Unit 13

Maintenance Engineering: Replacement

Introduction

In fact, in any system the efficacy (efficiency) of an item deteriorates with time. In such cases, either the old item should be replaced by a new item, or some kind of restorative action (maintenance) is necessary to restore the efficiency of the whole system. The cost of maintenance depends upon a number of factors, and a stage comes at which the maintenance cost is so large that it is more profitable to replace the old item. Thus, there is a need to formulate the most effective replacement policy.

The purpose of this chapter is to show what replacement models look like.

Definition

Replacement models are concerned with the problem of replacement of machines, individuals, capital assets, etc. due to their deteriorating efficiency, failure, or breakdown.

It is evident that the study of replacement is a field of application rather than a method of analysis. Actually, it is concerned with methods of comparing **alternative** replacement policies.

The various types of replacement problems can be broadly classified in following categories:

- Replacement of items whose efficiency deteriorates with time, e.g., machine, tools, etc.
- Replacement of items that fail suddenly and completely like electric bulbs & tubes.
- Replacement of human beings in an organization or staffing problem.
- Replacement of items may be necessary due to new researches and methods; otherwise, the system may become outdated.

Replacement Of Items That Deteriorates With Time

We begin here with the simplest replacement model where the deterioration process is predictable. More complex replacement models are studied in the subsequent sections.

This model is represented by:

- Increasing maintenance cost.
- Decreasing salvage value.

Assumption

- Increased age reduces efficiency

Generally, the criteria for measuring efficiency is the discounted value of all future costs associated with each policy.

Let
C = the capital cost of a certain item, say a machine
S(t) = the selling or scrap value of the item after t years.
F(t) = operating cost of the item at time t
n = optimal replacement period of the time

Now, the annual cost of the machine at time t is given by C - S(t) + F(t) and since the total maintenance cost incurred on the machine during n years is $\int_0^n F(t)\,dt$, the total cost T, incurred on the machine during n years is given by:

$$T = C - S(t) + \int_0^n F(t)\,dt$$

Thus, the average annual total cost incurred on the machine per year during n years is given by

$$TA = \frac{1}{n}\left[C - S(t) + \int_0^n F(t)\,dt \right]$$

To determine the optimal period for replacing the machine, the above function is differentiated with respect to n and equated to zero.

$$\frac{dTA}{dn} = \frac{-1}{n^2}\left[C - S(t) \right] \frac{-1}{n^2} \int_0^n F(t)\,dt + \frac{F(n)}{n}$$

Equating $\frac{dTA}{dn} = 0$, we get

$$F(n) = \frac{1}{n}\left[C - S(t) + \int_0^n F(t)\, dt \right]$$

That is, F(n) = TA

Thus, we conclude that an item should be replaced when the average cost to date becomes equal to the current maintenance cost.

Examples:-

Constant Resale Value

Example

The initial cost of a machine is US$. 7100 and scrap value is US$. 100. The maintenance costs found from experience are as follows:

Year	1	2	3	4	5	6	7	8
Maintenance	200	350	500	700	1000	1300	1700	2100

When should the machine be **replaced**?

Solution:

Year	Running cost	Cumulative running cost	Scrap value	Difference between initial cost and scrap value	Average investment cost / year	Average running cost / year	Average annual total cost
A	B	C	D	E	F = E/A	G = C/A	H = F + G
1	200	200	100	7000	7000	200	7200
2	350	200 + 350 = 550	100	7000	3500	275	3775
3	500	550 + 500 = 1050	100	7000	2333.33	350	2683.33
4	700	1050 + 700 = 1750	100	7000	1750	437.5	2187.50
5	1000	1750 + 1000 = 2750	100	7000	1400	550	1950
6	1300	2750 + 1300 = 4050	100	7000	1166.67	675	1841.67
7	1700	4050 + 1700 = 5750	100	7000	1000	821.42	1821.42

8	2100	5750 + 2100 = 7850	100	7000	875	981.25	1856.25

This table shows that the average annual total cost during the seventh year is minimum. Hence, the machine should be **replaced** after the **7th** year.

Falling Resale Value

Example

The initial cost of a machine is US$. 6100 and resale value drops as the time passes. Cost data are given in the following table:

Year	1	2	3	4	5	6	7	8
Maintenance	100	250	400	600	900	1200	1600	2000
Resale Value	800	700	600	500	400	300	200	100

When should the machine be replaced?

Solution:

Year	**Running cost**	**Cumulative running cost**	**Resale value**	**Difference between initial cost and resale value**	**Average investment cost / year**	**Average running cost / year**	**Average annual total cost**
1	100	100	800	5300	5300	100	5400
2	250	350	700	5400	2700	175	2875
3	400	750	600	5500	1833.33	250	2083.33
4	600	1350	500	5600	1400	337.5	1737.50
5	900	2250	400	5700	1140	450	1590
6	1200	3450	300	5800	966.67	575	1541.67
7	1600	5050	200	5900	842.85	721.42	1564.27
8	2000	7050	100	6000	750	881.25	1631.25

This table shows that the average annual total cost during the sixth year is minimum. Hence, the machine should be replaced after the 6th year.

Present Worth Factor

In this method, the present value of all future expenditures and revenues for each alternative is calculated. An item whose present worth factor is least is preferred.
Let

P = purchase cost of an item
A = annual running cost
n = life of an item in years
S = salvage value
r = annual interest rate

The present value can be calculated as follows:

P + A (Pwf for r% interest rate for n years) - S (Pwf for r% interest rate for n years)

For an illustration, consider the following problem.

Example

The China Moon restaurant is considering to purchase a new cooling system. Cost data are given in the following table:

	Cooling system A	Cooling system B	Cooling system C
Present investment (US$.)	12000	14000	17000
Total annual cost (US$.)	3000	2000	1500
Life (Years)	10	10	10
Salvage value (US$.)	500	1000	1200

On the basis of above data, select the best cooling system considering 12% normal rate of return per year.
Given
Pwf (total annual cost) @ 12% for 10 years = 5.650
Pwf (salvage value) @ 12% for 10 years = 0.322

Solution:

	Cooling system A	Cooling system B	Cooling system C
Present investment ($.)	12000	14000	17000
Total annual cost ($.)	3000 X 5.650	2000 X 5.650	1500 X 5.650
Salvage value ($.)	500 X 0.322	1000 X 0.322	1200 X 0.322
Total Cost	28789	24978	25088.6

Total Cost = Present investment + Total annual cost - Salvage value
Cooling system A = 12000 + 16950 - 161 = US$. 28789

Cooling system B = 14000 + 11300 - 322 = US$. 24978
Cooling system C = 17000 + 8475 - 386.4 = US$. 25088.6

Hence, cooling system B should be purchased because it has least total cost.

Replacement Of Items That Fail Completely

In some situations, failure of a certain item occurs all of a sudden, instead of gradual deterioration (e.g., failure of light bulbs, tubes, etc.). The failure of the item may result in complete breakdown of the system. The breakdown implies loss of production, idle inventory, idle labour, etc. Therefore, an organization must prepare itself against these **failures.**

Thus, to avoid the possibility of a complete breakdown, it is desirable to formulate a suitable replacement policy. The following two courses can be followed in such situations.

- **Individual replacement policy.** Under this policy, an item may be replaced immediately after its failure.
- **Group replacement policy**. Under this policy, the items are replaced in group after a certain period, say t, irrespective of the fact that items have failed or not. If any item fails before its preventive replacement is due, then individual replacement policy is used.

In situations where the items fail completely, the formulation of replacement policy depends upon the probability of failure. Mortality tables or Life testing techniques may be used to obtain a probability distribution of the failure of items in a system.

Mortality Tables

M(t) = Number of items surviving at time t
M(t - 1) = Number of items surviving at time (t - 1)
N = Total number of items in the system

The probability of failure of items during the interval t and (t - 1) is given by

$$\frac{M(t-1) - M(t)}{N}$$

The conditional probability that any item survived upto age (t - 1) and will fail in the next period is given by

$$\frac{M(t-1) - M(t)}{M(t-1)}$$

Example 1

Following mortality rates have been observed for certain type of light bulbs.

Time (weeks)	0	1	2	3	4	5	6	7	8	9	10
Number of bulbs still operating	100	94	82	58	40	28	19	13	7	3	0

Calculate the probability of failure.

Solution:

Here, t is the time (weeks) and M (t) is the number of bulbs still operating. The probability of failure can be calculated as shown in the following table.

Table

Time (t)	**M (t)**	**Probability of failure p_i = [M (t - 1)- M (t)] / N**
0	100	----
1	94	(100 - 94)/100 = 0.06
2	82	(94 - 82)/100 = 0.12
3	58	(82 - 58)/100 = 0.24
4	40	(58 - 40)/100 = 0.18
5	28	(40 - 28)/100 = 0.12
6	19	(28 - 19)/100 = 0.09
7	13	(19 - 13)/100 = 0.06
8	7	(13 - 7)/100 = 0.06
9	3	(7 - 3)/100 = 0.04
10	0	(3 - 0)/100 = 0.03