① Analysis of Stress:

1.1 State of stress at a point:

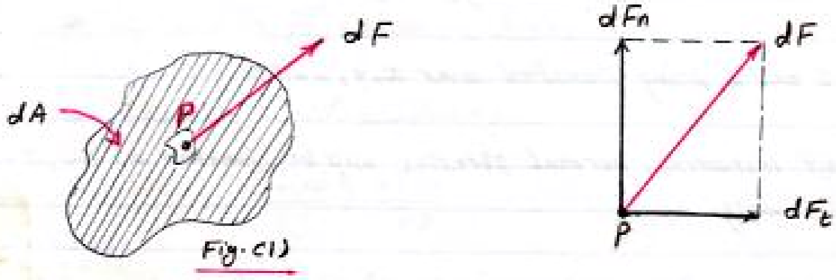


Fig. (1)

Consider an elemental force, dF, acting at a point p included in an elemental area dA.

The elemental force dF, can be reduced to the normal and tangential components dF_n and dF_t.

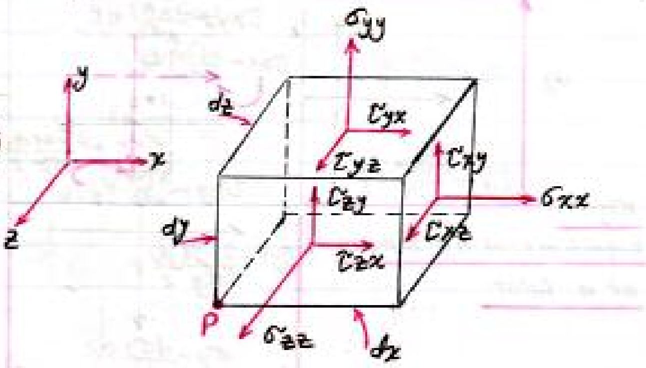
As $dA \to 0$ in the limit, stress is defined as follows:

$$\sigma = \frac{dF}{dA}, \quad \sigma_n = \frac{dF_n}{dA}, \quad \sigma_t = \frac{dF_t}{dA}$$

where σ is the total state of stress at point p and σ_n and σ_t are the normal and tangential components.

Now, in three dimensions:

Fig. (2) Stress element for a homogeneous state of Stress.



The nine components of stress which comprise the stress matrix are often described as σ_{ij} where:

$$\sigma_{ij} = \begin{vmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}$$

with i and j being iterated over x, y, and z.

In most instances, normal stresses will be denoted as σ_x, σ_y, and σ_z only.

Note that if moments are taking around point p, i.e.

$\sum M_p = 0$, this gives $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, and $\tau_{zy} = \tau_{yz}$.

therefore the nine stress components of the stress matrix is reduced to six.

Conventionally, tensile normal stresses are considered positive while compressive stresses are negative. So all normal stresses in the above figure are positive.

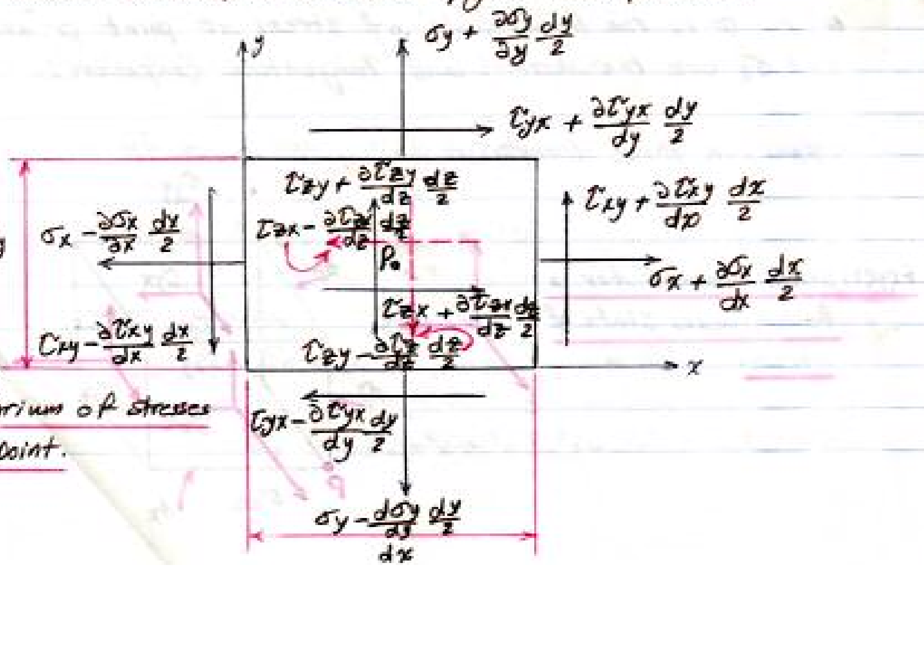


Fig-(3): Equilibrium of stresses at a point.

If the state of stress varies from point to point this type of change must be accommodated as shown in the above figure.

Using the x-y plane for refrence, considering a force summation in the x-direction, i.e.

$$\sum F_x = 0 = \sum (\text{stress})(\text{Area})$$

The following will produce,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \qquad (1.1a)$$

Similarly, an identical procedure, using the y and z directions, would produce.

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0 \qquad (1.1b)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \qquad (1.1c)$$

The above equations are called the equilibrium equations.

1.2 Three-dimensional Analysis of a homogeneous stress state:

often, it is important to determine the state of stress on a plane at some angle to those upon which the applied stresses act.

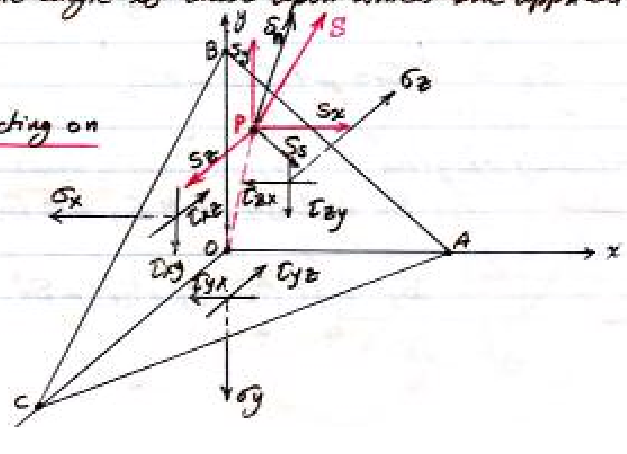
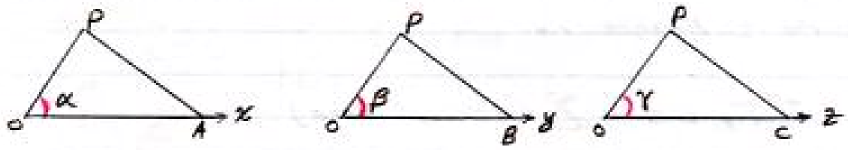


Fig.(4) Stress acting on plane ABC.

considering OP to be normal to ABC, its line of orientation with respect to the x-y-z coordinate system is defined by the three direction cosines shown below.



$$\cos\alpha = \frac{OP}{OA} = l \qquad \cos\beta = \frac{OP}{OB} = m \qquad \cos\gamma = \frac{OP}{OC} = n$$

Let the total stress acting on ABC is S, this would produce stress components S_x, S_y, S_z as shown in the figure. where

$$S^2 = S_x^2 + S_y^2 + S_z^2 \qquad (1.2)$$

If stress components perpendicular and parallel to ABC are of greater concern, we find

$$S^2 = S_n^2 + S_s^2 \qquad (1.3)$$

Now, taking a force balance in the x-direction $\Sigma F_x = 0$, gives

$$S_x = l\,\sigma_x + m\tau_{yx} + n\tau_{zx} \qquad (1.4a)$$

and from $\Sigma F_y = 0$ and $\Sigma F_z = 0$, we get

$$S_y = l\,\tau_{xy} + m\sigma_y + n\tau_{zy} \qquad (1.4b)$$

$$S_z = l\,\tau_{xz} + m\tau_{yz} + n\sigma_z \qquad (1.4c)$$

Vector analysis gives,

$$S_n = l\,S_x + m\,S_y + n\,S_z \qquad (1.5)$$

$$S_s = \sqrt{(S_x^2 + S_y^2 + S_z^2) - S_n^2} \qquad (1.6)$$

Also S_n can be determined from the direction cosines and the known stresses as follows:

$$S_n = \ell^2\sigma_x + m^2\sigma_y + n^2\sigma_z + 2(\ell m\,\tau_{yx} + mn\,\tau_{yz} + n\ell\,\tau_{zx})$$

In essence, no shear component acts on ABC and the direction cosines defining the line from the origin, O, that is now normal to ABC, however ℓ, m, and n can still be used with this in mind:

$$S_x = \ell S \ , \ S_y = mS \ , \ S_z = nS$$

The above relationships, if substituted into equ (1.4), produce the following

$$\ell(\sigma_x - S) + m(\tau_{yx}) + n(\tau_{zx}) = 0$$

$$\ell(\tau_{xy}) + m(\sigma_y - S) + n(\tau_{zy}) = 0 \qquad (1.7)$$

$$\ell(\tau_{xz}) + m(\tau_{yz}) + n(\sigma_z - S) = 0$$

These three homogeneous equations give real roots other than zero only if the determinant is zero. Setting the determinant to zero and expanding gives a cubic equation whose three roots are the principal stresses (i.e. the stresses on plane of zero shear stress).

Denoting the stress S as σ_p gives:

$$\sigma_p^3 - I_1\sigma_p^2 - I_2\sigma_p - I_3 = 0 \qquad (1.8)$$

Where I_1, I_2, and I_3 are called the Invariants, and they given by

$$I_1 = (\sigma_x + \sigma_y + \sigma_z) \qquad (1.9a)$$

$$I_2 = (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x) \qquad (1.9b)$$

$$I_3 = (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2) \qquad (1.9c)$$

Example (1.1) :

An applied stress state is described by

$$\sigma_{ij} = \begin{vmatrix} 20 & 3 & 8 \\ 3 & 15 & 5 \\ 8 & 5 & 10 \end{vmatrix}$$

(Note : all stresses are indicated as positive)

Determine the magnitude of the total state of stress, S, and its normal component, S_n, acting on a plane whose direction cosines are given by ($l = 0.707$, $m = 0.643$, and $n = 0.296$).

Solution :

Using eqn (1.4), gives

$$S_x = 20(0.707) + 3(0.643) + 8(0.296) = 18.44$$

$$S_y = 3(0.707) + 15(0.643) + 5(0.296) = 13.25$$

$$S_z = 8(0.707) + 5(0.643) + 10(0.296) = 11.83$$

From eqn (1.2),

$$S^2 = (18.44)^2 + (13.25)^2 + (11.83)^2 = 655.55$$

$$\therefore S = 25.6$$

And from equ (1.5),

$$S_n = 18.44(0.707) + 13.25(0.643) + 11.83(0.296) = 25.05$$

Example (1.2):

A given state is expressed as

$$\sigma_{ij} = \begin{vmatrix} 4 & 2 & 3 \\ 2 & 6 & 1 \\ 3 & 1 & 5 \end{vmatrix}$$

The unit of each stress are in MPa and all stresses are denoted as positive. Find the magnitudes of the principle stresses and the direction cosines defining the line of action of the largest principal stress with respect to the original x-y-z coordinate system.

Solution:

Using eqn (1.9), gives

$$I_1 = 4 + 6 + 5 = +15$$
$$I_2 = 4 + 1 + 9 - 24 - 30 - 20 = -60$$
$$I_3 = 120 + 12 - 4 - 54 - 20 = +54$$

Substitute in equ (1.8),

$$\sigma_p^3 - 15\sigma_p^2 + 60\sigma_p - 54 = 0$$

Solution of the above cubic equation gives the three principal stresses as:

$$\sigma_1 = +9, \quad \sigma_2 = +4.73, \quad \text{and } \sigma_3 = +1.27$$

To obtain the direction cosines of the largest principal stress, use equ (1.7), and subtitute the value of the largest principal stress, which is $\sigma_1 = +9$, we get:

$$l(4-9) + m(2) + n(3) = 0$$

$$l(2) + m(6-9) + n(1) = 0$$

$$l(3) + m(1) + n(5-9) = 0$$

which can be reduced to,

$$-5l + 2m + 3n = 0$$
$$2l - 3m + 1n = 0$$
$$3l - 1m - 4n = 0$$

Using any two of the above three equations plus the identity that

$$l^2 + m^2 + n^2 = 1$$

the values for l, m, and n are then found.

For this example since the stress is principal stress then $l = m = n$, i.e.

$$l^2 + m^2 + n^2 = 1 = 3l^2 = 3m^2 = 3n^2$$

i.e $l = \sqrt{\frac{1}{3}}$, $m = \sqrt{\frac{1}{3}}$, $n = \sqrt{\frac{1}{3}}$

which gives $\cos\alpha = l = \sqrt{\frac{1}{3}} \Rightarrow \alpha = 54.74^\circ = \beta = \gamma$

checking: $I_1 = \sigma_1 + \sigma_2 + \sigma_3 = 9 + 4.73 + 1.27 = +15$ o.k.
$I_2 = -(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = -(42.57 + 60 + 11.43) = -60$ o.k.
$I_3 = (\sigma_1\sigma_2\sigma_3) = +54$ o.k.

1.3 Stress Transformations:

To provide a systematic approach to the transformation of stress from one coordinate system to another. Consider the following situation, where forces F_x, F_y, and F_z act along the x, y, z refrence axes.

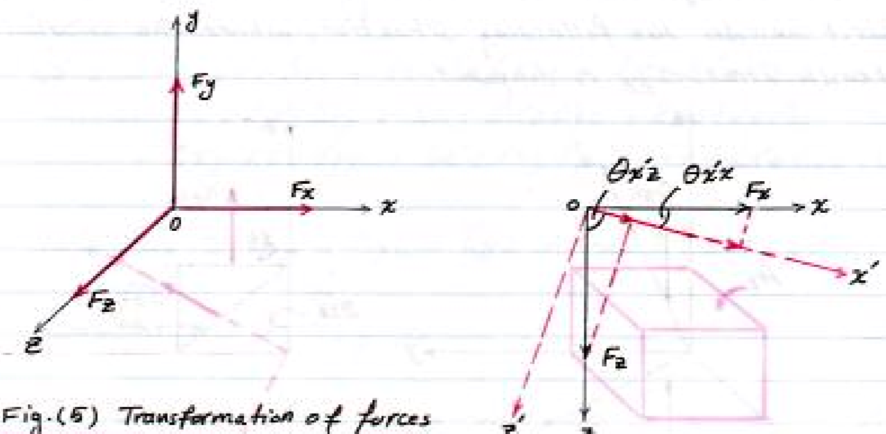


Fig.(5) Transformation of forces from one coordinate system to another.

Now, assume y' is the same as y such that the new x' and z' axes are in the same plane as x and z. The force component $F_{x'}$ is composed of the projections of F_x and F_z on the x' axis, thus.

$$F_{x'} = F_x \cos\theta_{x'x} + F_z \cos\theta_{x'z}$$

Since $\cos\theta_{x'x} = l$ and $\cos\theta_{x'z} = n$, then

$$F_{x'} = l F_x + n F_z$$

In the general situation, the force F_y would also contribute to $F_{x'}$, as follows:

$$F_{x'} = l F_x + m F_y + n F_z \qquad (1.10)$$

Similar relationships could be developed for $F_{y'}$ and $F_{z'}$ using the proper set of direction cosines for each transformation.
In general form:

$$F_i = \sum l_{ij} F_j \,, \quad j = x, y, z \,, \quad i = x', y', z' \,, \quad l_{ij} = \cos\theta_{ij}$$

For the transformation of matrix quantities such as stress, first consider the following situation, where the uniaxial tensile stress σ_{zz} is imposed.

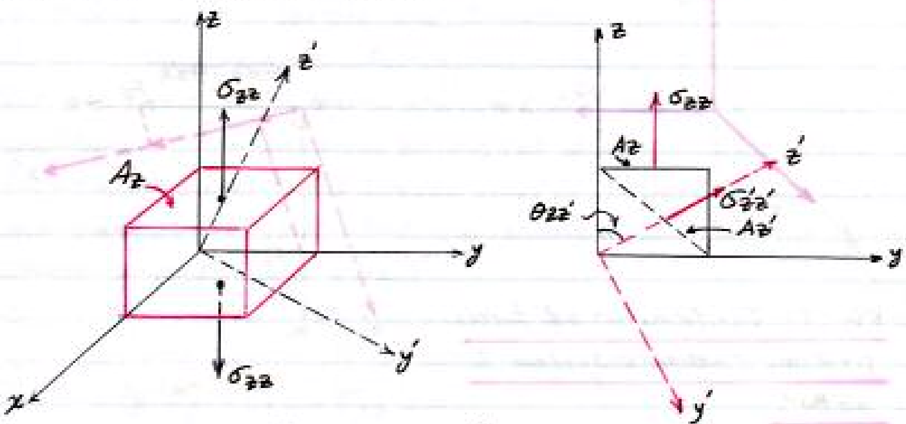


Fig. (6) Uniaxial stress transformation to an x', y', z' system.

The force in the z-direction is $F_z = \sigma_{zz} A_z$.

Thus $F_{z'} = F_z \cos\theta_{z'z}$ is the component of F_z acting along the z' axis.

The area $A_{z'}$ which is normal to z' is

$$A_{z'} = A_z / \cos\theta_{z'z}$$

Therefore, $\sigma_{z'z'} = \dfrac{F_{z'}}{A_{z'}} = \dfrac{F_z \cos\theta_{z'z}}{A_z / \cos\theta_{zz'}}$

$$\therefore \sigma_{z'z'} = \sigma_{zz} \cos^2\theta_{z'z}$$

For the fully generalised case, this type of transformation is expressed as

$$\sigma_{ij} = \sum l_{im} l_{jn} \sigma_{mn}$$

with m, n iterated over x, y, z and i, j iterated over x', y', z'.

Thus, the complete expression for $\sigma_{x'x'}$ becomes:

$$\begin{aligned}\sigma_{x'x'} = {} & l_{x'x} l_{x'x} \sigma_{xx} + l_{x'y} l_{x'y} \sigma_{yy} + l_{x'z} l_{x'z} \sigma_{zz} \\ & + l_{x'x} l_{x'y} \sigma_{xy} + l_{x'y} l_{x'z} \sigma_{yz} + l_{x'z} l_{x'x} \sigma_{zx} \\ & + l_{x'y} l_{x'x} \sigma_{yx} + l_{x'z} l_{x'y} \sigma_{zy} + l_{x'x} l_{x'z} \sigma_{xz} \qquad (1.11)\end{aligned}$$

where $\sigma_{xy} \equiv \tau_{xy}$, $\sigma_{yz} \equiv \tau_{yz}$, $\sigma_{zx} \equiv \tau_{zx}$, etc.

Rearrange the terms, gives

$$\begin{aligned}\sigma_{x'} = {} & l^2_{x'x} \sigma_x + l^2_{x'y} \sigma_y + l^2_{x'z} \sigma_z \qquad (1.12)(a) \\ & + 2 (l_{x'x} l_{x'y} \tau_{xy} + l_{x'x} l_{x'z} \tau_{xz} + l_{x'y} l_{x'z} \tau_{yz})\end{aligned}$$

Knowing that $l_{x'x} \equiv l$, $l_{x'y} \equiv m$, $l_{x'z} \equiv n$, we get

$$\sigma_{x'} = l^2 \sigma_x + m^2 \sigma_y + n^2 \sigma_z + 2 (lm\, \tau_{xy} + ln\, \tau_{xz} + mn\, \tau_{yz}) \qquad (1.12)(b)$$

Similarly from equ (1.11) by appropriate interchange of subscripts, equivalent expressions for $\sigma_{y'}$, $\sigma_{z'}$, $\tau_{x'y'}$, etc. can be developed.

Example (1.3):

Using the data given in Example (1.1), find the magnitute of the stress acting normal to ABC.

Solution:

Here, $\sigma_{x'}$ is the normal stress acting in the x'-direction which is defined by the direction cosines $l = 0.707$, $m = 0.643$

and $n = 0.296$, thus using equ (1.12 b), gives

$$\sigma_{x'} = 20(0.707)^2 + 15(0.643)^2 + 10(0.296)^2 + 2\big[(0.707)(0.643)(3)$$

$$+ (0.707)(0.296)(8) + (0.643)(0.296)(5)\big]$$

$$\sigma_{x'} = 25.05$$

1.4 Mohr's circle – Its use and limitation.

For any stress state that is equivalent to a biaxial or uniaxial conditions, Mohr's circle can be applied directly. The prime requirement is that shear stresses must be absent on at least one plane, thus, that plane is a principle plane and the normal stress is a principal stress.

Consider the biaxial stress state shown below, where for simplicity $\sigma_z = \tau_{xz} = \tau_{yz} = 0$.

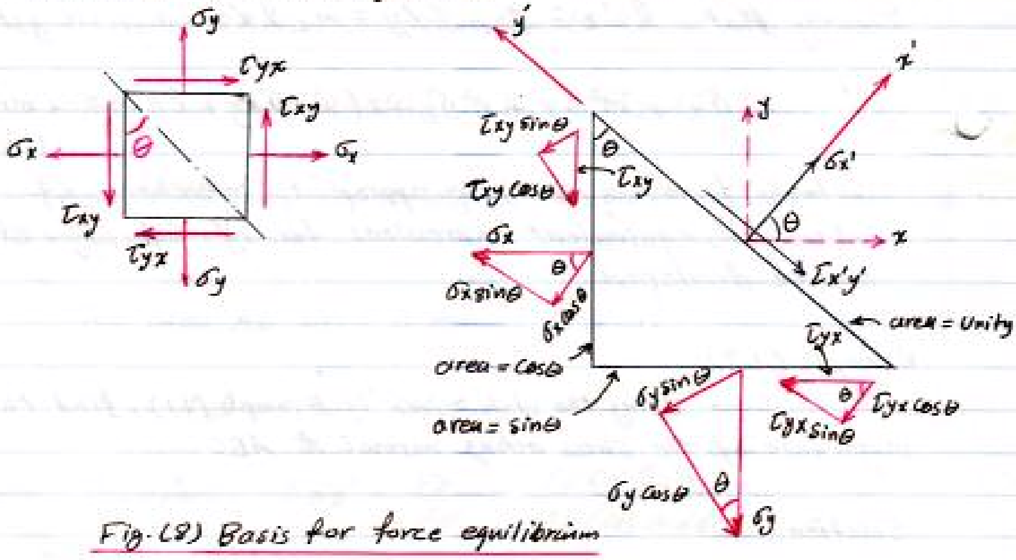


Fig. (8) Basis for force equilibrium Analysis.

Regarding Fig.(8), equilibrium of forces along the x' and y' axes must be zero.

Considering $\Sigma F_{x'} = 0$, we get,

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta \qquad (1.13)$$

with the aid of trigonometric identities, the above equation can be written as,

$$\sigma_{x'} = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy} \sin 2\theta \qquad (1.14)$$

The force balance in the y'-direction leads to,

$$\tau_{x'y'} = \tfrac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta \qquad (1.15)$$

Intrest often centres on those planes where $\tau_{x'y'}$ is zero. These planes then is the principal planes.

In order to find the two principal stresses in the x-y plane, $\tau_{x'y'}$ is set equal to zero, then the value of θ defines the principal directions with respect to the original coordinate system is then given by,

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad (1.16)$$

Note the third principal direction is perpendicular to the x-y plane.

From the above equation, we can get,

$$\sin 2\theta = \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} \qquad (1.17a)$$

$$\cos 2\theta = \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} \qquad (1.17b)$$

Substitution of these findings into equ (1.14) gives,

$$(\sigma_x')_p = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2\right]^{1/2} = A \pm B \quad (1.18)$$

These two expressions give the principle stresses in the x-y plane and from this point on such stresses will be denoted as σ_1, σ_2, and σ_3. From the above equation,

$$\sigma_1 = A + B \quad , \quad \sigma_2 = A - B$$

While σ_3 is known to act in the third z-direction.

With regard to the planes where $\tau_{x'y'}$ is maximum (i.e. τ_{max}), set,

$$\frac{d\tau_{x'y'}}{d\theta} = 0$$

This leads to,

$$\tan 2\theta = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}} \quad (1.19)$$

Using expressions similar to equ (1.17) and substituting into equ (1.15) gives:

$$\tau_{max} = \frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2\right]^{1/2} \quad (1.20)$$

It is also noted that comparisons of Equ (1.16) and (1.19) show that τ_{max} acts on planes oriented 45° from those exposed to the principal stresses.

The pervious developments have made no mention of a plot of Mohr's circle but the following will indicate the basis behined this concept. Rearranging Equ (1.14) and (1.15) to give,

$$\left[\sigma_x' - \frac{1}{2}(\sigma_x + \sigma_y)\right] = \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta$$

Squaring the above equations, then adding gives:

$$\left[\sigma_{x'} - \frac{1}{2}(\sigma_x + \sigma_y)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \qquad (1.21)$$

The above equation is a circle whose centre is at $\left[\frac{1}{2}(\sigma_x + \sigma_y), 0\right]$ and whose radius is $\left[(\sigma_x - \sigma_y)^2/4 + \tau_{xy}^2\right]^{1/2}$.

Every point on the circle defines the stress state acting on planes at any angle θ from the original x or y axis.

Example (1.4):

A stress state is described by

$$\sigma_{ij} = \begin{vmatrix} 20 & -4 & 0 \\ -4 & -15 & 0 \\ 0 & 0 & 10 \end{vmatrix}$$

The stress given as $\sigma_z = 10$ is a principal stress.

(a) Consider a plane located at an angle $\theta = 30^\circ$. Determine the normal and shear stresses acting on that plane.

(b) Determine the magnitude of the three principal stresses and the largest shear stress acting in the x-y plane.

Solution:

(a) From eqn (1.14) and (1.15)

$$\sigma_{x'} = \frac{1}{2}(20-15) + \frac{1}{2}\left[20 - (-15)\right]\cos 60^\circ + (-4)\sin 60^\circ$$

$$= 2.5 + \frac{1}{2}(35)(0.5) - 4(0.866) = +7.79 \text{ (tensile)}$$

$$\tau_{x'y'} = \frac{1}{2}[20-(-15)]\sin 60 - (-4)\cos 60$$

$$= \frac{1}{2}(35)(0.866) + 4(0.5) = 17.16$$

(b) Using Eqn (1.18)

$$\sigma_p = \frac{1}{2}(20-15) \pm \frac{1}{2}\sqrt{[20-(-15)]^2 + 4(-4)^2}$$

$$= 2.5 \pm \frac{1}{2}(35.9) = 2.5 \pm 17.95$$

So, $\sigma_1 = 20.45$, $\sigma_2 = 10$ (i.e σ_z) , $\sigma_3 = -15.45$

and from eqn (1.20) , $\tau_{max} = 17.95$.

Mohr's Circle Construction:

To illustrate Mohr's circle, the following figure show a physical plane and the corresponding Mohr's circle.

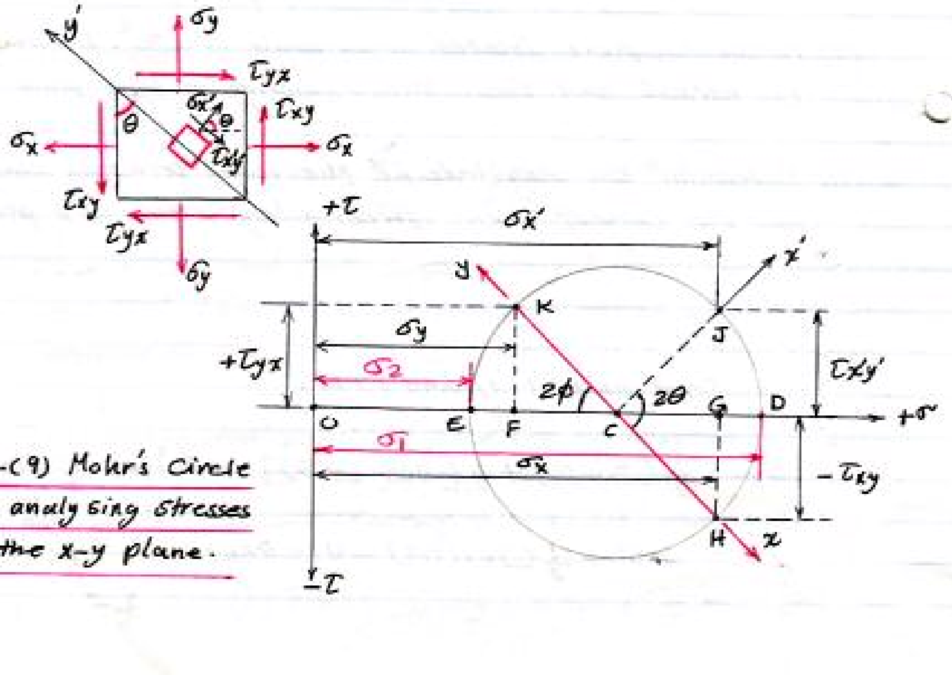


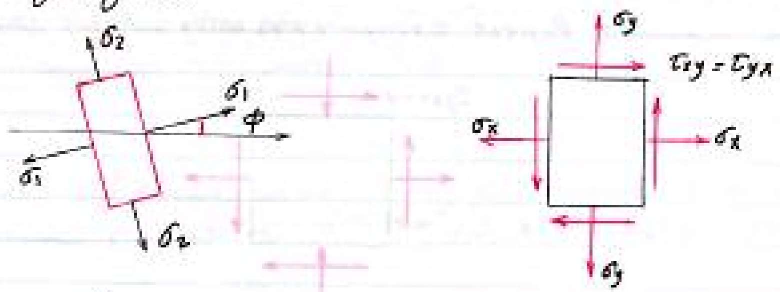
Fig-(9) Mohr's Circle for analysing stresses in the x-y plane.

The point having coordinate values of $(\sigma_x, -\tau_{xy})$ is shown as H, while K has coordinates of $(\sigma_y, +\tau_{yx})$ (recall that $\tau_{xy} = \tau_{yx}$).

The line yx drawn through H and K cuts the abscissa at C which is the centre of the circle whose radius is CK or CH.

The values of OD and OE provide the magnitudes of the principal stresses σ_1 and σ_2 while the angle 2ϕ is the angle between the x direction and principal direction 1.

The stress element involving principal stresses is shown in the following figure,



The above figure indicates that σ_1 acts along a line that is counterclockwise by ϕ degrees from the original x axis.

Returning to Fig.(9), the following useful relationships may be noted,

1. The centre of circle is located at point C and OC = OE + EC which is equals to,

$$OC = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{1}{2}(\sigma_x + \sigma_y)$$

2. The radius of the circle R = CD which is equals to,

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = [(\sigma_x - \sigma_y)^2/4 + \tau_{xy}^2]^{1/2} = \tau_{xy} / \sin 2\phi$$

with $\tan 2\phi = 2\tau_{xy}/(\sigma_x-\sigma_y)$.

3. $(\sigma_1-\sigma_2) = 2\tau_{max} = [(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2]^{1/2} = 2R$.

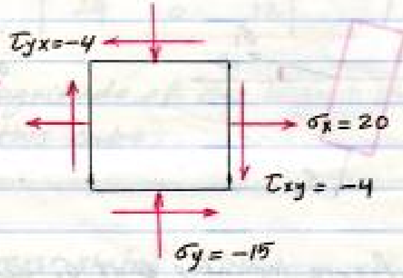
4. $\sigma_1 = OD = \frac{1}{2}(\sigma_x+\sigma_y) + \tau_{max}$.

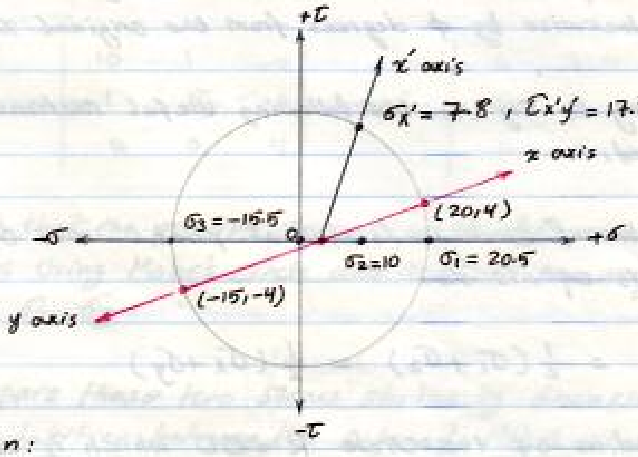
5. $\sigma_2 = OE = \frac{1}{2}(\sigma_x+\sigma_y) - \tau_{max}$.

6. $\tau_{xy} = R \sin 2\phi = [\frac{1}{2}(\sigma_1-\sigma_2)] \sin 2\phi$.

Example (1.5):

Repeat Example (1.4) using a Mohr's circle.





Solution:

From the stress matrix notation, τ_{xy} and τ_{yx} are negative so they act as shown in the sketch. For the construction of Mohr's circle τ_{xy} act clockwise and is defined as positive where as τ_{yx} is negative.

Mohr's Circle for stresses on an Arbitrary plane :

Now suppose the stress element is subjected to principal stresses as shown in the following figure and leads to the circle plot shown,

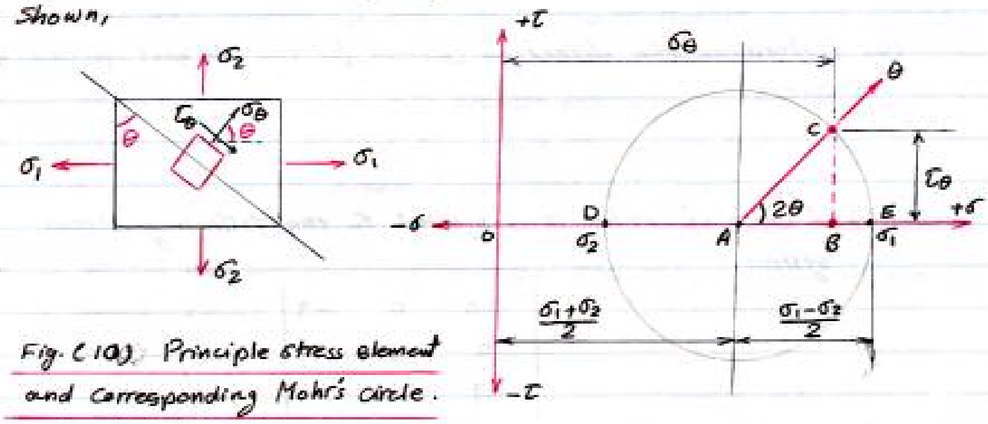


Fig. (10) Principle stress element and Corresponding Mohr's circle.

Regarding the stresses on an arbitrary plane oriented θ degrees counterclockwise from the first ("1") direction, note the following,

1. $\sigma_\theta = OB = [\frac{1}{2}(\sigma_1+\sigma_2)] + [\frac{1}{2}(\sigma_1-\sigma_2)] \cos 2\theta$

2. $\tau_\theta = CB = [\frac{1}{2}(\sigma_1-\sigma_2)] \sin 2\theta$

Problems :

(1.1) Given the following stress state, where all stresses are in MPa.

$$\begin{vmatrix} 2 & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & 1 & 0.65 \\ \frac{1}{5} & 0.65 & -\frac{1}{2} \end{vmatrix}$$

(a) Using the stress cubic equation, determine the magnitudes of the three principle stresses.

(b) Check the answers from (a) by use of the stress invariants I_1 and I_2.

(C) Determine the direction cosines for the largest principle stress.

(1.2) A cube of metal is subjected to the following stress system.

$$\begin{vmatrix} -20 & 0 & -9 \\ 0 & 45 & 0 \\ -9 & 0 & 12 \end{vmatrix}$$

Calculate the magnitude of the largest shear stress that exists in this cube.

(1.3) Consider the stress states,

$$\begin{vmatrix} 10 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 4 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} 6 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

(a) Show that the principle stresses can be determined for both states using Mohr's Circle and indicate the magnitude of σ_1, σ_2, σ_3.

(b) Compare these two stress states by discussing the similarities between the sets of Mohr's circles.