Differentials

The order of differential quantities frequently causes misunderstanding in the derivation of equations. Higher-order differentials may always be neglected compared with lower-order differentials when the mathematical limit is approached. For example, the element of volume ΔV of a right circular cone of altitude h and base radius r may be taken to be a circular slice a distance x from the vertex and of thickness Δ x. The expression for the volume of the element is

$$\Delta V = \frac{\pi r^2}{h^2}\left[x^2\Delta x + x(\Delta x)^2 + \frac{1}{3}(\Delta x)^3\right]$$

Note that, when passing to the limit in going from ΔV to dV and from Δ x to dx, the terms containing $(x)^2$ and $(x)^3$ drop out, leaving merely

$$dV = \frac{\pi r^2}{h^2}x^2 dx$$

which gives an exact expression when integrated.

Small-Angle Approximations

When dealing with small angles, we can usually make use of simplifying approximations. Consider the right triangle of Fig. 1/8 where the angle θ, expressed in radians, is relatively small. If the hypotenuse is unity, we see from the geometry of the figure that the arc length 1xθ and sinθ are very nearly the same. Also cosθ is close to unity. Furthermore, sinθ and tanθ have almost the same values. Thus, for small angles we may write

$$\sin\theta \cong \tan\theta \cong \theta \qquad \cos\theta \cong 1$$

provided that the angles are expressed in radians. These approximations may be obtained by retaining only the first terms in the series expansions for these three functions. As an example of these approximations, for an angle of 1°

$$1° = 0.017453 \text{ rad} \quad \tan 1° = 0.017455 \quad \sin 1° = 0.017452 \quad \cos 1° = 0.999848$$

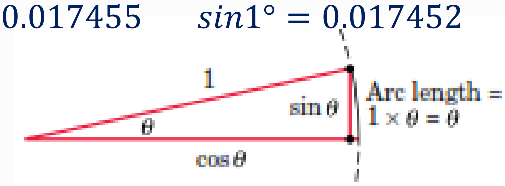


Figure 1/8

Formulating Problems and Obtaining Solutions

In statics, as in all engineering problems, we need to use a precise and logical method for formulating problems and obtaining their solutions. We formulate each problem and develop its solution through the following sequence of steps.

1. Formulate the problem:

(*a*) *State the given data.*

(*b*) *State the desired result.*

(*c*) *State your assumptions and approximations.*

2. Develop the solution:

(*a*) *Draw any diagrams you need to understand the relationships.*

(*b*) *State the governing principles to be applied to your solution.*

(*c*) *Make your calculations.*

(*d*) *Ensure that your calculations are consistent with the accuracy* justified by the data.

(*e*) *Be sure that you have used consistent units throughout your* calculations.

(*f*) *Ensure that your answers are reasonable in terms* of magnitudes, directions, common sense, etc.

(*g*) *Draw conclusions.*

Keeping your work neat and orderly will help your thought process and enable others to understand your work. The discipline of doing orderly work will help you develop skill in formulation and analysis. Problems which seem complicated at first often become clear when you approach them with logic and discipline.

Sample Problem 1/1

Determine the weight in newtons of a car whose mass is 1400 kg. Convert the mass of the car to slugs and then determine its weight in pounds.

Solution. From relationship 1/3, we have

① $$W = mg = 1400(9.81) = 13\ 730 \text{ N} \qquad \textit{Ans.}$$

From the table of conversion factors inside the front cover of the textbook, we see that 1 slug is equal to 14.594 kg. Thus, the mass of the car in slugs is

② $$m = 1400 \text{ kg}\left[\frac{1 \text{ slug}}{14.594 \text{ kg}}\right] = 95.9 \text{ slugs} \qquad \textit{Ans.}$$

Finally, its weight in pounds is

③ $$W = mg = (95.9)(32.2) = 3090 \text{ lb} \qquad \textit{Ans.}$$

As another route to the last result, we can convert from kg to lbm. Again using the table inside the front cover, we have

$$m = 1400 \text{ kg}\left[\frac{1 \text{ lbm}}{0.45359 \text{ kg}}\right] = 3090 \text{ lbm}$$

① Our calculator indicates a result of 13 734 N. Using the rules of significant-figure display used in this textbook, we round the written result to four significant figures, or 13 730 N. Had the number begun with any digit other than 1, we would have rounded to three significant figures.

② A good practice with unit conversion is to multiply by a factor such as $\left[\frac{1 \text{ slug}}{14.594 \text{ kg}}\right]$, which has a value of 1, because the numerator and the denominator are equivalent. Make sure that cancellation of the units leaves the units desired; here the units of kg cancel, leaving the desired units of slug.

③ Note that we are using a previously calculated result (95.9 slugs). We must be sure that when a calculated number is needed in subsequent calculations, it is retained in the calculator to its full accuracy (95.929834 · · ·) until it is needed. This may require storing it in a register upon its initial calculation and recalling it later. We must not merely punch 95.9 into our calculator and proceed to multiply by 32.2—this practice will result in loss of numerical accuracy. Some individuals like to place a small indication of the storage register used in the right margin of the work paper, directly beside the number stored.

Sample Problem 1/2

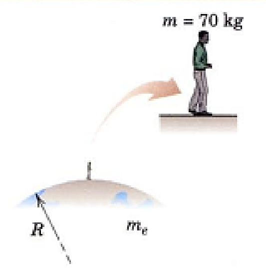
Use Newton's law of universal gravitation to calculate the weight of a 70-kg person standing on the surface of the earth. Then repeat the calculation by using $W = mg$ and compare your two results. Use Table D/2 as needed.

Solution. The two results are

① $$W = \frac{Gm_e m}{R^2} = \frac{(6.673 \cdot 10^{-11})(5.976 \cdot 10^{24})(70)}{[6371 \cdot 10^3]^2} = 688 \text{ N} \qquad \textit{Ans.}$$

$$W = mg = 70(9.81) = 687 \text{ N} \qquad \textit{Ans.}$$

The discrepancy is due to the fact that Newton's universal gravitational law does not take into account the rotation of the earth. On the other hand, the value $g = 9.81$ m/s^2 used in the second equation does account for the earth's rotation. Note that had we used the more accurate value $g = 9.80665$ m/s^2 (which likewise accounts for the earth's rotation) in the second equation, the discrepancy would have been larger (686 N would have been the result).



Helpful Hint

① The effective distance between the mass centers of the two bodies involved is the radius of the earth.

Sample Problem 1/3

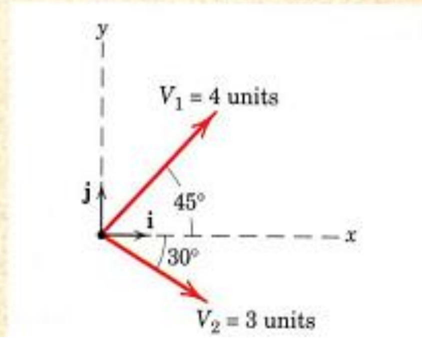
For the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ shown in the figure,

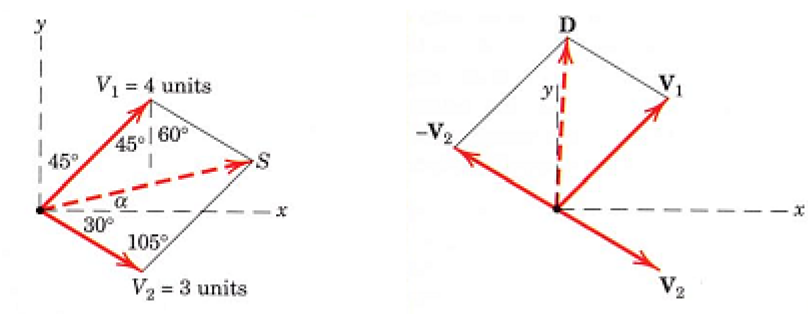
(*a*) determine the magnitude S of their vector sum $\mathbf{S} = \mathbf{V}_1 + \mathbf{V}_2$

(*b*) determine the angle α between $\mathbf{S}$ and the positive x-axis

(*c*) write $\mathbf{S}$ as a vector in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$ and then write a unit vector $\mathbf{n}$ along the vector sum $\mathbf{S}$

(*d*) determine the vector difference $\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2$





(*a*) (*b*)

Solution (*a*) We construct to scale the parallelogram shown in Fig. *a* for adding $\mathbf{V}_1$ and $\mathbf{V}_2$. Using the law of cosines, we have

$$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105°$$

$$S = 5.59 \text{ units} \qquad \textit{Ans.}$$

① (*b*) Using the law of sines for the lower triangle, we have

$$\frac{\sin 105°}{5.59} = \frac{\sin(\alpha + 30°)}{4}$$

$$\sin(\alpha + 30°) = 0.692$$

$$(\alpha + 30°) = 43.8° \qquad \alpha = 13.76°$$ *Ans.*

(*c*) With knowledge of both S and α, we can write the vector $\mathbf{S}$ as

$$\mathbf{S} = S[\mathbf{i} \cos \alpha + \mathbf{j} \sin \alpha]$$
$$= S[\mathbf{i} \cos 13.76° + \mathbf{j} \sin 13.76°] = 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units}$$ *Ans.*

② Then $$\mathbf{n} = \frac{\mathbf{S}}{S} = \frac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59} = 0.971\mathbf{i} + 0.238\mathbf{j}$$ *Ans.*

(*d*) The vector difference $\mathbf{D}$ is

$$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 = 4(\mathbf{i} \cos 45° + \mathbf{j} \sin 45°) - 3(\mathbf{i} \cos 30° - \mathbf{j} \sin 30°)$$
$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units}$$ *Ans.*

The vector $\mathbf{D}$ is shown in Fig. *b* as $\mathbf{D} = \mathbf{V}_1 + (-\mathbf{V}_2)$.

Helpful Hints

① You will frequently use the laws of cosines and sines in mechanics. See Art. C/6 of Appendix C for a review of these important geometric principles.

② A unit vector may always be formed by dividing a vector by its magnitude. Note that a unit vector is dimensionless.

Force Systems

Force

Before dealing with a group or system of forces, it is necessary to examine the properties of a single force in some detail. A force has been defined in Chapter 1 as an action of one body on another. In dynamics we will see that a force is defined as an action which tends to cause acceleration of a body. A force is a vector quantity, because its effect depends on the direction as well as on the magnitude of the action. Thus, forces may be combined according to the parallelogram law of vector addition.

The action of the cable tension on the bracket in Fig. 2/1*a* is represented in the side view, Fig. 2/1*b*, by the force vector **P** of magnitude ***P.*** The effect of this action on the bracket depends on *P, t*he angle , and the location of the point of application *A.* Changing any one of these three specifications will alter the effect on the bracket, such as the force in one of the bolts which secure the bracket to the base, or the internal force and deformation in the material of the bracket at any point. Thus, the complete specification of the action of a force must include its magnitude, direction, and point of application, and therefore we must treat it as a fixed vector.

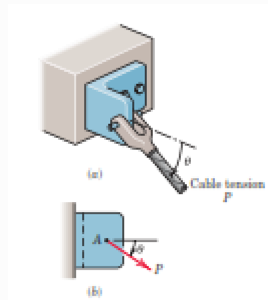


Figure 2/1

External and Internal Effects

We can separate the action of a force on a body into two effects, external and internal. For the bracket of *Fig. 2/1* **P** external to the bracket are the reactive forces (not shown) exerted on the bracket by the foundation and bolts because of the action of **P.** Forces external to a body can be either applied forces or reactive forces. The effects of **P** internal to the bracket are the resulting internal forces and deformations distributed throughout the material of the bracket. The relation between internal forces and internal deformations depends on the material properties of the body and is studied in strength of materials, elasticity, and plasticity.

Principle of Transmissibility

When dealing with the mechanics of a rigid body, we ignore deformation sin the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force **P** acting on the rigid plate in Fig. 2/2 may be applied at *A* or at *B* or at any other point on its line of action, and the net external effects of **P** on the bracket will not change. The external effects are the force exerted on the plate by the bearing support at *O* and the force exerted on the plate by the roller support at *C.*

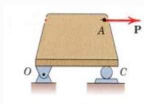


Figure 2/2

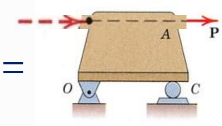


Figure 2/2

Forces are classified as either ***Contact*** or ***body forces***

Contact force: is produced by direct physical contact.

An example is the force exerted on a body by a supporting surface.

Body force: is generated by virtue of position of a body within a force field such as a gravitational, elastic, magnetic field. An example of a body force is your weight.

Also forces can be classified as ***Parallel Forces***, ***Coplanar Forces***, ***Collinear Forces***, ***Concurrent Forces***.

Forces may be further classified as either **concentrated** or **distributed**. Every contact force is actually applied over a finite area and therefore really a distributed force.

Action and Reaction

According to Newton's third law, the action of a force is always accompanied by an equal and opposite reaction. It is essential to distinguish between the action and the reaction in a pair of forces. To do so, we first isolate the body in question and then identify the force exerted on that body (not the force exerted by the body). It is very easy to mistakenly use the wrong force of the pair unless we distinguish carefully between action and reaction.

Concurrent Forces

Two or more forces are said to be concurrent at a point if their lines of action intersect at that point. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Fig. 2/3*a* have a common point of application and are concurrent at the point *A*. Thus, they can be added using the parallelogram law in their common plane to obtain their sum or resultant **R**, as shown in Fig. 2/3a. The resultant lies in the same plane as $\mathbf{F}_1$ and $\mathbf{F}_2$.

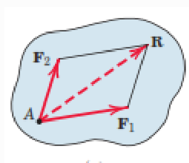
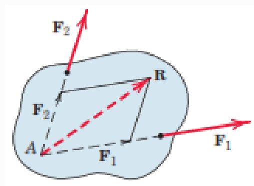


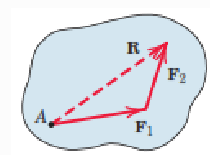
Figure 2/3

Suppose the two concurrent forces lie in the same plane but are applied at two different points as in Fig. 2/3*b*. By the principle of transmissibility, we may move them along their lines of action and complete their vector sum **R** at the point of concurrency *A*, as shown in *Fig. 2/3b.*



(*b*)

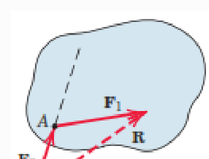
We can replace $\mathbf{F}_1$ and $\mathbf{F}_2$ with the resultant **R** without altering the external effects on the body upon which they act. We can also use the triangle law to obtain **R,** but we need to move the line of action of one of the forces, as shown in Fig. 2/3*c*.



(*c*)

If we add the same two forces as shown in Fig. 2/3*d*, we correctly preserve the magnitude and direction of **R,** but we lose the correct line of action, because **R** obtained in this way does not pass through *A*. Therefore this type of combination should be avoided. We can express the sum of the two forces mathematically by the vector equation:

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$

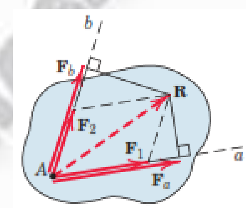


(*d*)

Vector Components

In addition to combining forces to obtain their resultant, we often need to replace a force by its vector components in directions which are convenient for a given application. The vector sum of the components must equal the original vector. Thus, the force **R** in Fig. 2/3a may be replaced by, or resolved into, two vector components $\mathbf{F}_1$ and $\mathbf{F}_2$ with the specified directions by completing the parallelogram as shown to obtain the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.

The relationship between a force and its vector components along given axes must not be confused with the relationship between a force and its perpendicular projections onto the same axes. Figure 2/3*e* shows the perpendicular projections $\mathbf{F_a}$ and $\mathbf{F_b}$ of the given force **R** onto axes *a* and *b*, which are parallel to the vector components $\boldsymbol{F_1}$ *and* $\boldsymbol{F_2}$ of Fig. 2/3*a*. *Figure 2/3e* shows that the components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore the vector sum of the projections $\mathbf{F_a}$ and $\mathbf{F_b}$ is not the vector **R,** because the parallelogram law of vector addition must be used to form the sum. The components and projections of **R** are equal only when the axes *a and b* are perpendicular.



(e)

A Special Case of Vector Addition

To obtain the resultant when the two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ are parallel as in Fig. 2/4, we use a special case of addition. The two vectors are combined by first adding two equal, opposite, and collinear forces **F** and **-F** of convenient magnitude, which taken together produce no external effect on the body. Adding $\mathbf{F_1}$ and **F** to produce **R**, and combining with the sum $\mathbf{R_2}$ of $\mathbf{F_2}$ and **-F** yield the resultant **R,** which is correct in magnitude**,** direction, and line of action. This procedure is also useful for graphically combining two forces which have a remote and inconvenient Point of concurrency because they are almost parallel. It is usually helpful to master the analysis of force systems in two dimensions before undertaking three-dimensional analysis. Thus the remainder of Chapter 2 is subdivided into these two categories.