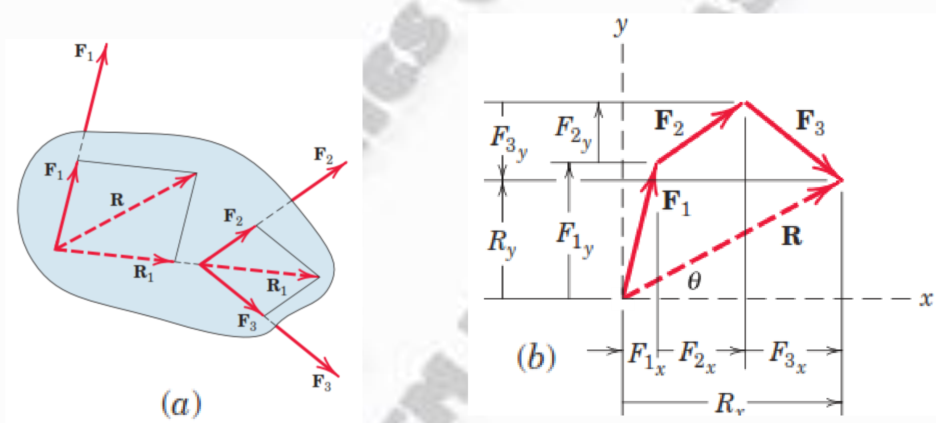
Resultants of system of forces

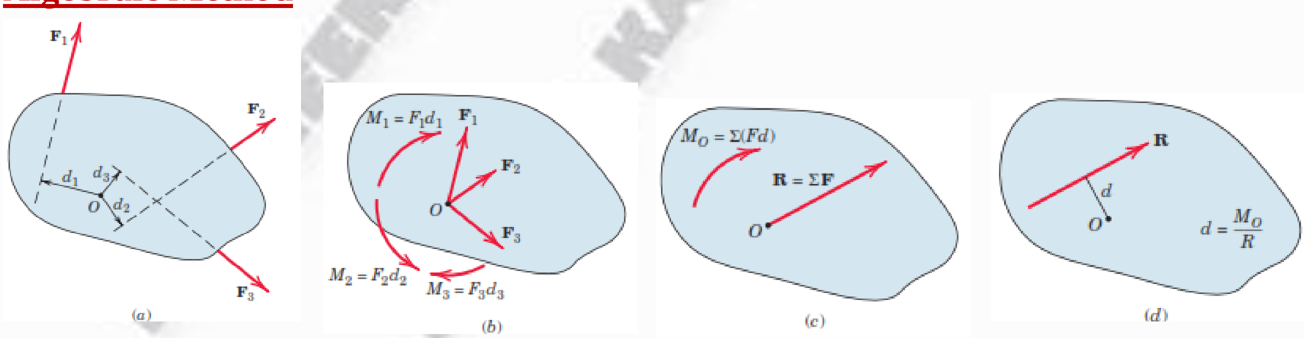
The most common type of force system occurs when the forces all act in a single plane, say, the *x-y* plane, as illustrated by the system of three forces $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$ in Fig. 2/13*a*. We obtain the magnitude and direction of the resultant force **R** by forming the force polygon shown in part *b* of the figure, where the forces are added head-to-tail in any sequence. Thus, for any system of coplanar forces we may write:



$$\mathbf{R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} + \cdots \sum \mathbf{F}$$

$$R_x = \sum F_x \qquad R_y = \sum F_y \qquad R = \sqrt{(\sum F_x)^2 + (\sum F_Y)^2} \qquad \theta = tan^{-1}\frac{R_y}{R_x} = tan^{-1}\frac{\sum F_y}{\sum F_x}$$

Algebraic Method



Principle of Moments

This process is summarized in equation form by

$$\left.\begin{array}{l} M_O = \sum \mathbf{F} \\ M_O = \sum \mathbf{M} = \sum (Fd) \\ Rd = MO \end{array}\right\} \qquad (2/10)$$

The first two of Eqs. 2/10 reduce a given system of forces to a force–couple system at an arbitrarily chosen but convenient point *O. The last* equation specifies the distance *d from point O to the line of action of* **R,** and states that the moment of the resultant force about any point *O* equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of *nonconcurrent* force systems; we call this extension the *principle of moments.*

For a concurrent system of forces where the lines of action of all forces pass through a common point *O, the moment sum SM* about that point is zero. Thus, the line of action of the resultant $\mathbf{R}=\sum\mathbf{F}$, determined by the first of Eqs. 2/10, passes through point *O.* For a parallel force system, select a coordinate axis in the direction of the forces. If the resultant force **R** for a given force system is zero, the resultant of the system need not be zero because the resultant may be a couple. The three forces in Fig. 2/15, for instance, have a zero resultant force but have a resultant clockwise couple $M = F_3 d$.

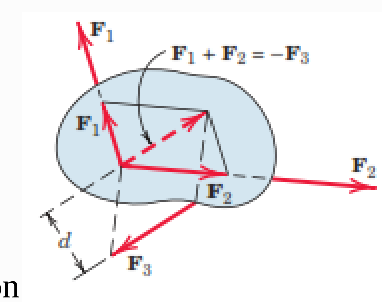
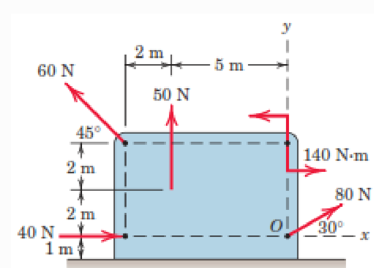


Figure 2/15

SAMPLE PROBLEM 2/9

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution: Point O is selected as a convenient reference point for the force–couple system which is to represent the given system.

$[R_x = \sum F_x] \qquad R_x = 40 + 80cos30° - 60cos45° = 66.9\ N$

$[R_y = \sum F_y] \qquad R_y = 50 + 80sin30° + 60\ sin45° = 132.4\ N$

$\left[R = \sqrt{F_x^2 + F_y^2}\right] \qquad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3\ N$

$\left[\theta = tan^{-1}\frac{R_y}{R_x}\right] \qquad \theta = tan^{-1}\frac{132.4}{66.9} = 63.2°$

$[M_o = \sum(Fd)] \qquad M_o = 140 - 50(5) + 60cos45°(4) - 60sin45°(7)$