## *Unit 7 Game Theory:   Simplex Method Application*

Let us consider the 3 x 3 matrix



As per the assumptions, A always attempts to choose the set of strategies with the non-zero probabilities say $p_1$, $p_2$, $p_3$ where $p_1 + p_2 + p_3 = 1$ that maximizes his minimum expected gain.

Similarly B would choose the set of strategies with the non-zero probabilities say $q_1$, $q_2$, $q_3$ where $q_1 + q_2 + q_3 = 1$ that minimizes his maximum expected loss.

**Step 1**

Find the minimax and maximin value from the given matrix

**Step 2**

The objective of A is to maximize the value, which is equivalent to minimizing the value $1/V$. The LPP is written as

$$\text{Min } 1/V = p_1/V + p_2/V + p_3/V$$

and constraints $\geq 1$

It is written as

$$\text{Min } 1/V = x_1 + x_2 + x_3$$

and constraints $\geq 1$

Similarly for B, we get the LPP as the dual of the above LPP

$$\text{Max } 1/V = Y_1 + Y_2 + Y_3$$

and constraints $\leq 1$

Where $Y_1 = q_1/V$, $Y_2 = q_2/V$, $Y_3 = q_3/V$

**Step 3**

Solve the LPP by using simplex table and obtain the best strategy for the players

**Example 1**

**Solve by Simplex method**

$$
A \begin{array}{c} \\ \\ \\ \end{array}
\overset{\text{B}}{\begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}}
$$

**Solution**



We can infer that $2 \leq V \leq 3$. Hence it can be concluded that the value of the game lies between 2 and 3 and the $V > 0$.

LPP

Max $1/V = Y_1 + Y_2 + Y_3$

Subject to

$\quad\quad 3Y_1 - 2Y_2 + 4Y_3 \leq 1$

$\quad\quad -1Y_1 + 4Y_2 + 2Y_3 \leq 1$

$\quad\quad 2Y_1 + 2Y_2 + 6Y_3 \leq 1$

$\quad\quad Y_1, Y_2, Y_3 \geq 0$

SLPP

Max $1/V = Y_1 + Y_2 + Y_3 + 0s_1 + 0s_2 + 0s_3$

Subject to

$\quad\quad 3Y_1 - 2Y_2 + 4Y_3 + s_1 = 1$

$\quad\quad -1Y_1 + 4Y_2 + 2Y_3 + s_2 = 1$

$\quad\quad 2Y_1 + 2Y_2 + 6Y_3 + s_3 = 1$

$\quad\quad Y_1, Y_2, Y_3, s_1, s_2, s_3 \geq 0$

| Basic Variables | $C_B$ | $Y_B$ | $Y_1$ | $Y_2$ | $Y_3$ | $S_1$ | $S_2$ | $S_3$ | Min Ratio $Y_B / Y_K$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | $C_j\rightarrow$ 1 | 1 | 1 | 0 | 0 | 0 | |
| $S_1$ | 0 | 1 | 3 | -2 | 4 | 1 | 0 | 0 | $1/3\rightarrow$ |
| $S_2$ | 0 | 1 | -1 | 4 | 2 | 0 | 1 | 0 | - |
| $S_3$ | 0 | 1 | 2 | 2 | 6 | 0 | 0 | 1 | 1/2 |
| | $1/V = 0$ | | ↑ -1 | -1 | -1 | 0 | 0 | 0 | |
| $Y_1$ | 1 | 1/3 | 1 | -2/3 | 4/3 | 1/3 | 0 | 0 | - |
| $S_2$ | 0 | 4/3 | 0 | 10/3 | 10/3 | 1/3 | 1 | 0 | 2/5 |
| $S_3$ | 0 | 1/3 | 0 | 10/3 | 10/3 | -2/3 | 0 | 1 | $1/10\rightarrow$ |
| | $1/V =1/3$ | | 0 | ↑ -5/3 | 1/3 | 1/3 | 0 | 0 | |
| $Y_1$ | 1 | 2/5 | 1 | 0 | 2 | 1/5 | 0 | 1/5 | |
| $S_2$ | 0 | 1 | 0 | 0 | 0 | 1 | 1 | -1 | |
| $Y_2$ | 1 | 1/10 | 0 | 1 | 1 | -1/5 | 0 | 3/10 | |
| | $1/V = 1/2$ | | 0 | 0 | 2 | 0 | 0 | 1/2 | |

$1/V =1/2$

$V = 2$


$y_1 = 2/5 * 2 = 4/5$

$y_2 = 1/10 * 2 = 1/5$

$y_3 = 0 * 2 = 0$


$x_1 = 0*2 = 0$

$x_2 = 0*2 = 0$

$x_3 = 1/2*2 = 1$


$S_A = (0, 0, 1)$

$S_B = (4/5, 1/5, 0)$

Value = 2

**Example 2**

$$
A \begin{array}{c} \\ \\ \end{array}
\begin{array}{ccc}
 & B & \\
1 & -1 & -1 \\
-1 & -1 & 3 \\
-1 & 2 & -1
\end{array}
$$

**Solution**

$$
A \begin{bmatrix}
1 & -1 & -1 \\
-1 & -1 & 3 \\
-1 & 2 & -1
\end{bmatrix}
\begin{array}{c} -1 \\ -1 \\ -1 \end{array}
$$

$$ \quad\; 1 \quad\;\; 2 \quad\;\; 3 $$

Maximin = -1

Minimax = 1

We can infer that $-1 \leq V \leq 1$

Since maximin value is -1, it is possible that value of the game may be negative or zero, thus the constant 'C' is added to all the elements of matrix which is at least equal to the negative of maximin.

Let C = 1, add this value to all the elements of the matrix. The resultant matrix is

$$
A \begin{bmatrix}
2 & 0 & 0 \\
0 & 0 & 4 \\
0 & 3 & 0
\end{bmatrix}
$$

LPP

Max $1/V = Y_1 + Y_2 + Y_3$

Subject to

$\qquad 2Y_1 + 0Y_2 + 0Y_3 \leq 1$

$\qquad 0Y_1 + 0Y_2 + 4Y_3 \leq 1$

$\qquad 0Y_1 + 3Y_2 + 0Y_3 \leq 1$

$\qquad Y_1, Y_2, Y_3 \geq 0$

SLPP

Max $1/V = Y_1 + Y_2 + Y_3 + 0s_1 + 0s_2 + 0s_3$

Subject to

$2Y_1 + 0Y_2 + 0Y_3 + s_1 = 1$

$0Y_1 + 0Y_2 + 4Y_3 + s_2 = 1$

$0Y_1 + 3Y_2 + 0Y_3 + s_3 = 1$

$Y_1, Y_2, Y_3, s_1, s_2, s_3 \geq 0$

| | | $C_j \rightarrow$ | 1 | 1 | 1 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|---|
| Basic Variables | $C_B$ | $Y_B$ | $Y_1$ | $Y_2$ | $Y_3$ | $S_1$ | $S_2$ | $S_3$ | Min Ratio $Y_B / Y_K$ |
| $S_1$ | 0 | 1 | 2 | 0 | 0 | 1 | 0 | 0 | $1/2 \rightarrow$ |
| $S_2$ | 0 | 1 | 0 | 0 | 4 | 0 | 1 | 0 | - |
| $S_3$ | 0 | 1 | 0 | 3 | 0 | 0 | 0 | 1 | - |
| | | $1/V = 0$ | $\uparrow$ -1 | -1 | -1 | 0 | 0 | 0 | |
| $Y_1$ | 1 | 1/2 | 1 | 0 | 0 | 1/2 | 0 | 0 | - |
| $S_2$ | 0 | 1 | 0 | 0 | 4 | 0 | 1 | 0 | - |
| $S_3$ | 0 | 1 | 0 | 3 | 0 | 0 | 0 | 1 | $1/3 \rightarrow$ |
| | | $1/V = 1/2$ | 0 | $\uparrow$ -1 | -1 | 1/2 | 0 | 0 | |
| $Y_1$ | 1 | 1/2 | 1 | 0 | 0 | 1/2 | 0 | 0 | - |
| $S_2$ | 0 | 1 | 0 | 0 | 4 | 0 | 1 | 0 | $1/4 \rightarrow$ |
| $Y_2$ | 1 | 1/3 | 0 | 1 | 0 | 0 | 0 | 1/3 | - |
| | | $1/V = 5/6$ | 0 | 0 | $\uparrow$ -1 | 1/2 | 0 | 1/3 | |
| $Y_1$ | 1 | 1/2 | 1 | 0 | 0 | 1/2 | 0 | 0 | |
| $Y_3$ | 1 | 1/4 | 0 | 0 | 1 | 0 | 1/4 | 0 | |
| $Y_2$ | 1 | 1/3 | 0 | 1 | 0 | 0 | 0 | 1/3 | |
| | | $1/V = 13/12$ | 0 | 0 | 0 | 1/2 | 1/4 | 1/3 | |

$1/V = 13/12$

$V = 12/13$

$y_1 = 1/2 * 12/13 = 6/13$

$y_2 = 1/3 * 12/13 = 4/13$

$y_3 = 1/4 * 12/13 = 3/13$

$x_1 = 1/2*12/13 = 6/13$

$x_2 = 1/4 * 12/13 = 3/13$

$x_3 = 1/3 * 12/13 = 4/13$

$S_A = (6/13, 3/13, 4/13)$

$S_B = (6/13, 4/13, 3/13)$

$Value = 12/13 - C = 12/13 - 1 = -1/13$