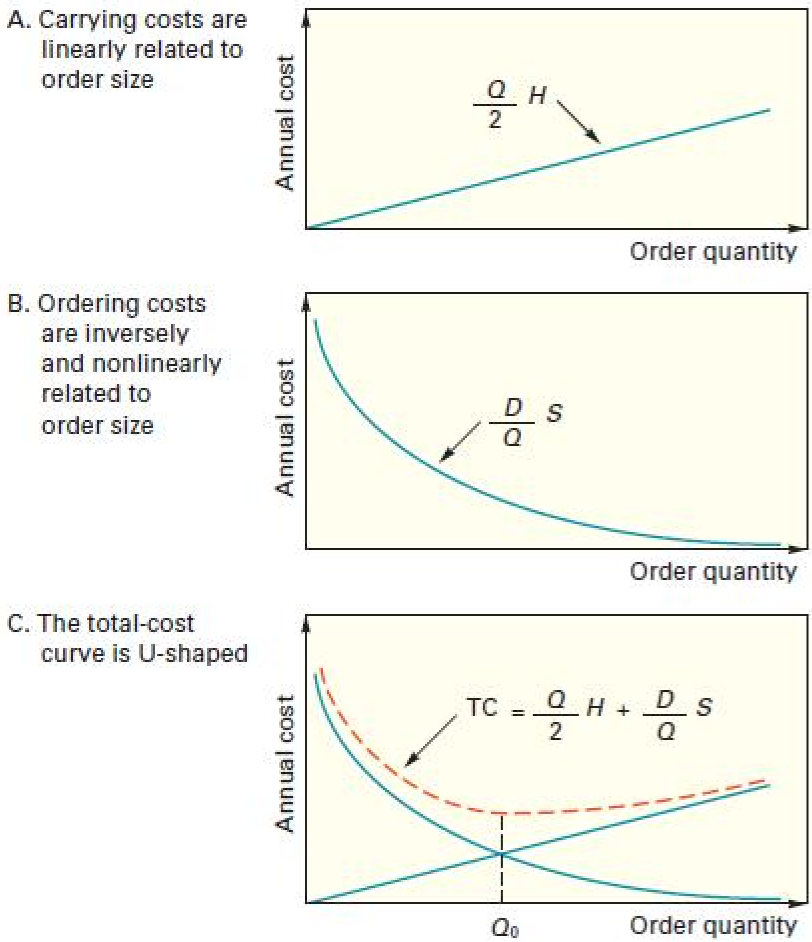

Where;
D = Annual Demand, usually in units per year.
S = Ordering cost.
H= Holding cost.
Q = Order size.

Because the **number of orders per year, D/Q**, decreases as Q increases, annual ordering cost is inversely related to order size, as Figure B illustrates.
The total annual cost (TC) associated with carrying and ordering inventory when Q units are ordered each time is

$$TC = \begin{matrix}\text{Annual}\\ \text{carrying}\\ \text{cost}\end{matrix} + \begin{matrix}\text{Annual}\\ \text{ordering}\\ \text{cost}\end{matrix} = \frac{Q}{2}H + \frac{D}{Q}S$$

(Note that D and H must be in the same units, e.g., months, years.) Figure C reveals that the total-cost curve is U-shaped (i.e., convex, with one minimum) and that it reaches its minimum at the quantity where carrying and ordering costs are equal. An expression for the optimal order quantity can be obtained using calculus.

$$Q_0 = \sqrt{\frac{2DS}{H}}$$

(HW: Derive this relation and prove that it indicates a minimum has been obtained.)

Thus, given annual demand, the ordering cost per order, and the annual carrying cost per unit, one can compute the optimal (economic) order quantity. The minimum total cost is then found by substituting Q_0 for Q in TC Formula.
The length of an order cycle (i.e., the time between orders) is

$$\text{Length of order cycle} = \frac{Q}{D}$$

WHEN TO REORDER WITH EOQ ORDERING

EOQ models answer the question of how much to order, but not the question of when to order.
The latter is the function of models that identify the reorder point (ROP) in terms of a quantity:
The reorder point occurs when the quantity on hand drops to a predetermined amount.
There are four determinants of the reorder point quantity:

a) Rate of demand
b) The lead time
c) The extent of demand and/or lead time variability
d) The degree of stock out risk acceptable to management

If demand and lead time are both constant, the reorder point is simply

ROP (Reorder Point) = Demand rate (units per day or week) x Lead Time (in days or weeks)

ROP = D x LT

Example

A local distributor for a national tire company expects to sell approximately 9,600 steel-belted radial tires of a certain size and tread design next year. Annual carrying cost is $16 per tire, and ordering cost is $75. The distributor operates 288 days a year.

a. What is the EOQ?
b. How many times per year does the store reorder?
c. What is the length of an order cycle?
d. What is the total annual cost if the EOQ quantity is ordered?

Answer

D = 9,600 tires per year
H = $16 per unit per year
S = $75

a. $Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(9{,}600)75}{16}} = 300 \text{ tires}$

b. Number of orders per year: $D/Q = \frac{9{,}600 \text{ tires}}{300 \text{ tires}} = 32.$

c. Length of order cycle: $Q/D = \frac{300 \text{ tires}}{9{,}600 \text{ tires/yr}} = 1/32$ of a year, which is $1/32 \times 288$, or nine workdays.

d. TC = Carrying cost + Ordering cost

$$\begin{aligned} &= (Q/2)H + (D/Q)S \\ &= (300/2)16 + (9{,}600/300)75 \\ &= \$2{,}400 + \$2{,}400 \\ &= \$4{,}800 \end{aligned}$$

Note that the ordering and carrying costs are equal at the EOQ, as illustrated in Figure C. Carrying cost is sometimes stated as a percentage of the purchase price of an item rather than as a dollar amount per unit. However, as long as the percentage is converted into a dollar amount, the EOQ formula is still appropriate.

Economic Production Quantity (EPQ)

The batch mode of production is widely used in production. Even in assembly operations, portions of the work are done in batches. The reason for this is that in certain instances, the capacity to produce a part exceeds the part's usage or demand rate. As long as production continues, inventory will continue to grow. In such instances, it makes sense to periodically produce such items in batches, or lots, instead of producing continually. The assumptions of the EPQ model are similar to those of the EOQ model, except that instead of orders received in a single delivery, units are received incrementally during production. The assumptions are

1. Only one item is involved.
2. Annual demand is known.
3. The usage rate is constant.
4. Usage occurs continually, but production occurs periodically
5. The production rate is constant.
6. Lead time does not vary.
7. There are no quantity discounts.

$$TC_{min} = \text{Carrying cost} + \text{Setup cost} = \left(\frac{I_{max}}{2}\right)H + (D/Q_0)S$$

where

I_{max} = Maximum inventory

The economic run quantity is

$$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}}$$

where

p = Production or delivery rate

u = Usage rate

The cycle time (the time between orders or between the beginnings of runs):

$$\text{Cycle time} = \frac{Q_0}{u}$$

Similarly, the run time (the production phase of the cycle):

$$\text{Run time} = \frac{Q_0}{p}$$

The maximum and average inventory levels are

$$I_{max} = \frac{Q_0}{p}(p - u) \qquad \text{and} \qquad I_{average} = \frac{I_{max}}{2}$$

Example

A toy manufacturer uses 48,000 rubber wheels per year for its popular dump truck series. The firm makes its own wheels, which it can produce at a rate of 800 per day. The toy trucks are assembled uniformly over the entire year. Carrying cost is $1 per wheel a year. Setup cost for a production run of wheels is $45. The firm operates 240 days per year.

Determine the:

a. Optimal run size.

b. Minimum total annual cost for carrying and setup.

c. Cycle time for the optimal run size.

d. Run time.

D = 48,000 wheels per year

S = \$45

H = \$1 per wheel per year

p = 800 wheels per day

u = 48,000 wheels per 240 days, or 200 wheels per day

a. $$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(48{,}000)45}{1}}\sqrt{\frac{800}{800-200}} = 2{,}400 \text{ wheels}$$

b. $$TC_{min} = \text{Carrying cost} + \text{Setup cost} = \left(\frac{I_{max}}{2}\right)H + (D/Q_0)S$$

Thus, you must first compute I_{max}:

$$I_{max} = \frac{Q_0}{p}(p - u) = \frac{2{,}400}{800}(800 - 200) = 1{,}800 \text{ wheels}$$

$$TC = \frac{1{,}800}{2} \times \$1 + \frac{48{,}000}{2{,}400} \times \$45 = \$900 + \$900 = \$1{,}800$$

Note again the equality of cost (in this example, setup and carrying costs) at the EOQ.

c. $$\text{Cycle time} = \frac{Q_0}{u} = \frac{2{,}400 \text{ wheels}}{200 \text{ wheels per day}} = 12 \text{ days}$$

Thus, a run of wheels will be made every 12 days.

d. $$\text{Run time} = \frac{Q_0}{p} = \frac{2{,}400 \text{ wheels}}{800 \text{ wheels per day}} = 3 \text{ days}$$

Thus, each run will require three days to complete.