

SHEAR CENTER FOR THIN-WALL BEAM CROSS SECTIONS

As defined in Chapter 7, the shear center is a point in the cross section of a beam through which the plane of loads must pass for the beam to be subjected to only bending and shear. No torsion is caused by transverse loads that act through the shear center. Locating the shear center for a beam cross section is a necessary step in the analysis of beam under loads that cause bending, shear, and possibly torsion. For a general cross section, the theory of elasticity may be used to locate the shear center (Boresi and Chong, 2000). However, in this chapter we use the methods of mechanics of materials to develop an approximate solution for the location of the shear center for thin-wall beam cross sections.

8.1 APPROXIMATIONS FOR SHEAR IN THIN-WALL BEAM CROSS SECTIONS

For a beam with a cross section that possess two or more axes of symmetry or antisymmetry, the bending axis is the same as the longitudinal centroidal axis, because for each cross section the shear center and centroid coincide. However, for cross sections with only one axis of symmetry, the shear center and centroid do not coincide, but both lie on the symmetry axis.

For example, consider the equal-leg angle section shown in Figure 8.1. Let the beam cross section be oriented so that the principal axes of inertia (X, Y) are directed horizontally and vertically. When the load P is applied at the centroid O of the cross section, the beam bends and twists (Figure 8.1*a*). However, the beam bends without twist if it is loaded by a force P that passes through the shear center C (Figure 8.1*b*). As is shown later, the shear center C coincides approximately with the intersection of the center lines of the two legs of the angle section.

To locate the shear center for a thin-wall cross section, we first make simplifying assumptions. They may be illustrated by reference to Figure 8.2. In Figure 8.2, the cross section shown is that of the beam in Figure 8.1*b* and is obtained by passing a cutting plane perpendicular to the bending axis through the beam. The view shown is obtained by looking from the support toward the end of the beam at which P is applied.

For equilibrium of the beam element so obtained, the shear stresses on the cut cross section must balance the load P . However, the shear stresses in the cross section are difficult to compute exactly. Hence, simplifying approximations are employed. Accordingly, consider a portion of the legs of the cross section, shown enlarged in Figure 8.2*b*. Let axes $x-y-z$ be chosen so that the $x-y$ axes are tangent and normal, respectively, to the upper leg, and let the z axis be taken perpendicular to the cross section (the plane of Figure 8.2*b*) and directed positively along the axis of the beam from the load P to the support. Then, the shear stress

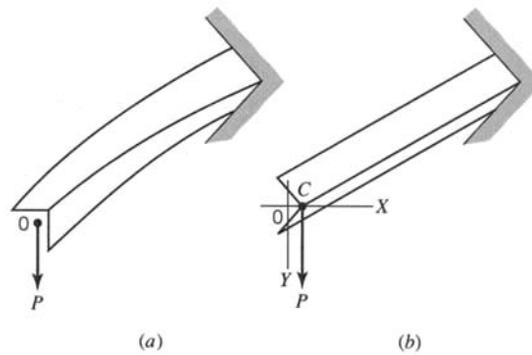


FIGURE 8.1 Effect of applying load through shear center. (a) Load P applied at point O produces twist and bending. (b) Load P applied at point C produces bending only.

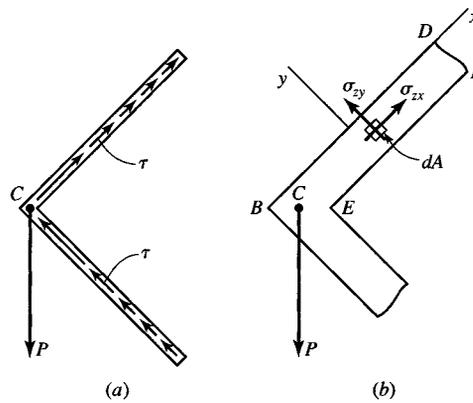


FIGURE 8.2 Shear stress distribution in an equal-leg angle section.

components in the cross section of the beam are σ_{zx} and σ_{zy} as shown. Since the shear stresses on the lateral surfaces of the beam are zero, $\sigma_{yz} = 0$. Hence, σ_{zy} vanishes at BD and EF (since $\sigma_{yz} = \sigma_{zy}$; see Eq. 2.4). Since $\sigma_{zy} = 0$ at BD and EF and the wall thickness between BD and EF is small (a thin wall) with respect to the length of the legs of the cross section, we assume that σ_{zy} does not change significantly (remains approximately zero) through the wall. The effect of σ_{zy} (the shear stress in the thickness direction) is ignored in the following discussion. In addition, it is assumed that the shear stress component σ_{zx} (along the legs) is approximately constant through the wall thickness and is equal to the average tangential shear stress τ in the wall (Figure 8.2a). With these approximations for σ_{zy} and σ_{zx} , we find that a reasonably accurate and simple estimate of the shear center location may be obtained.

8.2 SHEAR FLOW IN THIN-WALL BEAM CROSS SECTIONS

The average shear stress τ at each point in the walls of the beam cross section is assumed to have a direction tangent to the wall. The product of this shear stress and the wall thickness t defines the *shear flow* q ; thus,

$$q = \tau t \tag{8.1}$$

In the equation that will be derived for determining the shear flow q , we assume that the beam material remains linearly elastic and that the flexure formula is valid. Hence, we assume that the plane of the loads contains the bending axis of the beam and is parallel to one of the principal axes of inertia. It is convenient to consider a beam cross section that has one axis of symmetry (the x axis in Figure 8.3). If the load P is parallel to the y axis and passes through the shear center C'' , the x axis is the neutral axis for linearly elastic behavior and the flexure formula is valid. The derivation of the formula for q requires that both the bending moment M_x and total shear V_y be defined; load P is taken in the negative y direction so that both M_x and V_y are positive.

We wish to determine the shear flow q at point J in the cross section of the beam in Figure 8.3a at a distance $z + dz$ from load P . The free-body diagram necessary to determine q is obtained by three cutting planes. Cutting planes 1 and 2 are perpendicular to the z axis at distances z and $z + dz$ from the load P . Cutting plane 3 is parallel to the z axis and perpendicular to the lateral surface of the beam at J . The free body removed by the three cutting planes is indicated in Figure 8.3b. The normal stress σ_{zz} as given by the flexure formula acts on the faces made by cutting planes 1 and 2. The resulting forces on these faces of area A' are parallel to the z axis and are indicated in Figure 8.3b as H and H' , respectively. Since the forces H and H' are unequal in magnitude, equilibrium of forces in the z direction is maintained by the force $q dz$ on the face made by cutting plane 3. Therefore,

$$q dz = H' - H \quad (8.2)$$

Now, integration of σ_{zz} over the faces with area A' at sections 1 and 2 yields (with the flexure formula)

$$H = \int_{A'} \sigma_{zz} dA = \int_{A'} \frac{M_x y}{I_x} dA$$

and

$$H' = \int_{A'} (\sigma_{zz} + d\sigma_{zz}) dA = \int_{A'} \frac{(M_x + dM_x)y}{I_x} dA$$

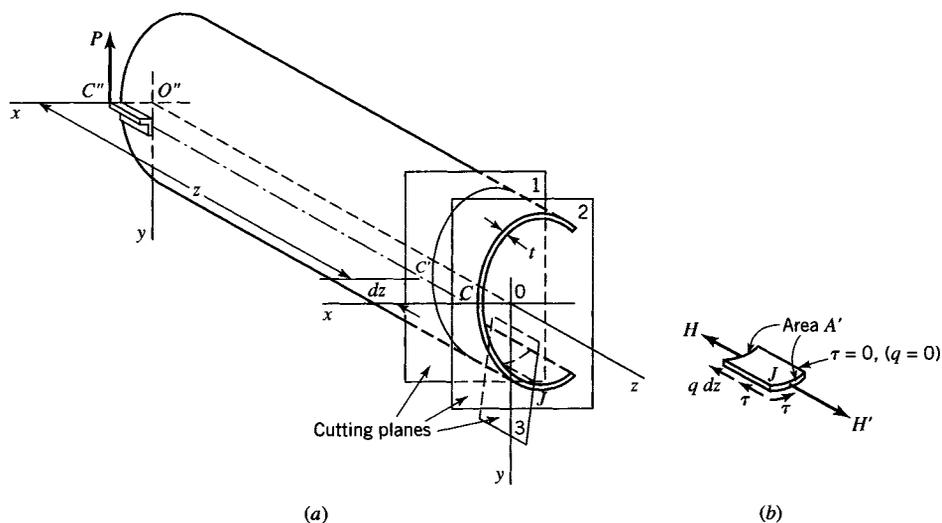


FIGURE 8.3 Shear flow in a beam having a symmetrical cross section.

Substituting these two relations into Eq. 8.2 and solving for q yields

$$q = \frac{dM_x}{dz} \frac{1}{I_{x A'}} \int y dA$$

According to beam theory, the total shear V_y in the cross section of a beam is given by $V_y = dM_x/dz$. Also, since $\int_{A'} y dA = A' \bar{y}'$, where \bar{y}' is the distance from the x axis to the centroid of A' , we may express q as

$$q = \frac{V_y A' \bar{y}'}{I_x}$$

Furthermore, since the value of the shear stress τ in the longitudinal section cut by plane 3 (Figure 8.3) is the same as the shear stress in the cross section cut by plane 2, the shear flow in the cross section at point J is

$$q = \tau t = \frac{V_y A' \bar{y}'}{I_x} = \frac{V_y Q}{I_x} \quad (8.3)$$

where t is the wall thickness at point J . The first moment of area A' , that is, $A' \bar{y}'$, is commonly denoted by Q .

Equation 8.3 is used to locate the shear center of thin-wall beam cross sections for both symmetrical and nonsymmetrical bending. The method is demonstrated in Section 8.3 for beam cross sections made up of moderately thin walls.

In many applications (e.g., girders), the beam cross sections are built up by joining stiff longitudinal stringers by thin webs. The webs are generally stiffened at several locations along the length of the beam. The shear center location for beams of this type is considered in Section 8.4.

8.3 SHEAR CENTER FOR A CHANNEL SECTION

A cantilever beam subjected to a bending load V at C' in a plane perpendicular to the axis x of symmetry of the beam is shown in Figure 8.4. We wish to locate the plane of the load so that the channel bends without twisting. In other words, we wish to locate the bending axis CC' of the beam or the shear center C of any cross section AB .

In Figure 8.4a let V be transformed into a force and couple at section AB by introducing, at the shear center C whose location is as yet unknown, two equal and opposite forces V' and V'' , each equal in magnitude to V . The forces V and V'' constitute the external bending couple at section AB , which is held in equilibrium by the internal resisting moment at section AB in accordance with the flexure formula, Eq. 7.1; the distribution of the normal stress σ_{zz} on section AB is shown in Figure 8.4a. The force V' is located at a distance e from the center of the web of the channel, as indicated in Figures 8.4a and 8.4b. Force V' is resisted by shear stress τ or shear flow q (Eq. 8.3) in cross section AB . Since the shear flow is directed along the straight sides of the channel, it produces forces F_1 , F_2 , and F_3 , which lie in the cross section as indicated in Figure 8.4b. Accordingly, by equilibrium

$$\sum F_x = F_2 - F_1 = 0 \quad (8.4)$$

$$\sum F_y = V' - F_3 = 0 \quad (8.5)$$

$$\sum M_A = V'e - F_1 h = 0 \quad (8.6)$$

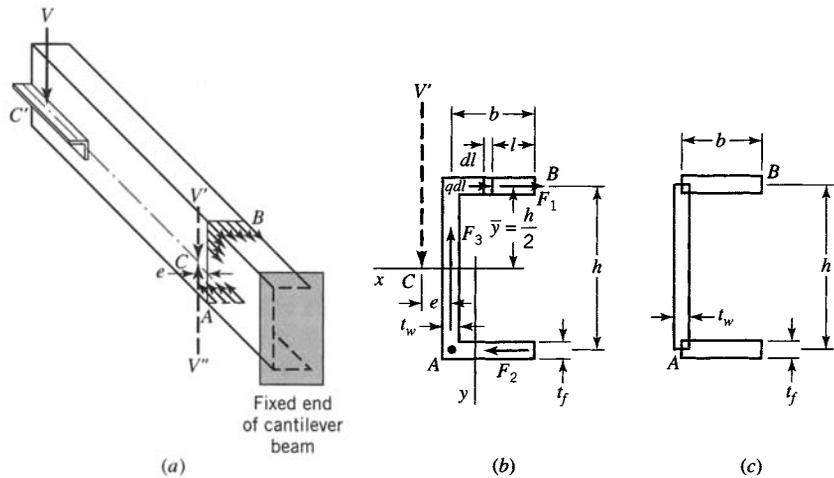


FIGURE 8.4 Shear center for a channel section. (a) Channel section beam. (b) Location of C . (c) Idealized areas.

The magnitude of the load V' is assumed to be known. Therefore, the determination of the distance e from the center line of the web to the shear center requires only that the force $F_1 (= F_2)$ be determined.

To determine F_1 , it is convenient to think of the beam cross section as made up of line segments (Figure 8.4a) with specified thicknesses. Since the forces F_1, F_2, F_3 are assumed to lie along the center line of the walls, the cross section is idealized as three narrow rectangles of lengths b, h , and b as indicated in Figure 8.4c; note that the actual and idealized cross-sectional areas are equal since the three areas overlap. However, the moments of inertia of the actual and idealized cross sections differ from each other slightly. The moment of inertia of the idealized area is

$$I_x = \frac{1}{12}t_w h^3 + 2bt_f \left(\frac{h}{2}\right)^2 + 2\frac{1}{12}bt_f^3$$

This result may be simplified further by neglecting the third term, since for the usual channel section t_f is small compared to b or h . Thus, we write

$$I_x = \frac{1}{12}t_w h^3 + \frac{1}{2}t_f b h^2 \quad (8.7)$$

The force F_1 may be found from the shear flow equation

$$F_1 = \int_0^b q \, dl = \frac{V_y}{I_x} \int_0^b A' \bar{y}' \, dl = \frac{V_y t_f h}{2I_x} \int_0^b l \, dl = \frac{V_y t_f b^2 h}{4I_x} \quad (8.8)$$

where q is given by Eq. 8.3. The distance e to the shear center of the channel section is determined by substituting Eqs. 8.7 and 8.8 into Eq. 8.6 with the magnitude of V' set equal to that of V_y . Thus, we find

$$e = \frac{b}{2 + \frac{1}{3} \frac{t_w h}{t_f b}} \quad (8.9)$$

Because of the assumptions employed and approximations used, Eq. 8.9 gives only an approximate location of the shear center for channel sections. However, the error is small for thin-wall sections. The approximate locations of the shear center for several other thin-wall sections with an axis of symmetry are given in Table 8.1.

TABLE 8.1 Locations of Shear Centers for Sections Having One Axis of Symmetry

	$\frac{e}{b} = \frac{1 + \frac{2b_1}{b} \left(1 - \frac{4b_1^2}{3h^2}\right)}{2 + \frac{h}{3b} + \frac{2b_1}{b} \left(1 + \frac{2b_1}{h} + \frac{4b_1^2}{3h^2}\right)}$
	$\frac{e}{b} = \frac{1 + \frac{2b_1}{b} \left(1 - \frac{4b_1^2}{3h^2}\right)}{2 + \frac{h}{3b} + \frac{2b_1}{b} \left(1 - \frac{2b_1}{h} + \frac{4b_1^2}{3h^2}\right)}$
	$\frac{e}{b} = \frac{1 - \frac{b_1^2}{b^2}}{2 + \frac{2b_1}{b} + \frac{t_w h}{3t_1 b}}, \quad b_1 < b$
	$\frac{e}{b} = \frac{\frac{b_1^2}{\sqrt{2}b^2} \left(3 - \frac{2b_1}{b}\right)}{1 + \frac{3b_1}{b} - \frac{3b_1^2}{b^2} + \frac{b_1^3}{b^3}}$
	$\frac{e}{R} = \frac{2(\sin \theta - \theta \cos \theta)}{\theta - \sin \theta \cos \theta}$ <p>For semicircle, $\theta = \frac{\pi}{2}$ and $\frac{e}{R} = \frac{4}{\pi}$</p>
	$\frac{e}{R} = \frac{12 + 6\pi \frac{b+b_1}{R} + 6\left(\frac{b}{R}\right)^2 + 12\frac{bb_1}{R^2} + 3\pi\left(\frac{b_1}{R}\right)^2 - 4\left(\frac{b_1}{R}\right)^3 \frac{b}{R}}{3\pi + 12\frac{b+b_1}{R} + 4\left(\frac{b_1}{R}\right)^2 \left(3 + \frac{b_1}{R}\right)}$ <p>For $b_1 = 0$: $\frac{e}{R} = \frac{4 + 2\pi \frac{b}{R} + 2\left(\frac{b}{R}\right)^2}{\pi + 4\frac{b}{R}}$</p> <p>For $b = 0$: $\frac{e}{R} = \frac{3\left[4 + \frac{2b_1\pi}{R} + \pi\left(\frac{b_1}{R}\right)^2\right]}{3\pi + 4\left(\frac{b_1}{R}\right)^3 + 12\frac{b_1}{R} + 12\left(\frac{b_1}{R}\right)^2}$</p>

EXAMPLE 8.1
Shear Center for
Channel with
Sloping Flanges

A 4-mm thick plate of steel is formed into the cross section shown in Figure E8.1a. Locate the shear center for the cross section.

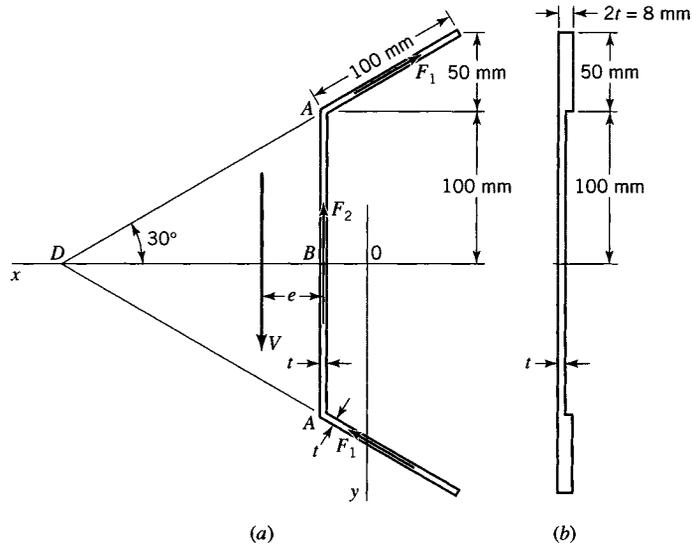


FIGURE E8.1

Solution

For simplicity in finding the moment of inertia, we approximate the actual cross section (Figure E8.1a) by the cross section shown in Figure E8.1b. The moment of inertia about the x axis for the cross section in Figure E8.1b closely approximates that for the actual cross section in Figure E8.1a and is

$$I_x = \frac{8(300)^3}{12} - \frac{4(200)^3}{12} = 15,330,000 \text{ mm}^4$$

After finding I_x we make no further use of Figure E8.1b. Because of the shear flow, forces F_1 and F_2 are developed in the three legs of the cross section. The magnitude of force F_1 requires integration; therefore, it is convenient to take moments about point D so that F_1 is not required. Since the shear flow from A to B to A varies parabolically, the average shear flow is equal to the shear flow at A plus $\frac{2}{3}$ of the difference between the shear flow at B and shear flow at A . Thus,

$$q_A = \frac{V}{I_x} A' \bar{y} = \frac{V}{I_x} (100)(4)(125) = 50,000 \frac{V}{I_x}$$

$$q_B = q_A + \frac{V}{I_x} (100)(4)(50) = 70,000 \frac{V}{I_x}$$

$$q_{\text{ave}} = q_A + \frac{2}{3}(q_B - q_A) = 63,330 \frac{V}{I_x}$$

$$F_2 = 200q_{\text{ave}} = 63,330 \frac{V}{I_x} (200) = 12,670,000 \frac{V}{I_x}$$

With point D as the moment center, the clockwise moment of V must equal the counterclockwise moment of F_2 . Thus, we have $(173.2 - e)V = 173.2 F_2$, and hence $e = 30.1$ mm.

EXAMPLE 8.2
Shear Center for
Unequal-Leg
Channel

A beam has a nonsymmetrical section whose shape and dimensions are shown in Figure E8.2a. Locate the shear center.

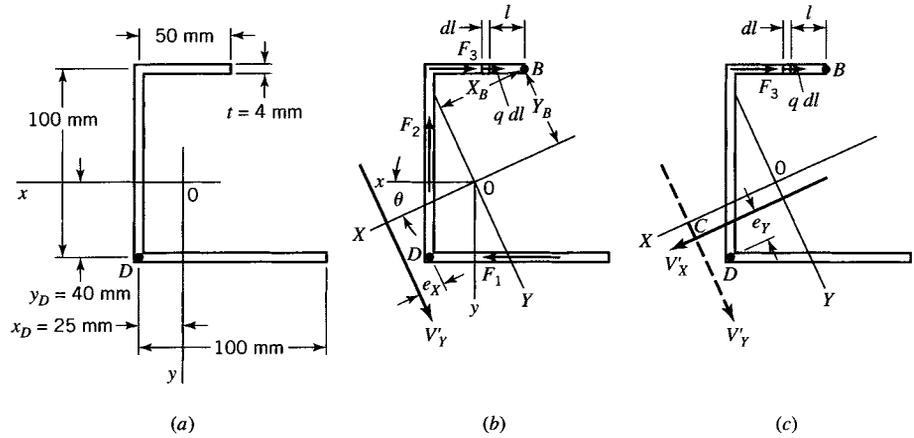


FIGURE E8.2

Solution

Centroidal x and y axes are chosen to be parallel to the sides of the thin-wall legs of the cross section. The origin O of the coordinates' axes is located at $x_D = 25$ mm and $y_D = 40$ mm. To apply the theory to nonsymmetrical sections, we use principal axes X - Y . As indicated in Appendix B, the orientation of the principal axes may be described in terms of I_x , I_y , and I_{xy} . These values are $I_x = 1.734 \times 10^6$ mm⁴, $I_y = 0.876 \times 10^6$ mm⁴, and $I_{xy} = -0.500 \times 10^6$ mm⁴. The angle θ between the x axis and X axis is obtained by the relation (Eq. B.12)

$$\tan 2\theta = -\frac{2I_{xy}}{I_x - I_y} = \frac{-2(-0.500 \times 10^6)}{1.734 \times 10^6 - 0.876 \times 10^6} = 1.166$$

from which $\theta = 0.4308$ rad. Since θ is positive, the X axis is located counterclockwise from the x axis. By using the equations in Appendix B, we find the principal moments of inertia to be $I_X = 1.964 \times 10^6$ mm⁴ and $I_Y = 0.646 \times 10^6$ mm⁴. The principal axes are shown in Figures E8.2b and E8.2c.

The shear center C is located by considering two separate cases of loading (without twisting) in two orthogonal planes of the loads. The intersection of these two planes of loads determines the shear center C . Thus, assume that the resultant V'_y of unbalanced loads on one side of the section in Figure E8.2b is parallel to the Y axis. Since V'_y is assumed to pass through the shear center, the beam bends without twisting and the X axis is the neutral axis; hence, the flexure formula and Eq. 8.3 apply. Because of the shear flow, forces F_1 , F_2 , and F_3 are developed in the three legs of the cross section (Figure E8.2b). Only the magnitude of F_3 is required if point D is chosen as the moment center. To determine F_3 , it is necessary that the shear flow q be determined as a function of l , the distance from point B . The coordinates of point B , the shear flow q , and force F_3 are determined as follows:

$$X_B = x_B \cos \theta + y_B \sin \theta = -25(0.9086) - 60(0.4176) = -47.77 \text{ mm}$$

$$Y_B = y_B \cos \theta - x_B \sin \theta = -60(0.9086) + 25(0.4176) = -44.08 \text{ mm}$$

$$q = \frac{V_Y}{I_X} A' \bar{Y}' = \frac{V_Y}{I_X} t l \left(|Y_B| + \frac{1}{2} l \sin \theta \right)$$

$$F_3 = \int_0^{50} q \, dl = \frac{V_Y t}{I_X} \int_0^{50} l \left(44.08 + \frac{0.4176}{2} l \right) dl = 0.1299 V_Y$$

Using the fact that $V'_Y = V_Y$ (the total shear at the section), we obtain the distance e_X from point D to force V'_Y , which passes through the shear center, from the equilibrium moment equation. Therefore,

$$V_Y e_X = 100F_3$$

or

$$e_X = 12.99 \text{ mm}$$

Next assume that the resultant of the unbalanced loads on one side of the section in Figure E8.2c is V'_X and it is parallel to the X axis. Since V'_X is assumed to pass through the shear center, the beam bends without twisting and the Y axis is the neutral axis. The shear flow q and force F_3 are given by

$$q = \frac{V_X}{I_Y} A' \bar{X}' = \frac{V_X}{I_Y} t l \left(|X_B| - \frac{1}{2} l \cos \theta \right)$$

$$F_3 = \int_0^{50} q \, dl = \frac{V_X t}{I_Y} \int_0^{50} l \left(47.77 - \frac{0.9086}{2} l \right) dl = 0.2525 V_X$$

Set $V'_X = V_X$ (the total shear at the section) and take moments about point D . Therefore,

$$V_X e_Y = 100F_3$$

$$e_Y = 25.25 \text{ mm}$$

In terms of principal coordinates, the shear center C is located at

$$X_C = x_D \cos \theta + y_D \sin \theta + e_X = 52.41 \text{ mm}$$

$$Y_C = y_D \cos \theta - x_D \sin \theta - e_Y = 0.66 \text{ mm}$$

The x and y coordinates of the shear center C are

$$x_C = X_C \cos \theta - Y_C \sin \theta = 47.35 \text{ mm}$$

$$y_C = Y_C \cos \theta + X_C \sin \theta = 22.49 \text{ mm}$$

8.4 SHEAR CENTER OF COMPOSITE BEAMS FORMED FROM STRINGERS AND THIN WEBS

Often, particularly in the aircraft industry, beams are built up by welding or riveting longitudinal stiffeners, called stringers, to thin webs. Such beams are often designed to carry large bending loads and small shear loads. Two examples of cross sections of such beams are shown in Figure 8.5. A beam whose cross section consists of two T-section stringers

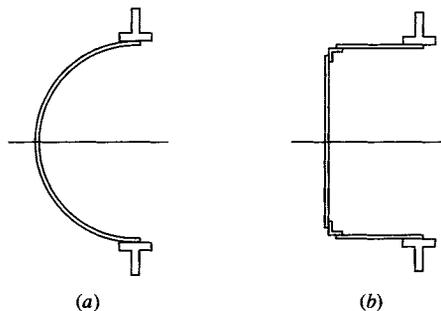


FIGURE 8.5 Beam cross sections built up of stringers and thin webs.

joined to a semicircular web is shown in Figure 8.5a, and a beam whose cross section consists of a vertical web joined to two angle-section stringers that, in turn, are joined to two horizontal webs that support two T-section stringers is shown in Figure 8.5b.

The calculation of the shear center location for beam cross sections similar to those shown in Figure 8.5 is based on two simplifying assumptions: 1. that the web does not support tensile or compressive stresses resulting from bending loads and 2. that the shear flow is constant in a web between pairs of transverse stiffeners. The actual webs of these composite beams are often so thin that they may buckle under small compressive stresses. Therefore, the webs should not be expected to carry compressive flexure stresses. In general, the webs can carry tensile flexure stresses. However, this capability is sometimes ignored in their design.

Since the web walls are usually very thin, the moment of inertia for symmetrical cross sections of composite beams is approximated by the relation

$$I_x = 2 \sum_{i=1}^n A_i \bar{y}_i^2 \quad (8.10)$$

where $2n$ is the number of stringers, A_i is the cross-sectional area of the stringer on one side of the neutral axis (x axis), and \bar{y}_i is the distance from the neutral axis to the centroid of the area A_i . Equation 8.10 discards the effect of the web. Hence I_x is underestimated. With this value of I_x , the computed flexure stresses are overestimated (higher than the true stresses).

Note: Transverse shear stresses are developed in the area A_i of each stringer so that the stringer carries part of the total shear load V_y applied to the beam. However, the part of V_y carried by each stringer is usually ignored. This error is corrected in part by assuming that each web is extended to the centroid of the area of each stringer, thus increasing the contribution of the web. The procedure is demonstrated in the following example.

EXAMPLE 8.3 Shear Center for Composite Beam

A composite beam has a symmetrical cross section as shown in Figure E8.3. A vertical web with a thickness of 2 mm is riveted to two square stringers. Two horizontal webs, with a thickness of 1 mm, are riveted to the square stringers and the T-section stringers. Locate the shear center of the cross section.

Solution

The centroid of each T-section is located 9.67 mm from its base. The distance from the x axis to the centroid of each T-section is

$$\bar{y}_2 = 100 + 10 + 1 + 9.67 = 120.67 \text{ mm}$$

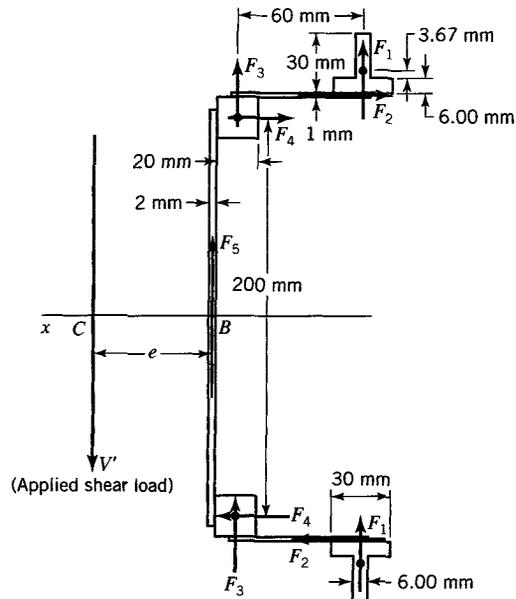
The approximate value of I_x (Eq. 8.10) is

$$\begin{aligned} I_x &= 2A_1 \bar{y}_1^2 + 2A_2 \bar{y}_2^2 = 2(400)(100)^2 + 2(324)(120.67)^2 \\ &= 17.44 \times 10^6 \text{ mm}^4 \end{aligned}$$

In these calculations, the shear flow q_1 is assumed to be constant from the centroid of the T-section to the centroid of the square stringers. The magnitude of q_1 is (Eq. 8.3)

$$q_1 = \frac{V_y}{I_x} A' \bar{y}' = \frac{V_y}{I_x} (324)(120.67) = 39.10 \times 10^3 \frac{V_y}{I_x}$$

where V_y is the total shear at the section. The forces F_1 , F_2 , and F_3 are given by the relations


FIGURE E8.3

$$F_1 = (9.67 + 0.5)q_1 = 397.6 \times 10^3 \frac{V_y}{I_x}$$

$$F_2 = 60q_1 = 2.346 \times 10^6 \frac{V_y}{I_x}$$

$$F_3 = (10 + 0.5)q_1 = 410.5 \times 10^3 \frac{V_y}{I_x}$$

The shear flow q_2 is also assumed to be constant between centroids of the square stringers. Hence,

$$q_2 = q_1 + \frac{V_y}{I_x}(400)(100) = 79.10 \times 10^3 \frac{V_y}{I_x}$$

The forces F_4 and F_5 are given by the relations

$$F_4 = (10 + 1)q_2 = 870.1 \times 10^3 \frac{V_y}{I_x}$$

$$F_5 = 200q_2 = 15.82 \times 10^6 \frac{V_y}{I_x}$$

These forces with V' (Figure E8.3) must satisfy equilibrium in the y direction; that is,

$$\sum F_y = V' - 2F_1 - 2F_3 - F_5 = 0$$

Hence,

$$V' = \frac{2(397.6 \times 10^3 V_y) + 2(410.5 \times 10^3 V_y) + 15.82 \times 10^6 V_y}{17.44 \times 10^6} = V_y$$

Thus, the applied shear load V' is equal to the total internal shear V_y in the section. The moment equilibrium equation for moments about point B determines the shear center location. Thus,

$$\begin{aligned} \sum M_B &= V'e + 2F_1(71) + 2F_3(11) - F_2(221) - F_4(200) = 0 \\ e &= [2.346 \times 10^6(221) + 870.1 \times 10^3(200) - 2(397.6 \times 10^3)(71) \\ &\quad - 2(410.5 \times 10^3)(11)] / (17.44 \times 10^6) \\ e &= 35.95 \text{ mm} \end{aligned}$$

This estimate of the location of the shear center C (Figure E8.3) may be in error by several percent because of the simplifying assumptions. Hence, if the transverse bending loads are placed at C , they may introduce a small torque load in addition to bending loads. In most applications, the shear stresses resulting from this small torque are relatively insignificant. In addition, it is questionable that the beam can be manufactured to such precise dimensions and that the loads can be placed with great accuracy. Thus, the need for greater accuracy in our computations is also questionable.

8.5 SHEAR CENTER OF BOX BEAMS

Another class of practical beams is the box beam (with boxlike cross section) (Figure 8.6). Box beams ordinarily have thin walls. However, the walls are usually sufficiently thick so that they will not buckle when subjected to elastic compressive stresses developed by bending. Box beams may be composed of several legs of different thickness (Figure 8.6) or they may be a composite of longitudinal stringers and very thin webs (Figure 8.7). The beams in Figures 8.6 and 8.7 are one-compartment box beams. In general, box beams may contain two or more compartments.

For convenience, let the x axis be an axis of symmetry in Figures 8.6 and 8.7. Let the beams be subjected to symmetrical bending. Hence, let the plane of the loads be parallel to the y axis and let it contain the shear center C . The determination of the location of the shear center requires that the shear stress distribution in the cross section be known. However, the shear stress distribution cannot be obtained using Eq. 8.3 alone, since area A' is not known. (A' is the area of the wall from a point of interest in the wall to a point in the wall where $q = 0$.) Consequently, an additional equation, specifically Eq. 6.67, is required to obtain the shear stress distribution for a cross section of a box beam. Since there is no twisting, the unit angle of twist in the beam is zero and hence Eq. 6.67 yields

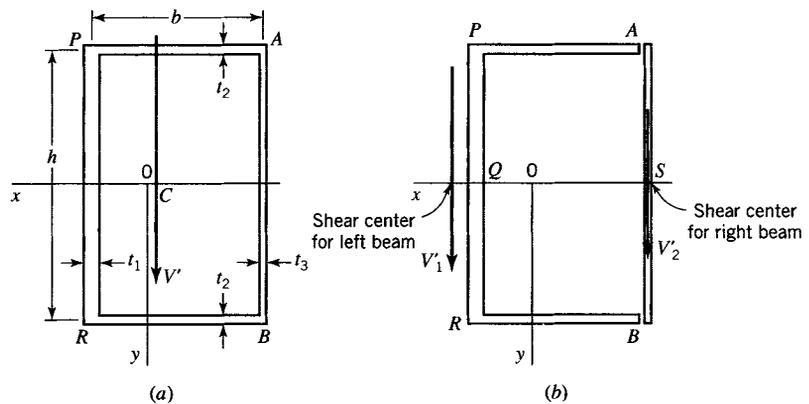


FIGURE 8.6 Box beam.

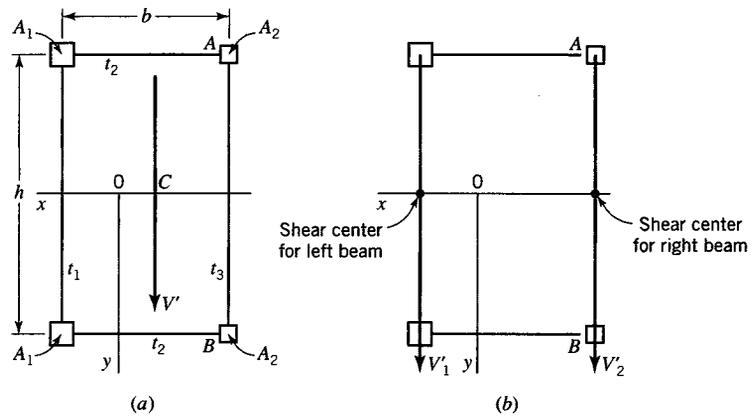


FIGURE 8.7 Ultra-thin-wall box beam with stringers.

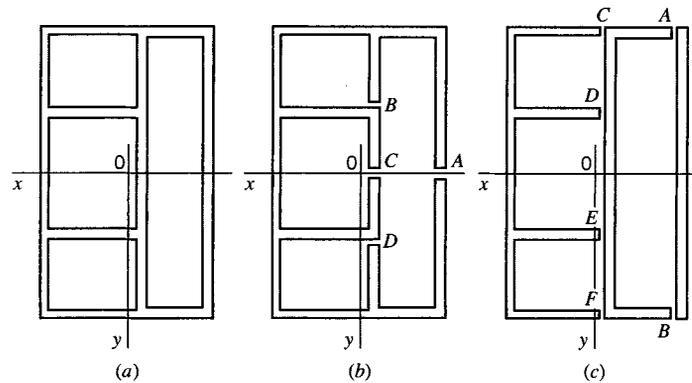
$$\int_0^l \frac{q}{t} dl = 0 \quad (8.11)$$

where dl is an infinitesimal length of the wall of the box beam cross section at a point where the thickness is t and the shear flow is q . The length l of the perimeter of the box beam cross section is measured counterclockwise from any convenient point in the wall.

The shear flow q_A at any point, say point A in Figure 8.6 or 8.7, is an unknown. If this shear flow is subtracted from the actual shear flow at every point of the box beam wall, the resulting shear flow at A (and, in this case, at B because of symmetry) is zero. We refer to such a point (of zero shear flow) as a *cut*. Then the resulting shear flow is the same as if the two beams (Figures 8.6b and 8.7b) have no shear resistance at points A and B but still have continuity of displacement at points A and B . Since the subtraction of q_A results in a subtraction of a zero force resultant, the subtraction produces no additional horizontal or vertical components of load on the cross section. The portions V'_1, V'_2 of the shear load V' acting on each of the two parts AB and BA (Figures 8.6b and 8.7b) are proportional to the moments of inertia of the two parts of the beam because the curvature of the two parts must be continuous at points A and B . For convenience, let $V' = I$ (in magnitude) so that $V'_1 = I_1$ and $V'_2 = I_2$. Then, the shear flow at any point in the wall of either of the two parts of the beams (Figure 8.6b) can be obtained using Eq. 8.3. The shear flow q_A is then added to the resulting shear flows for the two parts of the beam. The magnitude of q_A is obtained by satisfying Eq. 8.11. The force in each wall of the cross section can then be determined. The location of the shear center is obtained from the fact that the moment of these forces about any point in the plane of the cross section must be equal to the moment of the applied shear load V' about the same point.

For beams whose cross sections contain more than one compartment (Figure 8.8), this procedure must be repeated for a point in the wall of each compartment, such as at $A, B, C,$ and D in Figure 8.8b or at $A, B, C, D, E,$ and F in Figure 8.8c. The magnitudes of the shear flows that must be subtracted for each compartment are obtained by satisfying Eq. 8.11 for each compartment.

Nonsymmetrical box beam cross sections can also be treated by this procedure. In this case, it is desirable to refer the calculation to principal axes, say, X - Y . The method proceeds as follows: First, locate the plane of the loads for bending about the X axis; second, locate the plane of the loads for bending about the Y axis. The shear center of the cross


FIGURE 8.8 Multicompartment box beam.

section is given by the intersection of these two planes. The bending axis intersects each cross section of the box beam at the shear center.

EXAMPLE 8.4 Shear Center for Box Beam

For the box beam in Figure 8.6, let $b = 300$ mm, $h = 500$ mm, $t_1 = 20$ mm, and $t_2 = t_3 = 10$ mm. Determine the location of the shear center for the cross section.

Solution

The moment of inertia for the x axis is $I_x = 687.5 \times 10^6 \text{ mm}^4$. Cuts are taken at points A and B to divide the beam into two parts (Figure 8.6b). For convenience, let the magnitude of the shear load V' [N] for the box beam be equal to the magnitude of I_x so that $V'_1 = I_{x1}$ and $V'_2 = I_{x2}$. The shear flow q is determined at points P , Q (the midpoint of PR), and S (the midpoint of AB) for the two parts of the cut beam cross section (Figure 8.6b) as follows (with $V'_1 = V_1$, $V'_2 = V_2$):

$$q_P = \frac{V'_1 A' \bar{y}'}{I_{x1}} = (bt_2) \frac{h}{2} = 300(10)(250) = 750.0 \text{ kN/mm}$$

$$q_Q = q_P + \left(\frac{h}{2} t_1\right) \frac{h}{4} = 1,375.0 \text{ kN/mm}$$

$$q_S = \left(\frac{h}{2} t_3\right) \frac{h}{4} = 312.5 \text{ kN/mm}$$

The senses of the shear flows oppose those of V'_1 and V'_2 . For the left part of the beam (Figure E8.4a), the shear flow increases linearly from zero at B to q_P at R and decreases linearly from q_P at P to zero at A . The shear flow changes parabolically from q_P at R to q_Q at Q and back to q_P at P . For the right of the beam, the shear flow changes parabolically from zero at B to q_S at S and back to zero at A . Now, we add q_A (assumed positive in a counterclockwise direction) to the value of q at every point in the cross section (Figure E8.4b), and we require that Eq. 8.11 be satisfied. Starting at P , we find that

$$0 = \left[q_A - q_P - \frac{2}{3}(q_Q - q_P) \right] \frac{h}{t_1} + \left(q_A - \frac{q_P}{2} \right) \frac{b}{t_2} + \left(q_A + \frac{2}{3}q_S \right) \frac{h}{t_3} + \left(q_A - \frac{q_P}{2} \right) \frac{b}{t_2}$$

$$0 = 135.0q_A - \left(750.0 + \frac{2}{3} \times 625.0 \right) 25 - (375.0)30 + \left(\frac{2}{3} \times 312.5 \right) 50 - (375.0)30$$

$$q_A = 305.6 \text{ kN/mm}$$

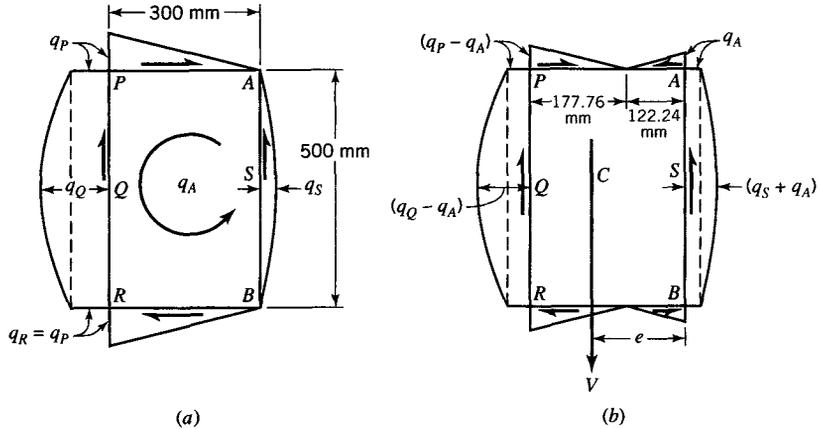


FIGURE E8.4

This value of q_A must be added to the values computed for the cross section with the cuts to give the shear flow (Figure E8.4b). The equilibrium equation for moments about point B gives

$$\begin{aligned}
 0 &= V'e - \left(444.4 + \frac{2}{3} 625\right)(500)(300) - \frac{444.4}{2}(177.76)(500) \\
 &\quad + \frac{305.6}{2}(122.24)(500) \\
 e &= \frac{139.57 \times 10^9 \text{ N} \cdot \text{mm}}{687.5 \times 10^6 \text{ N}} = 203 \text{ mm}
 \end{aligned}$$

The shear center C lies on the x axis at a point 203 mm to the left of the center line of the right leg of the box section. We can check the result by noting that, by Figures 8.6b and E8.4b,

$$\begin{aligned}
 V_1 &= \left[q_P - q_A + \frac{2}{3}(q_Q - q_P) \right](500) \\
 &= \left[750.0 - 305.6 + \frac{2}{3}(1375.0 - 750.0) \right](500) \\
 &= 430,533 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 V_2 &= \left(q_A + \frac{2}{3}q_S \right)(500) \\
 &= \left[305.6 + \frac{2}{3}(312.5) \right](500) \\
 &= 256,967 \text{ kN}
 \end{aligned}$$

or

$$V_1 + V_2 = 687,500 \text{ kN} = V$$

EXAMPLE 8.5
Shear Center
for a
Multicompartment
Box Beam

For the box beam shown in Figure E8.5a, determine the location of the shear center for the cross section. Let $a = b = 500$ mm, $t_1 = 5$ mm, $t_2 = 10$ mm, and $t_3 = 20$ mm.

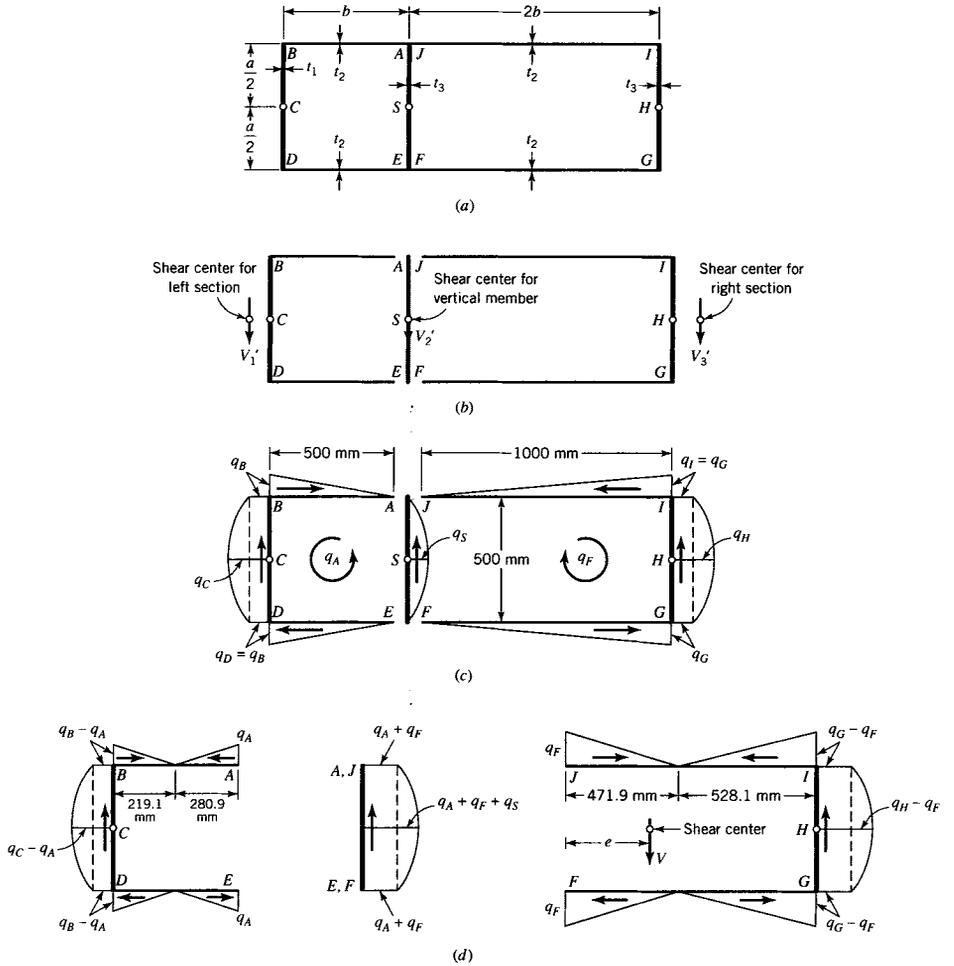


FIGURE E8.5

Solution

The moment of inertia about the x axis is $I_x = 2343.75 \times 10^6 \text{ mm}^4$. Cuts are taken to the left and right of the internal vertical member (Figure E8.5b). For convenience, let the magnitude of the shear load V for the box beam be equal to the magnitude of I_x , so that $V'_1 = I_{x1}$, $V'_2 = I_{x2}$, and $V'_3 = I_{x3}$. The shear flow for each part of the cut beam cross section (Figure E8.5b) is determined as follows (with $V'_1 = V_1$, $V'_2 = V_2$, and $V'_3 = V_3$):

For the left section,

$$q_B = \frac{V_1 A' \bar{y}'}{I_{x1}} = (bt_2) \frac{a}{2} = 1250 \text{ kN/mm}$$

$$q_C = q_B + \left(\frac{a}{2}t_1\right) \frac{a}{4} = 1406.25 \text{ kN/mm}$$

For the internal vertical member,

$$q_S = \frac{V_2 A' \bar{y}'}{I_{x2}} = \left(\frac{a}{2}t_3\right)\frac{a}{4} = 625 \text{ kN/mm}$$

For the right section,

$$q_G = \frac{V_3 A' \bar{y}'}{I_{x3}} = (2bt_2)\frac{a}{2} = 2500 \text{ kN/mm}$$

$$q_H = q_G + \left(\frac{a}{2}t_3\right)\frac{a}{4} = 3125 \text{ kN/mm}$$

The senses of the shear flows oppose those of $V\phi$, $V\phi$, and $V\phi$. For the left section (Figure E8.5c), the shear flow increases linearly from zero at E to q_B at D and decreases linearly from q_B to zero at A . The shear flow changes parabolically from q_B at D to q_C at C and back to q_B at B . For the internal vertical member, the shear flow changes parabolically from zero at E (or F) to q_S at S and back to zero at A (or J). The corresponding shear flows for the right section are shown in Figure E8.5c.

Next we add q_A to every point in the left cross section and q_F to every point in the right cross section, including the vertical internal member (Figure E8.5d). We require that Eq. 8.11 be satisfied for both the left and right sections.

For the left-hand section starting at point B , we find by Figure E8.5c

$$\begin{aligned} \theta_L = & \left[q_A - q_B - \frac{2}{3}(q_C - q_B) \right] \frac{a}{t_1} + \left(q_A - \frac{1}{2}q_B \right) \frac{b}{t_2} \\ & + \left(q_A + \frac{2}{3}q_S \right) \frac{a}{t_3} + \left(q_A - \frac{1}{2}q_B \right) \frac{b}{t_2} + q_F \frac{a}{t_3} = 0 \end{aligned}$$

Similarly for the right-hand compartment, we have starting at point I

$$\begin{aligned} \theta_R = & \left[q_F - q_G - \frac{2}{3}(q_H - q_G) \right] \frac{a}{t_3} + \left(q_F - \frac{1}{2}q_G \right) \frac{2b}{t_2} \\ & + \left(q_F + \frac{2}{3}q_S \right) \frac{a}{t_3} + \left(q_F - \frac{1}{2}q_G \right) \frac{2b}{t_2} + q_A \frac{a}{t_3} = 0 \end{aligned}$$

Inserting numerical values in Eqs. (a) and (b), we obtain

$$9q_A + q_F = 7500$$

$$q_A + 10q_F = 12,000$$

The solution of Eqs. (c) is

$$q_A = 702.3 \text{ kN/mm}$$

$$q_F = 1179.8 \text{ kN/mm}$$

The shear flow distribution is shown in Figure E8.5d.

The equilibrium equation for moments about point E (points E and F coincide, as do points A and J) gives

$$\begin{aligned} 0 = & V e + \left\{ q_B - q_A + \frac{2}{3}[q_C - q_A - (q_B - q_A)] \right\} (500)(500) \\ & + \frac{1}{2}(q_B - q_A)(219.1)(500) - \frac{1}{2}q_A(280.9)(500) \\ & + \frac{1}{2}q_F(471.9)(500) - \frac{1}{2}(q_G - q_F)(528.1)(500) \\ & - \left\{ q_G - q_F + \frac{2}{3}[q_H - q_F - (q_G - q_F)] \right\} (500)(1000) \end{aligned}$$

Substitution for the shear flows and $V = 2,343,750$ kN yields

$$e = \frac{759,894,167 \text{ kN} \cdot \text{mm}}{2,343,750 \text{ kN}} = 324.2 \text{ mm}$$

Hence, the shear center is located 324.2 mm to the right of the internal vertical member (Figure E8.5d).

As a check, note that by Figure E8.5d,

$$V_1 = \left\{ q_B - q_A + \frac{2}{3}[q_C - q_A - (q_B - q_A)] \right\} (500) = 325,960 \text{ kN}$$

$$V_2 = \left(q_A + q_F + \frac{2}{3}q_S \right) (500) = 1,149,350 \text{ kN}$$

$$V_3 = \left\{ q_G - q_F + \frac{2}{3}[q_H - q_F - (q_G - q_F)] \right\} (500) = 868,440 \text{ kN}$$

or

$$V_1 + V_2 + V_3 = 2,343,750 \text{ kN}$$

PROBLEMS

Section 8.3

8.1. Locate the shear center for the hat section beam shown in Figure A of Table 8.1 by deriving the expression for e .

8.2. Verify the relation for e for the cross section shown in Figure B of Table 8.1.

8.3. Locate the shear center for the nonsymmetrical I-beam shown in Figure C of Table 8.1 by deriving the expression for e .

8.4. Show that the shear center for the cross section in Figure D of Table 8.1 is located at distance e as shown.

8.5. Derive the relation for e for the circular arc cross section shown in Figure E of Table 8.1.

8.6. Derive the relation for e for the helmet cross section shown in Figure F of Table 8.1

8.7. An extruded bar of aluminum alloy has the cross section shown in Figure P8.7. Locate the shear center for the cross section. **Note:** Small differences in the value of e may occur because of differences in the approximations of I_x .

8.8. A 2.50-mm-thick plate of steel is formed into the cross section shown in Figure P8.8. Locate the shear center for the cross section.

8.9. A rolled steel channel has the dimensions shown in Figure P8.9. Locate the shear center for the cross section.

8.10. A beam has the cross section shown in Figure P8.10. Locate the shear center for the cross section. Express your answer relative to principal axes.

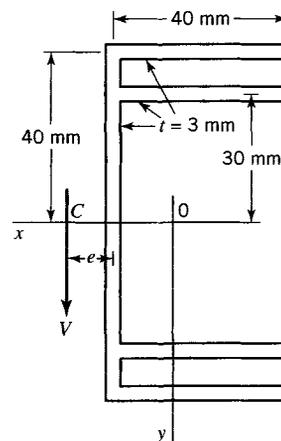


FIGURE P8.7

8.11. An extruded bar of aluminum alloy has the cross section shown in Figure P8.11. Locate the shear center for the cross section.

8.12. A 4-mm-thick plate of steel is formed into the cross section shown in Figure P8.12. Locate the shear center for the cross section.

8.13. A 5-mm-thick plate of steel is formed into the cross section shown in Figure P8.13. Locate the shear center for the cross section.

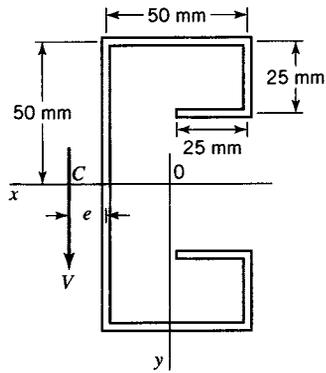


FIGURE P8.8

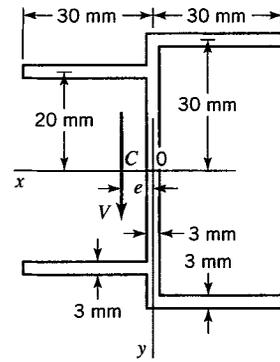


FIGURE P8.11

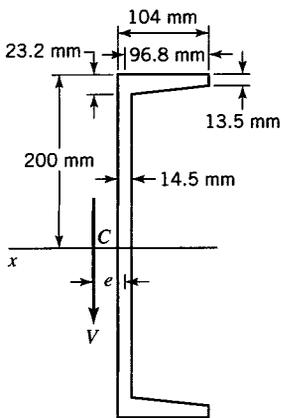


FIGURE P8.9

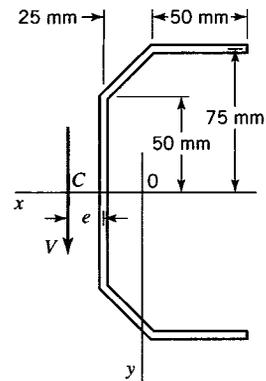


FIGURE P8.12

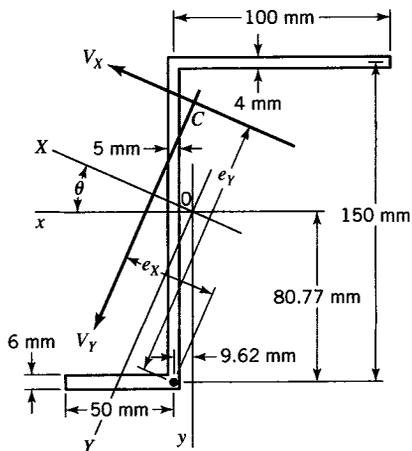


FIGURE P8.10

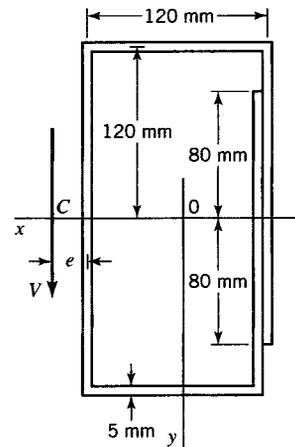


FIGURE P8.13

8.14. A 5-mm-thick plate of steel is formed into the semicircular shape shown in Figure P8.14. Locate the shear center for the cross section.

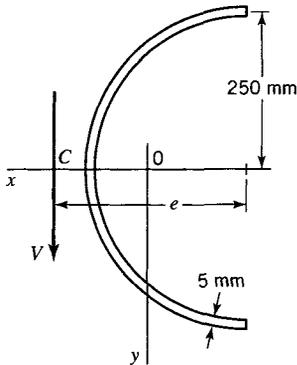


FIGURE P8.14

8.15. The horizontal top-most and bottom-most arms of the extruded bar of Figure P8.7 are removed. Locate the shear center for the modified section.

8.16. An aluminum alloy extrusion has the cross section shown in Figure P8.16. The member is to be used as a beam with the x axis as the neutral axis. Locate the shear center for the cross section.

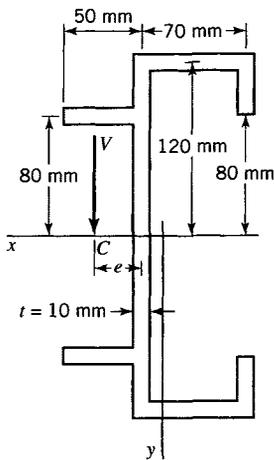


FIGURE P8.16

8.17. Locate the shear center for the beam cross section shown in Figure P8.17. Both flanges and the web have thickness $t = 3.00$ mm.

8.18. Locate the shear center for the beam cross section shown in Figure P8.18. The walls of the cross section have constant thickness $t = 2.50$ mm.

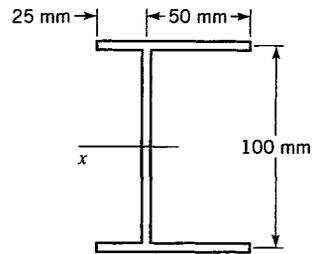


FIGURE P8.17

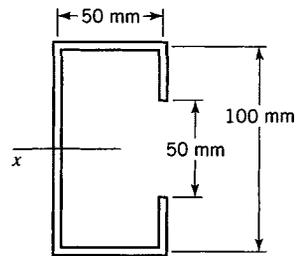


FIGURE P8.18

8.19. Locate the shear center for the beam cross section shown in Figure P8.19. The walls of the cross section have constant thickness $t = 2.00$ mm.

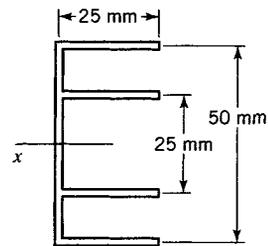


FIGURE P8.19

8.20. Locate the shear center for the beam cross section shown in Figure P8.20. The walls of the cross section have constant thickness $t = 2.00$ mm.

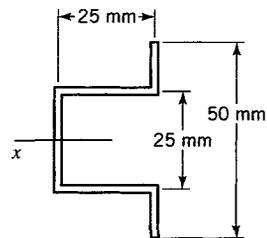


FIGURE P8.20

8.21. Locate the shear center for the beam cross section shown in Figure P8.21. The walls of the cross section have constant thickness $t = 2.00$ mm.

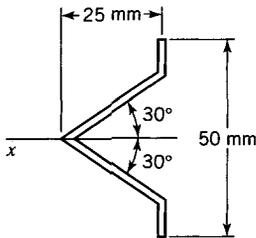


FIGURE P8.21

8.22. Locate the shear center for the beam cross section shown in Figure P8.22. The walls of the cross section have constant thickness $t = 2.00$ mm.

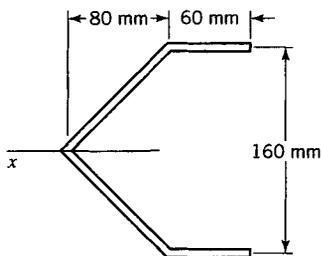


FIGURE P8.22

8.23. Locate the shear center for the beam cross section shown in Figure P8.23. The walls of the cross section have constant thickness $t = 2.50$ mm.

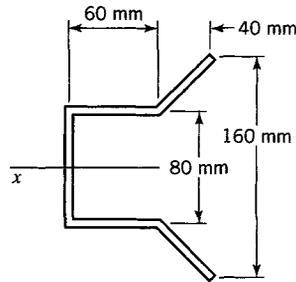


FIGURE P8.23

8.24. For the beam cross section shown in Figure P8.24, $b \gg t$. Show that the moment of inertia $I_x = 5.609b^3t$ and locate the shear center for the cross section.

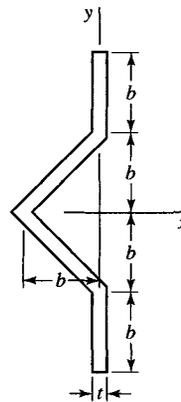


FIGURE P8.24

8.25. The channel shown in Figure P8.25 is subjected to non-symmetric bending. The associated shear forces, which act through the shear center, are $V_x = -2400$ N and $V_y = 1800$ N. Determine the distribution of the shear stress throughout the cross section. Make a sketch, to scale, of the shear stress distribution in the channel walls.

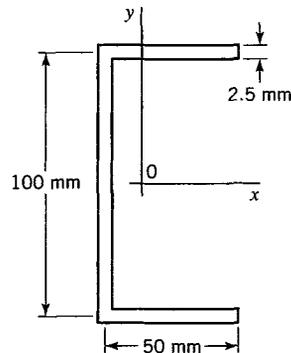


FIGURE P8.25

Section 8.4

8.26. A beam is built up of a thin steel sheet of thickness $t = 0.60$ mm bent into a semicircle as shown in Figure P8.26. Two 25-mm square stringers are welded to the thin web as shown. Locate the shear center for the cross section.

8.27. A beam has a symmetrical cross section (Figure P8.27). A vertical web with a thickness of 0.60 mm is welded to two 20 mm by 20 mm by 4 mm angle-section ($A = 146$ mm² and

centroid location 6.4 mm) stringers. The two horizontal webs have a thickness of 0.60 mm and are welded to the angle sections and 20 mm by 20 mm by 4 mm T-section stringers. Locate the shear center for the cross section.

8.28. A composite beam has a symmetrical cross section as shown in Figure P8.28. A vertical web with a thickness of 2 mm is welded to the center of the flange of two 50 mm by

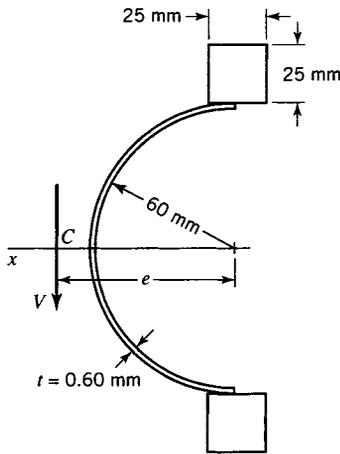


FIGURE P8.26

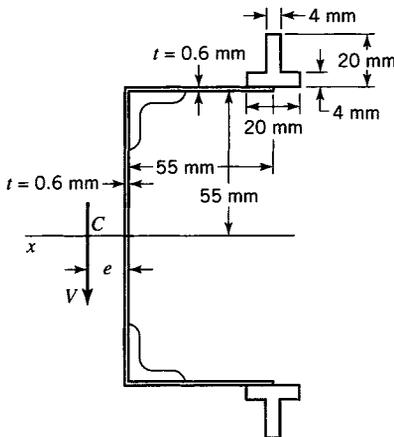


FIGURE P8.27

60 mm by 10 mm T-section stringers. Two horizontal webs, with a thickness of 1 mm, are welded to these stringers and to two additional T-section stringers. Locate the shear center of the cross section.

8.29. A composite beam has a symmetrical cross section as shown in Figure P8.29. A vertical web with a thickness of 2 mm is riveted to four rolled 30 mm by 30 mm by 5 mm angle sec-

Section 8.5

8.30. For the box beam in Figure 8.6, let $b = 100$ mm, $h = 200$ mm, $t_1 = 20$ mm, $t_2 = 10$ mm, and $t_3 = 5$ mm. Determine the location of the shear center for the cross section.

8.31. For the box beam in Figure 8.7, let $b = 200$ mm, $h = 400$ mm, $t_1 = t_2 = t_3 = 1$ mm, and $A_1 = 3A_2 = 900$ mm². Determine the location of the shear center for the cross section.

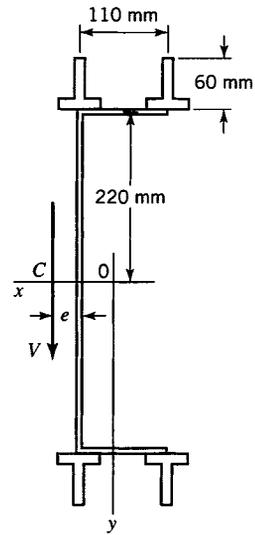


FIGURE P8.28

tions ($A = 278$ mm² and centroid location 7.7 mm). Two horizontal webs, with thickness of 1 mm, are riveted to the angles and to areas A_1 (25 mm by 25 mm) and A_2 (40 mm by 40 mm). Locate the shear center of the cross section.

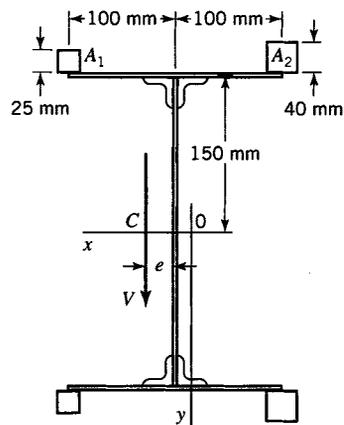


FIGURE P8.29

8.32. Let $t_1 = 2$ mm with other dimensions from Problem 8.31 remaining unchanged. Determine the location of the shear center.

8.33. A thin-wall box beam with the cross section shown in Figure P8.33 is used in the support structure of an airplane wing. Locate its shear center for the case $L_1 = L_2 = L_3 = L$ and $t_1 = t_2 = t_3 = t$.

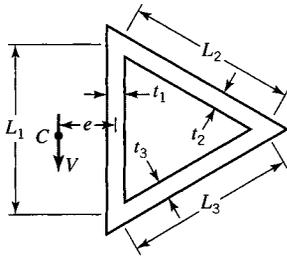


FIGURE P8.33

8.34. Determine the shear center location for the airplane wing box beam in Figure P8.33 for the case $L_1 = L$, $L_2 = L_3 = 1.5L$, $t_1 = t$, and $t_2 = t_3 = \frac{3}{4}t$.

8.35. Determine the shear center location for the airplane wing box beam in Figure P8.33 for the case $L_1 = L_2 = L_3 = 0.5$ m, $t_1 = 20$ mm, and $t_2 = t_3 = 15$ mm.

8.36. Determine the shear center location C for an aircraft semi-circular box beam whose cross section is shown in Figure P8.36.

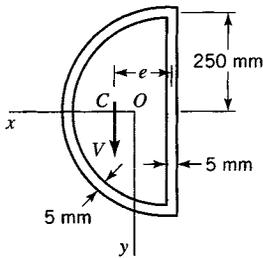


FIGURE P8.36

8.37. Determine the shear center C for an aircraft box beam whose cross section is shown in Figure P8.37.

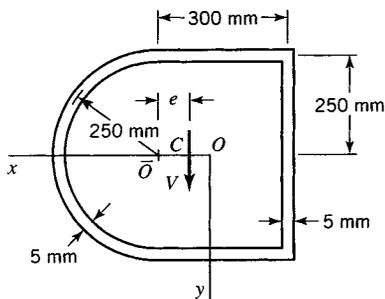


FIGURE P8.37

8.38. Locate the shear center C for the aircraft box beam whose cross section is shown in Figure P8.38.

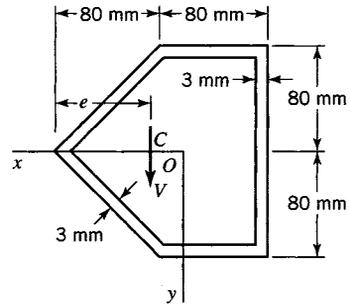


FIGURE P8.38

8.39. Locate the shear center C for the aircraft box beam whose cross section is shown in Figure P8.39.

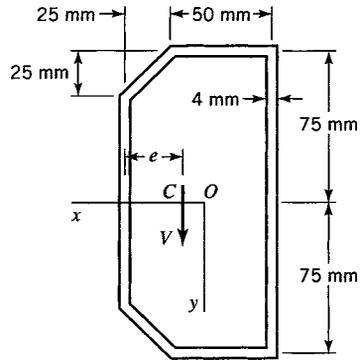


FIGURE P8.39

8.40. An aircraft box beam is built up of two thin steel sheets of thickness $t = 0.60$ mm, one bent into a semicircle of radius 60 mm and one a straight member of length 120 mm (Figure P8.40). Two 25-mm square stringers are welded to the sheets as shown. Locate the shear center C of the cross section.

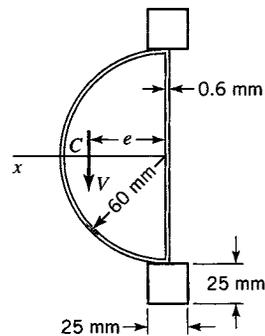


FIGURE P8.40

8.41. An aircraft box beam has a symmetrical cross section (Figure P8.41). It consists of two vertical webs and two horizontal flanges, each of thickness 0.60 mm, two 20 mm by 20 mm by 4 mm angle-section stringers ($A = 146 \text{ mm}^2$ and centroid located 6.4 mm from the outside face of the angle leg), and two 20-mm-wide by 20-mm-deep by 4-mm-thick T-section stringers. Locate the shear center C of the cross section.

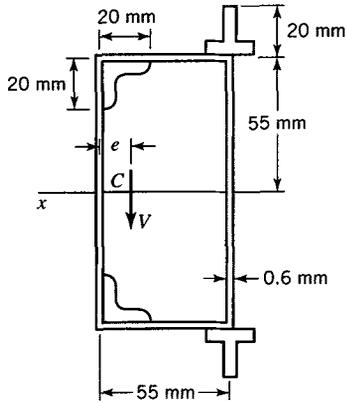


FIGURE P8.41

8.42. An aircraft box beam has a symmetrical cross section as shown in Figure P8.42. It consists of two 2-mm-thick vertical webs, two 1-mm-thick horizontal flanges, two 20 mm square stringers, and two 30 mm by 30 mm by 6 mm T-section stringers. Locate the shear center C of the cross section.

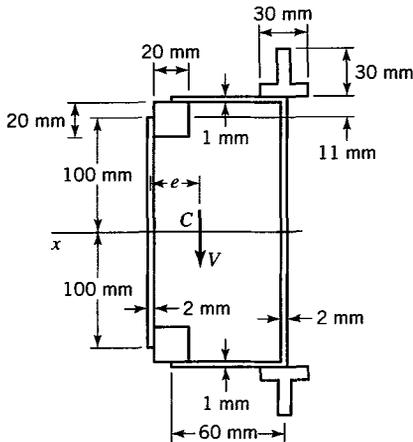


FIGURE P8.42

8.43. Locate the shear center C for the multicompartment box beam whose cross section is shown in Figure P8.43.

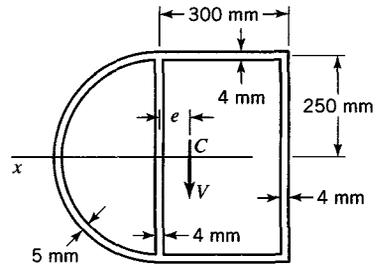


FIGURE P8.43

8.44. Locate the shear center C for the multicompartment box beam whose cross section is shown in Figure P8.44.

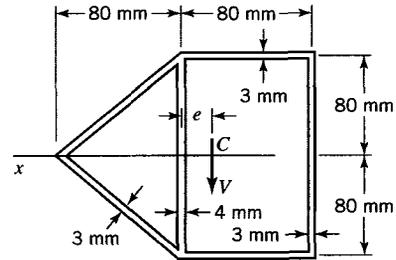


FIGURE P8.44

8.45. Locate the shear center C for the aircraft box beam whose cross section is shown in Figure P8.45.

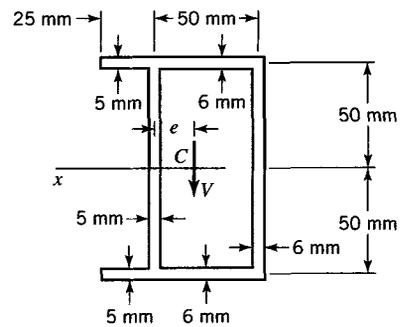


FIGURE P8.45

REFERENCE

BORESI, A. P., and CHONG, K. P. (2000) *Elasticity in Engineering Mechanics*, 2nd ed., Chapter 7. New York: Wiley-Interscience.