Example 2

Following mortality rates have been observed for a certain type of electronic component.

Month	0	1	2	3	4	5	6
% surviving at the end of the month	100	97	90	70	30	15	0

There are 10000 items in operation. It costs US$ 1 to replace an individual item and 35 cent per item, if all items are replaced simultaneously. It is decided to replace all items at fixed intervals & to continue replacing individual items as and when they fail. At what intervals should all items be replaced? Assume that the items failing during a month are replaced at the end of the month.

Solution:

Month	% surviving at the end of the month	Probability of failure p_i
0	100	----
1	97	(100 - 97)/100 = 0.03
2	90	(97 - 90)/100 = 0.07
3	70	(90 - 70)/100 = 0.20
4	30	(70 - 30)/100 = 0.40
5	15	(30 - 15)/100 = 0.15
6	0	(15 - 0)/100 = 0.15

The given problem can be divided into two parts.

I. Individual replacement.
II. Group replacement.

Case I

It should be noted that no item survives for more than 6 months. Thus, an item which has survived for 5 months is sure to fail during sixth month.

The expected life of each item is given by

= $\sum x_i p_i$, where x_i is the month and p_i is the corresponding probability of failure.
= (1 X 0.03) + (2 X 0.07) + (3 X 0.20) + (4 X 0.40) + (5 X 0.15) + (6 X 0.15)
= 4.02 months.

$\therefore$ Average number of replacement every month = N/(average expected life) = 10000/4.02 = 2487.5
= 2488 items (approx.).
Here average cost of monthly individual replacement policy = 2488 X 1 = US$. 2488/-, (the cost being US$ 1/- per item).

Case II

Let N_i denote the number of items replaced at the end of ith month.

Calculating values for N_i

N_0 = Number of items in the beginning = 10,000
N_1 = Number of items during the 1st month X probability that an item fails during 1st month of installation
= 10000 X 0.03 = 300

N_2 = Number of items replaced by the end of second month
=(Number of items in beginning X probability that these items will fail in 2nd month) + (Number of items replaced in first month X probability that these items will fail during second month)

$=N_0P_2 + N_1P_1$
=(10000 X 0.07) + (300 X 0.03) = 709

$N_3 = N_0P_3 + N_1P_2 + N_2P_1$
= (10000 X 0.20) + (300 X 0.07)+ (709 X 0.03) = 2042

$N_4 = N_0P_4 + N_1P_3 + N_2P_2 + N_3P_1$
= (10000 X 0.40) + (300 X 0.20)+ (709 X 0.07) + (2042 X 0.03) = 4171

$N_5 = N_0P_5 + N_1P_4 + N_2P_3 + N_3P_2 + N_4P_1$
= (10000 X 0.15) + (300 X 0.40)+ (709 X 0.20) + (2042 X 0.07) + (4171 X 0.03) = 2030

$N_6 = N_0P_6 + N_1P_5 + N_2P_4 + N_3P_3 + N_4P_2 + N_5P_1$
= (10000 X 0.15) + (300 X 0.15)+ (709 X 0.40) + (2042 X 0.20) + (4171 X 0.07) + (2030 X 0.03) = 2590.

From the above calculations, it is observed that N_i increases upto fourth month and then decreases. It can also be seen that N_i will later tend to increase and the value of N_i will oscillate till the system acquires a steady state.

The optimum replacement cycle under group replacement is given in the following table.

End of month	Total no. of items failed	Cumulative no. of failure	Cost of replacement after failure (US$ 1/ item)	Cost of all replacement (US $0.35/ item)	Total cost (US$)	Average cost per month (US$)
1	300	300	300	3500	3800	3800
2	709	1009	1009	3500	4509	2254.50
3	2042	3051	3051	3500	6551	2183.66
4	4171	7222	7222	3500	10722	2680.50
5	2030	9252	9252	3500	12752	2550.40
6	2590	11842	11842	3500	15342	2557.00

The above table shows that the average cost during the third month is Minimum. Thus, it would be economical to replace all the items every three months.

Staffing Problem

In the previous sections, we discussed about replacement problems, which were not related to human resources working in an organization. The replacement models can also be used to solve the problems of staff replacement. This section focuses on the problem of replacing staff in an organization. Staff replacement is essential due to the following factors:

- Inefficiency
- Resignation
- Retirement
- Unexpected events (like accident, death, etc.)

Assumption

- The life distribution for the service of staff in an organization is predetermined.

Example 1

A team of software developers at www.universalteacher.com is planned to rise to a strength of 50 persons, and then to remain at that level. Consider the following data:

Year	Total % who have left upto the end of the year
1	5
2	30
3	50
4	60
5	70
6	75
7	80
8	85
9	90
10	100

On the basis of above information, determine:

What is the recruitment per year necessary to maintain the strength? There are 8 senior posts for which the length of service is the main criterion. What is the average length of service after which new entrant can expect his promotion to one of these posts?

Solution:

Calculating values for table 1

- The values in column (c) are obtained by subtracting the corresponding elements of column (b) from 100.
- The values in column (d) are obtained by dividing the corresponding elements in column (b) by 100.
- The values in column (e) are obtained by dividing the corresponding elements in column (c) by 100.

Table 1

Year	No.of persons who leave at the end of the year	No. of persons in service at the end of year	Prob. of leaving at the end of the year	Prob. of in service at the end of the year
a	**b**	**c**	**d**	**e**
0	0	100	0	1.00
1	5	95	0.05	0.95
2	30	70	0.30	0.70
3	50	50	0.50	0.50
4	60	40	0.60	0.40
5	70	30	0.70	0.30
6	75	25	0.75	0.25
7	80	20	0.80	0.20
8	85	15	0.85	0.15
9	90	10	0.90	0.10
10	100	0	1.00	0
Total		**455**		

From table 1, we find that with a recruitment policy of 100 persons every year, the total number of persons serving in the organization would have been 455. Hence, if we want to maintain a strength of 50 persons then we should recruit

$$\frac{100 \times 50}{455} = \frac{1000}{91} = 10.98 = 11 \text{ persons/year}$$

Every year 11 persons should be **recruited** to maintain a strength of 50. Number of survivals after each year can be obtained by multiplying the various values of column (e) by 11.

Table 2

Year	Number of persons in service
0	11
1	10
2	8
3	6
4	4
5	3
6	3

7	2
8	2
9	1
10	0

Now there are 8 senior posts. From table 2, it can seen that there are 3 persons in service during the sixth year, 2 in seventh year, 2 in eighth year, and 1 in ninth year. Hence, promotions of new recruits will start by the end of sixth year and will continue upto seventh year.

Example 2

The Railway Ministry requires 200 private assistants, 300 private secretaries, and 50 section officers. Persons are recruited at the age of 21, if still in service, retire at the age of 60. Given the following life table, determine

- How many persons should be recruited every year ?
- At what age promotions should take place ?

Age	21	22	23	24	25	26	27	28
No. in service	1000	600	400	380	311	260	229	206
Age	29	30	31	32	33	34	35	36
No. in service	190	180	174	166	162	155	150	146
Age	37	38	39	40	41	42	43	44
No. in service	145	135	131	125	120	112	105	100
Age	45	46	47	48	49	50	51	52
No. in service	94	86	80	73	65	60	53	46
Age	53	54	55	56	57	58	59	60
No. in service	40	32	26	23	19	13	11	0

Solution:

If a policy of recruiting 1000 persons every year is followed, then the total number of employees in service between the age 21 to 59 years will be equal to 6403. But the requirement of organization is 550 (200 + 300 + 50) employees.

Therefore, to maintain a strength of 550 employees, the organization should recruit: (1000 X 550)/6403 = 86 (approx.) persons every year.

Private assistants

Out of a strength of 550, there are 200 private assistants. Hence, out of a strength of 1000 there will be
(200 X 1000)/550 = 364 private assistants.
From the above life table, 364 is available upto 24 years. Therefore, the promotion of private assistants will take place in 25th year.

Private Secretaries

Out of a strength of 1000 there will be
(300 X 1000)/550 = 545 private secretaries.

Section officers

Number of section officers = 1000 - (364 + 545) = 91.
From the above life table, we find that at the age of 46 only 86 will survive. Therefore, promotion of private secretaries will take place in 46th year.