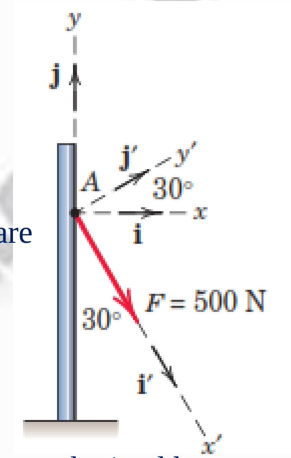
SAMPLE PROBLEM 2/3

The 500-N force **F** is applied to the vertical pole as shown. (1) Write **F** in terms of the unit vectors **i** and **j** and identify both its vector and scalar components. (2) Determine the scalar components of the force vector **F** along the *x'*- and *y'*-axes. (3) Determine the scalar components of **F** along the *x'*- and *y'*-axes.



Solution: ***Part (1)***. From Fig. a we may write **F** as

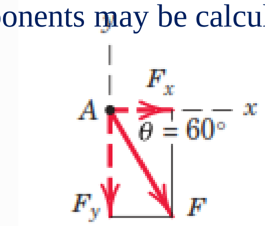
$$\mathbf{F} = (F\cos\theta)\boldsymbol{i} - (F\sin\theta)\boldsymbol{j} = (500\cos 60)\boldsymbol{i} - (500\sin 60)\boldsymbol{j}$$
$$= (250\boldsymbol{i} - 433\boldsymbol{j})N$$

The scalar components are F_x = 250 N and F_y = 433 N. The vector components are **F**$_x$=250**i** N and **F**$_y$=433**j** N.

Part (2). From *Fig. b* we may write **F** ***as*** F_x = 500i***' N,*** so that the required scalar components are

$$F_{x'} = 500\ N \qquad F_{y'} = 0$$

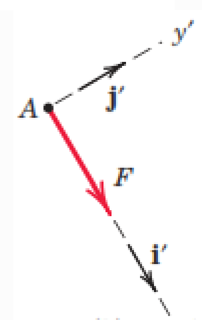
Part (3). The components of **F** in the ***x***- and ***y'***-directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig. *c*. The magnitudes of the components may be calculated by the law of sines. Thus,



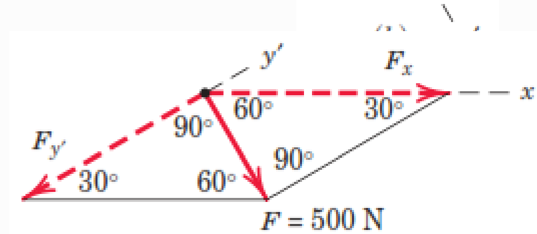
(a)

$$\frac{|F_x|}{\sin 90°} = \frac{500}{\sin 30°} \qquad |F_x| = 1000\ N$$

$$\frac{|F_{y'}|}{\sin 60°} = \frac{500}{\sin 30°} \qquad |F_{y'}| = 866\ N$$



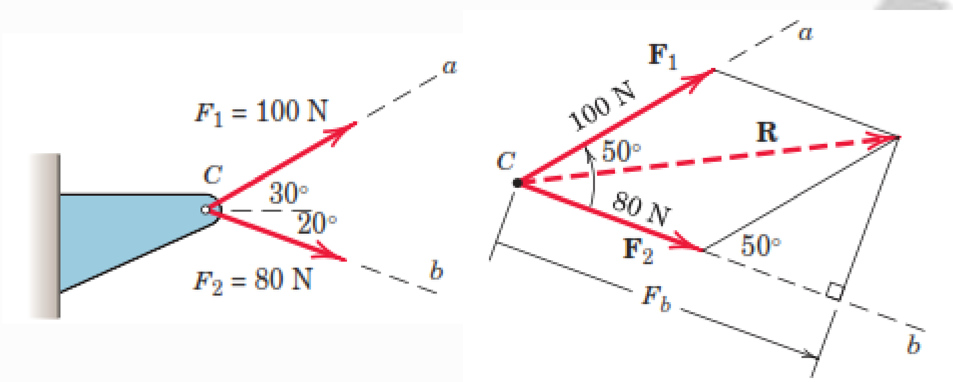
The required scalar components are then F_x=1000 N F_y=866 N



(c)

SAMPLE PROBLEM 2/4

Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bracket as shown. Determine the projection $\boldsymbol{F_b}$ of their resultant $\mathbf{R}$ onto the b-axis .



Solution. The parallelogram addition of $\mathbf{F}_1$and $\mathbf{F}_2$ is shown in the figure. Using the law of cosines gives us

$$\boldsymbol{R}^2 = (80)^2 + (100)^2 - 2(80)(100)cos130° \qquad \boldsymbol{R} = 163.4\ N$$

The figure also shows the orthogonal projection $\mathbf{F}_b$ of $\mathbf{R}$ onto the ***b-axis.*** Its length is

$$F_b = 80 + 100cos50° = 144.3\ N$$

Note that the components of a vector are in general not equal to the projections of the vector onto the same axes. If the *a-axis* had been perpendicular to the *b-axis*, then the projections and components of **R** would have been equal.

Moment

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* **M of** the force. Moment is also referred to as *torque*. As a familiar example of the concept of moment, consider the pipe wrench of Fig. 2/8*a*. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude *F* of the force and the effective length d of the wrench handle. Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.

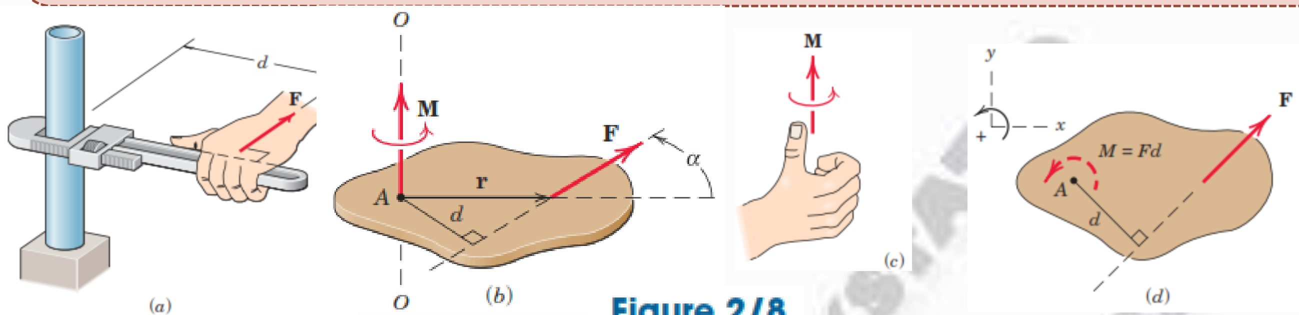


Figure 2/8

Moment about a Point

Figure 2/8*b* shows a two-dimensional body acted on by a force **F** in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm *d*, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

$$M = Fd \qquad (2/5)$$

The moment **M** obeys all the rules of vector combination and may be considered a sliding vector with a line of action coinciding with the moment axis. The basic units of moment in SI units are newton-meters (N m), and in the U.S. customary system are pound-feet (lb-ft). When dealing with forces which all act in a given plane, we customarily speak of the moment about a point. By this we mean the moment with respect to an axis normal to the plane and passing through the point. Thus, the moment of force **F** about point *A in Fig. 2/8d* has the magnitude *M* = *Fd* and is counterclockwise. Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (-) for clockwise moments, or vice versa. Sign consistency within a given problem is essential. For the sign convention of Fig. 2/8*d*, the moment of **F** about point *A* (or about the z-axis passing through point *A)* is positive. The curved arrow of the figure is a convenient way to represent moments in two-dimensional analysis.

The Cross Product

In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations. The moment of **F** about point *A of Fig. 2/8b* may be represented by the cross-product expression

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \qquad (2/6)$$