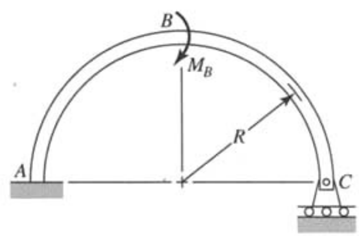

FIGURE P5.72

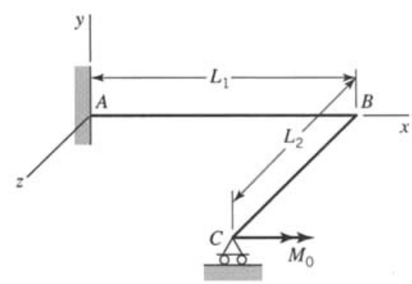

FIGURE P5.73

5.73. Member *ABC* in Figure P5.73 has a right angle bend at B, lies in the (x, z) plane, and has a circular cross section with diameter d that is small compared with either length L_1 or length L_2. The reaction at C prevents deflection in the y direction only. Determine the reaction at C when the moment M_0 is applied at C.

5.74. Member *AB* in Figure P5.74 is a quadrant of a circle lying in the (x, z) plane, has a circular cross section of radius r, which is small compared with R, and is supported by a spring (spring constant k) at B, whose action line is parallel to the y axis. Determine the force in the spring when torque T_0 is applied at B with action line parallel to the negative z axis.

5.75. The structure in Figure P5.75 has a uniform circular cross section with diameter d, which is small compared with either H or L. The structure is fixed at 0 and C and lies in the (x, z) plane. The load P is parallel to the y axis. Determine the magnitudes of the moment and torque at 0 and C.

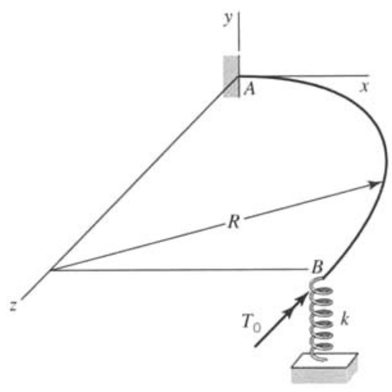

FIGURE P5.74

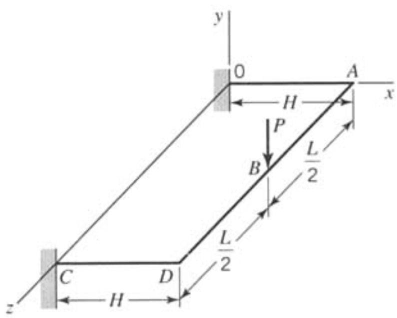

FIGURE P5.75

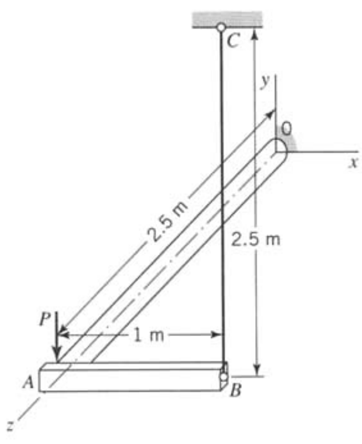

FIGURE P5.76

5.76. Each of the three members of the structure in Figure P5.76 is made of a ductile steel ($E = 200$ GPa and $\nu = 0.29$) with yield stress $Y = 420$ MPa. Member $0A$ has a diameter of 100 mm, is fixed at 0, and is welded to beam AB, which has a rectangular cross section with a depth of 75.0 mm and width of 50.0 mm. Tension member BC has a circular cross section with a diameter of 7.50 mm. All the members are unstressed when $P = 0$. Determine the value of P based on a factor of safety of $SF = 2.00$ against initiation of yielding. Neglect stress concentrations.

5.77. Member BCD and tension member BD in Figure P5.77 are made of materials having the same modulus of elasticity. Member BCD has a constant moment of inertia I, a cross-sectional area that is large compared to area A of tension member BD, and depth that is small compared to L. Determine the axial force in member BD.

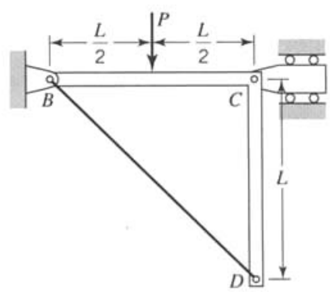

FIGURE P5.77

5.78. Member $BCDF$ in Figure P5.78 has the same moment of inertia I at every section. Determine the internal member forces N_D, V_D, and M_D at section D. Length L is large compared with the depth of the member.

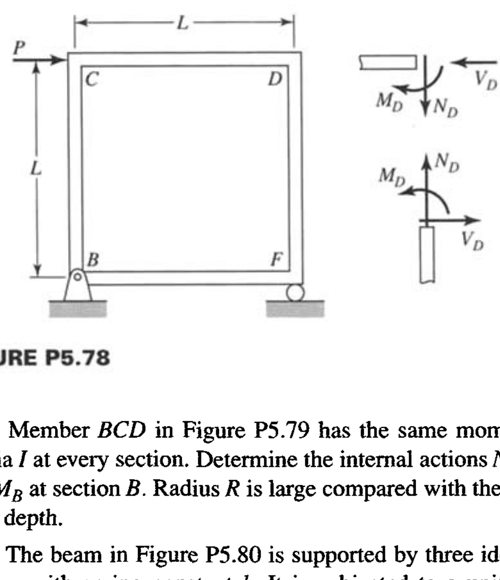

FIGURE P5.78

5.79. Member BCD in Figure P5.79 has the same moment of inertia I at every section. Determine the internal actions N_B, V_B, and M_B at section B. Radius R is large compared with the member's depth.

5.80. The beam in Figure P5.80 is supported by three identical springs with spring constant k. It is subjected to a uniformly distributed load w. Determine the force in each spring in terms of w, k, L, E, and I.

5.81. Show that the reaction at the roller support for the beam in Figure P5.81 is equal to $5Q/2$.

5.82. Determine the force in the spring (Figure P5.82) in terms of w, L, k, E, and I.

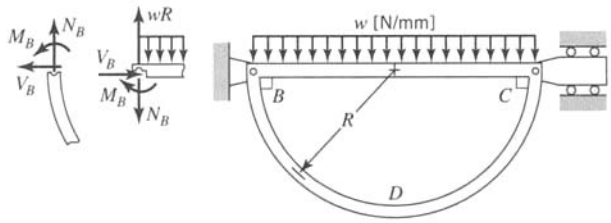

FIGURE P5.79

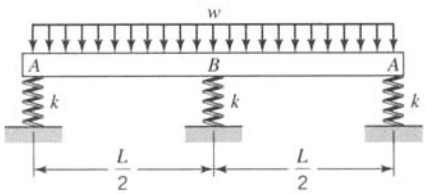

FIGURE P5.80

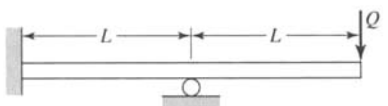

FIGURE P5.81

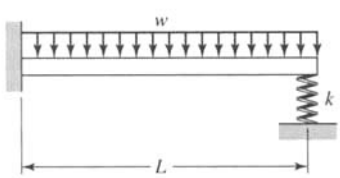

FIGURE P5.82

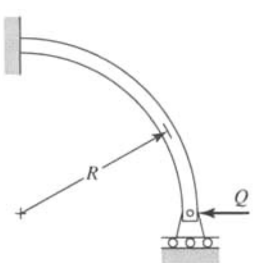

FIGURE P5.83

5.83. Show that the vertical reaction at the roller support for the curved member in Figure P5.83 is equal to $2Q/(3\pi - 8)$.

5.84. Determine the reaction at B for the beam in Figure P5.84 in terms of w, L_1, L_2, E, and I.

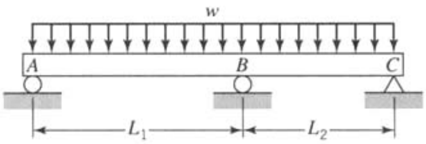

FIGURE P5.84

5.85. Determine the reaction at the roller support for the structure in Figure P5.85 in terms of Q, R, E, and I.

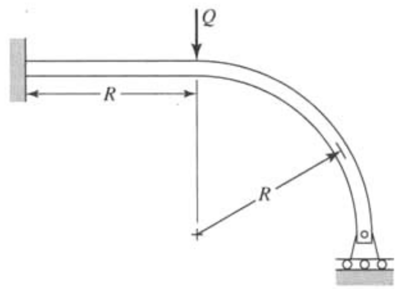

FIGURE P5.85

5.86. In Problem 5.81, determine the vertical deflection at the point where Q is applied.

5.87. In Problem 5.83, determine the horizontal deflection at the point where load Q is applied.

5.88. In Problem 5.85, determine the vertical deflection at the point where load Q is applied.

5.89. For the structure in Figure P5.89, determine the force in the spring in terms of M_0, k, L, E, and I.

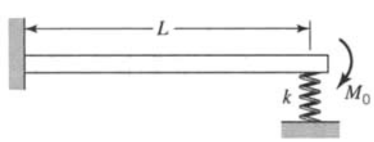

FIGURE P5.89

5.90. An I-beam is made of steel (E = 200 GPa) and is 5.0 m long (Figure P5.90). It has cross-section properties $I_x = 24.0 \times 10^6$ mm^4, $I_y = 1.55 \times 10^6$ mm^4, depth = 203.2 mm, flange width = 101.6 mm, and area = 3490 mm^2. The helical support spring has a constant k = 1.00 MN/m. For the case where Q = 30.0 kN, determine the force in the spring and the maximum bending stress in the beam.

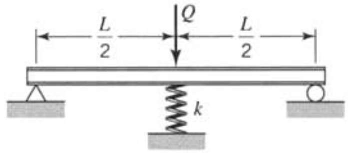

FIGURE P5.90

5.91. For the structure in Figure P5.91, derive a formula for the force in the bar in terms of w, E_1, I, and L_1 for the beam and E_2, A_2, and L_2 for the bar.

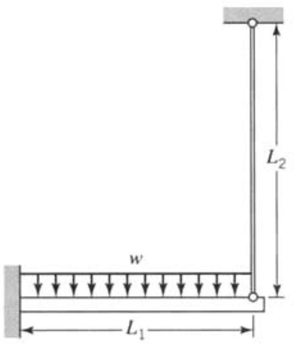

FIGURE P5.91

5.92. The beam in Problem 5.91 is made of steel (E = 200 GPa). It has a rectangular cross section with dimensions 90 mm deep, 30 mm wide, and a length L_1 = 2.00 m. The rod is made of an aluminum alloy (E = 72.0 GPa). It has a diameter of 5 mm and a length L_2 = 4.00 m. Determine the tension in the bar in terms of load w.

5.93. A shaft AB is attached to member $CDFH$ at A and fixed to a wall at B (Figure P5.93). The shaft has a diameter of 60 mm and the parts CD and FH of member $CDFH$ have square cross sections of 40 mm by 40 mm. The massive hub DF may be considered as rigid. A torque of magnitude 3 kN • m is applied to the midsection of the shaft as shown. All members are made of steel (E = 200 GPa and G = 77.5 GPa).

a. Determine the maximum bending stress in members CD and FH.

b. Determine the maximum shear stress in the shaft.

5.94. In Figure P5.94, the shaft is attached to a torsional spring at one end and fixed to a rigid wall at the other end. The shaft has an 80-mm diameter and shear modulus G = 77.5 GPa. The torsional spring constant is β = 200 kN • m/rad. A torque of magnitude 5 kN • m is applied to the midsection of the shaft as shown. Determine the maximum shear stress in the shaft.

5.95. A rectangular box-beam, 100 mm by 200 mm and with a wall thickness of 10 mm, is welded to a shaft of diameter 180 mm

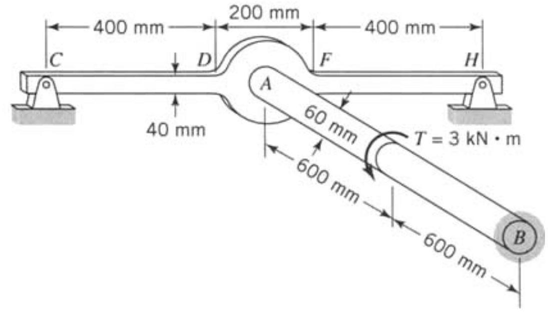

FIGURE P5.93

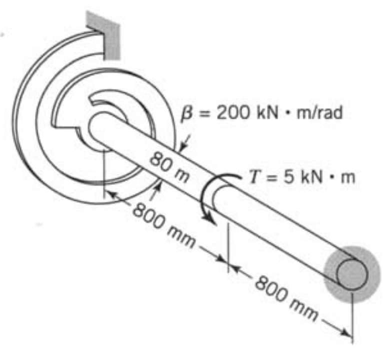

FIGURE P5.94

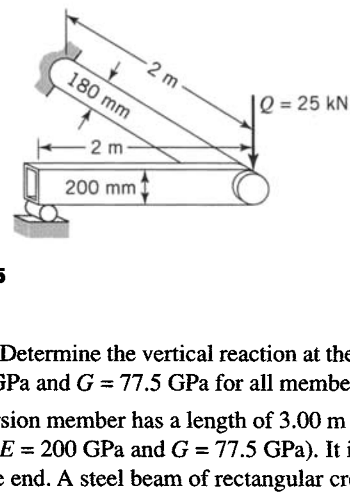

FIGURE P5.95

(Figure P5.95). Determine the vertical reaction at the roller support. (E = 200 GPa and G = 77.5 GPa for all members.)

5.96. A steel torsion member has a length of 3.00 m and diameter of 120 mm (E = 200 GPa and G = 77.5 GPa). It is fixed to a rigid wall at one end. A steel beam of rectangular cross section, 120 mm by 30.0 mm, is welded perpendicularly to the torsion member at its midsection (Figure P5.96). The beam is supported by a roller located 2.00 mm from the welded section. The free end of the member is subjected to a torque T = 16.0 kN • m. Determine the reaction at the roller.

5.97. A beam ABC is fixed at its ends A and C (Figure P5.97). Show that the shear V and moment M at C are given by the relations $V = Q(L^3 - 3a^2L + 2a^3)/L^3$ and $M = Q(aL^2 - 2a^2L + a^3)/L^2$.

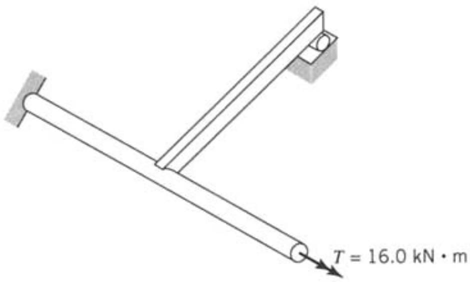

FIGURE P5.96

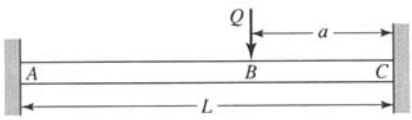

FIGURE P5.97

5.98. The beam in Problem 5.97 has a circular cross section of diameter $d = 150$ mm and length $L = 2.00$ m. For $a = L/3$, determine the magnitude of load Q that produces a maximum bending stress of 100 MPa.

5.99. The L-shaped beam $ABCD$ in Figure P5.99 has a constant cross section along its length. Show that the horizontal and vertical reactions H and V, respectively, of the pin at D are given by the expressions $H = 9Q/22$ and $V = 4Q/11$.

5.100. The L-shaped beam in Figure P5.99 has a square cross section, 80 mm by 80 mm, and a length $3L = 2400$ mm.

a. Determine the magnitude of load Q that produces a maximum bending stress of 120 MPa in the beam.

b. The beam is made of steel ($E = 200$ GPa). Determine the vertical deflection of the beam at point C.

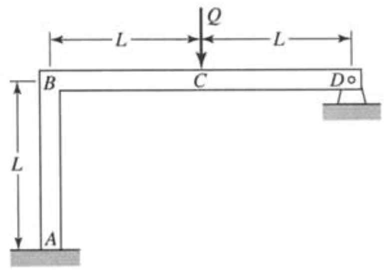

FIGURE P5.99

5.101. Consider the indeterminate beam of Example 5.15 (Figure E5.15). Let the beam material be elastic–perfectly plastic (Figure 4.4*a*). Let the cross section be rectangular with width b and depth h.

a. Determine the magnitude of the uniform load $w = w_Y$ that causes yielding to initiate and locate the section at which it occurs.

b. Determine the magnitude of the uniform load $w = w_P$ that causes a plastic hinge to form at the wall support.

c. Determine the magnitude of the uniform load $w = w_{PC}$ that causes the beam to form a plastic hinge to occur at the section of maximum positive moment. The load w_{PC} is called the *plastic collapse load* for the member.

d. Construct a moment diagram for the beam for load w_{PC}.

e. Draw a sketch of the deformed shape of the beam for $w = w_{PC}$.

Note: The elastic segments of the beam rotate about a plastic hinge as rigid bodies. For this reason, the response of the beam at $w = w_{PC}$ is like a mechanism that rotates kinematically about hinges. Therefore, the term *mechanism* or *kinematic mechanism* is used in plastic collapse analysis (limit analysis) to describe this process.

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