9.6 Examining the Initial Basic Feasible Solution for Non-Degeneracy

Examine the initial basic feasible solution for non-degeneracy. If it is said to be non-degenerate then it has the following two properties

- Initial basic feasible solution must contain exactly m + n – 1 number of individual allocations.

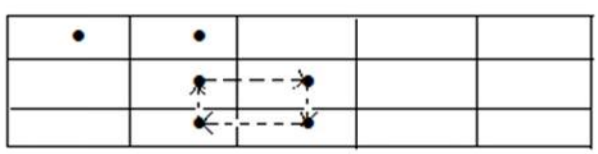
- These allocations must be in independent positions

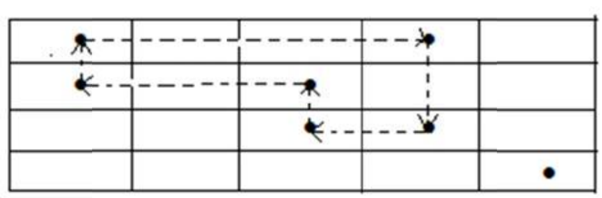
Independent Positions

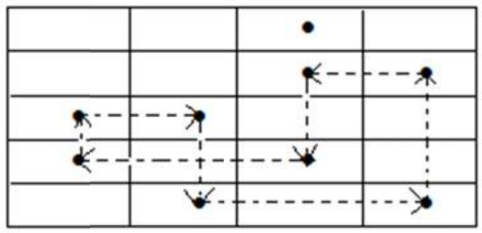
•	•	•		
		•	•	•
				•

•				•
			•	•
		•		•

Non-Independent Positions







9.7 Transportation Algorithm for Minimization Problem (MODI Method)

Step 1

Construct the transportation table entering the origin capacities a_i, the destination requirement b_j and the cost c_{ij}

Step 2

Find an initial basic feasible solution by vogel's method or by any of the given method.

Step 3

For all the basic variables x_{ij}, solve the system of equations $u_i + v_j = c_{ij}$, for all i, j for which cell (i, j) is in the basis, starting initially with some $u_i = 0$, calculate the values of u_i and v_j on the transportation table

Step 4

Compute the cost differences $d_{ij} = c_{ij} – (u_i + v_j)$ for all the non-basic cells

Step 5

Apply optimality test by examining the sign of each d_{ij}

- If all $d_{ij} \geq 0$, the current basic feasible solution is optimal
- If at least one $d_{ij} < 0$, select the variable x_{rs} (most negative) to enter the basis.
- Solution under test is not optimal if any d_{ij} is negative and further improvement is required by repeating the above process.

Step 6

Let the variable x_{rs} enter the basis. Allocate an unknown quantity Θ to the cell (r, s). Then construct a loop that starts and ends at the cell (r, s) and connects some of the basic cells. The amount Θ is added to and subtracted from the transition cells of the loop in such a manner that the availabilities and requirements remain satisfied.

Step 7

Assign the largest possible value to the Θ in such a way that the value of at least one basic variable becomes zero and the other basic variables remain non-negative. The basic cell whose allocation has been made zero will leave the basis.

Step 8

Now, return to step 3 and repeat the process until an optimal solution is obtained.

Example

Find an optimal solution

	W_1	W_2	W_3	W_4	Availability
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

Solution

1. **Applying vogel's approximation method for finding the initial basic feasible solution**

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	**5**(19)	(30)	(50)	**2**(10)	X	X
F_2	(70)	(30)	**7**(40)	**2**(60)	X	X
F_3	(40)	**8**(8)	(70)	**10**(20)	X	X
Requirement	X	X	X	X		
Penalty	X	X	X	X		

Minimum transportation cost is 5 (19) + 2 (10) + 7 (40) + 2 (60) + 8 (8) + 10 (20) = 779

2. **Check for Non-degeneracy**

The initial basic feasible solution has m + n – 1 i.e. 3 + 4 – 1 = 6 allocations in independent positions. Hence optimality test is satisfied.

3. **Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$**

					u_i
	• (19)			• (10)	$u_1 = -10$
			• (40)	• (60)	$u_2 = 40$
		• (8)		• (20)	$u_3 = 0$
v_j	$v_1 = 29$	$v_2 = 8$	$v_3 = 0$	$v_4 = 20$	

Assign a 'u' value to zero. (Convenient rule is to select the u_i, which has the largest number of allocations in its row)

Let $u_3 = 0$, then

$u_3 + v_4 = 20$ which implies $0 + v_4 = 20$, so $v_4 = 20$

$u_2 + v_4 = 60$ which implies $u_2 + 20 = 60$, so $u_2 = 40$

$u_1 + v_4 = 10$ which implies $u_1 + 20 = 10$, so $u_1 = -10$

$u_2 + v_3 = 40$ which implies $40 + v_3 = 40$, so $v_3 = 0$

$u_3 + v_2 = 8$ which implies $0 + v_2 = 8$, so $v_2 = 8$

$u_1 + v_1 = 19$ which implies $-10 + v_1 = 19$, so $v_1 = 29$

4. **Calculation of cost differences for non basic cells $d_{ij} = c_{ij} – (u_i + v_j)$**

c_{ij}

•	(30)	(50)	•
(70)	(30)	•	•
(40)	•	(70)	•

$u_i + v_j$

•	-2	-10	•
69	48	•	•
29	•	0	•

$d_{ij} = c_{ij} - (u_i + v_j)$

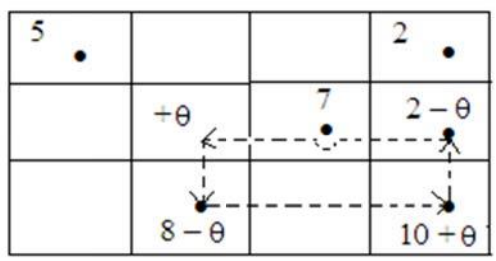
•	32	60	•
1	**-18**	•	•
11	•	70	•

5. **Optimality test**

$d_{ij} < 0$ i.e. $d_{22} = -18$

so x_{22} is entering the basis

6. **Construction of loop and allocation of unknown quantity Θ**



We allocate Θ to the cell (2, 2). Reallocation is done by transferring the maximum possible amount Θ in the marked cell. The value of Θ is obtained by equating to zero to the corners of the closed loop. i.e. min(8-Θ, 2-Θ) = 0 which gives Θ = 2. Therefore x_{24} is outgoing as it becomes zero.

5 (19)			2 (10)
	2 (30)	7 (40)	
	6 (8)		12 (20)

Minimum transportation cost is 5 (19) + 2 (10) + 2 (30) + 7 (40) + 6 (8) + 12 (20) = 743

7. **Improved Solution**

				u_i
• (19)			• (10)	$u_1 = -10$
	• (30)	• (40)		$u_2 = 22$
	• (8)		• (20)	$u_3 = 0$
$v_1 = 29$	$v_2 = 8$	$v_3 = 18$	$v_4 = 20$	v_j

c_{ij}

•	(30)	(50)	•
(70)	•	•	(60)
(40)	•	(70)	•

$u_i + v_j$

•	-2	8	•
51	•	•	42
29	•	18	•

$d_{ij} = c_{ij} - (u_i + v_j)$

•	32	42	•
19	•	•	18
11	•	52	•

Since $d_{ij} > 0$, an optimal solution is obtained with minimal cost 743

Solved problem

Is $x_{13} = 50$, $x_{14} = 20$, $x_{21} = 55$, $x_{31} = 30$, $x_{32} = 35$, $x_{34} = 25$ an optimal solution to the transportation problem.

					Available
	6	1	9	3	70
From	11	5	2	8	55
	10	12	4	7	90
Required	85	35	50	45	

Solution

						Available
			50(9)	20(3)		X
From	55(11)					X
	30(10)	35(12)		25(7)		X
Required	X	X	X	X		

Minimum transportation cost is 50 (9) + 20 (3) + 55 (11) + 30 (10) + 35 (12) + 25 (7) =2010

Check for Non-degeneracy

The initial basic feasible solution has m + n – 1 i.e. 3 + 4 – 1 = 6 allocations in independent positions. Hence optimality test is satisfied.

Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$

					u_i
			• (9)	• (3)	$u_1 = -4$
	• (11)				$u_2 = 1$
	• (10)	• (12)		• (7)	$u_3 = 0$
v_j	$v_1 = 10$	$v_2 = 12$	$v_3 = 13$	$v_4 = 7$	

Calculation of cost differences for non-basic cells $d_{ij} = c_{ij} – (u_i + v_j)$

c_{ij}

6	1	•	•
•	5	2	8
•	•	4	•

$u_i + v_j$

6	8	•	•
•	13	14	8
•	•	13	•

$d_{ij} = c_{ij} – (u_i + v_j)$

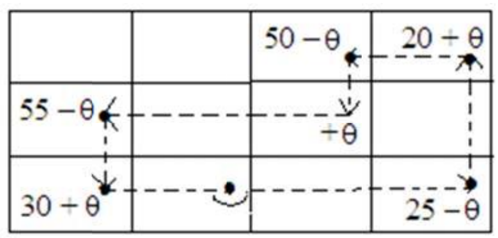
0	-7	•	•
•	-8	-12	0
•	•	-9	•

Optimality test

$d_{ij} < 0$ i.e. $d_{23} = -12$ is most negative

So x_{23} is entering the basis

Construction of loop and allocation of unknown quantity ϴ



min(50-ϴ, 55-ϴ, 25-ϴ) = 25 which gives ϴ = 25. Therefore x_{34} is outgoing as it becomes zero.

		25(9)	45(3)
30(11)		25(2)	
55(10)	35(12)		

Minimum transportation cost is 25 (9) + 45 (3) + 30 (11) + 25 (2) + 55 (10) + 35 (12) = 1710

II iteration

Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$

					u_i
			• (9)	• (3)	$u_1 = 8$
	• (11)		• (2)		$u_2 = 1$
	• (10)	• (12)			$u_3 = 0$
v_j	$v_1 = 10$	$v_2 = 12$	$v_3 = 1$	$v_4 = -5$	

Calculation of cost differences for non-basic cells $d_{ij} = c_{ij} - (u_i + v_j)$

c_{ij}

6	1	•	•
•	5	•	8
•	•	4	7

$u_i + v_j$

18	20	•	•
•	13	•	-4
•	•	1	-5

$d_{ij} = c_{ij} - (u_i + v_j)$

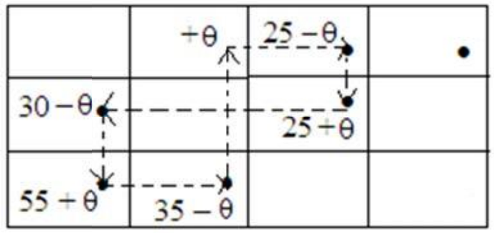
-12	-19	•	•
•	-8	•	12
•	•	3	12

Optimality test

$d_{ij} < 0$ i.e. $d_{12} = -19$ is most negative

So x_{12} is entering the basis

Construction of loop and allocation of unknown quantity ϴ



min(25-ϴ, 30-ϴ, 35-ϴ) = 25 which gives ϴ = 25. Therefore x_{13} is outgoing as it becomes zero.

	25(1)		45(3)
5(11)		50(2)	
80(10)	10(12)		

Minimum transportation cost is 25 (1) + 45 (3) + 5 (11) + 50 (2) + 80 (10) + 10 (12) =1235

III Iteration

Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$

					u_i
		• (1)		• (3)	u_1= -11
	• (11)		• (2)		u_2 = 1
	• (10)	• (12)			u_3 = 0
v_j	v_1 = 10	v_2 = 12	v_3 = 1	v_4 = 14	

Calculation of cost differences for non-basic cells $d_{ij} = c_{ij} – (u_i + v_j)$

c_{ij}

6	•	9	•
•	5	•	8
•	•	4	7

$u_i + v_j$

-1	•	-10	•
•	13	•	15
•	•	1	14

$d_{ij} = c_{ij} - (u_i + v_j)$

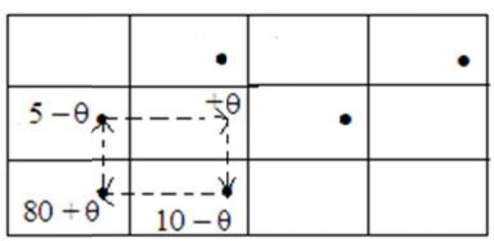
7	•	19	•
•	-8	•	-7
•	•	3	-7

Optimality test

$d_{ij} < 0$ i.e. $d_{22} = -8$ is most negative

So x_{22} is entering the basis

Construction of loop and allocation of unknown quantity Θ



min(5-Θ, 10-Θ) = 5 which gives Θ = 5. Therefore x_{21} is outgoing as it becomes zero.

	25(1)		45(3)
	5(5)	50(2)	
85(10)	5(12)		

Minimum transportation cost is 25 (1) + 45 (3) + 5 (5) + 50 (2) + 85 (10) + 5 (12) = 1195

IV Iteration

Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$

				u_i
	• (1)		• (3)	$u_1 = -11$
	• (5)	• (2)		$u_2 = -7$
• (10)	• (12)			$u_3 = 0$
$v_1 = 10$	$v_2 = 12$	$v_3 = 9$	$v_4 = 14$	v_j

Calculation of cost differences for non-basic cells $d_{ij} = c_{ij} - (u_i + v_j)$

c_{ij}

6	•	9	•
11	•	•	8
•	•	4	7

$u_i + v_j$

-1	•	-2	•
3	•	•	7
•	•	9	14

$d_{ij} = c_{ij} - (u_i + v_j)$

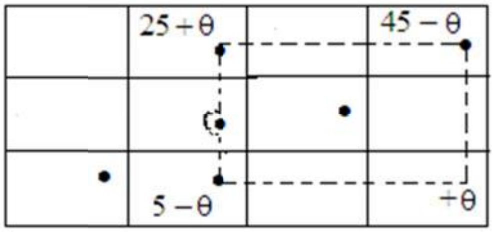
7	•	11	•
8	•	•	1
•	•	-5	-7

Optimality test

$d_{ij} < 0$ i.e. $d_{34} = -7$ is most negative

So x_{34} is entering the basis

Construction of loop and allocation of unknown quantity Θ



min(5-Θ, 45-Θ) = 5 which gives Θ = 5. Therefore x_{32} is outgoing as it becomes zero.

	30(1)		40(3)
	5(5)	50(2)	
85(10)			5(7)

Minimum transportation cost is 30 (1) + 40 (3) + 5 (5) + 50 (2) + 85 (10) + 5 (7) = 1160

V Iteration

Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$

					u_i
		• (1)		• (3)	u_1= -4
		• (5)	• (2)		u_2 = 0
	• (10)			• (7)	u_3 = 0
v_j	v_1 = 10	v_2 = 5	v_3 = 2	v_4 = 7	

Calculation of cost differences for non-basic cells $d_{ij} = c_{ij} – (u_i + v_j)$

c_{ij}

6	•	9	•
11	•	•	8
•	12	4	•

$u_i + v_j$

6	•	-2	•
10	•	•	7
•	5	2	•

$\mathbf{d_{ij} = c_{ij} - (u_i + v_j)}$

0	•	11	•
1	•	•	1
•	7	2	•

Since $d_{ij} > 0$, an optimal solution is obtained with minimal cost 1160. Further more $d_{11} = 0$ which indicates that alternative optimal solution also exists.

Exercise

I- Determine an initial basic feasible solution to the following transportation problems using the five given methods.

1.

	To					Availability
	2	11	10	3	7	4
From	1	4	7	2	1	8
	3	9	4	8	12	9
Requirement	3	3	4	5	6	

2.

	D_1	D_2	D_3	D_4	Availability
S_1	11	13	17	14	250
S_2	16	18	14	10	300
S_3	21	24	13	10	400
Requirement	200	225	275	250	

3.

		I	II	III	IV	Supply
	A	13	11	15	20	2000
From	B	17	14	12	13	6000
	C	18	18	15	12	7000
	Demand	3000	3000	4000	5000	

II- Determine the optimal solution of the above transportation problems.

III- Determine the optimal solution of the given transportation problem:

	To				Supply
	2	3	11	7	6
From	1	0	6	1	1
	5	8	15	10	10
Demand	7	5	3	2	17