CHAPTER 1

INTRODUCTION

In this chapter, we present general concepts and definitions that are fundamental to many of the topics discussed in this book. The chapter serves also as a brief guide and introduction to the remainder of the book. You may find it fruitful to refer to this chapter, from time to time, in conjunction with the study of topics in other chapters.

1.1 REVIEW OF ELEMENTARY MECHANICS OF MATERIALS

Engineering structures and machines, such as airplanes, automobiles, bridges, spacecraft, buildings, electric generators, gas turbines, and so forth, are usually formed by connecting various parts or members. In most structures or machines, the primary function of a member is to support or transfer external forces (loads) that act on it, without failing. Failure of a member may occur when it is loaded beyond its capacity to resist fracture, general yielding, excessive deflection, or instability (see Section 1.4). These types of failure depend on the nature of the load and the type of member.

In elementary mechanics of materials, members subjected to axial loads, bending moments, and torsional forces are studied. Simple formulas for the stress and deflection of such members are developed (Gere, 2001). Some of these formulas are based on simplifying assumptions and as such must be subjected to certain restrictions when extended to new problems. In this book, many of these formulas are used and extended to applications of more complex problems. But first we review, without derivation, some of the basic formulas from mechanics of materials and highlight the limitations to their application. We include a review of bars under axial load, circular rods subjected to torsion, and beams loaded in shear and bending. In the equations that follow, dimensions are expressed in terms of force [F], length [L], and radians [rad].

1.1.1 Axially Loaded Members

Figure 1.1 represents an axially loaded member. It could consist of a rod, bar, or tube,[1] or it could be a member of more general cross section. For such a member, the following elementary formulas apply:

[1]A rod or bar is considered to be a straight member with a solid cross section. A tube is a straight, hollow cylinder.

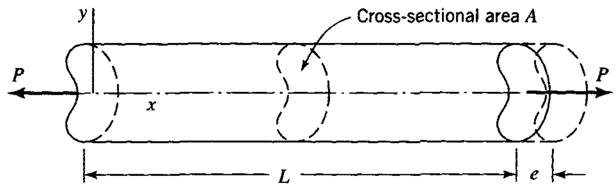

FIGURE 1.1 Axially loaded member.

Axial stress σ away from the ends of the member[2]

$$\sigma = \frac{P}{A} \quad \left[\mathrm{F/L^2}\right] \tag{1.1}$$

Elongation e of the member

$$e = \frac{PL}{AE} \quad [\mathrm{L}] \tag{1.2}$$

Axial strain ϵ in the member

$$\epsilon = \frac{e}{L} = \frac{P}{AE} = \frac{\sigma}{E} \tag{1.3}$$

In the above formulas:

P [F] is the axial load,
A [$\mathrm{L^2}$] is the cross-sectional area of the member,
L [L] is the length of the member, and
E [$\mathrm{F/L^2}$] is the modulus of elasticity of the material of the member.

Restrictions

i. The member must be prismatic (straight and of constant cross section).
ii. The material of the member must be homogeneous (constant material properties at all points throughout the member).
iii. The load P must be directed axially along the centroidal axis of the member.
iv. The stress and strain are restricted to the linearly elastic range (see Figure 1.2).

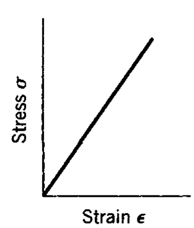

FIGURE 1.2 Linear stress–strain relation.

[2]At the ends, generally depending on how the load P is applied, a stress concentration may exist.

1.1.2 Torsionally Loaded Members

Figure 1.3 represents a straight torsional member with a circular cross section and radius r. Again the member could be a rod, bar, or tube. For such a member, the following elementary formulas apply:

Shear stress τ in the member

$$\tau = \frac{T\rho}{J} \quad \left[\mathrm{F/L^2}\right] \tag{1.4}$$

Rotation (angle of twist) ψ of the cross section B relative to cross section A

$$\psi = \frac{TL}{GJ} \quad [\mathrm{rad}] \tag{1.5}$$

Shear strain γ at a point in the cross section

$$\gamma = \rho\frac{\psi}{L} = \frac{\tau}{G} \tag{1.6}$$

In the above formulas (see Figure 1.3):

T [FL] is the torque or twisting moment,

ρ [L] is the radial distance from the center O of the member to the point of interest,

J [$\mathrm{L^4}$] is the polar moment of inertia of the cross section,

L [L] is the length of the member between A and B, and

G [$\mathrm{F/L^2}$] is the shear modulus of elasticity (also known as the modulus of rigidity) of the material.

Restrictions

i. The member must be prismatic and have a circular cross section.
ii. The material of the member must be homogeneous and linearly elastic.
iii. The torque T is applied at the ends of the member and no additional torque is applied between sections A and B. Also, sections A and B are remote from the member ends.
iv. The angle of twist at any cross section of the member is small.

1.1.3 Bending of Beams

A beam is a structural member whose length is large compared to its cross-sectional dimensions and is loaded by forces and/or moments that produce deflections perpendicular to its longitudinal axis. Figure 1.4*a* represents a beam of rectangular cross section

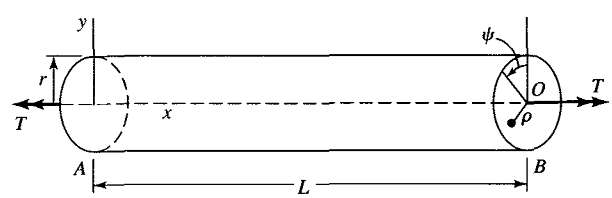

FIGURE 1.3 Circular torsion member.

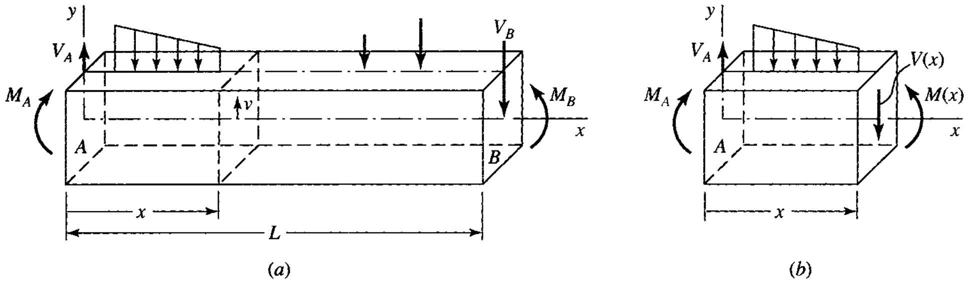

FIGURE 1.4 (*a*) Rectangular cross-section beam. (*b*) Section of length *x*.

subjected to forces and moments. A free-body diagram of a portion of the beam is shown in Figure 1.4*b*. For such a member, the following elementary formulas apply:

Stress σ acting normal to the cross section of the member at section x

$$\sigma = -\frac{M(x)y}{I}\left[\mathrm{F/L^2}\right] \tag{1.7}$$

The displacement v in the y direction is found from the differential expression

$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI} \tag{1.8}$$

Shear stress τ in the cross section at x for $y = y_1$ (see Figure 1.5)

$$\tau = \frac{V(x)Q}{Ib} \tag{1.9}$$

In the above formulas (see Figures 1.4*a*, 1.4*b*, and 1.5):

$M(x)$ [FL] is the positive bending moment at section x in the member,

y [L] is the vertical coordinate, positive upward, from the centroid to the point of interest,

I [$\mathrm{L^4}$] is the moment of inertia of the cross section,

E [$\mathrm{F/L^2}$] is the modulus of elasticity of the material of the member,

$V(x)$ [F] is the shear force at section x in the member,

Q [$\mathrm{L^3}$] is the first moment of the cross-sectional area (shaded in Figure 1.5) above the level $y = y_1$ and is given by

$$Q = \int_{y=y_1}^{y=a/2} y\,dA = \int_{y_1}^{a/2} yb\,dy = \frac{1}{8}b\left(a^2 - 4y_1^2\right)\ \left[\mathrm{L^3}\right] \tag{1.10}$$

and

b [L] is the width of the beam cross section at the level $y = y_1$.

Restrictions

i. Equation 1.7 is limited to bending relative to principal axes and to linear elastic material behavior.

ii. Equation 1.8 is applicable only to small deflections, since only then is d^2v/dx^2 a good approximation for the curvature of the beam.

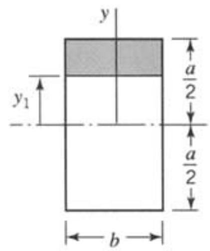

FIGURE 1.5 Beam cross section.

iii. Equations 1.9 and 1.10 are restricted to bending of beams of rectangular cross section relative to principal axes.

1.2 METHODS OF ANALYSIS

In this book, we derive relations between load and stress or between load and deflection for a system or a component (a member) of a system. Our starting point is a description of the loads on the system, the geometry of the system (including boundary conditions), and the properties of the material in the system. Generally the load–stress relations describe either the distributions of normal and shear stresses on a cross section of the member or the stress components that act at a point in the member. For a given member subjected to prescribed loads, the load–stress relations are based on the following requirements:

1. The equations of equilibrium (or equations of motion for bodies not in equilibrium)
2. The compatibility conditions (continuity conditions) that require deformed volume elements in the member to fit together without overlap or tearing
3. The constitutive relations

Two different methods are used to satisfy requirements 1 and 2: the method of mechanics of materials and the method of general continuum mechanics. Often, load–stress and load–deflection relations are not derived in this book by general continuum mechanics methods. Instead, the method of mechanics of materials is used to obtain either exact solutions or reliable approximate solutions. In the method of mechanics of materials, the load–stress relations are derived first. They are then used to obtain load–deflection relations for the member.

A simple member such as a circular shaft of uniform cross section may be subjected to complex loads that produce a multiaxial state of stress. However, such complex loads can be reduced to several simple types of load, such as axial, bending, and torsion. Each type of load, when acting alone, produces mainly one stress component, which is distributed over the cross section of the member. The method of mechanics of materials can be used to obtain load–stress relations for each type of load. If the deformations of the member that result from one type of load do not influence the magnitudes of the other types of loads and if the material remains linearly elastic for the combined loads, the stress components resulting from each type of load can be added together (i.e., the method of superposition may be used).

In a complex member, each load may have a significant influence on each component of the state of stress. Then, the method of mechanics of materials becomes cumbersome, and the use of the method of continuum mechanics may be more appropriate.

1.2.1 Method of Mechanics of Materials

The method of mechanics of materials is based on simplified assumptions related to the geometry of deformation (requirement 2) so that strain distributions for a cross section of the member can be determined. A basic assumption is that plane sections before loading remain plane after loading. The assumption can be shown to be exact for axially loaded members of uniform cross sections, for slender straight torsion members having uniform circular cross sections, and for slender straight beams of uniform cross sections subjected to pure bending. The assumption is approximate for other problems. The method of mechanics of materials is used in this book to treat several advanced beam topics (Chapters 7 to 10). In a similar way, we often assume that lines normal to the middle surface of an undeformed plate remain straight and normal to the middle surface after the load is applied. This assumption is used to simplify the plate problem in Chapter 13.

We review the steps used in the derivation of the flexure formula (Eq. 1.7 of Section 1.1) to illustrate the method of mechanics of materials and to show how the three requirements listed previously are used. Consider a symmetrically loaded straight beam of uniform cross section subjected to a moment M that produces *pure bending* (Figure 1.6*a*). (Note that the plane of loads lies in a plane of symmetry of every cross section of the beam.) We wish to determine the normal stress distribution σ for a specified cross section of the beam. We assume that σ is the major stress component and ignore other effects. Pass a section through the beam at the specified cross section so that the beam is cut into two parts. Consider a free-body diagram of one part (Figure 1.6*b*). The applied moment M for this part of the beam is in equilibrium with internal forces represented by the sum of the forces that result from the normal stress σ that acts over the area of the cut section. Equations of equilibrium (requirement 1) relate the applied moment to internal forces. Since no axial force acts, two integrals are obtained: $\int \sigma\, dA = 0$ and $\int \sigma y\, dA = M$, where M is the applied external moment and y is the perpendicular distance from the neutral axis to the element of area dA.

Before the two integrals can be evaluated, we must know the distribution of σ over the cross section. Since the stress distribution is not known, it is determined indirectly through a strain distribution obtained by requirement 2. The continuity condition is examined by consideration of two cross sections of the undeformed beam separated by an infinitesimal angle $d\theta$ (Figure 1.6*c*). Under the assumption that plane sections remain plane, the cross sections must rotate with respect to each other as the moment M is applied. There is a straight line in each cross section called the neutral axis along which the strains remain zero. Since plane sections remain plane, the strain distribution must vary linearly with the distance y as measured from this neutral axis.

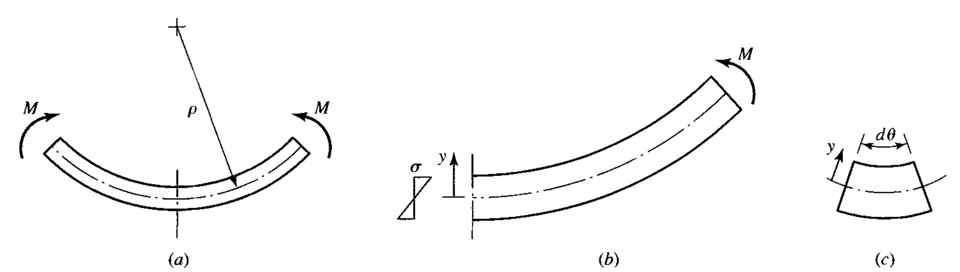

FIGURE 1.6 Pure bending of a long straight beam. (*a*) Circular curvature of beam in pure bending. (*b*) Free-body diagram of cut beam. (*c*) Infinitesimal segment of beam.

Requirement 3 is now employed to obtain the relation between the assumed strain distribution and the stress distribution. Tension and compression stress–strain diagrams represent the response for the material in the beam. For sufficiently small strains, these diagrams indicate that the stresses and strains are linearly related. Their constant ratio, $\sigma/\epsilon = E$, is the modulus of elasticity for the material. In the linear range the modulus of elasticity is the same in tension or compression for many engineering materials. Since other stress components are neglected, σ is the only stress component in the beam. Hence, the stress–strain relation for the beam is $\sigma = E\epsilon$. Therefore, both the stress σ and strain ϵ vary linearly with the distance y as measured from the neutral axis of the beam (Figure 1.6). The equations of equilibrium can be integrated to obtain the flexure formula $\sigma = My/I$, where M is the applied moment at the given cross section of the beam and I is the moment of inertia of the beam cross section.

1.2.2 Method of Continuum Mechanics and the Theory of Elasticity

Many of the problems treated in this book have multiaxial states of stress of such complexity that the mechanics of materials method cannot be employed to derive load–stress and load–deflection relations. Therefore, in such cases, the method of continuum mechanics is used. When we consider small displacements and linear elastic material behavior only, the general method of continuum mechanics reduces to the method of the theory of linear elasticity.

In the derivation of load–stress and load–deflection relations by the theory of linear elasticity, an infinitesimal volume element at a point in a body with faces normal to the coordinate axes is often employed. Requirement 1 is represented by the differential equations of equilibrium (Chapter 2). Requirement 2 is represented by the differential equations of compatibility (Chapter 2). The material response (requirement 3) for linearly elastic behavior is determined by one or more experimental tests that define the required elastic coefficients for the material. In this book we consider mainly isotropic materials for which only two elastic coefficients are needed. These coefficients can be obtained from a tension specimen if both axial and lateral strains are measured for every load applied to the specimen. Requirement 3 is represented therefore by the isotropic stress–strain relations developed in Chapter 3. If the differential equations of equilibrium and the differential equations of compatibility can be solved subject to specified stress–strain relations and specified boundary conditions, the states of stress and displacements for every point in the member are obtained.

1.2.3 Deflections by Energy Methods

Certain structures are made up of members whose cross sections remain essentially plane during the deflection of the structures. The deflected position of a cross section of a member of the structure is defined by three orthogonal displacement components of the centroid of the cross section and by three orthogonal rotation components of the cross section. These six components of displacement and rotation of a cross section of a member are readily calculated by energy methods. For small displacements and small rotations and for linearly elastic material behavior, Castigliano's theorem is effective as a method for the computation of the displacements and rotations. The method is employed in Chapter 5 for structures made up of axially loaded members, beams, and torsion members, and in Chapter 9 for curved beams.

1.3 STRESS-STRAIN RELATIONS

To derive load–stress and load–deflection relations for specified structural members, the stress components must be related to the strain components. Consequently, in Chapter 3 we discuss linear stress–strain–temperature relations. These relations may be employed in the study of linearly elastic material behavior. In addition, they are employed in plasticity theories to describe the linearly elastic part of the total response of materials.

Because experimental studies are required to determine material properties (e.g., elastic coefficients for linearly elastic materials), the study of stress–strain relations is, in part, empirical. To obtain needed isotropic elastic material properties, we employ a tension specimen (Figure 1.7). If lateral as well as longitudinal strains are measured for linearly elastic behavior of the tension specimen, the resulting stress–strain data represent the material response for obtaining the needed elastic constants for the material. The fundamental elements of the stress–strain–temperature relations, however, are studied theoretically by means of the first law of thermodynamics (Chapter 3).

The stress–strain–temperature relations presented in Chapter 3 are limited mainly to small strains and small rotations. The reader interested in large strains and large rotations may refer to Boresi and Chong (2000).

1.3.1 Elastic and Inelastic Response of a Solid

Initially, we review the results of a simple tension test of a circular cylindrical bar that is subjected to an axially directed tensile load P (Figure 1.7). It is assumed that the load is monotonically increased slowly (so-called static loading) from its initial value of zero load to its final value, since the material response depends not only on the magnitude of the load but also on other factors, such as the rate of loading, load cycling, etc.

It is customary in engineering practice to plot the tensile stress σ in the bar as a function of the strain ϵ of the bar. In engineering practice, it is also customary to assume that the stress σ is uniformly distributed over the cross-sectional area of the bar and that it is equal in magnitude to P/A_0, where A_0 is the original cross-sectional area of the bar. Similarly, the strain ϵ is assumed to be constant over the gage length L and equal to $\Delta L/L = e/L$, where $\Delta L = e$ (Figure 1.7b) is the change or elongation in the original gage length L (the

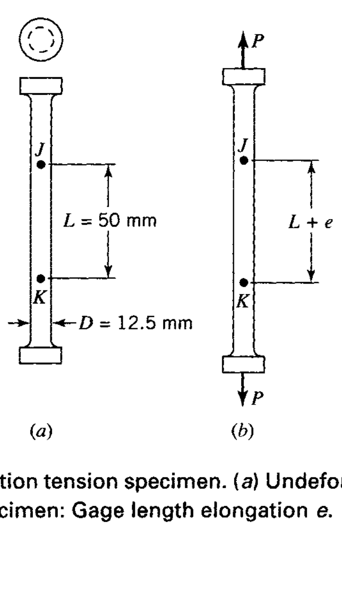

FIGURE 1.7 Circular cross section tension specimen. (a) Undeformed specimen: Gage length L; diameter D. (b) Deformed specimen: Gage length elongation e.

distance JK in Figure 1.7a). For these assumptions to be valid, the points J and K must be sufficiently far from the ends of the bar (a distance of one or more diameters D from the ends).

According to the definition of stress (Section 2.1), the true stress is $\sigma_t = P/A_t$, where A_t is the true cross-sectional area of the bar when the load P acts. (The bar undergoes lateral contraction everywhere as it is loaded, with a corresponding change in cross-sectional area.) The difference between $\sigma = P/A_0$ and $\sigma_t = P/A_t$ is small, provided that the elongation e and, hence, the strain ϵ are sufficiently small (Section 2.8). If the elongation is large, A_t may differ significantly from A_0. In addition, the instantaneous or true gage length when load P acts is $L_t = L + e$ (Figure 1.7b). Hence, the true gage length L_t also changes with the load P. Corresponding to the true stress σ_t, we may define the true strain ϵ_t as follows: In the tension test, assume that the load P is increased from zero (where $e = 0$) by successive infinitesimal increments dP. With each incremental increase dP in load P, there is a corresponding infinitesimal increase dL_t in the instantaneous gage length L_t. Hence, the infinitesimal increment $d\epsilon_t$ of the true strain ϵ_t resulting from dP is

$$d\epsilon_t = \frac{dL_t}{L_t} \tag{1.11}$$

Integration of Eq. 1.11 from L to L_t yields the true strain ϵ_t. Thus, we have

$$\epsilon_t = \int_L^{L_t} d\epsilon_t = \ln\left(\frac{L_t}{L}\right) = \ln\left(\frac{L+e}{L}\right) = \ln(1+\epsilon) \tag{1.12}$$

In contrast to the engineering strain ϵ, the true strain ϵ_t is not linearly related to the elongation e of the original gage length L. (Compare Eqs. 1.3 and 1.12.)

For many structural metals (e.g., alloy steels), the stress–strain relation of a tension specimen takes the form shown in Figure 1.8. This figure is the *tensile stress–strain diagram* for the material. The graphical stress–strain relation (the curve $0ABCF$ in Figure 1.8) was obtained by drawing a smooth curve through the tension test data for a certain alloy steel. Engineers use stress–strain diagrams to define certain properties of the material that are judged to be significant in the safe design of a statically loaded member. Some of

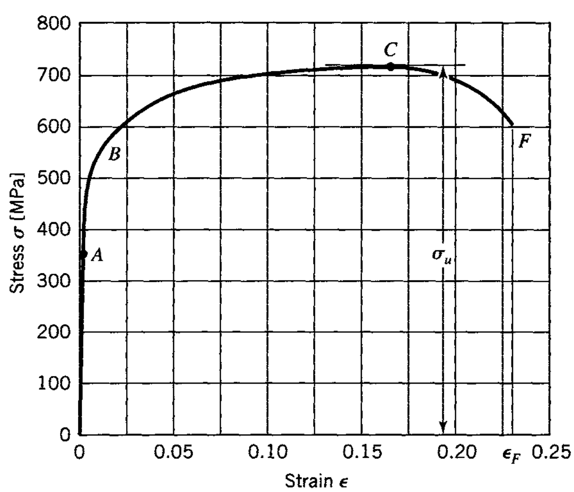

FIGURE 1.8 Engineering stress–strain diagram for tension specimen of alloy steel.

these special properties are discussed briefly in the following section. In addition, certain general material responses are addressed.

1.3.2 Material Properties

A tensile stress–strain diagram is used by engineers to determine specific material properties used in design. There are also general characteristic behaviors that are somewhat common to all materials. To describe these properties and characteristics, it is convenient to expand the strain scale of Figure 1.8 in the region $0AB$ (Figure 1.9). Recall that Figures 1.8 and 1.9 are based on the following definitions of stress and strain: $\sigma = P/A_0$ and $\epsilon = e/L$, where A_0 and L are constants.

Consider a tensile specimen (bar) subjected to a strain ϵ under the action of a load P. If the strain in the bar returns to zero as the load P goes to zero, the material in the bar is said to have been strained within the *elastic limit* or the material has remained *perfectly elastic*. If under loading the strain is linearly proportional to the load P (part $0A$ in Figures 1.8 and 1.9), the material is said to be strained within the limit of *linear elasticity*. The maximum stress for which the material remains perfectly elastic is frequently referred to simply as the *elastic limit* σ_{EL}, whereas the stress at the limit of linear elasticity is referred to as the *proportional limit* σ_{PL} (point A in Figures 1.8 and 1.9).

Ordinarily, σ_{EL} is larger than σ_{PL}. The properties of elastic limit and proportional limit, although important from a theoretical viewpoint, are not of practical significance for materials like alloy steels. This is because the transitions from elastic to inelastic behavior and from linear to nonlinear behavior are so gradual that these limits are very difficult to determine from the stress–strain diagram (part $0AB$ of the curves in Figures 1.8 and 1.9).

When the load produces a stress σ that exceeds the elastic limit (e.g., the stress at point J in Figure 1.9), the strain does not disappear upon unloading (curve JK in Figure 1.9). A *permanent strain* ϵ_P remains. For simplicity, it is assumed that the unloading occurs along the straight line JK, with a slope equal to that of the straight line $0A$. The

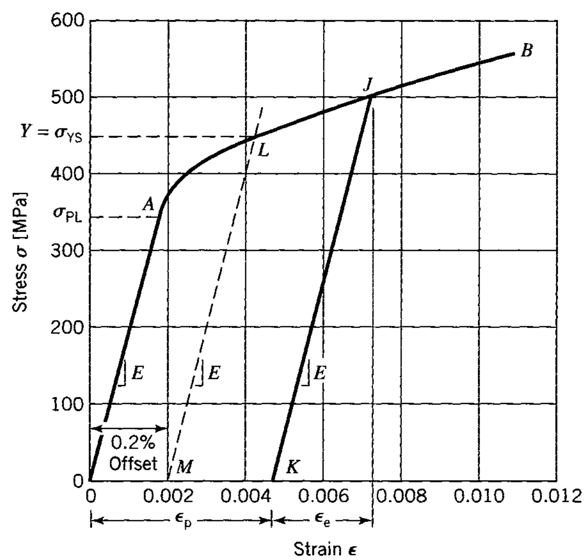

FIGURE 1.9 Engineering stress–strain diagram for tension specimen of alloy steel (expanded strain scale).

strain that is recovered when the load is removed is called the *elastic strain* ϵ_e. Hence, the total strain ϵ at point J is the sum of the permanent strain and elastic strain, or $\epsilon = \epsilon_P + \epsilon_e$.

Yield Strength

The value of stress associated with point L (Figure 1.9) is called the *yield strength* and is denoted by σ_{YS} or simply by Y. The yield strength is determined as the stress associated with the intersection of the curve $0AB$ and the straight line LM drawn from the offset strain value, with a slope equal to that of line $0A$ (Figure 1.9). The value of the offset strain is arbitrary. However, a commonly agreed upon value of offset is 0.002 or 0.2% strain, as shown in Figure 1.9. Typical values of yield strength for several structural materials are listed in Appendix A, for an offset of 0.2%. For materials with stress–strain curves like that of alloy steels (Figures 1.8 and 1.9), the yield strength is used to predict the load that initiates inelastic behavior (yield) in a member.

Ultimate Tensile Strength

Another important property determined from the stress–strain diagram is the *ultimate tensile strength* or *ultimate tensile stress* σ_u. It is defined as the maximum stress attained in the engineering stress–strain diagram, and in Figure 1.8 it is the stress associated with point C. As seen from Figure 1.8, the stress increases continuously beyond the elastic region $0A$, until point C is reached. As the material is loaded beyond its yield stress, it maintains an ability to resist additional strain with an increase in stress. This response is called *strain hardening*. At the same time the material loses cross-sectional area owing to its elongation. This area reduction has a softening (strength loss) effect, measured in terms of initial area A_0. Before point C is reached, the strain-hardening effect is greater than the loss resulting from area reduction. At point C, the strain-hardening effect is balanced by the effect of the area reduction. From point C to point F, the weakening effect of the area reduction dominates, and the engineering stress decreases, until the specimen ruptures at point F.

Modulus of Elasticity

In the straight-line region $0A$ of the stress–strain diagram, the stress is proportional to strain; that is, $\sigma = E\epsilon$. The constant of proportionality E is called the *modulus of elasticity*. It is also referred to as Young's modulus. Geometrically, it is equal in magnitude to the slope of the stress–strain relation in the region $0A$ (Figure 1.9).

Percent Elongation

The value of the elongation e_F of the gage length L at rupture (point F, Figure 1.8) divided by the gage length L (in other words, the value of strain ϵ_F at rupture) multiplied by 100 is referred to as the *percent elongation* of the tensile specimen. The percent elongation is a measure of the *ductility* of the material. From Figure 1.8, we see that the percent elongation of the alloy steel is approximately 23%.

An important metal for structural applications, mild or structural steel, has a distinct stress–strain curve as shown in Figure 1.10*a*. The portion $0AB$ of the stress–strain diagram is shown expanded in Figure 1.10*b*. The stress–strain diagram for structural steel usually exhibits a so-called upper yield point, with stress σ_{YU}, and a lower yield point, with stress σ_{YL}. This is because the stress required to initiate yield in structural steel is larger than the stress required to continue the yielding process. At the lower yield the stress remains essentially constant for increasing strain until strain hardening causes the curve to rise (Figure 1.10*a*). The constant or flat portion of the stress–strain diagram may extend over a strain range of 10 to 40 times the strain at the yield point. Actual test data indicate that the curve from A to B bounces up and down. However, for simplicity, the data are represented by a horizontal straight line.

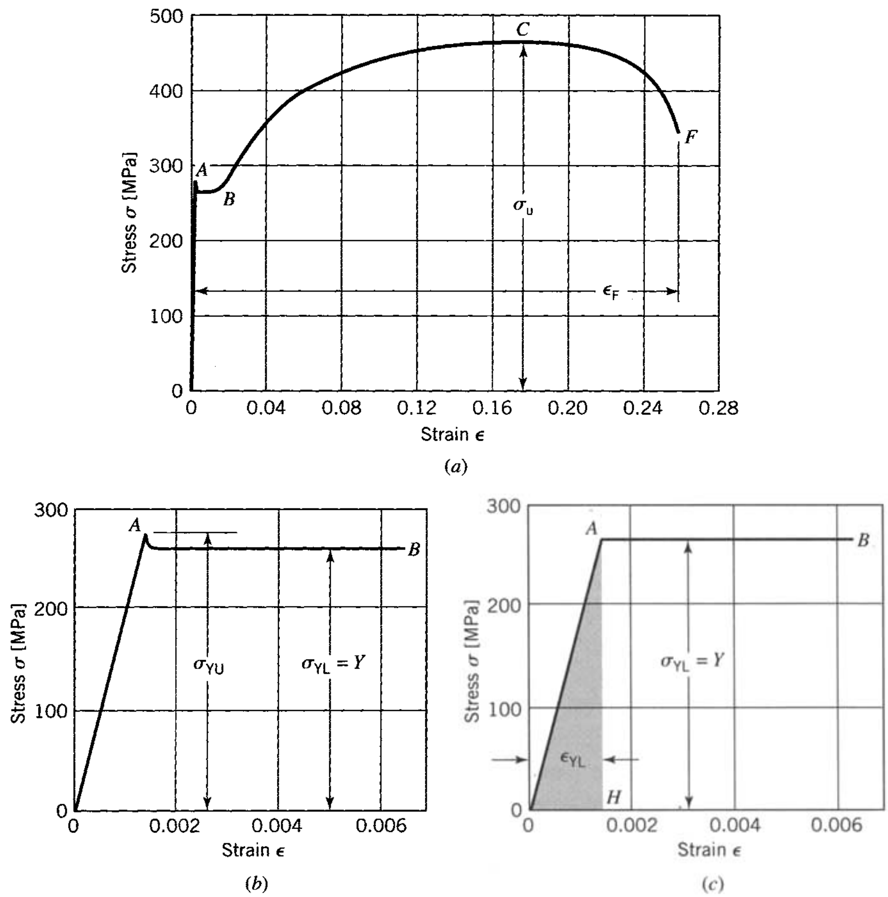

FIGURE 1.10 Engineering stress–strain diagram for tension specimen of structural steel. (*a*) Stress–strain diagram. (*b*) Diagram for small strain ($\epsilon < 0.007$). (*c*) Idealized diagram for small strain ($\epsilon < 0.007$).

Yield Point for Structural Steel

The upper yield point is usually ignored in design, and it is assumed that the stress initiating yield is the lower yield point stress, σ_{YL}. Consequently, for simplicity, the stress–strain diagram for the region $0AB$ is idealized as shown in Figure 1.10*c*. Also for simplicity, we shall refer to the yield point stress as the *yield point* and denote it by the symbol Y. Recall that the yield strength (or yield stress) for alloy steel, and for materials such as aluminum alloys that have similar stress–strain diagrams, was also denoted by Y (Figure 1.9).

Modulus of Resilience

The *modulus of resilience* is a measure of energy per unit volume (energy density) absorbed by a material up to the time it yields under load and is represented by the area under the stress–strain diagram to the yield point (the shaded area $0AH$ in Figure 1.10*c*). In Figure 1.10*c*, this area is given by $\frac{1}{2}\sigma_{YL}\epsilon_{YL}$. Since $\epsilon_{YL} = \sigma_{YL}/E$, and with the notation $Y = \sigma_{YL}$, we may express the modulus of resilience as follows:

$$\text{modulus of resilience} = \frac{1}{2}\frac{Y^2}{E} \tag{1.13}$$

Modulus of resilience is an important property for differentiating among materials for applications in which energy absorption is critical.

Modulus of Toughness

The *modulus of toughness* U_F is a measure of the ability of a material to absorb energy prior to fracture. It represents the strain energy per unit volume (strain-energy density) in the material at fracture. The strain-energy density is equal to the area under the stress–strain diagram to fracture (point F on curves $0ABCF$ in Figures 1.8 and 1.10*a*). The larger the modulus of toughness is, the greater is the ability of a material to absorb energy without fracturing. A large modulus of toughness is important if a material is not to fail under impact or seismic loads.

Modulus of Rupture

The *modulus of rupture* is the maximum tensile or compressive stress in the extreme fiber of a beam loaded to failure in bending. Hence, modulus of rupture is measured in a bending test, rather than in a tension test. It is analogous to the ultimate strength of a material, but it does not truly represent the maximum bending stress for a material because it is determined with the bending formula $\sigma = My/I$, which is valid only in the linearly elastic range for a material. Consequently, modulus of rupture normally overpredicts the actual maximum bending stress at failure in bending. Modulus of rupture is used for materials that do not exhibit large plastic deformation, such as wood or concrete.

Poisson's Ratio

Poisson's ratio is a dimensionless measure of the lateral strain that occurs in a member owing to strain in its loaded direction. It is found by measuring both the axial strain ϵ_a and the lateral strain ϵ_l in a uniaxial tension test and is given by the value

$$v = -\frac{\epsilon_l}{\epsilon_a} \tag{1.14}$$

In the elastic range, Poisson's ratio lies between 0.25 and 0.33 for most engineering materials.

Necking of a Mild Steel Tension Specimen

As noted previously, the stress–strain curve for a mild steel tension specimen first reaches a local maximum called the upper yield or plastic limit σ_{YU}, after which it drops to a local minimum (the lower yield point Y) and runs approximately (in a wavy fashion) parallel to the strain axis for some range of strain. For mild steel, the lower yield point stress Y is assumed to be the stress at which yield is initiated. After some additional strain, the stress rises gradually; a relatively small change in load causes a significant change in strain. In this region (BC in Figure 1.10*a*), substantial differences exist in the stress–strain diagrams, depending on whether area A_0 or A_t is used in the definition of stress. With area A_0, the curve first rises rapidly and then slowly, turning with its concave side down and attaining a maximum value σ_u, the ultimate strength, before turning down rapidly to fracture (point F, Figure 1.10*a*). Physically, after σ_u is reached, *necking* of the bar occurs (Figure 1.11). This necking is a drastic reduction of the cross-sectional area of the bar in the region where the fracture ultimately occurs.

If the load P is referred to the true cross-sectional area A_t and, hence, $\sigma_t = P/A_t$, the true stress–strain curve differs considerably from the engineering stress–strain curve in the region BC (Figures 1.10*a* and 1.12). In addition, the engineering stress–strain curves for

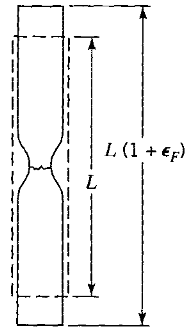

FIGURE 1.11 Necking of tension specimen.

tension and compression differ considerably in the plastic region (Figure 1.12), because of the fact that in tension the cross-sectional area decreases with increasing load, whereas in compression it increases with increasing load. However, as can be seen from Figure 1.12, little differences exist between the curves for small strains ($\epsilon_t < 0.01$).

Equation 1.12 remains valid until necking of the tension specimen occurs. Once necking beings, the engineering strain ϵ is no longer constant in the gage length (see Figures 1.7 and 1.11). However, a good approximation of the true strain may be obtained from the fact that the volume of the specimen remains nearly constant as necking occurs.

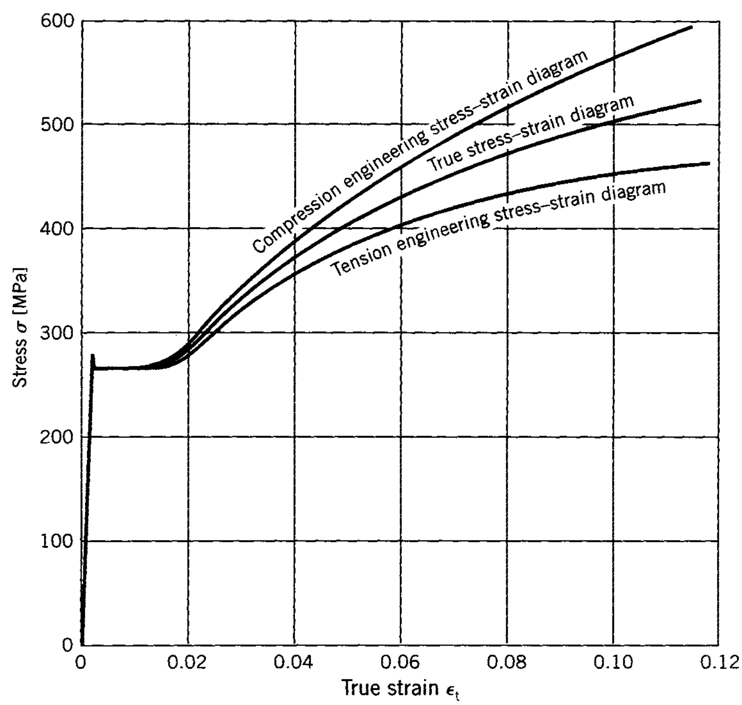

FIGURE 1.12 Comparison of tension and compression engineering stress–strain diagrams with the true stress–strain diagram for structural steel.

Thus, if the volume of the specimen in the gage length remains constant, before necking we have

$$A_0L = A_t(L+e) \tag{a}$$

or, after dividing by A_tL, we obtain for a bar of circular cross section and initial diameter D_0

$$\epsilon = \frac{e}{L} = \frac{A_0}{A_t} - 1 = \frac{D_0^2}{D_t^2} - 1 \tag{b}$$

where D_t is the true diameter of the bar.

Substitution of Eq. (b) into Eq. 1.12 yields

$$\epsilon_t = \ln\frac{A_0}{A_t} = \ln\frac{D_0^2}{D_t^2} = 2\ln\frac{D_0}{D_t} \tag{1.15}$$

By measurement of the true diameter at the minimum cross section of the bar in the necked region (Figure 1.11), we obtain a good approximation of the true strain in the necked region up to fracture.

Other Materials

There are many materials whose tensile specimens do not undergo substantial plastic strain before fracture. These materials are called *brittle materials*. A stress–strain diagram typical of brittle materials is shown in Figure 1.13. It exhibits little plastic range, and fracture occurs almost immediately at the end of the elastic range. In contrast, there are materials that undergo extensive plastic deformation and little elastic deformation. Lead and clay are such materials. The idealized stress–strain diagram for clay is typical of such materials (Figure 1.14). This response is referred to as *rigid-perfectly plastic*.

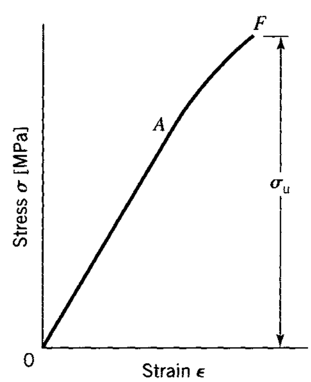

FIGURE 1.13 Stress–strain diagram for a brittle material.

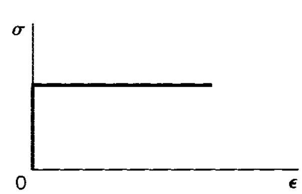

FIGURE 1.14 Stress–strain diagram for clay.

EXAMPLE 1.1
Material Properties for Alloy and Structural Steel

(a) By Figures 1.8 and 1.9, estimate the yield strength Y and the ultimate strength σ_u of a tension rod of alloy steel.

(b) By Figures 1.8 and 1.10*a*, estimate the modulus of toughness U_F for alloy steel and structural steel.

Solution

(a) By measurement of Figure 1.8,

$$\sigma_u \approx 700 + \frac{1}{6}(100) = 717 \text{ MPa}$$

By measurement of Figure 1.9,

$$Y \approx 450 \text{ MPa}$$

(b) By Figure 1.8, we estimate the number of squares (a square consists of 100-MPa stress by 0.025 strain) under the curve 0*ABCF* to be approximately 62. Hence,

$$U_F = 62(100)0.025 = 155 \times 10^6 \text{ N} \cdot \text{m/m}^3$$

Similarly, by Figure 1.10*a*, we estimate the number of squares (here a square consists of 100-MPa stress by 0.04 strain) under the curve 0*ABCF* to be 27. Hence,

$$U_F = 27(100)0.04 = 108 \times 10^6 \text{ N} \cdot \text{m/m}^3$$

EXAMPLE 1.2
Tension Rod: Modulus of Elasticity, Permanent Strain, and Elastic Strain

A rod of alloy steel (Figure 1.9) is subjected to an axial tension load that produces a stress of $\sigma = 500$ MPa and an associated strain of $\epsilon_{500} = 0.0073$. Assume that elastic unloading occurs.

(a) Determine the modulus of elasticity of the rod.

(b) Determine the permanent strain in the rod and the strain that is recovered as the rod is unloaded.

Solution

(a) By inspection of Figure 1.9, the stress and strain at point A are $\sigma_A = 343$ MPa and $\epsilon_A = 0.00172$. Hence, the modulus of elasticity is

$$E = \frac{\sigma_A}{\epsilon_A} = \frac{343}{0.00172} = 199 \text{ GPa} \tag{a}$$

(b) By Figure 1.9 and Eq. (a), the elastic strain (recovered strain) is

$$\epsilon_e = \frac{\sigma}{E} = \frac{500 \text{ MPa}}{199 \text{ GPa}} = 0.0025$$

The strain at $\sigma = 500$ MPa is (from Figure 1.9)

$$\epsilon_{500} = 0.0073$$

Hence, the permanent strain is

$$\epsilon_p = \epsilon_{500} - \epsilon_e = 0.0073 - 0.0025 = 0.0048$$

1.4 FAILURE AND LIMITS ON DESIGN

To design a structural system to perform a given function, the designer must have a clear understanding of the possible ways or modes by which the system may fail to perform its function. The designer must determine the possible *modes of failure* of the system and then establish suitable *failure criteria* that accurately predict the failure modes.

In general, the determination of modes of failure requires extensive knowledge of the response of a structural system to loads. In particular, it requires a comprehensive stress analysis of the system. Since the response of a structural system depends strongly on the material used, so does the mode of failure. In turn, the mode of failure of a given material also depends on the manner or history of loading, such as the number of cycles of load applied at a particular temperature. Accordingly, suitable failure criteria must account for different materials, different loading histories, and factors that influence the stress distribution in the member.

A major part of this book is concerned with 1. stress analysis, 2. material behavior under load, and 3. the relationship between the mode of failure and a critical parameter associated with failure. The critical parameter that signals the onset of failure might be stress, strain, displacement, load, and number of load cycles or a combination of these. The discussion in this book is restricted to situations in which failure of a system is related to only a single critical parameter. In addition, we will examine the accuracy of the theories presented in the text with regard to their ability to predict system behavior. In particular, limits on design will be introduced utilizing factors of safety or reliability-based concepts that provide a measure of safety against failure.

Historically, limits on the design of a system have been established using a *factor of safety*. A factor of safety SF can be defined as

$$SF = \frac{R_{\mathrm{n}}}{R_{\mathrm{w}}} \tag{1.16}$$

where R_{n} is the nominal resistance (the critical parameter associated with failure) and R_{w} is the safe *working* magnitude of that same parameter. The letter R is used to represent the *resistance* of the system to failure. Generally, the magnitude of R_{n} is based on theory or experimental observation. The factor of safety is chosen on the basis of experiments or experience with similar systems made of the same material under similar loading conditions. Then the safe working parameter R_{w} is determined from Eq. 1.16. The factor of safety must account for unknowns, including variability of the loads, differences in material properties, deviations from the intended geometry, and our ability to predict the critical parameter.

In industrial applications, the magnitude of the factor of safety SF may range from just above 1.0 to 3.0 or more. For example, in aircraft and space vehicle design, where it is critical to reduce the weight of the vehicle as much as possible, the SF may be nearly 1.0. In the nuclear reactor industry, where safety is of prime importance in the face of many unpredictable effects, SF may be as high as 5.

Generally, a *design inequality* is employed to relate load effects to resistance. The design inequality is defined as

$$\sum_{i}^{N} Q_i \le \frac{R_{\mathrm{n}}}{SF} \tag{1.17}$$

where each Q_i represents the effect of a particular working (or service-level) load, such as internal pressure or temperature change, and N denotes the number of load types considered.

Design philosophies based on *reliability concepts* (Harr, 1987; Cruse, 1997) have been developed. It has been recognized that a single factor of safety is inadequate to account for all the unknowns mentioned above. Furthermore, each of the particular load types will exhibit its own statistical variability. Consequently, appropriate load and resistance factors are applied to both sides of the design inequality. So modified, the design inequality of Eq. 1.17 may be reformulated as

$$\sum_{i}^{N} \gamma_i Q_i \leq \phi R_n \tag{1.18}$$

where the γ_i are the load factors for load effects Q_i and ϕ is the resistance factor for the nominal capacity R_n. The statistical variation of the individual loads is accounted for in γ_i, whereas the variability in resistance (associated with material properties, geometry, and analysis procedures) is represented by ϕ. The use of this approach, known as *limit-states design*, is more rational than the factor-of-safety approach and produces a more uniform reliability throughout the system.

A *limit state* is a condition in which a system, or component, ceases to fulfill its intended function. This definition is essentially the same as the definition of *failure* used earlier in this text. However, some prefer the term limit state because the term failure tends to imply only some catastrophic event (brittle fracture), rather than an inability to function properly (excessive elastic deflections or brittle fracture). Nevertheless, the term failure will continue to be used in this book in the more general context.

EXAMPLE 1.3
Design of a Tension Rod

A steel rod is used as a tension brace in a structure. The structure is subjected to dead load (the load from the structure itself), live load (the load from the structure's contents), and wind load. The effect of each of the individual loads on the tension brace is $D = 25$ kN, $L = 60$ kN, and $W = 30$ kN. Select a circular rod of appropriate size to carry these loads safely. Use steel with a yield strength of 250 MPa. Make the selection using (a) factor-of-safety design and (b) limit-states design.

Solution

For simplicity in this example, the only limit state that will be considered is yielding of the cross section. Other limit states, including fracture and excessive elongation, are ignored.

(a) In factor-of-safety design (also known as *allowable stress* or *working stress* design), the load effects are added without load factors. Thus, the total service-level load is

$$\sum Q_i = D + L + W = 115 \text{ kN} = 115{,}000 \text{ N} \tag{a}$$

The nominal resistance (capacity) of the tension rod is

$$R_n = YA_g = (250)A_g \tag{b}$$

where A_g is the gross area of the rod. In the design of tension members for steel structures, a factor of safety of 5/3 is used (AISC, 1989). Hence, the design inequality is

$$115{,}000 \leq \frac{250A_g}{\frac{5}{3}} \tag{c}$$

which yields $A_g \geq 767$ mm^2. A rod of 32 mm in diameter, with a cross-sectional area of 804 mm^2, is adequate.

(b) In limit-states design, the critical load effect is determined by examination of several possible load combination equations. These equations represent the condition in which a single load quantity is at its maximum lifetime value, whereas the other quantities are taken at an arbitrary point in time. The relevant load combinations for this situation are specified (ASCE, 2000) as

$$1.4D \tag{d}$$

$$1.2D + 1.6L \tag{e}$$

$$1.2D + 0.5L + 1.6W \tag{f}$$

For the given load quantities, combination (e) is critical. The total load effect is

$$\sum \gamma_i Q_i = 126 \text{ kN} = 126{,}000 \text{ N} \tag{g}$$

In the design of tension members for steel structures, a resistance factor of $\phi = 0.9$ is used (AISC, 2001). Hence, the limit-states design inequality is

$$126{,}000 \leq 0.9(250 A_g) \tag{h}$$

which yields $A_g \geq 560$ mm^2. A rod 28 mm in diameter, with a cross-sectional area of 616 mm^2, is adequate.

Discussion

The objective of this example has been to demonstrate the use of different design philosophies through their respective design inequalities, Eqs. 1.17 and 1.18. For the conditions posed, the limit-states approach produces a more economical design than the factor-of-safety approach. This can be attributed to the recognition in the load factor equations (d–f) that it is highly unlikely both live load and wind load would reach their maximum lifetime values at the same time. Different combinations of dead load, live load, and wind load, which still give a total service-level load of 115 kN, could produce different factored loads and thus different area requirements for the rod under limit-states design.

1.4.1 Modes of Failure

When a structural member is subjected to loads, its response depends not only on the type of material from which it is made but also on the environmental conditions and the manner of loading. Depending on how the member is loaded, it may fail by *excessive deflection*, which results in the member being unable to perform its design function; it may fail by *plastic deformation* (*general yielding*), which may cause a permanent, undesirable change in shape; it may fail because of a *fracture* (break), which depending on the material and the nature of loading may be of a *ductile type* preceded by appreciable plastic deformation or of a *brittle type* with little or no prior plastic deformation. *Fatigue failure*, which is the progressive growth of one or more cracks in a member subjected to repeated loads, often culminates in a brittle fracture type of failure.

Another manner in which a structural member may fail is by elastic or plastic instability. In this failure mode, the structural member may undergo large displacements from its design configuration when the applied load reaches a critical value, the *buckling load* (or *instability load*). This type of failure may result in excessive displacement or loss of ability (because of yielding or fracture) to carry the design load. In addition to the failure modes already mentioned, a structural member may fail because of environmental corrosion (chemical action).

To elaborate on the modes of failure of structural members, we discuss more fully the following categories of failure modes:

1. Failure by excessive deflection
   a. Elastic deflection
   b. Deflection caused by creep
2. Failure by general yielding
3. Failure by fracture
   a. Sudden fracture of brittle materials
   b. Fracture of cracked or flawed members
   c. Progressive fracture (fatigue)
4. Failure by instability

These failure modes and their associated failure criteria are most meaningful for simple structural members (e.g., tension members, columns, beams, circular cross section torsion members). For more complicated two- and three-dimensional problems, the significance of such simple failure modes is open to question.

Many of these modes of failure for simple structural members are well known to engineers. However, under unusual conditions of load or environment, other types of failure may occur. For example, in nuclear reactor systems, cracks in pipe loops have been attributed to stress-assisted corrosion cracking, with possible side effects attributable to residual welding stresses (Clarke and Gordon, 1973; Hakala et al., 1990; Scott and Tice, 1990).

The physical action in a structural member leading to failure is usually a complicated phenomenon, and in the following discussion the phenomena are necessarily oversimplified, but they nevertheless retain the essential features of the failures.

1. Failure by Excessive Elastic Deflection

The maximum load that may be applied to a member without causing it to cease to function properly may be limited by the permissible elastic strain or deflection of the member. Elastic deflection that may cause damage to a member can occur under these different conditions:

a. Deflection under conditions of stable equilibrium, such as the stretch of a tension member, the angle of twist of a shaft, and the deflection of an end-loaded cantilever beam. Elastic deflections, under conditions of equilibrium, are computed in Chapter 5.

b. Buckling, or the rather sudden deflection associated with unstable equilibrium and often resulting in total collapse of the member. This occurs, for example, when an axial load, applied gradually to a slender column, exceeds the Euler load. See Chapter 12.

c. Elastic deflections that are the amplitudes of the vibration of a member sometimes associated with failure of the member resulting from objectionable noise, shaking forces, collision of moving parts with stationary parts, etc., which result from the vibrations.

When a member fails by elastic deformation, the significant equations for design are those that relate loads and elastic deflection. For example, the elementary mechanics of materials equations, for the three members mentioned under condition (a), are $e = PL/AE$, $\theta = TL/GJ$, and $v = PL^3/3EI$. It is noted that these equations contain the significant property of the material involved in the elastic deflection, namely, the modulus of elasticity E (sometimes called the stiffness) or the shear modulus $G = E/[2(1 + \nu)]$, where ν is Poisson's ratio.

The stresses caused by the loads are not the significant quantities; that is, the stresses do not limit the loads that can be applied to the member. In other words, if a member of given dimensions fails to perform its load-resisting function because of excessive elastic deflection, its load-carrying capacity is not increased by making the member of stronger material. As a rule, the most effective method of decreasing the deflection of a member is by changing the shape or increasing the dimensions of its cross section, rather than by making the member of a stiffer material.

2. Failure by General Yielding

Another condition that may cause a member to fail is general yielding. General yielding is inelastic deformation of a considerable portion of the member, distinguishing it from localized yielding of a relatively small portion of the member. The following discussion of yielding addresses the behavior of metals at ordinary temperatures, that is, at temperatures that do not exceed the recrystallization temperature. Yielding at elevated temperatures (creep) is discussed in Chapter 18.

Polycrystalline metals are composed of extremely large numbers of very small units called crystals or grains. The crystals have slip planes on which the resistance to shear stress is relatively small. Under elastic loading, before slip occurs, the crystal itself is distorted owing to stretching or compressing of the atomic bonds from their equilibrium state. If the load is removed, the crystal returns to its undistorted shape and no permanent deformation exists. When a load is applied that causes the yield strength to be reached, the crystals are again distorted but, in addition, defects in the crystal, known as dislocations (Eisenstadt, 1971), move in the slip planes by breaking and reforming atomic bonds. After removal of the load, only the distortion of the crystal (resulting from bond stretching) is recovered. The movement of the dislocations remains as permanent deformation.

After sufficient yielding has occurred in some crystals at a given load, these crystals will not yield further without an increase in load. This is due to the formation of dislocation entanglements that make motion of the dislocations more and more difficult. A higher and higher stress will be needed to push new dislocations through these entanglements. This increased resistance that develops after yielding is known as *strain hardening* or *work hardening*. Strain hardening is permanent. Hence, for strain-hardening metals, the plastic deformation and increase in yield strength are both retained after the load is removed.

When failure occurs by general yielding, stress concentrations usually are *not* significant because of the interaction and adjustments that take place between crystals in the regions of the stress concentrations. Slip in a few highly stressed crystals does not limit the general load-carrying capacity of the member but merely causes readjustment of stresses that permit the more lightly stressed crystals to take higher stresses. The stress distribution approaches that which occurs in a member free from stress concentrations. Thus, the member as a whole acts substantially as an ideal homogeneous member, free from abrupt changes of section.

It is important to observe that, if a member that fails by yielding is replaced by one with a material of a higher yield stress, the mode of failure may change to that of elastic deflection, buckling, or excessive mechanical vibrations. Hence, the entire basis of design may be changed when conditions are altered to prevent a given mode of failure.

3. Failure by Fracture

Some members cease to function satisfactorily because they break (fracture) before either excessive elastic deflection or general yielding occurs. Three rather different modes or mechanisms of fracture that occur especially in metals are now discussed briefly.

a. **Sudden Fracture of Brittle Material.** Some materials—so-called brittle materials—function satisfactorily in resisting loads under static conditions until the material breaks rather suddenly with little or no evidence of plastic deformation. Ordinarily, the tensile stress in members made of such materials is considered to be the significant quantity associated with the failure, and the ultimate strength σ_u is taken as the measure of the maximum utilizable strength of the material (Figure 1.13).

b. **Fracture of Flawed Members.** A member made of a ductile metal and subjected to static tensile loads will not fracture in a brittle manner as long as the member is free of flaws (cracks, notches, or other stress concentrations) and the temperature is not unusually low. However, in the presence of flaws, ductile materials may experience brittle fracture at normal temperatures. Plastic deformation may be small or nonexistent even though fracture is impending. Thus, yield strength is not the critical material parameter when failure occurs by brittle fracture. Instead, *notch toughness*, the ability of a material to absorb energy in the presence of a notch (or sharp crack), is the parameter that governs the failure mode. Dynamic loading and low temperatures also increase the tendency of a material to fracture in a brittle manner. Failure by brittle fracture is discussed in Chapter 15.

c. **Progressive Fracture (Fatigue).** If a metal that ordinarily fails by general yielding under a static load is subjected to repeated cycles of stress, it may fail by fracture without visual evidence of yielding, provided that the repeated stress is greater than a value called the *fatigue strength*. Under such conditions, minute cracks start at one or more points in the member, usually at points of high *localized* stress such as at abrupt changes in section, and gradually spread by fracture of the material at the edges of the cracks where the stress is highly concentrated. The *progressive fracture* continues until the member finally breaks. This mode of failure is usually called a *fatigue failure*, but it is better designated as *failure by progressive fracture* resulting from repeated loads. (See Chapter 16.)

4. Failure by Instability (Buckling)

Some members may fail by a sudden, catastrophic, lateral deflection (instability or buckling), rather than by yielding or crushing (Chapter 12). Consider an ideal pin-ended slender column (or strut) subjected to an axial compressive load P. Elastic buckling of the member occurs when the load P reaches a critical value $P_{cr} = \pi^2 EI/L^2$, where E is the modulus of elasticity, I is the moment of inertia of the cross section, and L is the member length.

PROBLEMS

1.1. What requirements control the derivation of load–stress relations?

1.2. Describe the method of mechanics of materials.

1.3. How are stress–strain–temperature relations for a material established?

1.4. Explain the differences between elastic response and inelastic response of a solid.

1.5. What is a stress–strain diagram?

1.6. Explain the difference between elastic limit and proportional limit.

1.7. Explain the difference between the concepts of yield point and yield stress.

1.8. What is offset strain?

1.9. How does the engineering stress–strain diagram differ from the true stress–strain diagram?

1.10. What are modes of failure?

1.11. What are failure criteria? How are they related to modes of failure?

1.12. What is meant by the term factor of safety? How are factors of safety used in design?

1.13. What is a design inequality?

1.14. How is the usual design inequality modified to account for statistical variability?

1.15. What is a load factor? A load effect? A resistance factor?

1.16. What is a limit-states design?

1.17. What is meant by the phrase "failure by excessive deflection"?

1.18. What is meant by the phrase "failure by yielding"?

1.19. What is meant by the phrase "failure by fracture"?

1.20. Discuss the various ways that a structural member may fail.

1.21. Discuss the failure modes, critical parameters, and failure criteria that may apply to the design of a downhill snow ski.

1.22. For the steels whose stress–strain diagrams are represented by Figures 1.8 to 1.10, determine the following properties as appropriate: the yield point, the yield strength, the upper yield point, the lower yield point, the modulus of resilience, the ultimate tensile strength, the strain at fracture, the percent elongation.

1.23. Use the mechanics of materials method to derive the load–stress and load–displacement relations for a solid circular rod of constant radius r and length L subjected to a torsional moment **T** as shown in Figure P1.23.

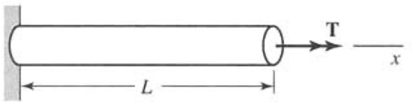

FIGURE P1.23 Solid circular rod in torsion.

1.24. Use the mechanics of materials method to derive the load–stress and load–displacement relations for a bar of constant width b, linearly varying depth d, and length L subjected to an axial tensile force **P** as shown in Figure P1.24.

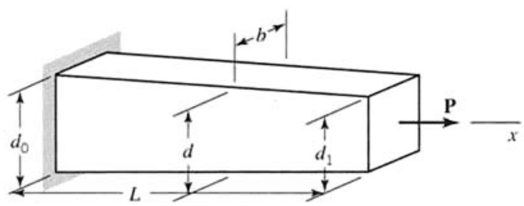

FIGURE P1.24 Tapered bar in tension.

1.25. A pressure vessel consists of two flat plates clamped to the ends of a pipe using four rods, each 15 mm in diameter, to form a cylinder that is to be subjected to internal pressure p (Figure P1.25). The pipe has an outside diameter of 100 mm and an inside diameter of 90 mm. Steel is used throughout (E = 200 GPa). During assembly of the cylinder (before pressurization), the joints between the plates and ends of the pipe are sealed with a thin mastic and the rods are each pretensioned to 65 kN. Using the mechanics of materials method, determine the internal pressure that will cause leaking. Leaking is defined as a state of zero bearing pressure between the pipe ends and the plates. Also determine the change in stress in the rods. Ignore bending in the plates and radial deformation of the pipe.

1.26. A steel bar and an aluminum bar are joined end to end and fixed between two rigid walls as shown in Figure P1.26. The cross-sectional area of the steel bar is A_s and that of the aluminum bar is A_a. Initially, the two bars are stress free. Derive general expressions for the deflection of point A, the stress in the steel bar, and the stress in the aluminum bar for the following conditions:

a. A load P is applied at point A.

b. The left wall is displaced an amount δ to the right.

1.27. In South African gold mines, cables are used to lower worker cages down mine shafts. Ordinarily, the cables are made of steel. To save weight, an engineer decides to use cables made of aluminum. A design requirement is that the stress in the cable resulting from self-weight must not exceed one-tenth of the ultimate strength σ_u of the cable. A steel cable has a mass density ρ = 7.92 Mg/m^3 and σ_u = 1030 MPa. For an aluminum cable, ρ = 2.77 Mg/m^3 and σ_u = 570 MPa.

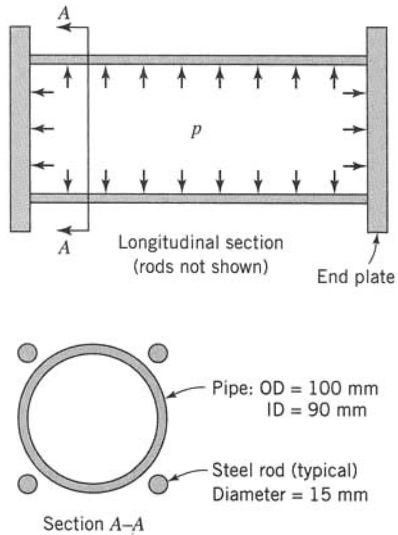

FIGURE P1.25 Pressurized cylinder.

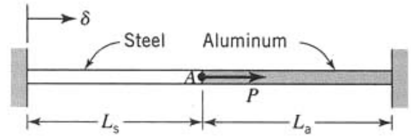

FIGURE P1.26 Bi-metallic rod.

a. Determine the lengths of two cables, one of steel and the other of aluminum, for which the stress resulting from the self-weight of each cable equals one-tenth of the ultimate strength of the material. Assume that the cross-sectional area A of a cable is constant over the length of the cable.

b. Assuming that A is constant, determine the elongation of each cable when the maximum stress in the cable is $0.10\sigma_u$. The steel cable has a modulus of elasticity E = 193 GPa and for the aluminum cable E = 72 GPa.

c. The cables are used to lower a cage to a mine depth of 1 km. Each cable has a cross section with diameter D = 75 mm. Determine the maximum allowable weight of the cage (including workers and equipment), if the stress in a cable is not to exceed $0.20\sigma_u$.

1.28. A steel shaft of circular cross section is subjected to a twisting moment T. The controlling factor in the design of the shaft is the angle of twist per unit length (ψ/L; see Eq. 1.5). The maximum allowable twist is 0.005 rad/m, and the maximum shear stress is τ_{max} = 30 MPa. Determine the diameter at which the maximum allowable twist, and not the maximum shear stress, is the controlling factor. For steel, G = 77 GPa.

1.29. An elastic T-beam is loaded and supported as shown in Figure P1.29a. The cross section of the beam is shown in Figure P1.29b.

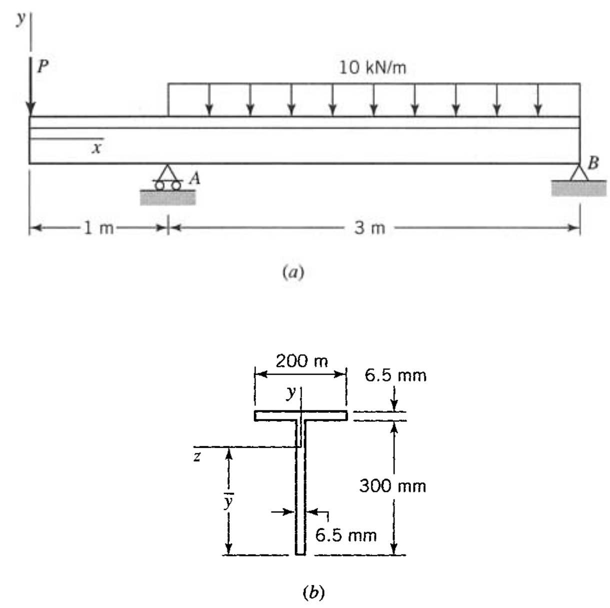

FIGURE P1.29 T-beam.

a. Determine the location $\bar{y}$ of the neutral axis (the horizontal centroidal axis) of the cross section.

b. Draw shear and moment diagrams for the beam.

c. Determine the maximum tensile stress and the maximum compressive stress in the beam and their locations.

1.30. Determine the maximum and minimum shear stresses in the web of the beam of Problem 1.29 and their locations. Assume that the distributions of shear stresses in the web, as in rectangular cross sections, are directed parallel to the shear force V and are uniformly distributed across the thickness ($t = 6.5$ mm) of the web. Hence, Eq. 1.9 can be used to calculate the shear stresses.

1.31. A steel tensile test specimen has a diameter of 10 mm and a gage length of 50 mm. Test data for axial load and corresponding data for the gage-length elongation are listed in Table P1.31. Convert these data to engineering stress–strain data and determine the magnitudes of the toughness U_F and the ultimate strength σ_u.

TABLE P1.31

Load (kN)	Elongation (mm)	Load (kN)	Elongation (mm)
0	0	36.3	0.40
3.1	0.01	38.8	0.50
6.2	0.02	41.2	0.60
9.3	0.03	44.1	1.25
12.4	0.04	48.1	2.50
15.5	0.05	50.4	3.75
18.6	0.06	51.4	5.00
21.7	0.07	52.0	6.25
24.7	0.08	52.2	7.50
25.8	0.09	52.0	8.75
26.1	0.10	50.6	10.00
29.2	0.15	45.1	11.25
31.0	0.20	43.2	11.66
34.0	0.30	—	—

1.32. Using an expanded strain scale, plot the stress–strain diagram for small strains using the data in Problem 1.31 and determine the modulus of elasticity E, the yield strength σ_{YS} for an offset of 0.2%, the proportional limit σ_{PL}, and the modulus of resilience.

1.33. A tensile test specimen of an aluminum alloy has a diameter of 20.0 mm and a gage length of 100 mm. In the tensile test, the axial load was found to remain proportional to the elongation up to an axial load of 75.4 kN. At that load the diameter of the tensile test specimen was 19.978 mm and the elongation over the gage length was 0.330 mm. Determine the modulus of elasticity E, Poisson's ratio v, and the proportional limit of the aluminum alloy.

1.34. The percentage reduction of area for the tensile test specimen in Problem 1.31 was found to be 55%. Compare the engineering fracture stress with the true fracture stress for the steel specimen.

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