Rectangular components are convenient for finding the sum or resultant **R** of two forces are concurrent. Consider two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ which are originally concurrent at a point *O*. *Figure 2/7* shows the line of action of $\mathbf{F}_2$shifted from *O* to the tip of $\boldsymbol{F_1}$ according to the triangle rule of Fig. 2/3. In adding the force vectors $\mathbf{F_1}$ and $\mathbf{F_2}$, we may write:

$$\mathbf{R} = \mathbf{F_1} + \mathbf{F_2} = (\boldsymbol{F_1}\mathbf{i} + \boldsymbol{F_1}\mathbf{j}) + (\boldsymbol{F_2}\mathbf{i} + \boldsymbol{F_2}\mathbf{j})$$

**Or**

$$R_x i + R_y j = (F_{1_x} + F_{2_x})i + \left(F_{1_y} + F_{2_y}\right)j$$

$$\left.\begin{aligned} R_x &= (F_{1_x} + F_{2_x}) = F_x \\ R_y &= \left(F_{1_y} + F_{2_y}\right) = F_y \end{aligned}\right\} \quad (2/4)$$

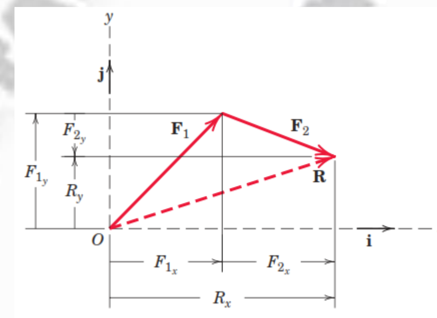
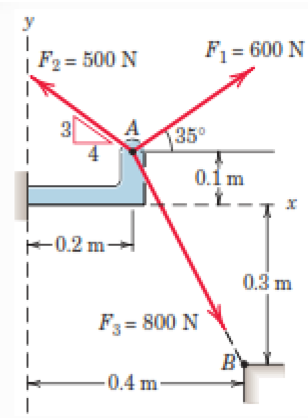



Figure 2/7

## SAMPLE PROBLE 2/1

The forces $\mathbf{F}_1$, $\mathbf{F_2}$, and $\mathbf{F_3}$, all of which act on point *A* of the bracket, are specified in three different ways. Determine the *x* and *y* scalar components of each of the three forces.



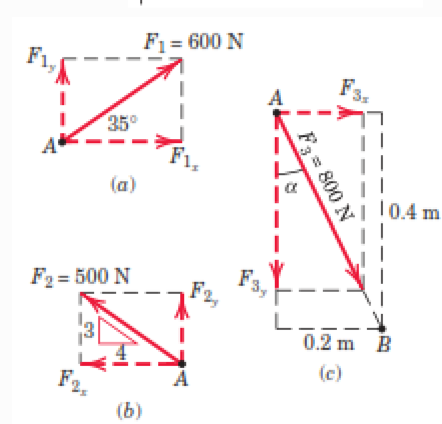

### Solution

The scalar components of $\mathbf{F_1}$, from Fig. *a*, are

$$F_{1_x} = 600\cos 35° = 491\ N$$
$$F_{1_y} = 600\sin 35° = 344\ N$$

The scalar components of $\mathbf{F_2}$, from Fig. *b*, are

$$F_{1_x} = -500\left(\frac{4}{5}\right) = -\,400\ N$$
$$F_{1_x} = 500\left(\frac{3}{5}\right) = 300\ N$$




Note that the angle which orients $\mathbf{F_2}$ to the *x-axis* is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the *x* scalar component of $\boldsymbol{F_2}$ is negative by inspection.

The scalar components of $\mathbf{F_3}$ can be obtained by first computing the angle α of Fig. *c*.

$$\alpha = tan^{-1}(0.2/0.4) = 26.6°$$

1 Then $F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6° = 358\ N$

$$F_{3_y} = -F_3 \cos\alpha = -\ 800 \cos 26.6° = -716\ N$$

Alternatively, the scalar components of $\mathbf{F_3}$ can be obtained by writing $\mathbf{F_3}$ as a magnitude times a unit vector $\mathbf{n}_{AB}$ in the direction of the line segment *AB* Thus,

2 $$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3\frac{\overrightarrow{AB}}{\overline{AB}} = 800\left[\frac{0.2i-0.4j}{\sqrt{(0.2)^2+(0.4)^2}}\right]$$

$$= 800[0.447i - 0.894j] = 358i - 716j$$

The required scalar components are then $F_{3_x} = 358\ N$ , $F_{3_y} = -716\ N$ which agree with our previous results.

## Helpful Hints

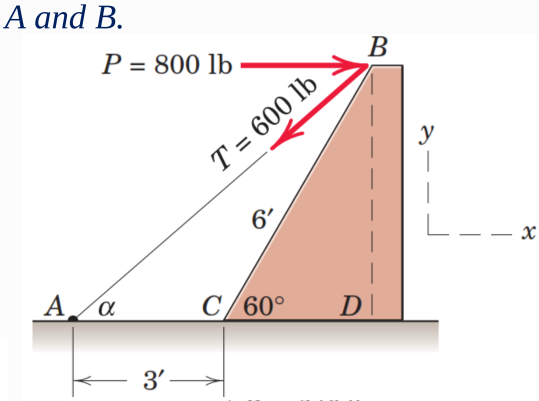
1 You should carefully examine the geometry of each component determination problem and not rely on the blind use of such formulas as

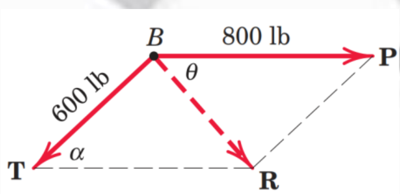
$$F_x = F\cos\theta \ \ and \ \ F_y = F\sin\theta$$

2 A unit vector can be formed by dividing any vector, such as the geometric position vector $\overrightarrow{AB}$, by its length or magnitude. Here we use the over arrow to denote the vector which runs from *A* to *B and the* over bar to determine the distance between *A and B.*

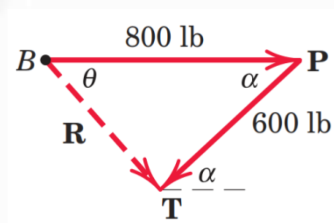
## SAMPLE PROBLEM 2/2

Combine the two forces P and T, which act on the fixed structure at B, into a single equivalent force R.

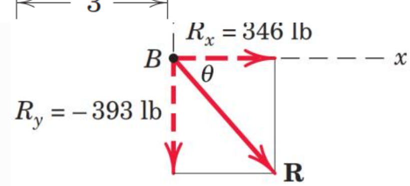


(a)



(b)



(c)

1 **Graphical solution:** The parallelogram for the vector addition of forces **T** and **P** is constructed as shown in Fig. ***a.*** The scale used here is 1 in. 800 lb; a scale of 1 in. 200 lb would be more suitable for regular-size paper and would give greater accuracy. Note that the angle $\alpha$ must be determined prior to construction of the parallelogram. From the given figure

$$tan = \frac{\overrightarrow{BD}}{AB} = \frac{6\,sin60°}{3 + 6\cos 60°} = 0.866 \qquad \alpha = 40.9°$$

Measurement of the length $R$ and direction θ of the resultant force **R** yields the approximate results $R = 525\ lb \qquad \theta = 49°$

2 **Geometric solution:** The triangle for the vector addition of **T** and **P is** shown in Fig. *b*. The angle α is calculated as above. The law of cosines gives

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)cos40.9° = 274300 \qquad \therefore R = 524\ lb$$

3 **Algebraic solution :** By using the *x*-*y* coordinate system on the given figure**,** we may write

$$R_x = \sum F_x = 800 - 600cos40.9 = 346\text{ lb}$$
$$R_y = \sum F_y = -600sin40.9 = -393\text{ lb}$$

The magnitude and direction of the resultant force **R** as shown in Fig. ***c*** are then

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(346)^2 + (-393)^2} = 524\ lb$$

$$\theta = tan^{-1}\left|\frac{F_y}{F_x}\right| = tan^{-1}\left(\frac{393}{346}\right) = 48.6°$$

The resultant **R** may also be written in vector notation as

$$\mathbf{R} = R_x\boldsymbol{i} + R_y\boldsymbol{j} = 346\boldsymbol{i} - 393\boldsymbol{j}\ lb$$